Plain Talk on Preferences

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Abstract

Assume the preferences of each individual are transitive but preferences for the same alternatives differ among some individuals. Condorcet (1785) showed that choices made by majority voting are not always transitive. Therefore, the choice among alternatives made by the group will not coincide with the preferences of any individual in the group. This is also Arrow's conclusion. Drop the assumption that all individuals have transitive preferences and the paradox loses much of its force. How groups resolve the perennial conflict between liberty and consensus remains unsolved by majority voting. JEL D7 & D8 Criteria for Decision Making by Individuals and Groups

1 Introduction

Given a choice between A and B, if I choose A and given a choice between B and C, if I choose B, then it appears logical that between A and C, I would choose A. The first sign that something is amiss was demonstrated more than 200 years ago by the Marquis de Condorcet in his voting paradox (1785). A obtains a majority over B, B obtains a majority over C, but C obtains a majority over A, not the reverse as transitivity would assert. Therefore, majority voting can be intransitive. Sporting fans know many examples. In a contest between A and B, A wins, in a contest between B and C, B wins, but in a contest between A and C, C, not A, wins. Winning is not a transitive relation. The outcomes of sporting contests can be intransitive, but few would describe this outcome as illogical. Preferences are contests between alternatives similar to sporting contests. Economic science can survive very well without transitive preferences, even without transitive revealed preferences encumbered by its baggage of twice differentiable ordinal utility indicators, pace Samuelson and Houthakker.

My purpose is to demonstrate that sensible preferences need not be transitive. I begin with two relations, dominance and implication, and
prove they are transitive. With the aid of these transitive relations, we can see why preferences about economic alternatives need not be transitive.

2 Diagms and Dominance

Cardinal utility attaches a number to objects similar to measuring temperature. Cardinal utility would be transitive if it resembled temperature because ordinary numbers are transitive. Whenever a single number would not suffice to describe economic alternatives, cardinal utility would not be transitive. Figures 1 and 2 illustrate domination, a simple concept amenable to straightforward numerical comparisons.

Figure 1
Definition of Domination. The coordinates of the pairs denote quantities of two goods. The pair \((x,y)\) dominates \((s,t)\) if \(x \geq s, y \geq t\) and there is at most one equality.

It follows from the definition that the pair \((x,y)\) does not dominate the pair \((s,t)\), if at least one of the following three assertions is true.

(1) \(x \leq s\) and \(y > t\),
(2) \(x > s\) and \(y \leq t\),
(3) \(x < s\) and \(y < t\).
In case (3) the pair \((s,t)\) dominates the pair \((x,y)\). Neither pair dominates the other if either (1) is true or (2) is true. It is impossible for both (1) and (2) to be true simultaneously.

**Theorem.** Domination is transitive.

**Proof.** If \((x,y)\) dominates \((s,t)\) and \((s,t)\) dominates \((u,v)\), then \((x,y)\) dominates \((u,v)\). Therefore, domination is transitive. \(\square\)

Figure 1 illustrates transitivity. Coordinates of AA are greater than the coordinates of BB so AA dominates BB. Coordinates of BB are all bigger than those of CC so BB dominates CC. Because the coordinates of AA are bigger than those of CC, there is an implication of transitivity. Indeed, CC dominates DD so that BB also dominates DD. However, the situation in Figure 1 is unusual. The one in Figure 2 is more typical.

![Figure 1](image1.png)

**Figure 2**

Consider Figure 2. The point A dominates the point B and the point C also dominates the point B but the relation between the points A and C is indeterminate according to dominance because neither dominates the other. Figure 2 shows three such cases; A, C and E. It follows that no implication of transitivity applies among these three points. Next take
the points B, D, and F, each dominated by one of the three points, A, C and E. However, A, C and E do not dominate each other and there is no implication of transitivity among them.

3 Implication is Transitive

In formal logic the assertion that proposition p implies proposition q is not true if p occurs but q does not occur.
Definition of Implication. \( p \implies q \) is equivalent to \((-p) \lor q\), that is, \(\neg p \lor q\).
Theorem. Implication is transitive.
Proof. By contradiction. Suppose \((-p) \lor q \land (-q) \lor r \land \neg((-p) \lor r)\). The last assertion is equivalent to p and \(\neg r\). But since p is true, the first assertion rules out q is false. Hence q must be true so that the second assertion rules out r is false so r must be true. This contradicts the third assertion. It follows that implication is transitive.

4 Inclusion Is Not Implication

Inclusion means 'belongs to'. It does not mean implication.
(1) If p belongs to the set B, then p belongs to the set A.
(2) If p belongs to the set C, then p belongs to the set B.
However, (1) and (2) do not imply (3).
(3) p belongs to the set C.
It would be different if every member of B were a member of A and every member of C were a member of B. It would follow that every member of C is a member of A. These are assertions about inclusion, not about implication. However, if inclusion were regarded as verbally equivalent to dominance, then it would imply transitivity but would add nothing to the analysis.

Pairwise comparisons depend on the application. In some 2-person games there may be 3 possible outcomes; A wins over B, B wins over A or the result is indecisive, a tie. If there are three players, A, B and C and A beats B, B beats C, then no logical deduction follows from the outcome of a game between A and C. Neither win nor lose is a transitive relation.
5 Valid Economics

Proponents of ordinal utility claim that ranking objects by means of pairwise comparisons can suffice as a foundation for the theory of demand provided pairwise comparisons are transitive. Indeed some proponents even go to the extreme position that intransitive preferences are not rational. This is nonsense. Transitivity and rationality have nothing to do with each other.

Valid economics never asks about preferences without stating what is held constant. At the very least, whether A is preferred to B requires that nothing apart from A and B affects the choice. A valid preference between A and B must not muddle in other unstated factors that can affect the preference. The choice must be between \((A,x)\) and \((B,x)\) where x means everything is the same for A and B including their cost. It would not be sensible to ask someone whether they prefer sleeping in a mansion to sleeping in a wigwam. To announce a preference without defining the cost is idle chatter. Suppose \((A,x)>(B,x)\). Presumably, the preference would change between the two pairs \((A,x)\) and \((B, x-\Delta x)\) or between the two pairs \((A,x+\Delta x)\) and \((B,x)\). There are \(\Delta x>0\) such that \((A,x)>(B,x)\) and \((A,x)<(B,x-\Delta x)\) or \((A,x+\Delta x)<(B,x)\). The conjunction of the object and its cost ignores what may underlay the preferences between the objects themselves. In reality it is impossible to hold everything constant apart from the two things A and B themselves. Anyone who believes it is really possible to state a preference between A and B begs this question.

Definition. A relation \(R\) defined on members of the set \(X\) can order all the members of \(X\) if the relation is reflexive, anti-symmetric and transitive (Halmos, 1974, p. 54).

Halmos calls a totally ordered set a chain. Every subset has a least element and a greatest element. Ordinary scalar numbers are transitive, complex numbers are intransitive. To impose an order on complex numbers mathematicians often use the norm, the length of the complex number that is a scalar valued function of a complex number.
6 The Voting Paradox: Condorcet and Arrow

Condorcet and Arrow raise the question of whether the choices of a group reflect the wishes of the individual members of the group. Arrow assumes that the preferences of every individual is transitive and this view seems implicit in Condorcet's analysis. It follows that if the choices made by the group are not transitive, then they cannot reflect the preferences of the members of the group.

Assume the preferences of each individual are transitive. Assume preferences for the same alternatives differ among some of the individuals. Otherwise, the problem could not arise. More than two centuries ago Condorcet showed that choices made by a group of individuals by means of majority voting are not always transitive. Therefore, the choice among alternatives made by the group will not coincide with the preferences of any individual. This is also Arrow's conclusion. Now drop the assumption that all individuals have transitive preferences. Hence the paradox loses much of its force. The problem of how groups resolve the conflict between liberty and consensus remains unsolved by majority voting.

References


