February, 2012

An Economy Is Not a Zero Sum Game: How Economics Uses Game Theory

Lester G Telser, *University of Chicago*

Part 1. An Economy Is Not a Zero-Sum Game: How Economics Uses Game Theory

L.G. Telser

2/9/12

1 Introduction

Unlike Gaul game theory is divided into only two parts, noncooperative and cooperative games. Noncooperative game theory closely matches actual games. The rules of most games set the number of players, 2 in chess and checkers, 4 in bridge, 2 teams of 5 in basketball, 9 in baseball, 11 in football and so on. In an actual game two or more players contest for victory. Players may be either individuals as in tennis, chess and ping pong or teams as in baseball. Players in opposing teams in real games are not supposed to cooperate. Teams strive to win. When a player in one team makes a secret deal with a rival team even through an intermediary, this is cheating. The Chicago Black Sox scandal in the 1919 World Series is a well known example of this. Non cooperation among contestants is the essence of a real game. How noncooperative game theory can be a useful tool for economic analysis is far from obvious. Before taking up this topic let us consider how economics applies the theory of cooperative games.

A market is the archetype of an economy. Traders in a market bargain among themselves. Two or more traders can tentatively agree on terms of exchange. They may seek other trading partners. Nothing is final until no one can get better terms. Edgeworth calls this process contracting and recontracting. Those traders who reach a tentative agreement are a coalition. The search for better terms of trade causes competition among coalitions for traders. Competition is the essence of a free enterprise economy. To call this process a cooperative game is decidedly misleading but since the usage is entrenched, I shall accept it.

A free enterprise economy is not a state of anarchy. It needs suitable rules that can advance the interests of the whole community. These rules require humane treatment of workers, fair treatment of investors, a definition of property rights, institutions to resolve disputes, maintenance of law and order, preservation of a healthful environment and many social services that may not or cannot respond suitably to private financial incentives. All this is understand as the background for the application of core theory.

The number of participants in an economy is not fixed. This is a fundamental difference between actual games and the economy. It makes little sense in most games to ask what would happen if the number of players were to increase. Would you still have a game of chess with 3 players? To ignore this difference between economies and games travels on the road to irrelevance if not outright error. It is better to study the economy on the hypothesis that the 'number' of participants corresponds to a continuous variable, not to a finite integer, 2, 5, 9, 11 etc.

A coalition with two or more members supposes they will meet their obligations. Traders in a market abide by the final, agreed upon terms of their trades. Suppliers of a firm deliver what they had promised. Lenders repay their loans with interest. Franchisees satisfy the terms of their franchises. All of this raises the problem of how to enforce the terms of agreement among the members of a coalition. Here is where noncooperative game theory enters the scene. The archetypes for economic applications of noncooperative game theory are the principal-agent problem and the prisoners' dilemma. The latter applies to players of equal status, the former to those of unequal status because the agent works for the principal. If there are many agents, then some or all may join forces to advance their own interests at the expense of the principal's.

Like the Prisoners' Dilemma a cartel faces the problem of how to enforce collusion agreed upon by its members. A disloyal cartel member gets a short term gain at the expense of the loyal members of the cartel who do adhere to the terms of the cartel agreement. Widespread cheating by cartel members breaks down the cartel altogether. Nevertheless theorists of cartels are well advised to use models based on cooperative games. Here, too, the more individuals are in the cartel, the greater the variety of ways to cheat. Promotion and advertising may also be subject to collusion. Firms may promote their own products and agree to refrain from denigrating rival products. Cheating may take the form of larger outlays on promotion or disparagement of rivals. Core theory is well suited to handle complicated varieties of cheating.

The theory of the core did not originate in game theory. Two economists share the credit for starting core theory, Menger's Principles of Economics (1871) and Edgeworh's Mathematical Psychics (1881). Both start with an analysis of a market with a finite number of traders but Edgeworth went farther. He described the ultimate outcomes as the number of traders becomes large.

A reader may object to my criticism by pointing to models of oligopoly championed by distinguished pioneers in economic theory starting with Cournot (1838). Cournot's clebrated theory of oligopoly begins with two idntical firms selling bottled mineral water side by side in a small town in France. They post the same price. At the common price each gets the same positive net revenue. Were they to merge, their net revenue would more than double. Were there a third firm the net revenue of each would be lower. The Cournot model is not a counter-example to my thesis, actually it is a confirmation of it. To assume a given number of firms in the market begs the question of what determines this number. Yet Cournot himself shows that the net revenue varies inversely with the number of sellers in the market and is closer to zero the bigger the number.

Cournot' s theory is an exercise in comparative statics, not a dynamic theory of entry. It describes a sequence of outcomes for n firms such that given n, each firm chooses its own output independently to maximize its net revenue. The price that clears the market depends on the total quantity offered by the n firms and varies inversely with this total. To Cournot the number of firms in the industry is a parameter.

Not until Chamberlin's theory of monopolistic competition (1931) was there an attempt to describe the equilibrium number of firms in an industry. Chamberlin introduces a novel feature. Not only does each firm affect the price at which it can sell its output so that the larger is the quantity, the lower

the price, but also the level of demand facing each firm varies inversely with the number of firms in the industry. He shows there is a particular number such that the net revenue of each firm is zero and the marginal revenue equals the marginal cost of the quantity chosen by each firm. It follows that the total output of the industry is inefficient because marginal revenue is below the price. Buyers and sellers leave money on the table. The equilibrium in monopolistic competition is not in the core.

2 Corporations Are Joint Ventures

The prevailing form of a joint venture in the economy is a corporation. The owners of a corporation are its shareholders. A corporation is a semi private good in the language of economics because its net revenue belongs to its owners who can prevent others, apart from government, from taking any part of it.

All revenues and remunerations are measured in money. There are n types of investors, t_i of type i, i = 1, 2, ... , n. Each t_i is a positive real number. An individual investor is infinitesimally small but the cumulative number of the n types is positive and not small. Groups of investors may combine to finance a joint venture. Such a group is represented by an n-vector s={s_1, s_2, ... , s_n}, provided $0 \leq s_i \leqq t_i$. The joint venture that the investors contemplate promises a gross return f(s). Not all types of potential investors may wish to join this group. If type j does not join, then s_j=0. Of course, at least one coordinate of s must be positive or there is nothing to discuss. The pool of all potential investors is described by

(1) T(t) = {s: $0 \leqq s \leqq t$}.

For now take t as fixed.

Next consider the remuneration to the investors. Assume each type i investor gets the same return x_i. The total return to type i investors in the joint venture s is $x_i\, s_i$. The total payoff to all investors contemplating the formation of the joint venture f(s) would be $\sum_{i=1}^{n} x_i\, s_i$, or, more concisely, the scalar product xs. The joint venture f(s) retains earnings given by

(2) f(s)-xs.

This us the gross return minus the payment to its investors. For the moment take x as fixed.

Conditions that the function f (.) must satisfy emerge in the course of the analysis. At the outset it is important to record that f(s) for semi private goods must be convex in s.

3 A [The?] Standard Model

The next step seeks the best joint venture by means of the solution of the following maximum problem.

(1) max f(s)-xs with respect to s∈T(t).

This formulation of the problem faces some difficulties. Why should investors maximize retained earnings of a joint venture instead of their own return? Is it because retained earnings are 'capital gains' taxed at a lower rate than 'income' called dividends? But then retained earnings would be a maximum at x=0 and the objective of the maximum would be f(s) with respect to s∈T(t). This would require an explanation of how to remunerate investors. Suppose their remuneration were proportional to their investment. A type i investor would get $w_i\, s_i$/ws. Let $\theta_i = w_i\, s_i$/ws so that $\sum_{i=1}^{n} \theta_i = 1$. Consequently, f(s)-$\sum_{i=1}^{n} \theta_i$f(s)=0 no matter what joint venture is chosen by any group.

Another approach pursues a different route. Assume a potential type i investor obliges f(s) to pay not less than a_i so $x_i \geq a_i$. But then for the sake of simplicity we may as well assume $x_i = a_i$ and use the original version of the maximum problem in (1). A necessary condition for the maximum, assuming f(s) has partial derivatives, $f_i = \partial_{s_i}\, f(s)$ and that f(s) is at least locally concave is that

(2) $f_i - x_i \leq 0$ and $(f_i - x_i)\, s_i = 0$.

A convex function defined on a closed convex set such as T(t) has no maximum in the interior of such a set. The only candidates for a maximum are at the extreme points of T(t). Therefore, a positive coordinate of any candidate for a maximum must be the corresponding coordinate of t itself.

Even if f were not convex and were an increasing function of s so that $f_i > 0$, it would not follow that the best joint venture would include all the potential investors. Suppose in this case the best joint venture is f(s*), s*≤t, so that

(3) $f_i(s*) - x_i = 0$.

There is a fresh difficulty in the form of an old question about the relation between f(s*) and $f_s(s^*)$s*. Write the contribution to net earnings, r, as follows:

(4) r=f(s*)-$f_s(s^*)$s*.

No group of investors would pick a project that promises a loss. Only f(s) such that r is nonnegative would be considered. If f(s) were homogeneous of degree one, constant returns to scale, then r would be zero no matter what s is.

We now face the problem of how to distribute the positive residual, r, among the investors. Any scheme to distribute the residual that depends on s in any way must determine what joint venture the group would choose. There is only one avenue to escape this conclusion. The division of the residual among the various investor types must not depend on who they are. Say there are m types of investors who pick f(s) as their best joint venture, the solution of the maximum problem in (3). Draw m random numbers, y_i, i=1,2 ,..., m. Set $z_i = y_i \big/ \sum_{j=1}^{m} y_j$ so that z_i equals the share of type i in the sum. Note that this share does *not* depend on type i's relative size in s, $s_i \big/ \sum_{j=1}^{m} s_j$. It follows that for this procedure where the random draw takes place after the joint venture has been chosen, the distribu-

tion of the residual cannot affect the choice of a joint venture by a coalition of investors. Although there is no risk in this model to begin with, it must lead inevitably to the creation of some kind of risk. Otherwise, it would interfere with the best choice by a group. Diversification supplies investors the means of managing how much of this risk they are willing to bear.

A positive residual poses another complication. There are investors outside s*, t-s*, who, it would seem, have an incentive to start their own joint venture lured by the prospect of a positive net revenue. In short, the model fails to answer some pertinent economic questions. How can a situation persist in which a joint venture gets a positive net return and some investors remain on the side lines. Something else waits on the sidelines, free enterprise.

4 Enter Free Enterprise

A model of free enterprise determines whether the group of all investors can finance their joint venture f(t). This is possible only if the group can offer better terms to all potential investors than they could obtain from other joint ventures s≤t. The formal model poses and solves this problem.

Each type i investor gets the same return x_i . Their total return is $x_i\, t_i$. The total return to all n types of investors is the sum $\sum_{i=1}^{n} x_i\, t_i$=xt. The most they can get from their joint venture is f(t). This gives the first constraint in the model of free enterprise.
(1) xt≤f(t).

A coalition of investors is an s such that s≤t which does not include all n types and can get f(s). Since their total return as members of the grand coalition t would be xs, they would not be willing to participate in the pool financing f(t) unless the grand coalition would offer them terms at least as good as what they could get on their own by financing f(s). Therefore, competition to t from s is shown by
(2) xs≥f(s) for all s in T(t).

To see whether f(t) can survive in the face of this competition for its investors from all possible rival joint ventures, consider the following minimum problem:
(3) min xt with respect to x≥0 subject to (2).
This objective is a surprise because it does not impose feasibility in the form of (1). Indeed to discover whether f(t) is feasible in the face of this competition compare min xt to f(t).
(4) If min xt≤f(t), then f(t) can survive, otherwise min xt > f(t) so f(t) cannot survive.
The term non empty core has become used to describe survival and empty core to describe non survival. Which situation applies depends on the properties of the functions that produce the returns to the joint ventures.

Before turning to this aspect of the analysis, let us pause to study what competition means in this model of free enterprise. The inequalities (2) do not say that the groups of investors, the s's, actually form the joint ventures f(s). It says they *can* do so. Because they can form f(s), they can influence the terms they can demand as members of t. Nor is this all. A type i investor can explore many alternatives and tentatively propose joining them. Joint ventures compete for investors none of whom makes a final decision until it is no longer possible for any of them to get better terms than x. The

solution of the minimum problem shows the least that would be acceptable to all the investors in light of all their potential alternatives. While the solution of the minimum problem produces certain basic equations among the inequalities in (2), these joint ventures do not actually form. These equations pose the most formidable alternatives to the joint venture f(t).

We are now ready to deduce some conditions that f(.) must satisfy so that f(t) can survive competition from other joint ventures. There is the joint venture t-s such that

(4) $x(t-s) \geq f(t-s)$.

Add an inequality in (2) for some s to (4) and get

(5) $xs + x(t-s) \geq f(s)+f(t-s)$.

If x is feasible so that it satisfies inequality (1), then

(6) $f(t) \geq xt = xs+x(t-s) \geq f(s)+f(t-s)$.

Now relax the assumption that t is given. Consequently, (6) must hold for all t and for all $s \leq t$. Therefore,

(7) $f(t) \geq f(s)+f(t-s)$.

This says that f must be a superadditive function.

Similarly, we can deduce another important property of f(s). Choose s < t and $\lambda > 1$ so that $\lambda s = t$. Now (2) implies

(8) $xt = \lambda xs \leq f(t) = f(\lambda s)$.

From (2) we have

(9) $xs \geq f(s) \Longrightarrow \lambda xs \geq \lambda f(s)$.

Together (8) and (9) imply

(10) $\lambda f(s) \leq \lambda xs \leq f(t) = f(\lambda s$.

so f(.) must be superhomogeneous. A summary of these results is the

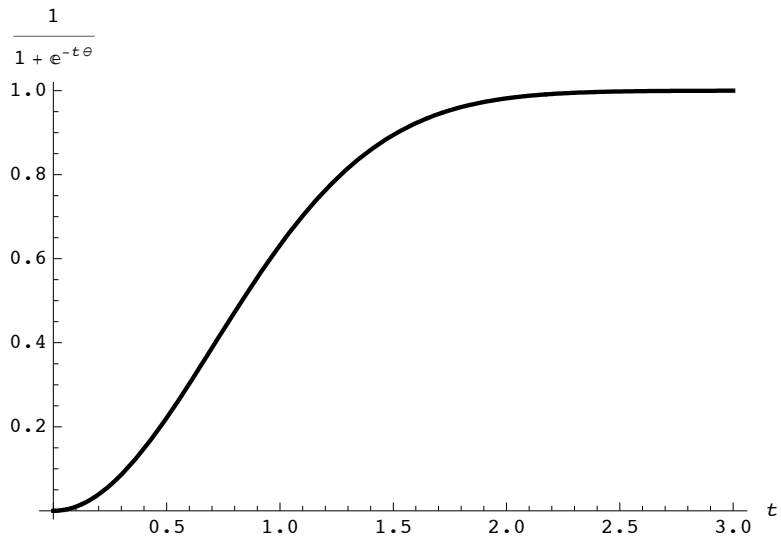
Proposition. If the inequalities in (1) and (2) hold for all positive t, then f(t) is superadditive and superhomogeneous.

The stability of the grand coalition t to finance f(t) depends on the nature of the function f. There are several alternatives; stability for all t, for a countably infinite number of t's, for t's in some intervals or for no t's whatever. When it is not possible to find an x that can satisfy (1) and (2), the coalition t cannot finance f(t) so the joint venture will not form.

A reasonable model of joint ventures assumes conditions on the shape of f(t) that imply an upper bound on the stability of certain grand coalitions. The range of t where there is stability is a familiar condition in economic theory. It is where f(t) has nondecreasing returns to scale, namely, $W=\{t: f_t(t)\, t - f(t) \geq 0\}$. There is also a set $U=\{t: f_t(t)\, t - f(t) < 0\}$. No t in U can yield a non empty core because there are decreasing returns to scale so that every t in U is unstable. The stable joint ventures in W have finite upper bounds.

An example of a function that sets an upper bound on the size of stable joint ventures is $f(t) = 1 - e^{-t^2}$ shown in the following figure. No joint venture will be stable for this function above its infection point where $f''(t)=0$ at $t = 1/\sqrt{2}$. Many other functions also yield stable grand coalitions. The graph in the figure imposes no minimum size for a proposed joint venture. By shifting the graph

to the right so that f(t) = 0 for all t such that $0 \le t \le t_o$ and f(t)>0 for all $t > t_o$, we set a minimum size at t_o for a proposed joint venture. This would also raise the upper bound.



Reconsider the two parts of game theory, cooperative and noncooperative. Use a model based on the core to study a cartel. Individual firms or groups of firms in the cartel can gain by by offering potential customers better terms than those from firms inside the cartel but outside this group of disloyal cartel members. Such rivalry destabilizes the grand coalition of all the firms that form the cartel. Hence the core is empty. Core theory has the advantage to a student of a cartel that it supplies a systematic framework to see how a cartel can break down. It does not confine attention to cheating by only one firm as is often the case for noncooperative models of this situation.

■ References

Chamberlin, Edward H. 1933. The Theory of Monopolistic Competition. Cambridge, MA.: Harvard University Press.
Cournot, Augustin. 1838. Researches into the Mathematical Principles of the Theory of Wealth. trans. Nathaniel Bacon. New York: Kelley.
Edgeworth, F.Y. 1881. Mathematical Psychics. London: Keegan-Paul.
Menger, Carl, 1871. Principles of Economics. trans. James Dingwall and Bert Hoselitz. Grove City, PA: Libertarian Press.
Telser, Lester G. 1987. A Theory of Efficient Cooperation and Competition. Cambridge, U. K. : Cambridge University Press.