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Two Certainties: Death, Taxes and an Honorable Mention, Financial Crises

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Abstract. If the probability of a financial crisis were a fixed positive number, no matter how small, then the law of large numbers says a financial crisis will occur eventually. In my model the probability of a crisis approaches but never attains zero. It yields the surprising result that the probability of a financial crisis does not exceed 0.63212=1-1/e. Financial Institutions and Services. JEL G2.

1 Introduction

One might think the probability of a financial crisis can approach certainty because of the law of large numbers. If this were true, then financial crises would join the two certainties, death and taxes. The law of large numbers says that if the probability of a financial crisis were a fixed positive number, no matter how small, then there would be a financial crisis eventually. However, this conclusion rests on a false premise that the probability of a financial crisis is a fixed number. In my model this probability tends toward zero but never attains it. The probability of a financial crisis never exceeds 0.63212, or, more precisely, 1 – 1/e.

2 Ordinary Insurance

Let a private insurance company with m policy holders offer each insurance against loss at the price A. Assume the loss is random variable, either Y or 0. Since each customer pays A to the insurance company, it receives m A. If n customers incur the loss, then the insurance company pays them n Y. The company's total receipts can cover the total payments, apart from its costs of doing business, if m A = n Y. Because its receipts are m A for sure, but its actual payments, n Y, are not known in advance, the insurance company incurs risk. It may have too little on hand to pay n Y or it may have too much. This poses the problem of how the insurance company can meet its obligations.

If the loss to a customer of the insurance company occurs as an independent random event, then there is a cornucopia of mathematical literature to handle the problem provided there is some way of estimating the probability of loss. The problem is harder if the random events are not independent. Perhaps the bad event affects almost all the customers whenever it happens. The simplest way to study what can happen is by means of numerical examples.

3 Numerical Examples

Suppose the insurance company has 1,000 policy holders. If a loss of $1,000 affects 1 person per 1,000, then an individual's expected loss is $1. If the insurance company collects $1 yearly from each customer, then, because the law of large numbers applies, almost surely it will have enough to pay the customer who suffered the loss.

Next consider the situation with correlated adverse random events. Suppose the bad event affects all the policy holders so that each loses $1,000. The total obligation of the insurance company is $1,000,000 to be divided among its 1000 policy holders. Consequently, it must collect $1,000 per customer to meet its obligations. Hence insurance is not possible.

A modified example covering several years is instructive. About once every 20 years each of the 1,000 policy holders loses $1,000 so the total loss of $1,000,000 occurs about 1 year out of 20. Nobody knows in advance when this loss will happen. Moreover, assume a loss can happen in any year independently of the past outcomes. To assume independence is the most favorable case for feasibility of insurance supplied by a private company. Thus bad years resemble independent random events across years. The probability of loss in any year is only 1 in 20 and the expected loss to an individual policy holder is 1/20 of 1,000 = $50 annually. However, if the insurance company collects $50 yearly from each customer, then it is very likely that over the next 20 years, the insurance company will not be able to meet its obligations. The probability of a good year is 19/20. The probability that one bad year will occur in the next 20 years equals $1 - (19/20)^{20} = 0.6415$. This means that the probability the insurance company will default in the next 20 years because it has not collected enough from its policy holders to satisfy its obligations is 0.6415.

Let the insurance company double the premium from $50 to $100 yearly. It will thereby accumulate enough to pay the claims for one bad year in 10 instead of in 20 years. Consequently, it incurs a risk of default only if the crisis occurs some time in the next 10 years instead of the next 20 years. The risk of default in this case is $1 - (19/20)^{10} = 0.401263$, still high. Raise the premium to $200 annually so the risky interval becomes 5 years. Even so, the probability of default in the next 5 years is $1 - (19/20)^{5} = 0.2262$, still above what a private insurance company would accept.

These examples apply to financial crises because they form a chain reaction of failures. A chain reaction occurs because the assets of one company are the liabilities of some other company. When companies fail, those who held liabilities of the failing firms had reckoned these liabilities as their own assets. Hence they incur the loss of these assets. Consequently, financial crises affect nearly everyone. Even if financial crises seem to occur as independent random events 1 year out of 50 or 60 and individuals do not all suffer alike, few escape unscathed. Just as no private insurance company could survive in a business of insuring against losses from war, so too no private insurance company could survive in a business of insuring their policy holders against losses from financial crises.

4 The Principal Result

At first blush one may believe the obstacle to provision of insurance by a private company appears because the law of large numbers does not apply to a small number of cases. The pertinent number is the number of years, not the number of individuals exposed to independent bad events. This is not why the law of large numbers does not apply in the present situation. The true reason lies deeper.

The law of large numbers says that sooner or later an event will occur no matter how unlikely if the probability of its occurrence is a *fixed* positive number as small as you please. This does not hold in my model. In my model the probability of crisis is not a fixed number, it is 1/T. It depends on how long it takes to save enough to cover the loss from a crisis. My model assumes a crisis can occur with probability 1/T in any year before enough funds have been accumulated to cover all the damage. The probability of no crisis in any year during the vulnerable interim is 1-1/T. The probability of a crisis approaches zero as T increases. The number of years increases at the same rate as the probability of crisis decreases. Consequently, the law of large numbers does not apply to this model.

The numerical examples in a more abstract setting point to a remarkable general result. Each individual in a group is exposed to equal loss from a financial crisis that occurs about once in T years. They form a cooperative that collects an amount annually that is enough after T years to cover their total loss. Since a financial crisis can happen before the passage of T years, the funds may not be enough to cover the total loss and the cooperative would default. Nobody knows in advance when a crisis may happen. It

can occur in any year as a random event independent of the past. The cooperative decides to hire a consultant who can calculate the probability of a crisis before T years have passed.

Summarizing, the probability of a crisis in any year is 1/T and of no crisis in any year is 1 - 1/T. Thus in any year either there is a crisis or there is no crisis. After T years have passed, the cooperative will have accumulated enough funds to cover the total cost of a crisis. We want to know the chance of at least one crisis before the Tth year. This probability is $1-(1 - 1/T)^T$. The probability of no crisis in T years is $(1 - 1/T)^T$. Formula (1) gives the probability of at least one crisis in T years.

(1) $1-(1 - 1/T)^T$.

Formula (1) includes a well known expression.

(2) $(1 - 1/T)^T \to 1/e$ as $T\to\infty$.

Therefore, in the limit the probability of default is 1 – 1/e = 0.632121. Even when T is as small as 20 for which the exact probability of default given above is 0.6415, it is tolerably close to the limit. Therefore, convergence to the limit is rapid as Figure 1 shows.

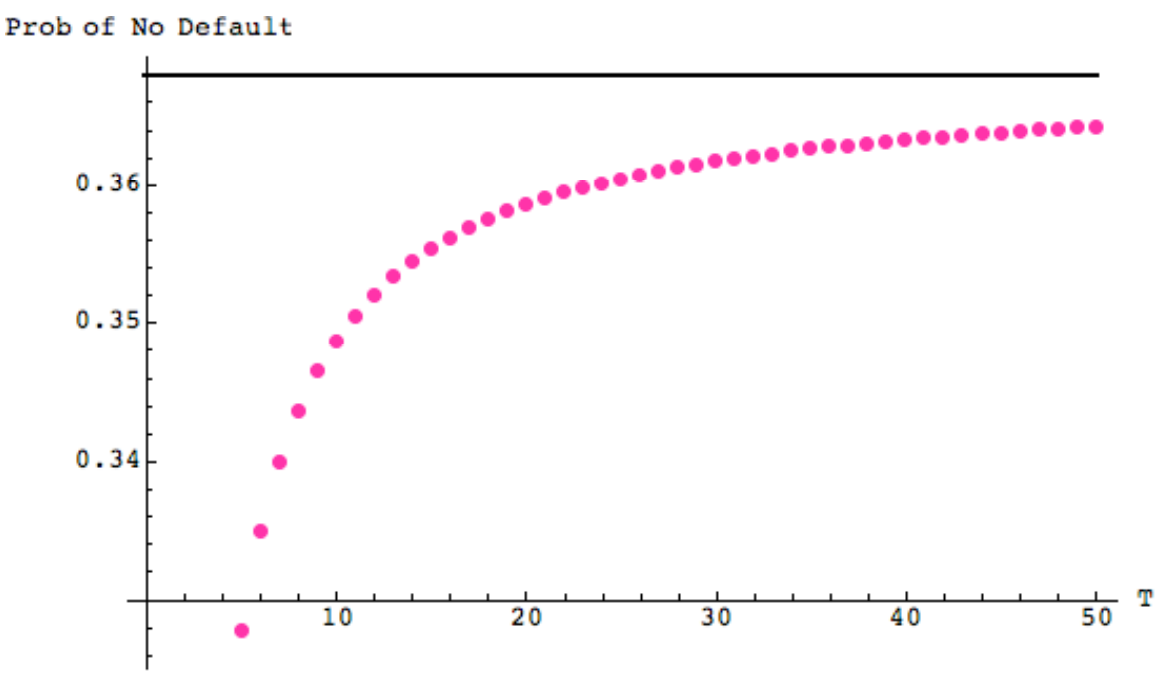

Figure 1

Table 1: Schedule of Loans, Repayments and Savings during T Periods

Period	Loan to Coop at Beginning of Period	Coop Savings at End of Period	Coop Repays at End of Period	Probability of Loss
1	T	1	T	1/T
2	T-1	2	T-1	(1/T)(1-1/T)
3	T-2	3	T-2	$(1/T)(1-1/T)^2$
…	…	…	…	
T-1	T-(T-2) = 2	T-1	2	$(1/T)(1-1/T)^{T-2}$
T	T-(T-1) = 1	T	1	$(1/T)(1-1/T)^{T-1}$

One may object that this result overlooks an alternative. A cooperative may borrow enough funds to tide itself over the interim when it lacks enough funds to survive a crisis. Table 1 has some pertinent details. Let the cooperative borrow T at the beginning of the first period, T-1 at the beginning of the second, T-2 at the beginning of the third and so on to the Tth period. Table 1, Column 5 shows the probability of loss for each period.

Now a subtle point enters that needs emphasis. In this model the total loss, call it X, is *given*. If each policy holder pays A annually and there are m policy holders, the cooperative collects m A = x annually. It sets aside the amount x in each t to cover this given total loss X. Because every policy holder incurs a loss when there is a crisis and the total loss exceeds the amount collected annually, the time it takes to cover the total loss T= X/x *must* exceed 1. That T > 1 is important for calculating the expected loss of a financial crisis. To ask what happens as T increases is the same as to ask what happens as x decreases. The probability of at least one crisis approaches the limit, 1 –1/e, and x approaches zero. It can be shown that the expected loss equals (X-x)(1-2 q^T). As T increases, the expected loss approaches (1 –2/e)X = 0.2642 X.

Should a crisis occur during the first T periods, a lender would face the same loss as the cooperative had it not borrowed. Borrowing to cover the loss from default only shifts the burden of loss from the shoulders of the borrower to the lender but does not remove the loss.

A more serious issue is implicitly present in this analysis. Describing the risk in financial terms is misleading. The story of Joseph the Provider rescues us from this error. To satisfy the requirements of the 7 lean years Joseph advised the Pharaoh to store grain during the 7 fat years presumably because no outside source could be relied upon to supply grain when the harvest was small. The same advice applies to the cooperative. Ultimately, the task of overcoming the effects of a financial crisis lies in the hands of the cooperative itself. It cannot be shifted elsewhere.

5 Real vs Nominal Loss

A simple example shows the distinction between real and nominal loss. Let a firm start at time t=0 with an inventory X(0). If it sells x per period, then the total inventory would last for X(0)/x periods. Let X(t) denote the inventory that remains at the end of period t. Consequently, $X(t) = X(0) - t\,x$ and $X(T) = 0$. Assume the price of the commodity is constant. Because the owner of the inventory obtains receipts equal to x in each period until the inventory is exhausted, he will have enough funds by the end of period T to buy as much inventory as he had at the outset.

Let something unexpected happen in period t*, $0 < t^* < T$. The demand for this commodity collapses so $x = 0$ for $t^* \leq t \leq T$. The firm has unsold inventory equal to $X(t^*) = X(0) - x\,t^*$ that has no commercial value whatever. Consequently, the firm cannot recover the cost of its initial inventory. Some of the resources used to produce the initial inventory also incurs a loss owing to the unanticipated drop in demand. While it may be possible to furnish funds to pay for these resources, nevertheless the loss is real not nominal.

Figure 2 shows the rise and fall of the annual rate of housing starts monthly, seasonally adjusted from January 1959 to August 2010. Housing is the lion's share of consumer outlays on durables. It is financed by mortgages held as assets by various financial institutions. If mortgages default, then the worth of these assets becomes zero. The holders incur financial losses. There is a real loss to the economy from the resources used to construct these houses. This housing was collateral on defaulted mortgages. Even so, this high annual rate of housing construction was a spur to high economic activity of the economy for more than 20 years starting around 1990. Figure 2 shows the spectacular collapse of housing starts beginning in 2008. Figure 3 shows this in logs (percent).

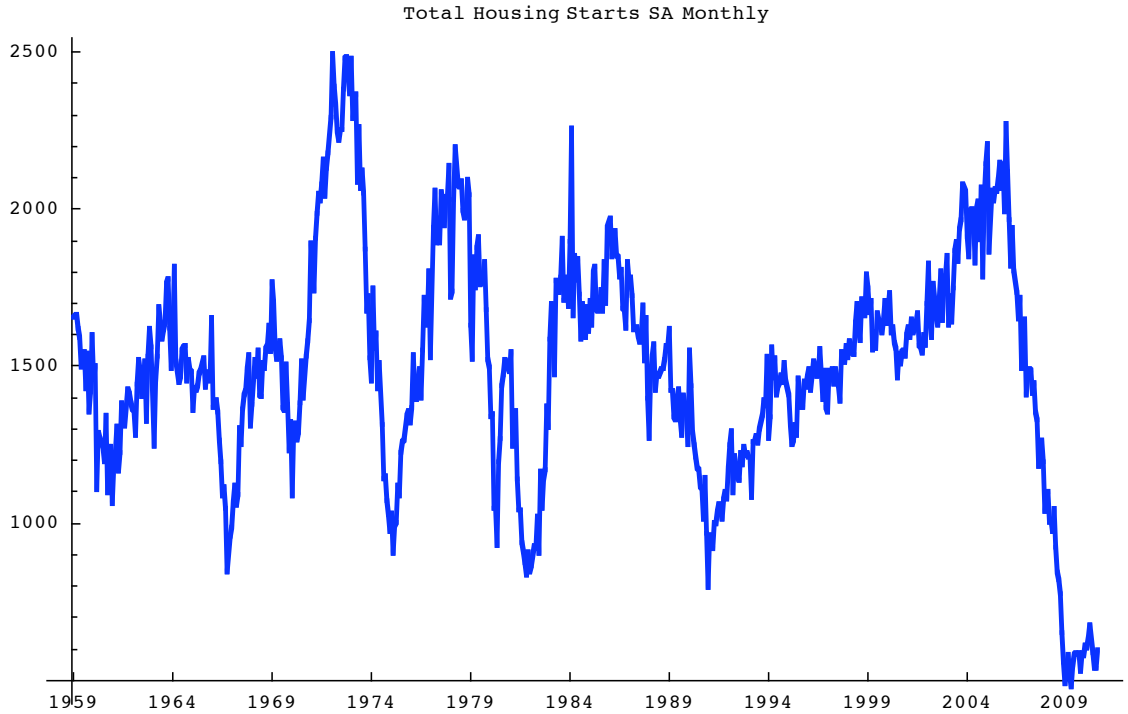

Figure 2

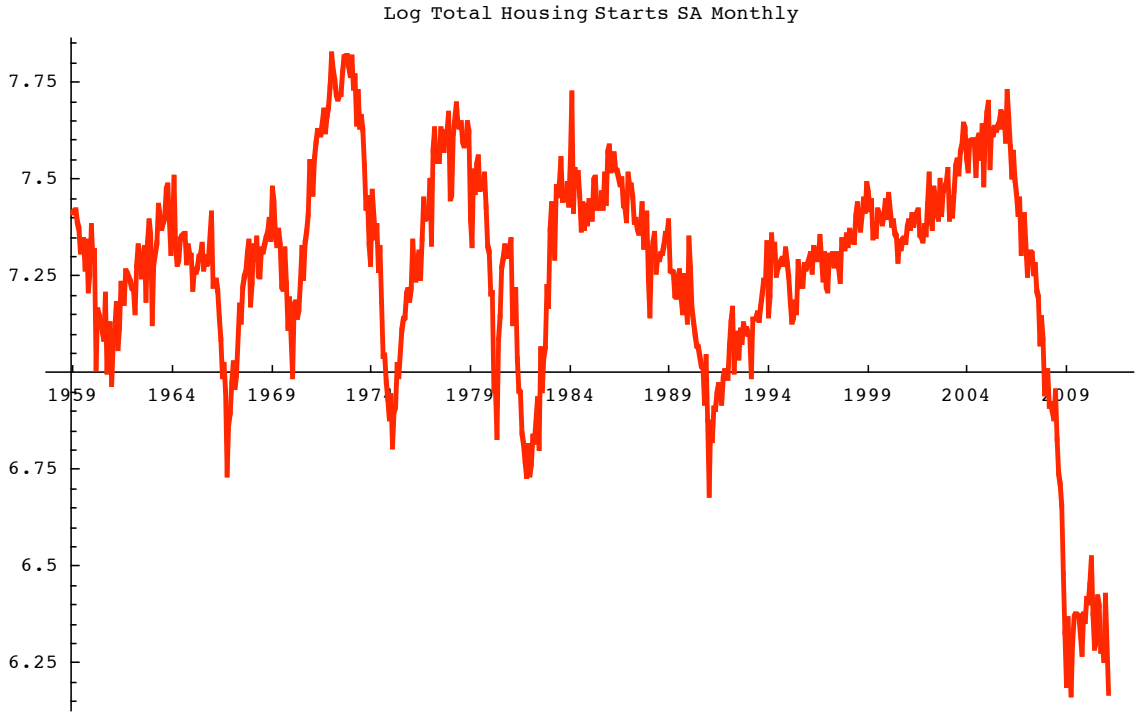

Figure 3

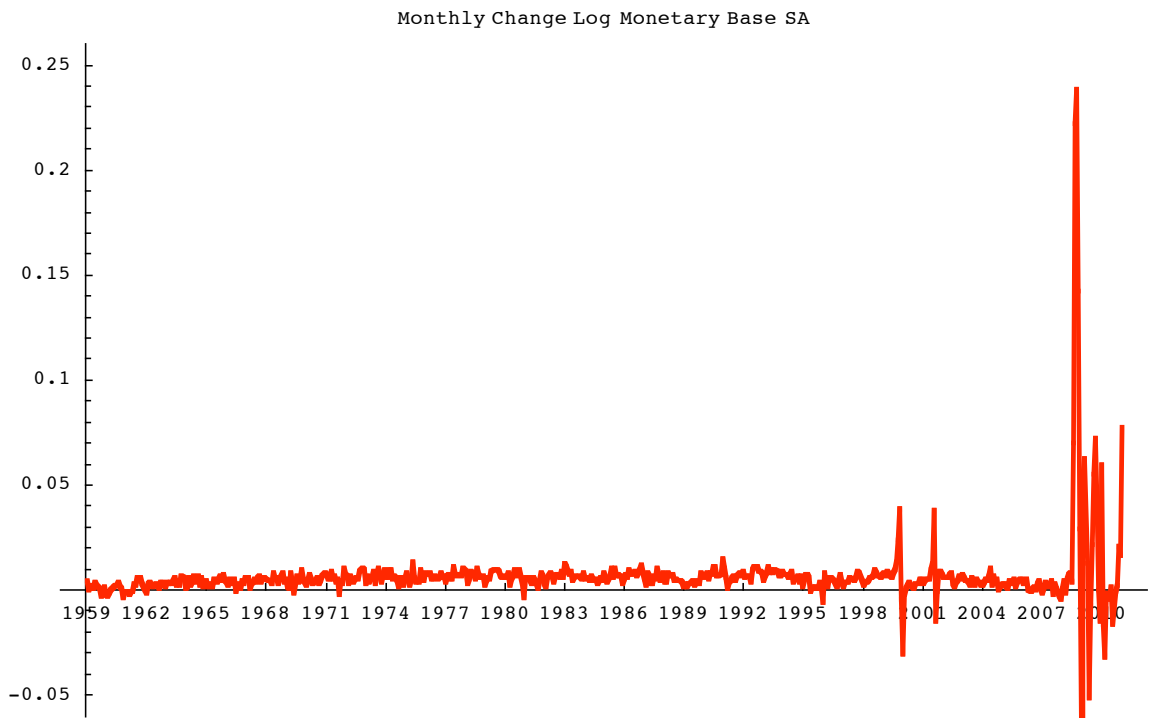

Figure 4

Figure 4 could hardly be more dramatic. It shows the monthly change of the seasonally adjusted log of the monetary base from January, 1959 to February, 2011. The huge changes in 2008 dwarf the two sizeable changes a decade before. This figure shows one aspect of how the monetary authorities responded to the financial crisis beginning in September, 2008.

6 Summary

This story has a moral. History teaches that a financial crisis precedes a real crisis. A real crisis requires a real solution. If a financial crisis were predictable, then it would be preventable. If a financial crisis were not predictable, then, as I have shown in my model, it is not insurable by any private company. To say otherwise is a delusion.