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Competition in the Core

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The most lucid introduction to a book can only be written after the book is finished.

Many economists who study rivalry among participants in markets and industries feel they must confine their choice to models to those based on non cooperative game theory. They reject core theory because they believe that its use of coalitions renders it useless to study competition. This view is mistaken. On the contrary, core theory and models of competition go hand in hand. This has been true since the birth of core theory in 1881.

Edgeworth is the founder of core theory. His book, Mathematical Psychics, published in 1881 has almost all the key ingredients of core theory . It abounds in formidable mathematics and acute reasoning. The more accessible parts are in every price theory course. Millions of students learn about indifference curves, the Edgeworth box, contract curves and that a competitive equilibrium lies at the tangency between indifference curves of two traders at a certain well defined point on the contract curve. Recontracting is a more subtle concept in Edgeworth's analysis of pure exchange. Traders make tentative agreements among themselves. The final settlement, using Edgeworth 's term, has the property that nobody can find anybody who would offer better terms than is available in the final settlement. I must emphasize that this salient feature of core models of a market first appears in the pages of Edgeworth's small monograph.

Some students may have been told that the contract curve for two traders shrinks to a point when the number of traders is infinite. Few learn Edgeworth's intricate reasoning that proves this result. I must confess that while this aspect of Edgeworth's monograph was duly recognized, for decades it had little effect on economics.

The seeds of renewed interest were sown in 1944 when von Neumann and Morgenstern published The Theory of Games and Economic Behavior. The influence of Austrian economics on their treatment of economics is unmistakable. The names of Menger, father and son, v. Böhm-Bawerk and the Italian Pareto appear in their index but nowhere is the name of any English or Scottish economist. The seeds of revived interest in Edgeworth's model still lay dormant.

Before we can find the cause of revival we must go back to 1928 when v. Neumann published his famous article on 2-person zero-sum games. This article contains and proves his Minimax Theorem. It asserts that a discrete game has a unique equilibrium called a saddle value such that each player has a mixed strategy that guarantees him the best possible expected return even under the least favorable conditions. Any departure from this saddle value redounds to the advantage of your opponent at a cost to yourself.

A non-zero sum game does not have a saddle value nor does a game with 3 or more players. Nash's non cooperative equilibrium applies to such games (1950, 1951). It too requires mixed strategies in general but very often it also has a serious drawback. There are many non cooperative equilibria. Therefore, neither the participants nor the economist who wants a model capable of making

accurate predictions about their actions can say what will happen because they do not know which of the many non cooperative equilibria will prevail. Moreover, even when there is a unique non cooperative equilibrium, it is usually inefficient. Inefficiency is a trait that a non cooperative equilibrium shares with the text book treatment of monopoly. Inefficiency means it would be possible for at least one person to become better off without making anybody else worse off. Consequently, the theory of non cooperative equilibria makes room for superior alternative models. Indeed, these models are implicit in the 1944 monograph.

Most economic games of interest to economists are not zero sum. All players can gain in non zero sum games. The outcome of a game is a distribution of the returns among the players. A central economic problem is to determine whether the total gain is the maximum feasible. If it is, then the outcome is Pareto optimal. In a Pareto optimum no body can get more unless somebody gets less. Because of this, theories about games with more than two players should embrace the possibility that coalitions may form. Whether an individual would be willing to join one depends on the terms it could offer him. Beginning with its interest in economic applications in 1944, game theory paid explicit attention to forming and joining coalitions as part of a player's strategy. Players not only figure out how to play the game but they also figure out with whom to combine. These combinations, called coalitions in game theory, are tentative. They correspond to recontracting in Edgeworth's model of pure exchange.

The undominated distributions of the gains, if they exist, constitute the core of the game. They correspond to the final settlement in Edgeworth's model of pure exchange. A distribution is undominated if no coalition can enforce and obtain better returns for its members. A necessary but not sufficient condition for an undominated distribution is Pareto optimality.

Lloyd Shapley was first to detach the notion of the core from the stable set, the version of a solution of a game given by v. Neumann and Morgenstern. Shapley's contribution is the first big step toward the revival of interest in Edgeworth's theories of pure exchange. Martin Shubik (1959) found the link to Edgeworth. He showed the relation of the core to Edgeworth's market games. Herbert Scarf (1962) proved theorems on convergence of the imputations in the core to a competitive equilibrium. Shapley proved a general theorem that a Walrasian equilibrium belongs to the core. Therefore, a nonempty core is necessary for the existence of a Walrasian equilibrium (Telser, 2007, pp 105-6). Aumann (1964)and Hildenbrand (1974) proved general theorems relating the core to a Walrasian equilibrium for a continuum of traders.

A Walrasian equilibrium focuses on whether there are single-unit prices capable of clearing a market. Fixed costs, long term contracts, indivisibilities and externalities require suitable terms well beyond the capabilities of Walrasian prices. In the presence of such complications core theory is well stocked to show how the market can have an efficient equilibrium under more general arrangements than a Walrasian equilibrium would permit. A nonempty core describes an efficient general equilibrium although it is not a Walrasian equilibrium.

Freedom of contracting is the essence of Edgeworth's theory. It coincides with most people's view of what competition means. The recontracting mechanism produces terms of sale acceptable to buyers and sellers. It does not assume given prices unaffected by the participants in the market. It explains what produces the terms of sale and how they change with the number of participants.

In their first encounter with formal economics students are usually taught to abandon this common view of competition and to accept a different view – prices beyond the control of individuals - as the hallmark of competition. Because these students' transactions are limited to supermarkets and large retailers on the internet, they accept the assumption of given prices because it is consistent with their own experience. It is easy to teach model of consumption by household and production by firms assuming given prices. Supply and demand become exercises in simple geometry uncomplicated by analysis of what lies behind them.

A Walrasian equilibrium is the high tech version of all this. An economy in which everybody is infinitesimally small, there are decreasing marginal rates of substitution between commodities in demand and factors of production in supply is ripe for powerful mathematical machinery that delivers a Walrasian equilibrium. It is interpreted as *the* competitive equilibrium. In reality this view of competition is worse than useless, it is a dangerous misconception when it becomes a guide to economic policy.

Understanding how a free enterprise economy functions is a major task for economics. Such an economy combines cooperation and rivalry. Complicated long term arrangements among the participants become necessary to finance and serve their requirements. They form coalitions, shifting alliances, that compete for members and resources. Individuals seek coalitions that can offer them the best terms. Within a coalition members cooperate according to complicated arrangements. For those willing to look, the visible presence of a mixture of rivalry and cooperation is plain.

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