From the SelectedWorks of Lester G Telser

November, 2010

III Ventures with a Finite Sequence of Payments of Uncertain Duration

Lester G Telser, *University of Chicago*

Available at: https://works.bepress.com/lester_telser/26/

III. Ventures with a Finite Sequence of Payments of Uncertain Duration

L. G. Telser

Abstract. This part describes a model of the valuation of ownership shares joint ventures with limited liability such as corporations. Asset Pricing JEL G12

1 Setting the Stage.

Individuals determine their own valuations of a venture. These valuations are the v's in A(v), the cdf of valuations of the current share holders. At least half the valuations are the same in a stable cdf according to the principal theorem in Part II. This does not require that at least half the present owners follow the same avenue to a common valuation. People can reach a common valuation on the basis of different analyses. Nevertheless we may assume for the sake of simplicity that at least half the share holders do follow the same avenue toward their valuation.

Definition. A venture is a sequence of a finite number of payments. This number is a random variable T with probability $p^T q$ where q = 1-p and 0 < p < 1.
A venture promises to pay y in every period for as long as possible. If payment ceases after even one non payment, then the venture defaults. The total return of a venture is the random variable T y given y and random T. The expected value of a venture is conditional on y. These payments have diverse interpretations such as dividends, earnings, interest payments and so forth. To assume default after only one missed payment is severe but it does not affect the main aspects of my model and makes it easier to analyze.

2 The Present Value of a One Period Venture

I can consume a dollar's worth of goods and services today or tomorrow. Because a bird in hand is worth two in the bush so that consumption tomorrow is less sure than consumption today, it follows that consumption today is worth more than consumption tomorrow. A lender foregoes consumption today in exchange for a borrower's promise of repayment tomorrow. There is a positive probability, p<1, that consumption planned today to take place tomorrow will occur and a probability q=1-p that consumption will not occur tomorrow.

Let μ denote the expected value of 1 unit of consumption tomorrow.
(1) $\mu = p\ 1 + q\ 0 < 1$.
The amount repaid a lender tomorrow, x, for a loan of 1 to a borrower today must be ample enough to compensate the lender for the probability of non payment. Therefore, the payment promised tomorrow satisfies
(2) $p\ x = 1$ and $x = 1/p$.
Because p is a probability between zero and 1, we may assume p = 1/(1+r) with r > 0. Hence x = 1+

r >1. Thus the borrower promises to pay 1+ r tomorrow for a loan of 1 today. The present value of a dollar tomorrow = 1/(1+r) is less than 1.

■ 3 Expected Value of a Multi-period Venture

Table 1 shows revenue sequences for a venture such that q is the probability of stopping in any period and p is the probability of continuing in any period. The duration of a venture is uncertain. The owner of the revenue stream gets y in each period until the venture stops. The first row of Table 1 shows what happens if the venture stops at the end of the initial period so that the revenue is zero forever. Each revenue sequence has a finite number of non zero terms. This number of terms that defines the venture's duration is a random variable.

period	0	1	2	3	-	T
0	0	0	0	0	0	0
1	y	0	0	0	0	0
2	y	y	0	0	0	0
…	…	…	…	…	…	…
T	y	y	y	…	…	y

Table 1

period	0	1	2	3	…	T
probability	q	pq	p^2 q	p^3 q	…	p^T q
revenue	0	y	2 y	3 y	…	T y

Table 2

From the data in Table 2 we can calculate the expected value of the revenue sequences in Table 1. The probability of a sequence with T terms is $p^T q$. These probabilities sum to 1 because $\sum_{T=0}^{\infty} p^T q = q/(1-p)$ and p+q=1. Let Y denote total revenue.

(1) Probability$\{Y = \text{Ty}\} = p^T q$.

Let E(V:y) denote the expected value of the venture conditional on y. It equals the sum of the products of the corresponding terms in rows 2 and 3 in Table 2.

(2) $E(V:y) = 0\,q + 1\,y\,p\,q + 2\,y\,p^2\,q + 3\,y\,p^3\,q + \ldots + T\,y\,p^T\,q + \ldots$.

(3) $= p\,q\,y\left[p + 2\,p + 3\,p^2 + \ldots + T\,p^{T-1} + \ldots\right]$.

The expression in brackets in equation (3) equals $\partial_p\, p/(1-p) = 1/(1-p)^2$.

(4) $E(V:y) = p\,q\,y/q^2 = y\,p/q$,

Let E(H) denote the expected duration of the venture. Hence E(H) = p/q . This is by setting y = 1 in formula (2).

(5) E(V:y)= y E(H).

Theorem 1. The expected value of the uncertain venture, E(V:y), equals y multiplied by the expected duration of the venture.

For example, if the real rate of return, r, were 0.03, then the probability of payment in each period would be p =1/(1+r) = 0.970874. The probability of default = 0.029126. Because $1 - p = q = 1 - \frac{1}{1+r}$, the relation between the real rate of return and the expected duration of the process given by equation (5) implies

(6) $E(\text{Duration}) = \frac{1}{1+r} \div \frac{r}{1+r} = \frac{1}{r}$.

Theorem 2. The expected duration varies inversely with the real rate of return.

■ 4 The Probability and Size of Payment

Assumption. *Either* the probability and size of payment vary inversely *or* the expected value of a venture can be as large as you please.

Because I assume every uncertain venture has a finite value, I require y and p vary inversely. The simplest version that meets my requirement is

(1) y p = 1.

Hence p = 1/y and q = 1-p =1 - 1/y.

(2) E(V:y) = y (p/q) =1/q.

(3) E(H) = p/q = 1/(y-1)

If p = 1/(1+r) so that y/(1+r)=1, then y = 1+r.

However, an inverse relation between y and p is necessary but not sufficient for a finite expected value of a venture. For example, let $y = e^{1/p}$ so that $E(y) = p\, e^{1/p}$. Hence E(y) becomes unboundedly large as p approaches 0 although p and y do vary inversely.