January 3, 2010

Are Speculators Foolish in Keynes' Greater Fool Model

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- **Summary**

  Keynes' presents a model of speculation in his General Theory (1936, pp. 154-8). These models are often called either the Greater Fool Theory or the Keynesian beauty contest. A speculator may buy something at a price he regards as too high because he believes he can find a buyer willing to pay an even higher price. This need not be foolish as Keynes shows without using algebra. However, as the author of A Treatise on Probability and as FDR once described Keynes after a meeting with him 'as some sort of mathematician', one should not doubt Keynes' skill in algebra. Still some may find it useful to see the mundane algebra behind this Keynesian model of speculation.

- **A Simple Version of the Model**

  Let \( r \) denote the rate of return at the end of a period. Somebody who buys an item at a price of 1 at the beginning of the period and sells it at a price of \( 1+r \) at the end of the period gets a net return of \( r \). The gross return after holding the object for \( t \) periods would be \( (1+r)^t \). Let \( p \) denote the probability of finding a buyer in a period. Let \( q = 1-p \) denote the probability of not finding a buyer in a period. Assume finding or not finding a buyer is a random event, independent of the past and identically distributed in each period. For the price to rise for an unbroken sequence of \( t \) periods so that a buyer appears in each of these periods but not in period \( t+1 \) is a random event with probability \( p^t q \). This means the market for the object collapses after \( t+1 \) periods because no one wants to buy it at any price. The expected return for \( T = t+1 \) periods, denoted by \( E(R,T) \), is given by the following formula:

  \[
  (1) \quad E(R,T) = \frac{\sum_{t=0}^{T} ((1+r)p)^t q}{\sum_{t=0}^{T} p^t q}
  \]

  Although \( E(R,T) \) is finite for any finite \( T \), the series is convergent if and only if \( (1+r)p < 1 \).

  \[
  (2) \quad E(R,T) \to (1-p)/(1-(1+r)p) = q/(q-rp) \text{ as } T \to \infty.
  \]

  The limit is positive if and only if the denominator, \( q - rp = 1-p(1+r) \), is positive. Therefore, convergence is equivalent to a positive expected gross return.

  The duration of this random process is itself a random variable. Call the duration \( H \) for horizon. The formula for the expected horizon is

  \[
  (3) \quad E(H) = \sum_{t=0}^{\infty} (t+1) p^t q.
  \]

  The simplest way to evaluate this expression is by means of the following generating function \( f(s) \)

  \[
  (4) \quad f(s) = \sum_{t=0}^{\infty} (sp)^t q = q/(1-sp).
  \]

  \[
  (5) \quad E(H) = f'(s)_{s=1} = p/q.
  \]

  Tables 1 and 2 show both the expected gross return at \( T = E(H) \) for plausible values of \( p \) and \( r \) and the loss if the market collapses at the expected horizon.
Table 1: Results If Return = 0.1

<table>
<thead>
<tr>
<th>p</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>r</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>E (R, T)</td>
<td>1.1770</td>
<td>1.30435</td>
<td>1.6667</td>
</tr>
<tr>
<td>E (H)</td>
<td>1.5</td>
<td>2.33</td>
<td>4</td>
</tr>
<tr>
<td>Loss at E (H)</td>
<td>-0.4114</td>
<td>-0.30613</td>
<td>-0.175692</td>
</tr>
</tbody>
</table>

Table 2: Results If Return = 0.05

<table>
<thead>
<tr>
<th>p</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>r</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>E (R, T)</td>
<td>1.0811</td>
<td>1.1321</td>
<td>1.25</td>
</tr>
<tr>
<td>E (H)</td>
<td>1.5</td>
<td>2.33</td>
<td>4</td>
</tr>
<tr>
<td>Loss at E (H)</td>
<td>-0.4079</td>
<td>-0.3068</td>
<td>-0.1945</td>
</tr>
</tbody>
</table>

Figure: Expected Return as a Function of the Expected Horizon for p = 0.6 and r = 0.1

How long can this process continue in the view of a conservative speculator? At first blush one might think the process is more likely to expire the longer it has gone on because it would seem that the supply of greater fools should diminish with the passage of time. However, the model denies this. It describe the process as a sequence of independent, identically distributed, random events. Independence means that the process can stop at any time with the same probability q no matter how long it has been going on and without regard to the level of the current price. Nor is this all. Let a speculator buy the object at the price \((1 + r)^t\) at the beginning of period t and hope to sell it at the beginning of the next period. The price at the beginning of period \(t+1\) will be \((1 + r)^{t+1}\) with probability p and will be 0 with probability q. A price of zero means the supply of greater fools is exhausted, nobody wants to buy the object at any positive price. Hence the expected profit at the beginning of period t is \((1 + r)^{t+1} - (1 + r)^t\) with probability p and is \(-(1 + r)^t\) with probability q. Hence a potential buyer at the beginning of period t expects the net return to be

\[
(1/14/10)
\]

For a convergent series it is necessary and sufficient that \((1+r)p < 1\). Therefore, in case of convergence the greatest fool, who is the last buyer, must anticipate a loss. The expected duration of the
process is always given by formula (3) for any period. The last row in Tables 1 and 2 shows the expected loss if the last buyer entered the market in that period which equals the expected horizon. The longer the duration of the rise, the bigger is the loss to the last buyer notwithstanding independence of the random events describing the process.

Now we are ready for the climax of the story. We study the net return as a function of the time when a buyer enters the market and plans to hold the object for s more periods. This poses the problem of when to enter and how long to stay. The price when he enters is \((1+r)^t\) and when he exits is \((1+r)^{t+s}\) with probability \(p\) or is 0 with probability \(q\). Hence the expected net return is

\[
\text{(7)} \quad p^s [(1+r)^{t+s} - (1+r)^t] + q(1+r)^t = p^s (1 + r)^{t+s} - (1 + r)^t = (1 + r)^t [(p(1+r)^s - 1)].
\]

If there is convergence so that \(p(1+r) < 1\), formula (7) says the expected loss is bigger, the later the buyer enters the market and the longer he stays in.

The formula says something else much more important. Suppose you are in the market at the end of period \(t\) holding the object at the current price that is \((1+r)^t\). You could sell at this price and lock in whatever profit you now have or you can stay in for one more period. Formula (7) shows your expected loss is minimal if \(s = 1\). Indeed, if the 'game' is fair, so that \(p(1+r) = 1\) and \(s = 1\), then your expected loss is zero and you have a chance of winning big or losing big. This is the Petersburg paradox. Take \(r = 2\) and \(p = .5\) so your expected loss is always zero (Keynes, 1921, pp. 316-23). It does not stretch credibility to imagine there are people willing to play one more round in this situation. They decide sequentially one period at a time. At each period the expected profit is zero, but the stake rises pari passu, as Keynes would say, with the gain and the loss.

- **Conclusion**

My purpose of this exercise is to call attention to phenomena that deserve a place in the realm of economics as a science assuming economists want to explain how people behave.

- **References**