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lumpyCost II

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A fleet of passenger airplanes with different capacities illustrates lumpy cost. The cost of using an airplane depends on its capacity, crew size and trip distance. The cost is zero if the plane is idle. Variable cost includes its fuel consumption that is a continuously increasing function of distance flown. Crew size may depend on the number of passengers but it is a step-wise increasing, not a continuous function. The term avoidable cost is accurate because it is avoided if the airplane is idle. The fixed cost depends on the composition and size of the fleet, not on its activity. An efficient fleet depends on the level and variability of passenger demand. How to attain this is suggested by a famous number theory problem, le problème de Bàchèt.

A two pan balance weighs an object in one pan by placing known weights in the other pan until the two pans balance. Any amount up to 15 grams could be weighed using 15 one gram weights. A better solution uses weights that are powers of 2 starting with a 1 gram weight.

$$1 + 2 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 1.$$

Thus 4 weights, 1, 2, 4, 8, suffice to weigh any amount up to 15 grams instead of 15 one gram weights. Provided the cost of a weight does not double with size, it gives the least cost solution. Therefore, the least cost of a fleet of passenger airliners depends on the probability distribution of the number of passengers and on the cost of airliners of different capacities, power and so on.

Diagrams illustrate the economics of lumpy costs. We start with some formulas. Lumpy cost A is defined as follows:

$$A = \begin{pmatrix} a = 0 \text{ if } x = 0 \\ a > 0 \text{ if } x > 0 \end{pmatrix}$$

Variable cost is linear in output x, so variable cost = g x with positive g. The alternatives are a plant with avoidable (lumpy) cost and zero variable cost versus a plant with linear variable cost and zero avoidable cost. Assume the fixed cost is the same for both. A critical level of output is determined by the following inequalities.

$A/x > g$ if $x < x_o$ and $A/x \leq g$ if $x \geq x_o$.

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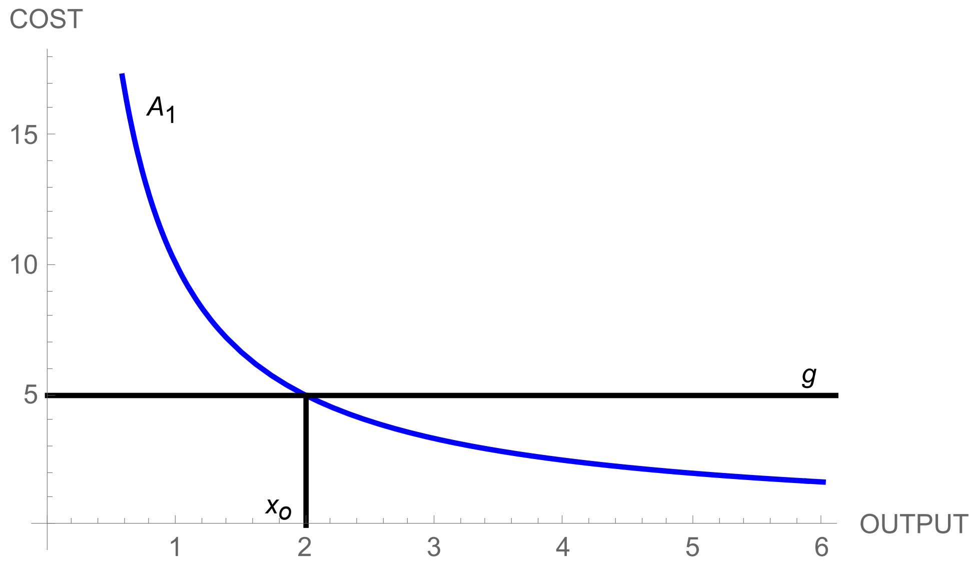

Figure 1

Figure 1 shows that the linear cost & zero avoidable cost plant, Firm 0, is cheaper for outputs smaller than x_0 and is more costly than the plant with positive avoidable cost and no variable cost for outputs bigger than x_o, Firm 1. The area under the hyperbola is the avoidable cost A_1. The avoidable cost plant A_1 is cheaper for outputs above x_0. Without an upper bound on the capacity of the avoidable cost plant, it would be the least cost producer for all outputs above x_0. Total variable cost increases linearly and exceeds the avoidable cost for all outputs above x_0. The situation changes if an avoidable cost plant has a finite upper bound on its capacity.

The situation is more complicated when avoidable cost plants have limited capacities. The bigger is the plant, the bigger its avoidable cost. The best solution takes this into account. The simplest case has two avoidable cost plants, A_1 and A_2. The first avoidable cost plant, A_1, has capacity k_1. The second avoidable cost plan, A_2, has capacity k_2. Let $A_2 > A_1$ and $k_2 > k_1$. Neither plant incurs a variable cost.

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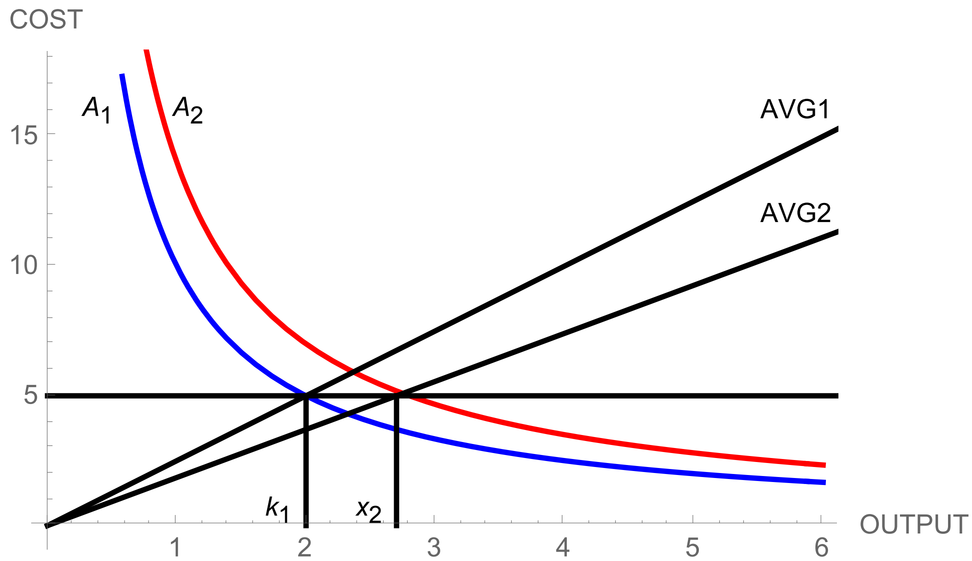

Figure 2

Figure 2 illustrates the case with two avoidable cost plants and no variable costs. It shows two lines through the origin, AVG1, with slope g_1 and AVG2 with slope g_2. Output k_1 is the capacity of Firm 1. Output k_2 is the capacity of Firm 2. At the capacity of plant 1, the unit cost, A_1 / k_1 is below the unit cost of plant A_2/k_1 because $A_2 > A_1$. Unit cost of plant A_2 / x is below A_1/x for all $x \geq x_2$

Next consider the least cost of producing more than the capacity of plant A_1, $x > k_1$. For $k_1 < x \leq k_2$, only plant A_2 is active and A_1 is idle. It does not pay for A_1 to resume activity when $x > k_2$ until $x > k_2 + x_1$. Only at this output can A_1 cover its avoidable cost. Finally, it is only when

$$k_1 + k_2 \geq x \geq k_2 + x_1$$

that both avoidable cost plants, A_1 and A_2 are active and that no output comes from linear variable cost plants without avoidable costs.

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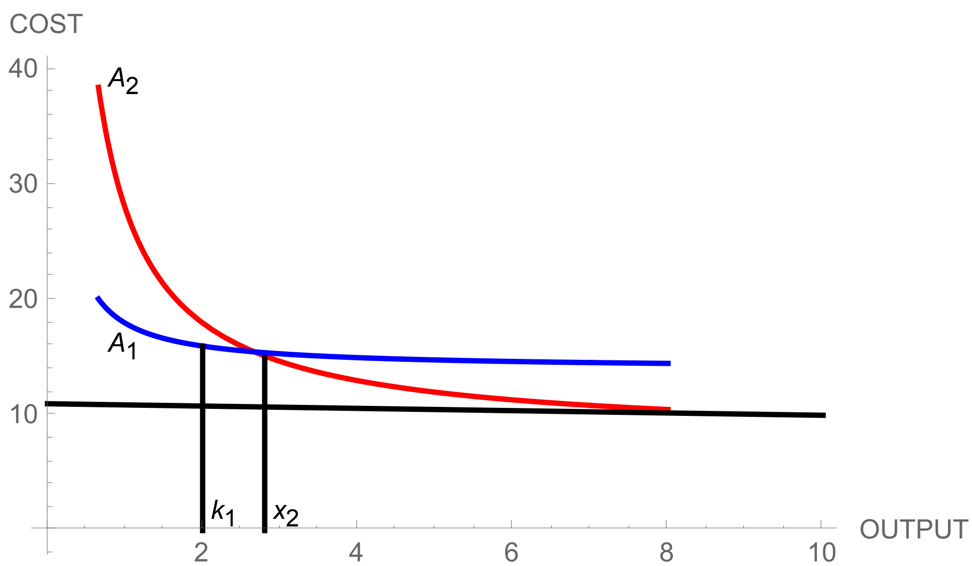

Figure 3

The hyperbolas that show unit cost can intersect only if the firms incur variable cost. If so, equation (1) describes the unit cost of a firm.
(1) (A + g x)/x = A/x + g → g as x →∞.
The asymptote of unit cost is g, the variable cost per unit output. Now output in the interval shown in (2) raises complications.
(2) $k_1 < x < x_2$
While the unit cost of Firm 1 is below the unit cost of Firm 2 for x in this range, output in this range is not feasible for Firm 1 since it is above its capacity, k_1. Firm 2 could produce in this range but at a higher unit cost than Firm 1.

Summary of Optimal Least Cost Production

The variable cost regime is best for x when $0 \le x < x_1$.
Avoidable cost regime A_1 is best when $x_1 \le x < k_1 = x_2$.
Avoidable cost regime A_2 is best when $x_2 \le x < k_2$= plant A_2 capacity.
All output is made by both avoidable cost plants when $k_2 + x_1 \le x \le k_2 + k_1$ and none from linear variable cost plants.

Solar panels are an important application of lumpy cost. Suppose individual households install their own solar panels on their roofs. Say that g is the cost of a solar panel so the total cost of x panels is g x, the linear variable cost function. Electric power generation corresponds to avoidable, i.e., lumpy cost.

A generator produces a given amount of electricity when it is active and none when it is idle. The total electricity generated by an electric utility comes from a collection of generators of various capacities. To satisfy a given demand for electricity at the least total cost, an electric utility decides which generators to use and which to keep idle. Although the actual amount of electric power generated does vary continuously, the amount generated is a sum of the capacities of the active generators. It must meet the required output, the amount demanded, but it cannot exceed the sum of the capacities of the active generators. Unused electricity goes back to the utility because storage of electric power is too costly. This application of lumpy cost to describe the generation of electric power shows that solar panels are efficient only if the required amount of electric power is small, such as x_1 in Figure 2. At power required above x_1, solar panels are inefficient for generating electricity.

Demand

Demand now enters. Unless buyers are willing to pay enough to cover the total cost of supplying them, there would be no output. If they are willing to cover the least cost of satisfying a given demand, then this raises the question of at what demand levels. My analysis answers this question for a simple case. Assume all buyers are alike and each is willing to pay g per unit of output. Now the linear function g x measures total revenue instead of total variable cost. If the value per unit equals g, then the linear variable cost regime would be active up to the output x_1. In the avoidable cost regime depicted in Figure 2, no supply is forthcoming unless the quantity demanded is at least x_1. However, this solution does not take into account the fixed cost problem. Perhaps two-part prices could solve it. Producers would require each buyer to pay a fixed amount independent of quantity that each buys so that the total amount of these receipts could cover the fixed cost of all the plants in the industry. Two-part prices face enforcing complications that are outside the scope of this essay.

Program
