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Technology Integration in Secondary Mathematics Classrooms: Effect on Students' Understanding

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Abstract: Technology use in secondary mathematics courses has the potential to bring about broad changes in learning environment and teaching pedagogy, allowing students to communicate and collaborate in new ways and to conjecture, justify, and generalize findings. However, this potential is only realized when teachers use technology in ways encouraging these outcomes (Galbraith, 2006). The purpose of this study is to examine the integration of technology in secondary mathematics classrooms and to evaluate the effectiveness of its use in relation to students' learning outcomes. This self-study research was conducted in honors geometry and AP calculus classes. Data sources included transcripts of classroom discourse, student responses to both forced-response and open-ended surveys, and teacher journals. It was found that when students were the primary users of technology, they were more engaged in learning and higher attained higher levels of understanding, especially in relating different mathematical representations and generalizing patterns.

About the authors: Megan Sheehan is a mathematics teacher at Bloomington High School, Illinois. She has written a project-based curriculum for Algebra 1 and enjoys using technology in teaching to help students make mathematical connections. Leah A. Nillas is an assistant professor at Illinois Wesleyan University. She serves as a university supervisor for mathematics student teachers and teaches curriculum courses in mathematics and high school science, stand-alone technology course, student teaching seminar, and a course on teacher research.

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"Our generation is so used to technology. We know the basics and it is good to explore because we will need that knowledge in the future... We get to be hands on and are interacting, not just sitting there listening to the teacher talk." – *Renee*

"[Technology] helped the activities to apply to more different learning styles. I'm more of a visual learner and being able to use technology to visually manipulate concepts is very beneficial to me…Being able to look at concepts from multiple angles, graphical and symbolic, helps me grasp a better understand (sic) the concept as a whole... I enjoy exploring problems from a different angle, and that's very beneficial for me." – *Emily*

Wells and Lewis (2006) reported that nearly ten percent of public schools in the United States have access to the internet. Eighty-three percent of public schools with Internet access have offered professional development to teachers on how to integrate the use of Internet (e.g., Web learning materials, use of digital libraries, museums, and multimedia) in their teaching. Ten percent of public schools lend laptop to their students and nineteen percent of public schools provide handheld computers to their students or teachers for use in instruction.

As technology continues to improve and become less expensive, it is increasingly incorporated into the classrooms. Students such as *Renee* and *Emily* are finding that technology can help them learn actively and provide visualization of seemingly abstract mathematical ideas. The National Council of Teacher and Mathematics (NCTM) has designated technology as “essential” and highly recommended its use in teaching mathematics, particularly for “graphing, visualizing, and computing” (NTCM, 2006). However, research studies show mixed results on the effect of technology use on students’ achievement. A 2005 study of the effects of student use of computer software and applets found that technology greatly heightened levels of achievement (Almeqdadi), while another in 2005 found no significant difference between users and non-users (Buykkoroglu, et al.). Analysis of the literature suggests that the impact of technology integration in the classrooms is highly dependent on the manner in which it is used. For this reason, it is essential that teachers and researchers examine in particular how technology is used in teaching.

The purpose of this study is to examine the impact of the integration of technology in teaching mathematics. The research questions addressed are: What are students’ perceptions on the use of technology in learning mathematics? How does technology affect students’ engagement to learning? The specific technologies examined in this study are graphing calculators, dynamic geometry software (i.e., Geometer’s Sketchpad), and interactive white boards. *Engagement* is used throughout this study to refer to the degree to which students are actively involved in learning. The impact of technology on learning is examined through students’ classroom discourse and through students’ reflections on their learning experiences.

The overall trends in the literature suggest that technology integration may lead to many benefits for students, but also stress the necessity of examining how technology is used to attain a full understanding of its impact on students’ learning.

Review of Literature

Technology: Increases Motivation and Improves Learning

Ruthven and Hennessy (2002) found, through conducting interviews with mathematics teacher in seven schools, that regular access to technology and familiarity with software and hardware made student success more likely. Teachers also reported that using technology led to increased participation and productivity by providing a new and different learning environment, removing instructional constraints (such as drawing graphs by hand), allowing students to “tinker” and experiment, improving student motivation and engagement, facilitating classroom routines, improving pace and productivity, accentuating mathematical features, increasing student attention, and helping students form and establish ideas. Ruthven and Hennessy’s study supported a claim that from the teachers’ perspective, technology use led to a variety of benefits for the students, including a deeper conceptual understanding and increased motivation.

Through a comprehensive literature review, Galbraith (2006) expanded on the effects of technology use in learning mathematics. He focused on three areas of research: mathematical outcomes resulting from technology use and how technology contributed to these outcomes, attitudes and beliefs expressed by students about using technology to learn mathematics, and how the learning environment changed when technology was used to study mathematics. In regards to student outcomes, Galbraith focused on problems on technology integration, both in terms of software intricacies and student behavior. For example, students tended to trust technology over their own knowledge or the context of the problem and often misinterpreted graphic representations of functions. Galbraith claimed that technology use had the potential to bring about broad changes (e.g., enable students to communicate and collaborate in new ways, facilitate the ability to conjecture, justify, and generalize findings) in learning environment and teaching pedagogy. Galbraith also pointed out the need for further research on impacts of technology. This need provided rationale for our research on the impact of technology use on students' engagement and learning.

In her summary about technology use in middle grades mathematics and science classrooms, Fies (2007) noted an increase in students' motivation and engagement, improved mathematical reasoning, and access to a variety of activities with interactivity, and use of multiple representations. Muir (2007b), on the other hand, noted that technology can be used on two levels: to imitate actions that were performed prior to the use of technology or to allow the performance of actions which originate because of the use of technology. The first type of usage could lead only to "automation, ease of access, ease of modification, and looking good". The second could lead to "innovation in teaching and learning," with a shift to constructivist teaching methods and cooperative learning (Muir, 2007b, p. 1). Muir (2007a) expanded further on this idea in a separate study, seeking to explore the variation in the literature; many studies pointed to a lack of effect, while others found a myriad of positive effects. He found that many of the studies that found no effect were conducted in schools where students used technology infrequently and/or teachers received little to no training on its use. In contrast, studies (e.g., Almeqdadi, 2005; Funkhouser, 2002; Hannafin, Burruss, & Little, 2001; Serhan, 2004) in which the teachers focused on integrating technology into a new teaching pedagogy found that technology use could engage and motivate students, help them make connections, and lead to more constructivist, student-centered teaching. Like Galbraith (2006), Muir (2007a, 2007b) observed that how technology is used is integral component in determining its impact on students' learning. McGraw and Grant (2005) focused on the classroom structure in which technology is used to classify lessons as Type 1 or Type 2. In "Type 1" lessons, students were given detailed instructions in investigating the relationships between different mathematical concepts. All students reached more or less the same conclusions because they were guided in their investigation, but they did not have the opportunity to engage in conjecturing, problem-solving, and decision-making. In "Type 2" lessons, students were provided more varied options of what to explore. These lessons allowed students to engage in true conjecturing and decision-making and could lead to a class discussion and pooling of conjectures and knowledge.

Finally, Garofalo, Drier, Harper, Timmerman, and Shockey (2000) categorized technology use by looking at who was the primary user: the teacher or the student. They suggested that technology was used most appropriately and effectively when students were the primary users. These categorizations enabled a varied approach analyzing how technology was used, allowing for versatility, and the use of multiple perspectives in analyzing how its usage impacted students.

Calculator: Facilitates Use of Multiple Representations

The most recent comprehensive overview of the effects of using calculators in mathematics classrooms was Ellington's (2003) meta-analysis conducted between 1983 and 2002 examining the effects of calculator use on students' achievement on mathematical assessments. Over 70% of these studies examined students in grades 8-12. She concluded that:

> When calculators were included in instruction but not testing, the operational skills and ability to select the appropriate problem-solving strategy improved for the participating students. Under these conditions, there were no changes in students' computational skills and skills used to understand mathematical concepts. When calculators were part of *both* testing and instruction, the operational skills, computational skills, skills necessary to understand mathematical concepts, and problem-solving skills improved for participating students. Under these conditions, there were no changes in students' ability to select the appropriate problem-solving strategies. Students who use calculators while learning mathematics reported more positive attitudes toward mathematics than their non-calculator counterparts on surveys taken at the end of the calculator treatment. (Ellington, 2003, p. 455)

The strength of the results was especially high when students were in middle school and high school and when graphing calculators were used. Overall, Ellington's study suggested that the use of calculators was certainly not harmful and may be beneficial in a variety of ways.

Researchers have continued to study the impact of calculator usage, expanding on Ellington's findings. Serhan (2004) examined the effect of calculator use on students' understanding of the concept of derivative at a point in undergraduate mathematics classes by comparing pretest and posttest scores for students in comparable courses using the same text at two different universities. Students in the experimental group received instruction that used graphing calculators to emphasize the connections between symbolic, visual, and numeric representations, while students in the control group received traditional instruction and were not allowed to use graphing calculators. Analysis of the pretest and posttest scores showed that there was not a significant difference between the pretest scores of the two groups but that the scores of the experimental group were significantly higher on the posttest. Interview analysis also showed that students in the experimental group were better able to interpret numerical representations and to understand and explain the connection between the average rate of change and the instantaneous rate of change Thus Serhan's (2004) research indicated that students who used calculators during instruction were more likely to form a deeper conceptual understanding of the topic they were studying,

In contrast to Serhan's (2004) quantitative study, Simonsen and Dick (1997) examined investigating math teachers' opinions on the impact the use of graphing calculators had in their classrooms. They conducted phone interviews of teachers who had received classroom sets of graphing calculators with CAS capabilities one year previously. The data trends revealed that teachers' perceptions of advantages were instructional, such as the identification of connections and the availability of feedback, while their perceptions of disadvantages were mainly logistical, such as lack of access and problems with calculator security. Particularly in classes that used the graphing calculators regularly, teachers felt that their use led to more student-centered classroom dynamics, increased cooperative and discovery learning, and increased student discussion, involvement, and enthusiasm. However, teachers also identified a need for increased preparation time and a critical need for further professional development and support.

Despite different approaches, Ellington, Serhan, and Simonsen and Dick all found that graphing calculators seemed to positively impact students' development and understanding of connections between different representations and concepts. The benefits discussed by the teachers interviewed by Simonsen and Dick also correspond very well to those discussed by the teachers interviewed by Ruthven and Hennessy (2002), adding further support to their generalizability.

Interactive Whiteboards: Provides Opportunity for Instructional Interactivity

Interactive White Boards (IWBs) link a projected display with a connected computer, allowing direct manipulation of the computer through the touch-sensitive white board. While their availability is just beginning to grow in the United States, they are much more common in England and Wales, where approximately 77% of mathematics teachers have an IWB in their classroom (Kennewall, Tanner, Jones, & Beauchamp, 2008). Only a limited amount of research has been conducted to explore the impact of IWB usage, although this gap has begun to be filled in recent years.

Kennewall et al. (2008) examined the effects of technology, especially interactive white boards (IWBs), in promoting England's national goal of interactive teaching, teaching that encourages student contribution and autonomy in directed learning. The authors presented a case study of one lesson in which an IWB was used in an elementary level mathematics class. The analysis showed that portions of the lesson did achieve a high degree of interactivity, but that the students only practiced their newly learned strategy during the whole-class portion, not during the group or individual portions. This suggested that the interactivity afforded by the IWB was not enough in and of itself to encourage student learning; rather, it was how the teacher and students used the IWB – in other words, the degree of instructional interactivity – that determined the degree of learning that occurred.

Miller and Glover (2007) performed a more comprehensive investigation of IWB use in secondary mathematics classes in which they videotaped and analyzed lessons shortly after IWBs were first installed and again two terms later in seven secondary schools across England. Their research showed that most teachers moved towards greater interactivity in their lessons but that the greatest gains and pedagogical changes occurred in schools where teachers had access to local "experts" to guide them through this process and where teachers were able to discuss and share resources and strategies effectively. Miller and Glover's research suggested that access to a local expert who could address technological as well as pedagogical concerns could lead to a greater degree of interactivity. It was also important that teachers formed a community or forum to discuss and share their concerns and successes.

Dynamic Geometry Software: Engages Students to Explore Mathematics

Dynamic geometry softwares such as *Geometer's Sketchpad* (GSP), *Cabri II*, *The Geometric Supposer*, and *GeoGebra* allow users to create and manipulate geometric figures and may allow for interactive visual representations of algebraic concepts as well. As these softwares have become more widely used, researchers have begun to examine how their use impacts students' achievement and attitudes towards mathematics.

Vincent's (2005) eighth-grade students used an interactive geometry software to explore a series of "linkages" and their underlying geometric structures, first conjecturing the geometry involved and then constructing (often informal) proofs that their conjectures were valid. Vincent noted that, for her students, proof emerged as a valued activity because it offered a verification of the truth of their conjectures and an explanation as to why the linkage works the way it does. Students were then able to extend this valuing of proof to further geometric proof and investigations. Vincent concluded that the software use helped students achieve an understanding of and appreciation for geometric proof, while Scher (2005) noted that the software use led to assessment difficulties for teachers but also the opportunity for a meaningful class discussion that may lead to a deeper conceptual understanding. He asked students to construct geometric figures using interactive geometric software. He presented three students' attempts to create a square and analyzed their comments and actions, recognizing that it was difficult to classify the degree to which a construction was valid, even using a seemingly well-defined system of classification. In practice, students' work frequently straddled the line between categories and was often completely different than what teachers might have expected or predicted. Scher suggested that these differences might form the basis for a class discussion of what makes a construction "right" and the advantages and disadvantages of the different constructions that the students attempted. Thus,

In a study by Hannafin et al. (2001), Students used GSP for a two-week unit on geometry, working in pairs to complete a series of activities at their own pace. Students were observed and interviewed; the teacher was also interviewed, both formally and informally. The two overarching themes that arose in the interviews were power and learning. In terms of power, the students were excited to gain power of the pace of their learning, but had mixed feelings about being assigned to work with a partner. The teacher struggled with losing direct power over her students and was very concerned on its impact on their long-term understanding and retention. Students were excited to be using computers and learning geometry, and felt that they had mastered the material, while the teacher was concerned that many rushed through the activities and would not retain the knowledge they would need to do well on the standardized test the following year. Like Simonsen and Dick (1997), Muir (2007b), and Becker (2000), Hannafin et al. noted the impact of the teacher's philosophy and the pressure to cover curriculum and prepare students for testing as well as the facilitation of a student-centered learning environment.

In addition to these qualitative studies, Funkhouser (2002) and Almeqdadi (2005) performed quantitative studies using control groups which also examined the use of geometric software. Funkhouser examined both achievement and attitude effects of constructivist, computer-augmented geometry instruction. One high school geometry class was taught using traditional methods, while a treatment group used the *Geometric Supposer* software throughout the entire school year; both groups took pre- and post-tests on geometry content and attitudes toward mathematics. The results showed a significantly higher achievement among the treatment group but did not show significantly better attitudes towards mathematics. Funkhouser suggested that technology use "*in and of itself*" might not lead to significantly more positive attitudes, and also that the attitudinal effect observed in shorter-term studies may be due to the novelty of technology use and may "wear off" over time (2002, p. 172). Almeqdadi, on the other hand, focused only the achievement impact of the use of geometric software. Almeqdadi's study took place in 9th-grade, all male, geometry classes in Jordan, in which the control group used only the textbook while the experimental group used GSP once a week over the course of a semester; both groups were taught by the same teacher and given the same pre- and post-tests. The results of the post-tests showed a higher gain by the experimental group and a higher mean score overall, along with a higher deviation. Almeqdadi inferred from this data analysis that the students in the experimental group did achieve a higher degree of understanding of the topic through the use of the GSP. Like Funkhouser, Almeqdadi found that the use of geometric software led to increased student achievement but noted that long-term technological use does not seem to lead to changed attitudes towards mathematics.

Summary of the Literature

The overall trends in the literature suggested that technology use in education did indeed lead to many benefits for students. According to Simonsen and Dick (1997), Hannafin et al. (2001), Ellington (2003), Serhan (2004), and Almeqdadi (2005), technology might instigate increased student achievement, participation, individualization, and conceptualization and might also help students to see connections among their studies and between their studies and the 'real world.' These benefits might not come without a price for teachers, however, who have to adopt a more constructivist, student-centered teaching approach in order to see students fully gain these benefits. Teachers might need to spend more time learning to use the technology, planning, and assessing students' work, and they must understand that how technology is used is more important than its use in and of itself; only when technology is used effectively will the full advantages be realized.

Given the rapid pace of current technological change, there is no way to predict what schools may look like in even ten or fifteen years. Heid (2005), for example, suggested that students may no longer use actual books or paper; schools themselves may even become obsolete as distance learning becomes more popular. Whatever the future may bring, it is important for educators to continually assess and reassess their understanding of the benefits and drawbacks offered by technology use, allowing them to adapt their teaching to be as effective as possible.

Methodology

Our research was designed as a *self-study* research which is a *situated inquiry* driven by our question to understand the impact of technology on students' learning of mathematics. Self-study research is a multifaceted research process of one's personal experience, employing collection and analysis of several data sources (Samaras & Freese, 2006). Though reflective, the process is also collaborative in nature. We collaborated together in planning, designing, collecting, analyzing and reporting the data collected from the classrooms. We collected multiple data: surveys, video recordings of class sessions, instructional materials, and teacher reflections. We focused on the technology integration experience of Megan in teaching mathematics.

The participants in this study were students in AP Calculus and Honors Geometry classes. The majority of these students were ninth- and twelfth-grade students, but the sample also included several tenth- and eleventh-grade students. The majority of the students were Caucasian, but the population also included several students of Asian origin and one African-American student. The students all attended the same high school, located in an urban, mixed income community. Students generally had access to a computer with the Internet at home, and all had some experience using technology, particularly word-processing software and calculators, during previous classes.

Students completed a short-response survey which was given to them on the first day of school. Throughout the semester, we collaborated (with support from the cooperating teacher) in planning, implementing and documenting mathematics lessons using different technology. For each lesson, data collected included: a lesson plan, video recording of the lessons, written observation notes I wrote on her teaching, as her university supervisor, of her teaching, and a post-teaching reflection, Megan wrote after teaching the lesson. The topics of the lessons discussed in this article included algebraic calculation of limits, graphical representations of derivatives, and the product and quotient rules for derivatives. Throughout the semester, we met together to discuss details and reflections on Megan's teaching and to focus our data collection and reflection process.

At the conclusion of the semester, students were asked to complete an open-ended survey focusing specifically on technology use. This survey asked students to discuss their thoughts about the use of technology over the course of the semester as well as their opinions about technology use in general. These questions were written specifically to address the research questions of this study.

We used a systematic iterative process of analyzing the qualitative data employing *conversation analysis* (Silverman, 1998, 2001) in analyzing transcripts of classroom video recording. We reviewed lesson plans and post teaching lessons for each lesson and noted how technology was used in each lesson. Data from the first surveys was quantitatively analyzed and responses from open-ended survey were coded in terms of common themes. The results of these analyses are discussed in the following section.

Results of the Study

Students' Perceptions about Technology Use in Learning Mathematics

Student responses on the first day of school indicated their initial opinions regarding the use of technology. These opinions, discussed in depth below, revealed generally positive attitudes toward the use of technology.

Figure 1 shows students' perceptions on how technology use influences their motivation and engagement. Students generally disagreed with that using computer makes mathematics more mechanical and boring. 58.5% of students either disagreed or strongly disagreed. A small percentage of students (18.3.%) agreed or strongly agreed with the statement, Finally, nearly a one-fourth of students (23.2%) neither agreed nor disagreed with this statement, These responses, in particular the 58.5% of students who disagreed or strongly disagreed, were consistent with the findings of Ruthven and Hennessy (2002) and of Fies (2007), who reported that mathematics teachers noted increased student participation, motivation, and engagement when technology was used in class.

Figure 1

Responses:
"Using computers makes mathematics more mechanical and boring."

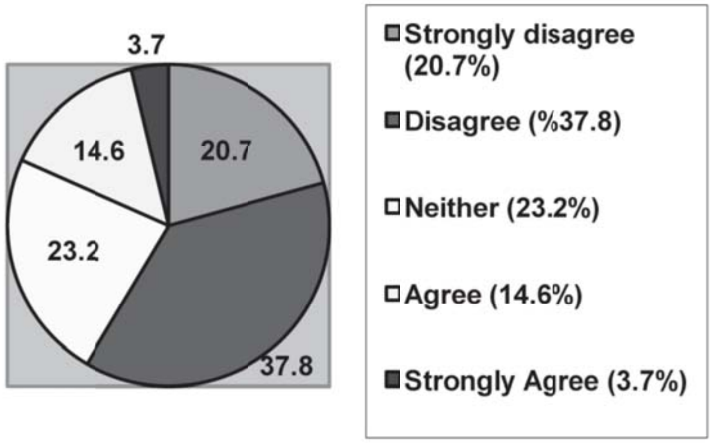
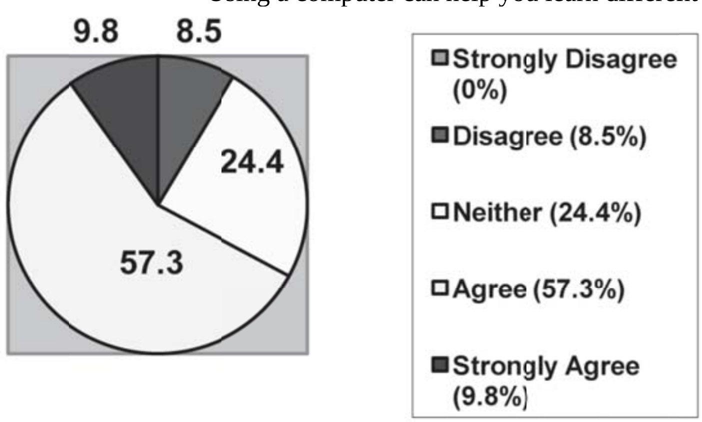


Figure 2 indicates positive opinions of technology use among students in their responses to the statement: Using a computer can help you learn different mathematical concepts. A majority of students, 67.1%, either agreed or strongly agreed with this statement. Similar percentages of students indicated negative or neutral feelings toward technology as with the first question – 8.5% disagreed and 24.4% neither agreed nor disagreed –These responses, especially those from the 67.1% of students who agreed or strongly agreed, were consistent with those given by teachers in Ruthven and Hennessy's (2002) study, who agreed with these students that technology can help students learn mathematics.

Figure 2

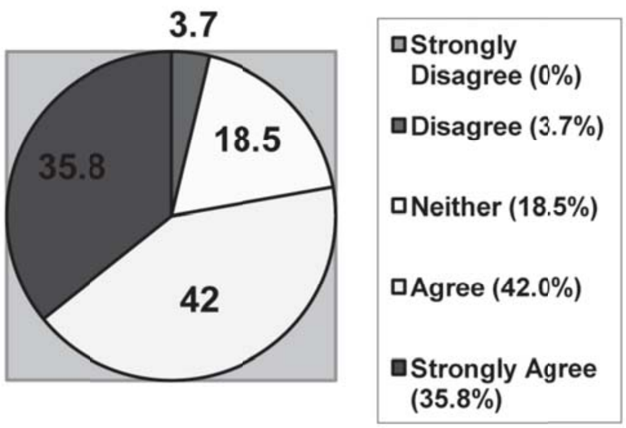
Response to:
"Using a computer can help you learn different mathematical concepts."



Finally, Figure 3 illustrates a possible reason for the first two responses. When responding to the statement "I expect to use computers in my job," students overwhelming agreed, with 77.8% of students agreeing or strongly agreeing and only 3.7% disagreeing. Since students recognized that they would need to be comfortable using computers as a tool in the future – as suggested by the Center for Educational Networking (2005), who listed use of a personal computer as one of the “new basic skills” demanded by employers – it made sense for them to use computers as a tool during their education so that they could develop this skill.

Figure 3

Responses:
“I expect to use computers in my job.”



In general, as suggested by Ruthven and Hennessy (2002) and Fies (2007), students recognized that technology could be a useful tool to help them learn mathematics and that they were more likely to be motivated to learn and engage in learning when technology is being used. The responses also suggested one possible underlying cause of these perceptions, the realization that technology use would likely be a component of their futures. These perceptions were confirmed through students’ actions and comments recorded in the following subsection, which took place during a lesson in which students used calculators as a learning tool.

In the discussion that follows, Megan discuss the classroom discourse during the three lessons she taught using technology.

Technology’s Impact on Students’ Engagement and Learning Conjecturing through mathematical patterns.

After students had spent several days exploring the definition of a *limit* and analyzing the existence of a limit from a table or a graph, students used calculators to investigate the behavior of the function

$$f(x) = \sin\frac{\pi}{x} \text{ when } x \text{ is near 0.}$$

This activity was designed to show students how a table of values can be misleading and thus motivate the need for an algebraic technique for evaluating limits. Students used calculators in this activity to be able to quickly evaluate function values for a variety of input values, allowing them to focus on the patterns that arose within these values rather than the computation of the values.

First, the entire class filled in the two tables of values shown below, after which the following dialogue occurred.

x	.1	.0 1	.00 1	.000 1	.0000 1	.00000 1	$\lim_{x \to 0^+} f(x)$
$f(x)$							

x	- .1	- .01	- .00 1	- .00 01	- .0000 1	- .00000 1	$\lim_{x \to 0^-} f(x)$
$f(x)$							

Student A: Zero.
Student B: I'm getting zeroes for all of them.
Student A: I have a feeling they're all zeroes.
Student B: I have a feeling the same thing.

Megan: Okay, *Student B* and *Student A* are suggesting that maybe these are all zeroes. Is anyone else finding the same thing?

Students (*in chorus*): Yeah.

Megan: Okay, so we have all zeroes in the first table. What about the second table?

Students (*in chorus*): Same.
Student C: Same thing.

Megan: We're getting the same thing, does everyone agree?

Students: Yes.

Megan: Okay, so we all get zeroes. This one. (*Pause.*) Is this the limit? (*Pause.*) Are we approaching zero from the right or the left in this top table here?

Students (*in chorus*): Left; Neither; Right.

> Megan: We're going in from this side, so we're going in from the right. This is our right-hand limit. And what do we think; do we have a limit here? Do we have a right-hand limit here?
>
> Students (*in chorus*): Yes.
>
> Megan: It looks like it's what?
>
> Students (*in chorus*): Zero.
>
> Megan: Okay. And this one down here is our left-hand limit, right?
>
> Student D: And it's also zero.
>
> Megan: And we're also getting zero, that's right. So do we think we have a limit in this function here?

In this lesson, the use of calculators allowed my students to focus on the numerical values they were finding and the pattern that was being formed. As observed by Jones (2000), technology use allowed students to focus on the end goal, in this case recognizing a pattern, rather than on the computational steps required to reach this goal. Similarly, Ruthven and Hennessy (2002) noted that technology use could accentuate mathematics, allowing students to concentrate on the bigger conceptual picture rather than minor computational details. Students were able to quickly calculate values and recognize that they were getting zero for each answer. This led them to the apparent, though incorrect, answer that the limit was zero.

At this point in time, I suggested that maybe there was something more happening with this function and that further exploration might show a different picture. In groups, students completed additional tables of values, with each group working on a different pair of tables. While students were working, the following discussions occurred:

> Megan: We're thinking that our limit is zero, right? Because our left-hand limit knows what our right-hand limit is doing, and vice-versa.
>
> Student B: Correct.
>
> Megan: Okay, I'm here to tell you that there might be something else going on in this function. Turn your papers over.
>
> Student E: Oh boy,
> Student F: Is it a hole? [*Note: A hole is a point in a graph where a limit exists but the function is undefined. In other words, it is exactly as it sounds: a hole along the curve of the graph.*]
>
> Megan: It could be a hole, (*Giving directions*.)

Student G: So we all have different numbers?

Megan: You all have different numbers. (*Pause.*)

Student B: I have a feeling they're all going to be negative root three over two!

Student D: I'm getting weird numbers.

Megan: That's okay; just write down the weird numbers you're getting.

Student D: I don't know if it's right.

This discussion also illustrated that the use of the calculator allowed my students to easily focus on the pattern that occurred rather than on computation. At the same time, one student (Student D) commented, I'm getting weird numbers, expressing his concern that, in his tables, a clear pattern was not emerging. This concern and questioning contrasted with the research of Galbraith (2006), who suggested that students tend to blindly trust what calculators tell them, without evaluating whether an answer makes sense. In this case, after affirming that the answers were correct calculations, the use of the calculator allowed this student (Student D) to question what these unexpected answers meant about the function.

Finally, each group of students wrote their tables on the board so that the class could gain a more complete picture of the behavior of this function. The following was part of the discussion that followed:

Megan: Okay, so we still had a bunch of zeroes, meaning what? If you were interpreting based on this table, what do you think?

Student H: The limit would be zero.

Megan: Zero. But now, let's look up here, at this one over here, the sevens'. Whose table was this? Yours? What happened?

Student F: Umm, there's, there's a pattern. There's up and down.

Megan: Ok, there's some sort of pattern. Do we have a limit?

Student F: Not really.

Teacher: We don't really seem to have a limit. They're not going to a specific value, right?

Student F: Uh-uh.

Megan: And the left and the right definitely do not agree.

Student F: They’re, like, opposites of each other.

Megan: They’re opposites of each other, but they don’t agree, in terms of like a limit behavior.

Student F: No.

Megan: Okay. What about this one right here?

Student A?
Student A: The limit does not exist.

Megan: Why not?

Student A: Because they are not (*inaudible*).

Megan: Okay, so the left-hand limit is root three over 2.

Student B: The left-hand limit is approaching a different number than the right-hand limit.

Megan: And the right-hand limit. So they do not agree, and our limit does not exist.

Megan: Okay, so we really don’t have a clear trend in that one or in this one here. What do we think is going on here? Does anyone have any ideas?

Student I: It’s like (*moving hand up and down really quickly, i.e., implying oscillating behavior*).

Student B: Yeah, is it like the oscillating function?

Megan: Maybe it is oscillating. Okay our graphing calculator magically got fixed. You figured out the setting problem. (Student F: Woo hoo!) Graph it and see what happens.

Megan: So, what do you see on your graph?

Student B: An oscillating pattern.

Megan: We’re going back and forth really quickly between zero and one, right? We’re oscillating. So, this is an example of why, if you look at the top of your page, why can’t we just trust the table. Because sometimes the table can be misleading. If you stopped after the first table on the front page, you would think you had a limit of zero. Is that right?

Student I: Correct.

> Megan: We all thought at that point that our limit was zero. But it turns out that that's not quite the case. So we need something we can do besides looking at the table in order to evaluate limits.

During this discussion, my students used the tables they created to try to make sense of this function's behavior. They looked at pairs of tables and discussed the interpretation that would follow from these tables, and then tried to predict what was actually happening with this function, before graphing it on their calculators to verify that their hypothesis was indeed correct. I planned this discussion to be highly student-centered, a structure which Simonsen and Dick (1997) suggested as facilitated by technology use, that is, allowing students to be active participants in exploring and in creating a shared body of knowledge.

Throughout this activity, students used their calculators to compute complex calculations quickly and easily. This allowed them to quickly compute a large number of values and then spend more time analyzing and discussing what these values told us about the behavior of the function. Thus, even without using the graphing tool on their calculators, students were using their calculators to try to visualize what was happening, and were instinctively relating numerical and graphical representations. This was consistent with the findings of Serhan (2003), who found that technology use helped students learn to relate different mathematical representations and to gain comfort in doing so.

Additionally, the use of technology led to a high rate of student engagement, as suggested by Simonsen and Dick (1997). The transcript showed that a large number of students participated in the sense-making discussion, including several students who were typically engaged to a lesser degree. Because students worked in small groups to complete their tables and then shared their data with the class, all of the students were actively involved in collecting this data. Each group then discussed what their pair of tables illustrated about the graph. Thus, the cooperative nature of the activity contributed to the engagement of students, a structure which was facilitated by the students' use of technology. At the same time, students' reactions illustrated that they saw this activity as a sort of game or a mystery to solve, and that they were thus interested in the activity and also engaged for that reason. Thus, the use of calculators in this lesson allowed students to reach a deeper understanding, relate different mathematical representations, and participate actively. Similar results will be seen in the next lesson, analyzing the use of dynamic geometry software.

Visualizing mathematical concepts.

The second lesson took place shortly after students began to explore the idea of a derivative, the instantaneous rate of change of a function at a point. Students learned the definition of a derivative and practiced finding the derivative using this definition. During this lesson, students began to explore in depth the graphical representation of a derivative. They worked individually or in small groups to complete a lab activity, using GSP to explore the relationships between the graph of a function and the graph of its derivative.

Students began by exploring the derivatives of polynomials. The first characteristic they were asked to investigate and describe was functional behavior that caused the derivative to be zero. While students were trying to answer this question, the following discussions occurred:

> Student I: Any time where it's at zero, like a straight line.
>
> Megan: Okay. Is that the only time?
>
> Student I: Well, I mean, like, the tangent line is straight.
>
> Megan: Oh, the tangent line is straight – a horizontal line?
>
> Student I: Yeah.
>
> Megan: Okay. When does that happen in terms of the function?
>
> Student I: When the line goes straight. When it goes up and then down [*trying to think of the word*] at a, umm, mountain or valley?
>
> Megan: Okay, that works. Maximum and minimum would be the words you probably were looking for. So, what's happening with the function at that point?
>
> Student F: Umm, it's at a vertex, kind of? Like the vertex of a parabola.
>
> Megan: Okay, that's not really a vertex
>
> Student F: It's changing directions? [*Noticing that they had written down x-values where the derivative was zero*]
>
> Megan: I don't want any *x*-values here. I want to know what the graph looks like there.

During this discussion, my students were trying to classify the behavior that they were observing. Although many of them struggled to find the exact vocabulary to describe what they were noticing (the use of words such as mountain, valley, and vertex instead of maximum and minimum), students were able to identify that this was the behavior that caused the derivative to be zero. Consistent with the research of Vincent (2005), the dynamic geometry software enabled students to identify patterns and make mathematical conjectures. As seen in the previous discussions, students, through manipulation facilitated by the dynamic software, were able to identify the types of function behavior that occurred when the derivative equaled zero and conjecture that such behavior would always cause the derivative to equal zero.

Later, students were asked to investigate whether the derivative was ever undefined. Several students, in manipulating the parameters of their polynomial function, assigned infinite value to one or more of the parameters, thus making it no longer a polynomial function, but causing the discontinuity of the derivative that they were seeking. The students sought to clarify what this meant in the following discussion:

> Student F: So, there are no places where it [*the polynomial function*] doesn't have it [*a derivative*]. I mean, well, I seem to have found one.
>
> Student J: She wins. [*Looking at her function*]
>
> Megan: How did you get it to do that? [*Explaining that one or more of her parameters was infinite*]
>
> Megan: Oh, well, yeah, if you have infinite, but that's not really a polynomial function. No, it really isn't. A polynomial function needs real coefficients, not infinite ones.
>
> Student J: So, basically, as long as it has, umm, real coefficients, then…
>
> Student F: In other words, as long as it's a real polynomial, then no [*the derivative is never undefined*].

In this discussion, students struggled to reconcile what they know about polynomial functions with what they were seeing on their computer screen. Consistent with the findings of Galbraith (2006), they instinctively trusted the representation of the software, believing that they had found a way for the derivative of a polynomial to be undefined. However, with some guidance and further thinking, they were able to recognize that the function on their screen was not actually a polynomial and thus to overcome this tendency to trust technology over their own critical thinking skills. However, this same discussion also illustrated how features of technology, in this case the ability to give a parameter infinite value, could lead students to incorrect conclusions. Thus, as suggested by Simonsen and Dick (1997) and Galbraith (2006), software flaws or technicalities could occasionally lead students astray.

Although the transcript for this video was incomplete because much of the student-teacher and student-student dialogue was inaudible, the video illustrated a very high rate of student engagement in this lesson. The teacher talked with almost every student at one point or another, giving students direct guidance or helping them clarify their understanding. The students were on task throughout the period, working to complete their project. As suggested by Hannafin et al. (2001), Ruthven and Hennessy (2002), Fies (2007), and Muir (2007b), the student-centered nature of this activity led to increased motivation to participate.

These student discussions illustrated that the students were reaching conceptual understanding while working on this project. For example, students were able to observe the pattern that the derivative was zero when the function was reached a maximum or minimum and were able to generalize this pattern to the understanding that this would always occur. Thus, as suggested by Almeqdadi (2005), Funkhouser (2005), and Serhan (2007), the use of dynamic geometry software did allow students to reach higher levels of understanding, leading to increased achievement. However, as discussed by Hannafin et al. (2001), some students did attempt to complete the lab as quickly as possible and thus may have missed the opportunity to achieve deeper conceptual understanding.

Finally, this lesson would be classified differently than the previous lesson. In this lesson, students were the primary users of technology, which Garofalo et al. (2000) suggested was likely to lead to increased engagement and understanding. In addition, students used technology as a cognitive tool and were given only general instructions about the types of investigations to use (a “type 2” lesson), structures which Peressini and Knuth (2005) and McGraw and Grant (2005) both recognized as leading to better student understanding and achievement. Thus, based on all three classifications, it was not surprising that this lesson led to increased student understanding, as demonstrated through their discussions.

In summary, in this lesson, as in the previous one, technology use led to increased student understanding and engagement. However, these findings contrasted with the analysis of the third lesson, in which an interactive white board was used.

Engaging in interactive activities.

This lesson took place after students had begun to explore derivatives. Students had previously learned to evaluate derivatives using limits as well as the sum, constant multiple, and power rules. In this lesson, students considered for the first time how to take the derivative of a product or quotient of two functions whose individual derivatives are known. Recognizing a common student misconception that the derivative of a product was the product of the individual derivatives, the teacher asked students to verify if this conception was true. Using the polynomial $x^2\ x^3$, students first found the derivative by simplifying the given function, then by testing the proposed “product rule,” and found that this “rule” did not work.

The following discussion occurred after students discovered that this desired “rule” was incorrect. Prior to and during this discussion, the teacher used an interactive white board (IWB) as a display tool to facilitate the discussion.

> Megan: So, here’s my question. [*Drawing arrow between f ‘(x) and g’(x)h’(x).*] Do these two match?
>
> Student K (others follow): No.
>
> Megan: No. So *this* [*writing ‘NO!’ under guess that f ‘(x) might equal g’(x) times h’(x)*] is *not* our rule.
>
> Various students: Ohhh. No.
>
> Megan: But what I want you to do is look at these four pieces of information [*circling g(x), h(x), g’(x), and h’(x)*] and see if you can come up with a way to get $5x^4$. (*Pause*.) So, I would look first at the powers. How can we get an x to the fourth? What pieces can we multiply together?
>
> Student L: x^2 and x^2.

Megan: Okay, the $2x$ and the x^3, and the x^2 and the x^2. Those will both give us powers of x^4, right? Student K, is that what you were going to say?

Student K: Oh, I was going to answer, I was(*inaudible*).

Megan: Okay. So if we do $g(x)$ times $h'(x)$, we get what?

Student M: $3x^4$

Megan: $3x^4$, thank you. And if we do $g'(x)$ times $h(x)$?

Student N: $2x^4$.

Megan: We get $2x^4$. And we're looking for $5x^4$, so we're going to...

Student O: You add them.

Megan: If we add these,

Students: Oh wow.

Student K: Oh, duh.

Megan: We get $5x^4$. Sweet, right?

Student P: I'd rather do it the other way.

Megan: You'd rather do it the other way, but the other way doesn't work. So we have to remember this way.

Student P: No, I mean, like, the original way. [*Referring to simplifying f(x) and then using the power rule.]*

Megan: Okay, right. And if you were looking at this function [*pointing to f(x)*], I hope you *would* do this way.

Student P: Then why are we learning this?

Megan: Because not all the times is our function going to look like that, where we can multiply them together.

Student P: Do you have an example of that?

Megan: I do have an example of that. [*Skipping several pages ahead on Smart Board*] For example, $3x\cos(x)$. You can't do the old way, you have to do it the new way.

Student P: Ah, gotcha. That makes sense.

In this discussion, the IWB fulfilled basically the same purpose as a traditional white board would. The only time special features of the IWB were used was when the teacher skipped several pages to display an example in response to a student's question; such an example could easily have been given without showing it on an IWB. Additionally, this lesson did not lead to a large degree of student engagement; only 6 out of 18 students were actively engaged and participating in this discussion.

This reiterated the findings of Kennewall et al. (2008), who found that the use of an IWB did not necessarily lead to more interactive teaching. While this discussion did achieve a degree of interaction between student and teacher, the same degree of interactivity could have been achieved without an IWB; furthermore, this discussion was still highly teacher-centered and could conceivably have been much more interactive. In addition, given this teacher's lack of access to professional development regarding the use of IWBs or to a local "expert" on the use of IWBs, this result was consistent with the findings of Miller and Glover (2007), who suggested that teachers with access to these resources were much more likely to achieve a greater level of interactivity in their teaching.

Finally, the limited success of this lesson was consistent with the classification suggestions of Garofalo et al. (2000), McGraw and Grant (2005), and Peressini and Knuth (2005). In this lesson, the teacher was the primary user of technology, technology was used as a communication tool, and students were given detailed instructions about what to consider, making this a type 1 lesson. Thus it was not surprising that this lesson structure was less successful in producing increased student engagement.

In summary, the use of an interactive white board in this lesson was less productive and successful than the use of calculators and dynamic geometry software in the preceding lessons. However, as discussed in the next subsection, students' overall opinions about the use of technology throughout the semester were positive.

Students' Perceptions of Technology Use after the Semester

At the conclusion of the semester, students completed an open-ended survey about technology use in mathematics class. Five of the questions on this survey were identified as relating directly to the research questions of this study, and responses to those five questions were coded to identify common themes among student responses. These themes, identified by corresponding question, are listed in Table 1 on the following page. After a discussion of overall themes, several themes will be examined in greater detail, including an analysis of individual student response.

Students reported that technology helped them to achieve greater understanding in various ways, many of which were parallel to the findings of previous research. Similarly to Ruthven and Hennessy (2002), students recognized that the use of technology enabled them to complete activities faster, thus increasing their productivity. Students also identified that the use of technology facilitated learning, made content easier, presented an opportunity for additional practice, and helped their grades. Ruthven and Hennessy (2002), Serhan (2004), Funkhouser (2005), Almeqdadi (2005), Galbraith (2006), and Ellington (2006), all noted similar results from their research. Many students also remarked that the use of technology helped them to visualize the mathematical content and thus led to increased understanding for them as visual learners, as suggested by the literature (e.g., Serhan 2004; Fies 2007). Finally, students also recognized that technology facilitated the use of cooperative learning, a structure which they found beneficial, as noted by Simonsen and Dick (1997) and Galbraith (2006). However, students also noted additional reasons that technology use was valuable, reasons that were not found in the literature reviewed. For example, students recognized that they would be highly likely to use technology in college and in their careers; thus, it made sense to them to learn how to use it and to gain comfort doing so. This was consistent with the results of the initial survey, in which 77.8% of students reported that they believed they would use computers in their job. Students also found it helpful to apply mathematical concepts to real-life situations, and they felt that technology helped them to do so.

Students also reported a few challenges that arose through the use of technology. Because of classroom time constraints, as well as limited access to computer labs, students had only a limited amount of time to complete labs and other activities. Many students reported that they felt rushed, and tried to complete the activities as quickly as possible, minimizing the amount of learning that they achieved. This reaffirmed the access concerns discussed by Simonsen and Dick (1997) and the time concerns noted by Hannafin et al. (2001) and suggested that teachers must plan carefully so that students would not feel a pressure to rush through an activity. Students also felt that many of the activities were repetitive; while completing the process of looking for a pattern, students were able to quickly identify the pattern and generalize it to additional situations (e.g., Galbraith 2006), which then made the time spent confirming this generalization in additional cases seem pointless. Finally, students did experience some technological difficulties while using technology in class, suggesting that teachers might need additional training to be able to carefully plan activities and quickly troubleshoot any problems that may arise, as discussed by Simonsen and Dick (1997).

Finally, students reported that the use of technology helped increase their motivation and engagement. Students identified that doing something different and fun was one benefit of using technology during math class, thus implying that they were more motivated and engaged when using technology. This was consistent with the findings of Hannafin et al. (2001), Ruthven and Hennessy (2002), and Fies (2007), as well as with the results of the initial survey, in which 58.5% of students disagreed with the statement that using computers makes math boring. Thus, students recognized that using technology helped them to be more engaged in their math classes.

The subsections below highlight student responses to four of the themes which were most relevant to the research questions of this study: visual representations, something different/fun, applying concepts, and facilitating learning/ increased understanding.

Representing visually. In particular, many students identified that the use of technology helped them to visualize the concepts that we were discussing, giving them another way of viewing and understanding the material. For example, one student said, “I could visualize things after [completing the labs/projects] … it shows a different way of doing things,” while another agreed, "They helped me learn because you got to see what all the graphs looked like… [it was] beneficial to have graphs to look at that represent the numbers." Similarly, a third student stated that completing labs and projects using technology "shows the patterns and helps with the understanding for visual learners." These students all recognized that the use of technology facilitated their understanding through providing a visual representation of the concepts that they were studying, as Fies (2007) suggested. In addition, several students also discussed the fact that using technology helped them to make connections between various representations; one said, "You can see what's actually happening alongside the algebra," while another related that using technology "helped the activities to apply to more different learning styles. I'm more of a visual learner and being able to use technology to visually manipulate concepts is very beneficial to me … being able to look at concepts from multiple angles 'graphical and symbolic' helps me grasp a better understand the concept as a whole (sic)." Thus, like Serhan (2004), these students recognized that technology use helped them to make connections between different mathematical representations, allowing them to achieve a deeper mathematical understanding.

Engaging in something different and fun. Many students commented that labs and projects using technology were more fun than a traditional math class, including one who

Table 1: Themes from Coding of Student Survey Responses

Question	Themes	Themes Students Responses (Number of Occurrences)
1: Do you think the use of technology in these labs and projects was necessary, or could the activity have been completed to the same degree without the use of technology?	Facilitated learning	16.9% (12)
	Made it faster	15.5% (11)
	Made it easier	15.5% (11)
	Served as visual representation	9.9% (7)
	Times are changing	5.6% (4)
2: In general, did these labs/projects help you to better understand what we were studying at that point in time? Why or why not?	Served as visual representation	21.1% (15)
	Opportunity for practice	21.1% (15)
	Opportunity to apply concepts	11.3% (8)
	Something different/fun	8.5% (6)
3: What did you most enjoy about these labs/projects? What did you find most beneficial or helpful?	Something different/fun	33.8% (24)
	Facilitated understanding	26.8% (19)
	Served as visual representation	18.3% (13)
	Opportunity for cooperative learning	14.1% (10)
	Opportunity for real-life application	7.0% (5)
	Helped grade	7.0% (5)
4: What did you least enjoy about these projects? What did you find	Concerns about time	35.2% (25)
	Repetition	12.7% (9)

least beneficial or helpful?	Technological difficulties	7.0% (5)
5: In general, do you find that technology use (computers, projectors, calculators, et cetera) in math classes is helpful? Why or why not?	Makes it faster/easier	32.1% (18)
	Serves as visual representation	16.1% (9)
	Makes it more accurate	8.9% (5)
	Will use in future/job	8.9% (5)

said "It was a different way to learn the info. (sic), which made it easier or more fun to do." Students noted that, because such lessons were more fun, they were more engaged and thus learned more from these lessons. For example, one student commented, "it was more interesting to do so I paid more attention," while another said, "It was more fun, and I was more willing to learn." Students thus confirmed the findings of Simonsen and Dick (1997), Fies (2007), and Muir (2007b). Examining student responses illuminated some of the underlying causes behind the increased engagement and motivation noted by Ruthven and Hennessey. In particular, students identified the 'hands-on' nature typical of much technology use as being an integral motivator; one said, "it was hands on experience and it was more interesting than listening to a lecture," while another similarly stated, "we get to be hands on and are interacting, not just sitting there listening to the teacher talk … They were fun! … We also got to go to the computer lab & not sit in the classroom all class period." Hence students were clearly motivated by the opportunity to actively participate in math class, as well as by the novelty of an experience that did not happen every day.

Applying concepts. Students also identified that using technology helped them to make connections between mathematics topics and between math and the real world. For example, one student said, "It was a picture or pattern shown in a way that help me (sic) make connections… I like the connections made." This student's response affirmed the results of Simonsen and Dick (1997) and Serhan (2004), who found that technology use facilitated students' ability to see connections between different concepts and representations. However, students also identified an increased capability to see connections between mathematics and the world around them. One student said that using technology "Gave us the chance to apply the concepts we were learning to the real world," while another shared that he or she "Liked applying the content to life - it helped me understand some concepts better." One student even went so far as to say, "I realized that everything contains some geometry." Clearly, using technology helped students to recognize the importance of mathematics in the world as a whole; this recognition gave students a reason to learn math, thus leading them to be more engaged in the studies.

Facilitating learning and increased understanding.

Finally, many students reported that the completion of projects and labs using technology helped facilitate their learning and led them to achieve increased understanding, confirming the results of Funkhouser (2005) and Almeqdadi (2005). For several students, such as those who said, "I was actually looking at the material," and "They forced me to think about the topics," this was because completing such an activity required active participation and thinking, which might not have always been typical behavior for these students. Consistent with the findings of Ruthven and Hennessy (2002) and Fies (2007), these students were more engaged when they were using technology, and they recognized that this increased engagement led them to achieve a fuller understanding. Other students identified the required additional practice as the cause for their increase in understanding; one said, “The more you work with a topic the more it sticks,” while another noted “These labs were very helpful at the time we did them since it was more and more practice for the lessons we were learning.” For these students, additional time spent engaging actively with mathematical content was the key to achieving a better understanding. Finally, some students noted that the process of discovery facilitated by technology use was the key to their knowledge. One student noted “seeing for myself how they work” as a benefit of technology use, while another said, “It better helped me get the concept of what we were learning by doing it on my own.” For these students, technology assisted the process of discovery through conjecturing and generalization.

Conclusions

In summary, the results of this study suggest that when students are primary users of technology, its use can lead to increased engagement and increased understanding. In particular, increased engagement results from technology making a lesson interesting, different, and fun for most students, which increases students’ motivation to participate. In addition, technology helps students to see connections between mathematics and the ‘real world’; once students understand why math is important, they are more likely to engage in learning it. Technology use can lead to deeper student understanding by helping students to manipulate and link different representations and by allowing them to focus on concepts instead of calculations. Finally, technology enables students to participate in the process of exploring, identifying patterns, conjecturing, and generalizing, helping them to develop critical thinking skills necessary for success in math and in life.

The implications for my teaching are that I must plan the use of technology carefully in order to fully reap these benefits. Most importantly, the activities I plan should be student-centered and students should be the primary users of technology to the greatest degree possible. In addition, activities should allow for exploration, conjecturing, and generalization and should use multiple representations and real-world connections. As much as possible, technology should be used as a cognitive learning tool, not just as a presentation tool, and students should have enough time to complete activities so that they do not have to rush. Finally, teachers should realize that using technology does not necessarily make their teaching more effective –in must be used well in order to be beneficial.

We recommend that future research is needed to expand the results of this study to additional student populations and settings. In addition, it is important that we continue to investigate how technology can be used most effectively. In particular, it would be useful to compare different categories of use in order to evaluate their success. Finally, it is essential to continue to investigate new technology as technology continues to evolve at a rapid pace. In particular, additional research is needed to discover the best ways to use IWBs and to help teachers use these tools more effectively.

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