

**Illinois Wesleyan University**

---

**From the SelectedWorks of Lawrence N. Stout**

---

2003

# Open Problems from the Linz2000 Closing Session

Lawrence N. Stout, *Illinois Wesleyan University*



SELECTEDWORKS™

Available at: [http://works.bepress.com/lawrence\\_stout/9/](http://works.bepress.com/lawrence_stout/9/)

## Open Problems from the Linz2000 Closing Session

L. Stout

The final session was devoted to presentation of open problems pointed out in the talks. Here is a listing of what we came up with:

- (Stout): Given a function  $f : L_1 \rightarrow L_2$  which preserves  $\bigvee$  the functor  $f^\leftarrow : \mathbf{Set}(L_2) \rightarrow \mathbf{Set}(L_1)$  is defined by

$$f^\leftarrow(A, \alpha : A \rightarrow L_2) = \left( A, a \mapsto \bigvee \{l_1 \in L_1 \mid f(l_1) \leq \alpha(a)\} \right).$$

Under what conditions on  $f$  does this preserve implication?

- (Hajek): Referring to the logics discussed in Hajek's book *Fuzzy Logic*, the complexity of some problems are known. Fill in the question marks in the following table:

	General Tautology	Standard Tautology	General Satisfiability	Standard Satisfiability
BL	$\Sigma_1$	$\Pi_2$ -hard	$\Pi_1$	?
t	known	?	?	?
G	known	?	?	?
$\Pi$	known	$\Pi_2$ -hard	?	?

- (Jenei) Characterize left continuous indecomposable t-norms with strong induced negation.
- (Klement and Mesiar) Characterize all left continuous t-norms.
- (Klement and Mesiar) Characterize all generated t-norms.
- (E. Walker): How many elements are there in the free Kleene Algebra and the free deMorgan Algebra? Use join irreducibles. The following are known:

$n$	Kleene	deMorgan
1	6	6
2	84	156

- (Gottwald)
  - Prove or disprove that MTL is the logic of left-continuous t-algebras.
  - Compare the equational theories of the classes of prelinear residuated lattices and left continuous t-algebras
- (Gottwald) Develop proof theory, sequent calculi or natural deduction calculi for fuzzy logics like ML, MTL, BTL.
- (Gottwald) Treat the topic of rule interpolation (in fuzzy control) by the use of fuzzy funafication.
- (Barone) What is the correct of M-valued sets in the bicategory **Rel**?
- (C. Walker) If  $\eta$  is strong negation, are the following inequalities independent?

$$\eta(ab)\eta(a\eta(b)) \geq \eta(a),$$

$$\eta(\eta(a)b)\eta(a\eta(b)) \geq \eta(\eta(ab)\eta(\eta(a)\eta(b))).$$

12. (Mesiar) A pseudo-t-norm is a function  $T : [0, 1]^2 \rightarrow [0, 1]$  which is associative in both components and has 1 as a neutral element (commutativity may be violated). Known examples are based on  $T$  which is 0 on some areas and min on the remainder or corresponding ordinal sums. Are there other non-commutative pseudo-t-norms; in other words, are there non-commutative pseudo-t-norms with no non-trivial idempotent elements?
13. (Mesiar, originally posed by de Baets) Characterize all t-norms such that for all  $x, y$ , and  $z$  in  $[0, 1]$   $T(x, y) + z \geq T(x, z) + T(y, z)$ . Note that then necessarily  $T \geq T_{\perp}$  (the Łukaciewicz t-norm). Further if  $T$  is a copula then it satisfies the inequality. However, there are examples of  $T \geq T_{\perp}$  not satisfying the inequality as well as examples satisfying it which are not copulas. For continuous t-norms it will suffice to characterize all Archimedean solutions.
14. (Vicenik, recalled by Klement) Characterize all additive generators of t-norms. That is, mappings  $f : [0, 1] \rightarrow [0, 1]$  such that  $T : [0, 1]^2 \rightarrow [0, 1]$  with  $T(x, y) = f^{(-1)}(f(x) + f(y))$  is associative. Here  $f^{(-1)} : [0, \infty] \rightarrow [0, 1]$  has  $f^{(-1)}(x) = \sup\{z \in [0, 1] \mid f(z) > x\}$ .
15. (Vicenik, recalled by Klement) A uninorm  $U : [0, 1]^2 \rightarrow [0, 1]$  is commutative, associative, non-decreasing, and has a neutral element in  $(0, 1)$ .
  - (a) Characterize all distributive uninorms over a given (continuous) t-conorm  $S$ . I.e.,  $U(x, S(y, z)) = S(U(x, y), U(x, z))$  for all  $x, y, z$  in  $[0, 1]$ .
  - (b) The same for conditional distributivity, where distributivity is required when  $S(y, z) < 1$ .