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# Open Problems from the Linz2000 Closing Session

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The final session was devoted to presentation of open problems pointed out in the talks. Here is a listing of what we came up with:

1. (Stout): Given a function $f : L_1 \to L_2$ which preserves $\bigvee$ the functor $f^{\leftarrow} : \mathbf{Set}(L_2) \to \mathbf{Set}(L_1)$ is defined by

$$f^{\leftarrow}(A, \alpha : A \to L_2) = \left(A, a \mapsto \bigvee\{l_1 \in L_1 | f(l_1) \leqslant \alpha(a)\}\right).$$

   Under what conditions on $f$ does this preserve implication?
2. (Hajek): Referring to the logics discussed in Hajek's book *Fuzzy Logic*, the complexity of some problems are known. Fill in the question marks in the following table:

| | General Tautology | Standard Tautology | General Satisfiability | Standard Satisfiability |
|---|---|---|---|---|
| BL | $\Sigma_1$ | $\Pi_2$-hard | $\Pi_1$ | ? |
| t | known | ? | ? | ? |
| G | known | ? | ? | ? |
| $\Pi$ | known | $\Pi_2$-hard | ? | ? |

3. (Jenei) Characterize left continuous indecomposable t-norms with strong induced negation.
4. (Klement and Mesiar) Characterize all left continuous t-norms.
5. (Klement and Mesiar) Characterize all generated t-norms.
6. (E. Walker): How many elements are there in the free Kleene Algebra and the free deMorgan Algebra? Use join irreducibles. The following are known:

| $n$ | Kleene | deMorgan |
|---|---|---|
| 1 | 6 | 6 |
| 2 | 84 | 156 |

7. (Gottwald)
   (a) Prove or disprove that MTL is the logic of left-continuous t-algebras.
   (b) Compare the equational theories of the classes of prelinear residuated lattices and left continuous t-algebras
8. (Gottwald) Develop proof theory, sequent calculi or natural deduction calculi for fuzzy logics like ML, MTL, BTL.
9. (Gottwald) Treat the topic of rule interpolation (in fuzzy control) by the use of fuzzy funafication.
10. (Barone) What is the correct of M-valued sets in the bicategory **Rel**?
11. (C. Walker) If $\eta$ is strong negation, are the following inequalities independent?

$$\eta(ab)\eta(a\eta(b)) \geqslant \eta(a),$$

$$\eta(\eta(a)b)\eta(a\eta(b) \geqslant \eta(\eta(ab)\eta(\eta(a)\eta(b))).$$

12. (Mesiar) A pseudo-t-norm is a function $T:[0,1]^2 \to [0,1]$ which is associative in both components and ahs 1 as a neutral element (commutativity may be violated). Known examples are based on $T$ which is 0 on some areas and min on the remainder or corresponding ordinal sums. Are there other non-commutative pseudo-t-norms; in other words, are there non-commutative pseudo-t-norms with no non-trivial idempotent elements?
13. (Mesiar, originally posed by de Baets) Characterize all t-norms such that for all $x, y$, and $z$ in [0,1] $T(x,y)+z \geqslant T(x,z)+T(y,z)$ Note that then necessarily $T \geqslant T_{Ł}$ (the Łukaciewcz t-norm). Further if $T$ is a copula then it satisfies the inequality. However, there are examples of $T \geqslant T_{Ł}$ not satisfying the inequality as well as examples satisfying it which are not copulas. For continuous t-norms it will suffice to characterize all Archimedian solutions.
14. (Viceník, recalled by Klement) Characterize all additive generators of t-norms. That is, mappings $f:[0,1]\to[0,1]$ such that $T:[0,1]^2\to[0,1]$ with $T(x,y)=f^{(-1)}(f(x)+f(y))$ is associative. Here $f^{(-1)}:[0,\infty]\to[0,1]$ has $f^{(-1)}(x)=\sup(z\in[0,1]|f(z)>x)$.
15. (Viceník, recalled by Klement) A uninorm $U:[0,1]^2\to[0,1]$ is commutative, associative , non-decreasing, and has a neutral element in (0,1).
    (a) Characterize all distributive uninorms over a given (continuous) t-conorm S. I.e., $U(x,S(y,z))=S(U(x,y),U(x,z))$ for all $x,y,z$ in [0,1].
    (b) The same for conditional distributivity, where distributivity is required when $S(y,z)<1$.