Reasoning & Proof in the High School Common Core

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Common Core Mathematical Practice Standard #3

Construct viable arguments and critique the reasoning of others

– Make conjectures and then explore truth of them
– Use what you know to construct arguments
– Reason with cases, counterexamples, etc.
– Negotiate mathematical ideas with others

http://www.sde.idaho.gov/site/common/
2000 NCTM Principles & Standards

Instruction should enable students to:

– Recognize reasoning and proof as fundamental aspects of mathematics
– Make and investigate mathematical conjectures
– Develop and evaluate mathematical arguments
– Select and use various types of reasoning and methods of proof

http://www.nctm.org/standards/content.aspx?id=26861
Common Core “Reasoning & Proof” 
High School Standards

• Proof—15
• Explain—15
• Derive—7

About 25% of the HS standards...
2014 NCTM Principles to Actions

IMPLEMENT TASKS THAT PROMOTE REASONING AND PROBLEM SOLVING

*Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.*
Plan for Today

• Illustrative Mathematics tasks that promote reasoning (and some proof)
  – From Number, Algebra, Functions, & Geometry
  – Consider the reasoning that students might use
  – Make observations about task structure

• Research brief on the use of tasks

• Final thoughts
Rational or Irrational

In each of the following problems, a number is given. If possible, determine whether the given number is rational or irrational.

a. $4 + \sqrt{7}$

b. $\frac{\sqrt{45}}{\sqrt{5}}$

c. $\frac{6}{\pi}$

d. $\sqrt{2} + \sqrt{3}$

e. $\frac{2 + \sqrt{7}}{2a + \sqrt{7a^2}}$

f. $x + y$, where $x$ and $y$ are irrational numbers

https://www.illustrativemathematics.org/illustrations/608
Rational or Irrational

Standards

• N-RN.A.2: Rewrite expressions involving radicals and rational exponents using the properties of exponents.

• N-RN.B: Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
Coordinates of Equilateral Triangles

Is there an equilateral triangle ABC so that segment AB lies on the x-axis and A, B, and C all have integer x and y coordinates?

https://www.illustrativemathematics.org/illustrations/1687
Coordinates of Equilateral Triangles

Standards

• G-GPE.B: Use coordinates to prove simple geometric theorems algebraically

• N-RN.B: Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

• F-TF.C.9: Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
Similar Circles

For this problem, \((a, b)\) is a point in the \(x\)-\(y\) coordinate plane and \(r\) is a positive number.

a. Using translation and a dilation, show how to transform the circle with radius \(r\) centered at \((a, b)\) into the circle of radius 1 centered at \((0, 0)\).

b. Explain how to use your work in part a. to show that any two circles are similar.

https://www.illustrativemathematics.org/illustrations/1368
Similar Circles

Standard

G-C.A.1: Prove that all circles are similar.
Solving Two Equations in Two Unknowns

Lisa is working with the system of equations

\[ x + 2y = 7 \]
\[ 2x - 5y = 5 \]

She wonders...will she always get the same solution if she uses a different elimination method?

https://www.illustrativemathematics.org/illustrations/1903
Solving Two Equations in Two Unknowns

Standard

A-REI.C.5: Prove that given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
Research on Using Tasks

1. Not all tasks provide the same opportunities for student thinking and learning.

2. Student learning is greatest in classrooms where tasks consistently encourage high-level student thinking.

3. Tasks with high cognitive demands are the most difficult to implement well.

(See NCTM, 2014, p. 17)
Final Thoughts

• Reasoning through a particular example (better yet, a series of examples) **first** is a productive way to prepare for considering the more general case

• This is not easy!

• A good place to start is having students explain HOW they solved a problem. Then invite discussion about WHY that strategy works.
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THANK YOU
References


Illustrative Mathematics.  
https://www.illustrativemathematics.org/