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Location Decisions of Competing Platforms

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Early entrants in markets with network effects usually occupy a “central location” and serve agents with “intermediate tastes,” whereas later entrants are niche players. Why would the first entrant choose to become a “general” network, given that later entrants will not have enough room for differentiation, resulting in a more intense competition for market share? In a Hotelling model with two rival networks, we show that for intermediate values of the network externality parameter the location equilibrium is indeed asymmetric: the first entrant locates at the center whereas the second entrant chooses an extreme (niche) location.

1. Introduction

We examine “product location” decisions of two competing networks. It is well-known that in the absence of network externalities firms choose maximum differentiation (d’Aspermont et al., 1979) in order to mitigate the ensuing competition. Most of the literature on network markets has ignored the issue of product selection and usually assumes maximum horizontal differentiation when the market has features of spatial competition.1 In this paper, we endogenize location decisions of

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networks and we show that maximum horizontal differentiation does not always hold.

We formulate a model with two competing networks that make their location decisions sequentially.² Our most interesting and novel result is that for intermediate values of the (direct) network externality parameter the location equilibrium is asymmetric: the first mover locates at the center of the market, whereas the follower locates at an extreme, with both networks having strictly positive market shares. This result may help to explain product location decisions that are observed in practice.³

More specifically, early entrants in markets with network effects usually occupy a “central location” and serve agents with “intermediate tastes,” whereas later entrants are niche players. In the market for online video web sites, YouTube, which was launched before Hulu, carries a huge number of diverse videos and clips (suitable for more general audiences), whereas Hulu serves those who watch commercial movies and TV shows.⁴ Other examples include online dating services, online auctions, and social networks.⁵

Our duopoly model can also be used to understand the geographic locations of physical markets.⁶ When externalities are low, markets locate at the periphery (maximum differentiation), that is, far apart from each other. For stronger externalities, there are two active markets,

2. A pure strategy equilibrium may fail to exist when networks locate simultaneously.
3. Gabszewicz et al. (2002), Peitz and Valletti (2008), and Kind et al. (2007) also investigate location choices of competing networks. Gabszewicz et al. (2002) develop a model of media (newspapers) competition that features readers who have horizontal preferences with respect to the political ideology of a newspaper and advertisers who are vertically heterogeneous. They show that when advertising revenue is important the two newspapers will choose the same political ideology (minimum differentiation). Without the advertising side the two newspapers differentiate maximally with respect to their political messages. An assumption that is made is that readers are indifferent to advertisements. Peitz and Valetti (2008) and Kind et al. (2007) build on the Gabszewicz et al. (2002) model by assuming that readers/viewers are not neutral about ads. They show that differentiation need not be maximal. We differ from the aforementioned papers in that: (i) we focus on direct network externalities (one-sided market) and more importantly (ii) none of these papers obtains an asymmetric location equilibrium.
4. Consistent with our model predictions, YouTube has a higher market share than Hulu. According to “Disney’s Hulu Deal Raises Questions About YouTube Model,” Wall Street Journal, April 30, 2009, YouTube had 100 million viewers in March of 2009, whereas in the same time period Hulu had 41 million viewers.
5. First entrants in the online dating services market catered to the average man and woman and later entrants targeted specific groups, e.g., professionals and millionaires. Ebay—an early entrant in the online auction market—sells a broad variety of goods and services, whereas later entrants cover niche markets, such as automobile auctions and real estate. Facebook, an early entrant, is a general social network, whereas many of the later entrants target specific groups in the population.
6. Jin and Rysman (2009) investigate the pricing decisions of sportcard conventions. These conventions try to attract consumers and dealers. An important decision of these conventions is where to locate, given the competition they face from rival conventions.
one at the center and the other at the periphery. One implication of our model is that as the network externalities become stronger market shares will become more asymmetric and eventually one network will dominate.

The driving force behind the product selection decisions is demand creation. When network externalities are weak, price competition dominates demand creation. No network wants to be a “general” network because if one network locates at the most attractive (central) location product differentiation is reduced and this creates stiff price competition. In contrast, when externalities are not weak, demand creation is important. A network in this case benefits by being “general” and attracting many agents with intermediate preferences. The rival network serves a niche market.\(^7\)

The rest of the paper is organized as follows. We present the model in Section 2. In Section 3, we solve the model and in Section 4 we solve the social planner’s problem (first-best). We conclude in Section 5. All proofs can be found in the Appendix.

2. The Description of the Model

The market consists of two horizontally differentiated (and incompatible) networks, \(k = A, B\). There is a continuum of agents that is uniformly distributed on the \([0, 1]\) interval. Network \(A\) is located at point \(a\) and network \(B\) is located at point \(b\), with \(0 \leq a \leq b \leq 1\). We assume that transportation cost is quadratic in the distance \(d\) an agent has to “travel” from his location to the location of the network, \(td^2\), where the parameter \(t > 0\) measures the per-unit cost of travel. We assume that each agent joins only one network (single-homing). Each agent who joins a given network cares about the number of agents that will join the same network. Denote by \(n_k\) the number of participants that network \(k\) attracts. The maximum willingness to pay of an agent who joins network \(k\) is given by \(V + \alpha n_k\), where \(V\) is a stand-alone benefit each agent receives independent of the number of participants on network \(k\). The parameter \(\alpha > 0\) measures the intensity of network externality. The indirect utility of an agent who is located at point

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\(^7\) The tension between demand creation and price competition is at the heart of most models with endogenous location decisions, regardless of whether the market exhibits network externalities or not. For instance, one can mitigate the intensity of price competition, and hence create a less than maximum differentiation, by changing the transportation cost functions, as in Economides (1986), or by allowing for multiple purchases, as in Kim and Serfes (2006).
$x \in [0, 1]$ is given by

$$U = \begin{cases} V + \alpha n^e_A - t (a - x)^2 - p_A, & \text{if he joins network } A, \\ V + \alpha n^e_B - t (b - x)^2 - p_B, & \text{if he joins network } B, \end{cases}$$

(1)

where $p_k$ is network $k$’s lump-sum charge (fee-based) and $n^e_k$ denotes the expectations agents have about the number of agents that will join network $k$.\(^8\) We assume that $V$ is high enough which ensures that the market is covered. Marginal cost is zero. We assume that horizontal differentiation is more important than the network externality, $t > \alpha$.

Given that, in reality, location decisions happen sequentially, we impose a sequential location timing, with network $A$ being the first mover.\(^9\) Then, networks set their prices simultaneously. Finally, agents, after observing network locations and prices, decide which network to join.

### 3. Analysis

We look for a subgame-perfect Nash equilibrium. We solve the game backwards.

#### 3.1 Stage 3: Agent Decisions and Market Shares

The marginal agent can be found as follows:

$$V + \alpha n^e_A - t(a - x)^2 - p_A - (V + \alpha n^e_B - t(b - x)^2 - p_B) = 0$$

$$\Rightarrow \hat{x} = \frac{p_B - p_A + t(b^2 - a^2) - \alpha (n^e_B - n^e_A)}{2t(b - a)}. \quad (2)$$

The fraction of agents that joins network $A$ is $n_A = \hat{x}$ and the fraction that joins network $B$ is $n_B = 1 - \hat{x}$. In equilibrium, it must be that expectations are confirmed, that is, $n_A = n^e_A$ and $n_B = n^e_B$. Using (2), this defines a system of two equations in two unknowns, $n_A$ and $n_B$. By solving the system, we obtain the market shares as a function of prices and parameters

$$n_A = \frac{p_B - p_A + t(b^2 - a^2) - \alpha}{2t(b - a) - \alpha}, \quad (3)$$

\(^8\) More generally, networks can be fee-based and/or ad-based. We assume away the possibility of ad revenues. Casadesus-Masanell and Zhu (2010) endogenize this decision.

\(^9\) When networks locate simultaneously, for a range of parameter values, a pure strategy equilibrium, in location decisions, does not exist. More details can be found in Serfes and Zacharias (2009).
and
\[ n_B = \frac{p_A - p_B - b^2 t + a^2 t - 2at + 2bt - \alpha}{2(t(b - a) - \alpha)}. \tag{4} \]

### 3.2 Stage 2: Networks’ Pricing Decisions

Network \( k \) chooses \( p_k \) to maximize its profits \( \pi_k = p_k n_k \), where \( n_k, k = A, B, \) are given by (3) and (4). The profit functions are strictly concave in a network’s own price if
\[ \alpha < t(b - a). \tag{5} \]

If (5) is satisfied, then the first-order conditions are also sufficient for profit maximization. The equilibrium prices then are given by
\[ p_A = \frac{t(b - a)}{3}(2 + b + a) - \alpha \text{ and } p_B = \frac{t(b - a)}{3}(4 - b - a) - \alpha. \tag{6} \]

The equilibrium market shares, by substituting (6) into (3) and (4), are given by
\[ n_A = \frac{t(b - a)(2 + b + a) - 3\alpha}{6(t(b - a) - \alpha)} \text{ and } n_B = \frac{t(b - a)(4 - b - a) - 3\alpha}{6(t(b - a) - \alpha)}. \]

Note that if (5) is satisfied, then the denominators in the above expressions are positive. For an interior equilibrium, we need the market shares to be in \((0, 1)\). It turns out that \( n_k \in (0, 1) \) if and only if
\[ \alpha < \min \left\{ \frac{t(b - a)}{3}(4 - b - a), \frac{t(b - a)}{3}(2 + b + a) \right\} \tag{7} \]
or, equivalently, for \( n_A \) to be less than one (which implies that \( n_B \) is greater than zero) we must have
\[ \alpha < \frac{t(b - a)}{3}(4 - b - a) \Leftrightarrow b > \tilde{b} \tag{8} \]
\[ \equiv -\frac{1}{t}(-2t + \sqrt{-3t\alpha + 4t^2 - 4t^2a + t^2a^2}) \]
\( b \) also has to be less than the positive root, which is always greater than one and it becomes irrelevant) and for \( n_A \) to be greater than zero (which implies that \( n_B \) is less than one) we must have\(^{10} \)
\[ \alpha < \frac{t(b - a)}{3}(2 + b + a) \Leftrightarrow b > \hat{b} \tag{9} \]
\[ \equiv -\frac{1}{t}(t - \sqrt{3t\alpha + t^2 + 2t^2a + t^2a^2}) \]

10. Observe that when condition (7) for an interior equilibrium is satisfied, then the concavity condition (5) automatically holds.
(\(b\) also has to be less than the negative root, which is always less than zero and it becomes irrelevant). For any given location \(a\) of network \(A\) the market tips, either in favor of \(A\) or in favor of \(B\), when network \(B\) locates close enough to network \(A\), as the above thresholds indicate. The interior equilibrium profits as a function of the networks’ locations, after we substitute (6) into the profit functions, are\(^{11}\)

\[
\pi_A(a, b) = \frac{(t(b - a)(2 + b + a) - 3\alpha)^2}{18(t(b - a) - \alpha)}
\]

and

\[
\pi_B(a, b) = \frac{(t(b - a)(4 - b - a) - 3\alpha)^2}{18(t(b - a) - \alpha)}.
\]

(10)

It can be easily verified that when \(\alpha = 0\) (no network externality) the equilibrium profits reduce to those in d’Aspermont et al. (1979), where it is each network’s dominant strategy to locate at the extreme points (maximum differentiation). As we show next, this is not always the case when \(\alpha > 0\).

3.3 Stage 1: Networks’ Location Decisions

Networks choose their locations \(a\) and \(b\) to maximize profits as they are given by (10). We assume that when the market tips all agents join the network that is closer to the middle point, \(\frac{1}{2}\). If they are equidistantly located from the middle, \(a = 1 - b\), then we assume that all agents join network \(A\).

Fix the location of network \(A\) at a specific point \(a\). Let us examine the profits of network \(B\) when it moves from \(b = 1\) to \(b = a\). Two effects arise: (i) price competition intensifies and (ii) the network attracts more agents. Initially, the intensified competition effect is stronger, but eventually demand creation dominates. The latter is due to the network externality. The Lemma below summarizes the result.

**Lemma 1:** The profit function of network \(B\), for any fixed location \(a\) of network \(A\), exhibits a U-shape.

To gain a better intuition about the properties of the profit functions as a function of the locations, let us look at Figures A3 and A4, where the profit of network \(B\) is depicted as its location \(b\) varies for a fixed location \(a\) of network \(A\). The difference between the two figures is that in Figure A4 the location of network \(A\) is closer to the most attractive point (the center) than in Figure A3. When \(A\) is closer to the

\(^{11}\) We will deal with the tipping solution next when we will analyze the location game.
Location Decisions of Competing Networks

Low network externality: Maximum differentiation
Intermediate network externality: Asymmetric location equilibrium
High network externality: Only one network is active

Figure 1 summarizes the equilibrium locations as a function of the network externality $\alpha$. These outcomes are presented formally in Proposition 1.

Proposition 1 (Location equilibria): The subgame perfect Nash equilibrium is characterized as follows:

- (Maximum differentiation). If $\alpha \in [0, \frac{t}{3}]$, then networks differentiate maximally, that is $a = 0$ and $b = 1$. The equilibrium profits are
  \[
  \pi_A(a = 0, b = 1) = \pi_B(a = 0, b = 1) = \frac{t - \alpha}{2}.
  \]

- (Asymmetric location equilibrium). If $\alpha \in (\frac{t}{3}, \frac{5t}{12})$, then the first mover (network A) locates at the center $a = \frac{1}{2}$ and the follower (network B) locates at an extreme $b = 1$. Both networks have strictly positive market shares. The equilibrium profits are
  \[
  \pi_A(a = 1/2, b = 1) = \frac{(7t - 12\alpha)^2}{144(t - 2\alpha)} \quad \text{and} \quad 
  \pi_B(a = 1/2, b = 1) = \frac{(5t - 12\alpha)^2}{144(t - 2\alpha)}.
  \]

- (Tipping location equilibrium). If $\alpha \in [\frac{5t}{12}, t)$, then the first mover (network A) locates at the center $a = \frac{1}{2}$ and the follower (network B) locates at an extreme $b = 1$. Network B’s market share is zero. The equilibrium profits are
  \[
  \pi_A(a = 1/2, b = 1) = \alpha - \frac{t}{4} \quad \text{and} \quad \pi_B(a = 1/2, b = 1) = 0.
  \]

The intuition for the asymmetric location equilibrium is as follows. When a network is moving closer to the center, horizontal
differentiation is reduced. The price of the network that is moving closer to the center is falling (in a sharing equilibrium) but its market share is increasing. Network effects have the following impact: prices are declining faster but market share is rising faster, relative to no network externalities. For strong enough network externalities the market share effect dominates the intensified competition effect. In this case, the first entrant has an incentive to locate at the center in order to dominate the market and maximize its market share and the associated network benefit. This leaves no room to the follower who finds it optimal to locate at an extreme.\textsuperscript{12} The intuition for the other two cases (maximum differentiation and tipping equilibria) follows straightforwardly from the intuition for the asymmetric equilibrium.

One important issue in the literature is the trade-off between “standardization” and variety, e.g., Farrell and Saloner (1986). Having only one network in our model can be viewed as having one technical standard, at the expense of product variety. So, Proposition 1 predicts that for intermediate network externalities, there will be two “standards” in equilibrium, with one being superior to the other (in terms of market share). Note also that in our model the “type” of the standard is not given but it is being determined endogenously. When externalities are low, neither standard is superior to the other. Finally, when externalities are strong, there will be only one standard in equilibrium.\textsuperscript{13}

4. Welfare Analysis

A social planner chooses the locations $a$ and $b$ of the two networks and the number of agents from each group that should join a network to maximize the difference between aggregate network externality and aggregate transportation cost. We denote by $x$ the number of agents that

12. The leader network in our model locates strategically and affects the location decision of the follower. This result shares similarities with the results in Economides et al. (2004). In a Hotelling model with sequential entry and no network effects, they show that firms may strategically position themselves to foreclose entry and entering first may not yield the highest profit. It is also possible that firms do not locate equidistantly from each other, so in that sense the location equilibrium can exhibit asymmetry.

13. Tyagi (2000) examines location decisions of two firms that enter sequentially and have different costs. If the second mover has lower cost, then it locates close to the most attractive location, whereas the first mover locates far away from the most attractive location. Our asymmetric location equilibrium result has a similar flavor, but it is the first mover in our model that locates at the most attractive location. Moreover, the underlying mechanisms between the two models are different and in our model firms are \textit{ex ante} symmetric.
join network A. Total welfare is given by

\[ W = \int_0^x (\alpha z - t(a - z)^2)dz + \int_x^1 (\alpha(1 - z) - t(b - z)^2)dz = \alpha x^2 + tax^2 - ta^2x + \frac{\alpha}{2} - \alpha x - \frac{t}{3} + tb - tbx^2 - tb^2 + tb^2. \] (11)

The next Proposition summarizes the solution to the social planner’s problem.

**Proposition 2 (First-best):** For \( \alpha \leq t/4 \), the optimal locations are \( a = 1/4 \) and \( b = 3/4 \) and the agents are split equally between the two networks. Total welfare is equal to \( W = \alpha/4 - t/48 \). For \( \alpha \geq t/4 \), all agents join one network which is located at the middle point \( 1/2 \). Total welfare is equal to \( W = \alpha/2 - t/12 \).

The intuition is simple. Aggregate network externality is maximized when all agents join one network. On the other hand, total transportation cost is minimized when networks are located at the first and third quartiles. When the externality is weak, transportation cost is relatively more important and the social planner splits the agents equally between the two networks. For strong externalities one network is chosen to dominate the market, because externalities are now relatively more important.

Comparing the first-best with the noncooperative outcome, we can see that the two coincide only when network externalities are strong, that is, \( \alpha \geq 5t/12 \). For low externalities (\( \alpha \in [0, t/4] \)), horizontal differentiation is higher than in the first-best, that is, “too much” differentiation. The two networks differentiate maximally, while the social planner wants them at the first and third quartiles, as in a model with no network effects. When \( \alpha \in [t/4, t/3] \), the social planner wants only one network to be active and locate at the center, whereas in the market equilibrium both networks are active and maximally differentiated. When \( \alpha \in [t/3, 5t/12] \), market differentiation is still higher than the first-best.

**5. Conclusion**

We examine location decisions of two horizontally differentiated competing networks. Our model can yield both symmetric and asymmetric location equilibria, depending on the strength of the network externality. There are two effects when a network moves closer to the location of the rival: (i) networks become less differentiated and prices tend to decline and (ii) its market share increases. Due to network externality,
the market share effect can be strong, and hence there is a tendency for less than maximum differentiation. In particular, we show that when the externality is weak, the principle of maximum differentiation holds. For intermediate externality, we obtain an asymmetric location equilibrium, where the first mover locates at the center and the follower at an extreme location. Both networks have strictly positive market shares. Our model offers an explanation for the coexistence (see the Introduction for examples) of “general” networks (first mover) that cater to agents with intermediate tastes with niche networks (follower) that serve agents with extreme tastes.

We would like to highlight the role of “product” selection. It is true that even with fixed locations at the extremes the market tips when the externalities are strong enough. This is typical in models with network externalities. Nevertheless, when locations are fixed, we do not obtain asymmetric market structures where both networks are active, which is the case when locations are endogenized (see Proposition 1). In that sense, a testable implication of our model is that market shares evolve “more continuously” as the degree of network externalities (or the degree of differentiation) varies, starting from symmetric market structures when externalities are low, then moving to asymmetric market structures when externalities become stronger. Eventually, for very strong externalities, the market is dominated by one network.

In this paper, we have made the assumption that networks make sales only after both networks are in the market. Alternatively, we could formulate the following two-period game. In period 1, network A enters the market and consumers decide whether to make a purchase from network A or no purchase at all. In the second period, network B enters and consumers have to make a choice between A and B. Let us assume that consumers consume in each period. If externalities are strong enough, the first entrant should locate in middle (now it has an additional incentive to position itself in the middle, given its monopoly status in the first period) and the follower at an extreme. But even if the network externality is very low, the first entrant may still wish to locate in the middle. In this case, it faces a trade-off between higher first-period profits and lower second-period profits due to more intense competition. It may very well be the case, depending on the intensity of competition and “how soon” the second entry happens, that one effect dominates the other and the asymmetric location configuration emerges even with very low (or zero) network externalities. However, if entry happens fast, relative to the length of time the two firms coexist in the market, then the asymmetric equilibrium becomes less likely in the absence of network effects.
Appendix

Proof of Lemma 1. Thresholds (8) and (9) that determine when tipping occurs are very important at this stage. It turns out that \( \hat{b} \geq \tilde{b} \) if and only if
\[
a \leq \bar{a} \equiv \frac{1}{2} - \frac{\alpha}{2t}.
\]
Symmetrically, we can define the thresholds for network \( A \) for a fixed location \( b \) of network \( B \). When \( \hat{b} \geq \tilde{b} \) the binding threshold is \( \hat{b} \). The opposite is true when \( \hat{b} \leq \tilde{b} \).

As we have assumed, when the market tips it is the network that is closer to the middle point, 1/2, that attracts all the agents. If they are equidistantly located from the middle, \( a = 1 - b \), then we assume that all agents join network \( A \). In what follows, we assume that \( a \leq b \).

When \( a \geq \bar{a} \), as network \( B \) decreases \( b \) (that is, moves toward the center) it tips the market in favor of \( A \) (which is located closer to the center) when \( b = \hat{b} \geq 1 - a \). For \( b \leq 1 - a \), the market tips in favor of \( B \), since now \( B \) is located closer to the center. Figure A1 depicts this case.

When \( a \leq \bar{a} \), as network \( B \) decreases \( b \) it attracts all agents when \( b = \tilde{b} \leq 1 - a \). After this point, the market remains tipped in favor of \( B \) until \( b = a \). Figure A2 depicts this case.

The derivative of network \( B \)'s profit function (10) with respect to \( b \) is
\[
\frac{\partial \pi_B(a, b)}{\partial b} = \frac{(4ta - 4tb + 5\alpha - 4tab - 4b\alpha + 3t b^2 + ta^2)(-4tb + 4ta + 3\alpha + t b^2 - ta^2)t}{18(bt - \alpha - at)^2}.
\]

14. It can be shown that \( \tilde{b} \geq 1 - a \) if and only if \( a \geq \bar{a} \).

15. It can be shown that \( \hat{b} \leq 1 - a \) if and only if \( a \leq \bar{a} \). In addition, \( \tilde{b} = 1 \), when
\[
a \geq 2 - \frac{\sqrt{t^2 + 3t \alpha}}{t}
\]
and \( \hat{b} = 1 \) when
\[
a \leq -1 + \frac{\sqrt{4t^2 - 3t \alpha}}{t}.
\]
Moreover, it can be shown that
\[
\bar{a} \leq 2 - \frac{\sqrt{t^2 + 3t \alpha}}{t} \leq -1 + \frac{\sqrt{4t^2 - 3t \alpha}}{t}.
\]
When \( a < \bar{a} \), the relevant threshold for tipping is \( \hat{b} \) and when \( a > \bar{a} \) it is \( \tilde{b} \). So, when \( a < \bar{a} \) according to the analysis above \( \hat{b} < 1 \) and when \( a > \bar{a} \), \( \tilde{b} < 1 \) as long as \( a < 2 - (\sqrt{t^2 + 3t \alpha})/t \).
FIGURE A1. TYPE OF EQUILIBRIUM AS THE LOCATION b OF NETWORK B VARIES FROM 1 TO a, WITH THE LOCATION OF NETWORK A FIXED AT a ≥ ă

FIGURE A2. TYPE OF EQUILIBRIUM AS THE LOCATION b OF NETWORK B VARIES FROM 1 TO a, WITH THE LOCATION OF NETWORK A FIXED AT a ≤ ă

Note that

\[
\frac{\partial \pi_B(a, b = 1)}{\partial b} = \frac{t(\alpha - t + ta^2)(4ta + 3\alpha - 3t - ta^2)}{18(t(1-a) - \alpha)^2} > 0
\]

if \( a < (2t - \sqrt{3t\alpha + t^2})/t \). Footnote 16 establishes that the derivative of the profit function at \( b = 1 \) is always strictly positive when the market is

16. There are four roots when we solve

\[
\frac{\partial \pi_B(a, b = 1)}{\partial b} = \frac{t(\alpha - t + ta^2)(4ta + 3\alpha - 3t - ta^2)}{18(t(1-a) - \alpha)^2} = 0
\]

with respect to \( a \). One is negative and the other is greater than one, so we rule them out. The remaining two are

\[
r_1 = \frac{1}{t}(2t - \sqrt{3t\alpha + t^2}) \text{ and } r_2 = \frac{1}{t}(\sqrt{t^2 - t\alpha}).
\]

It can be shown that \( r_2 \geq r_1 \) if and only if \( t \geq \alpha \), which we have assumed holds. In footnote 15, we showed that

\[
\tilde{a} \leq r_1 \leq -1 + \frac{\sqrt{4t^2 - 3t\alpha}}{t}.
\]

Therefore, when \( a > r_1 \) the relevant threshold is \( \tilde{b} \), but the market tips when \( a > r_1 \) (since \( \tilde{b} = 1 \) and hence for any location \( b \) of network B we must have \( b \leq \tilde{b} \)). This suggests that the other root, \( r_2 \), is irrelevant. Finally, the slope is positive when \( a \leq r_1 \), which suggests that, as long as the market is shared, network B’s profit decreases locally when it moves closer to A’s location starting from \( b = 1 \).
shared, implying that profits \textit{locally} decrease when firm \( B \) locates closer to firm \( A \).

For any \( b \) now, the derivative \( \partial \pi_B(a, b) / \partial b \) becomes zero at

\[
b = b_1 \equiv \frac{1}{t} \left( 2t - \sqrt{4t^2 - 3t\alpha - 4t^2a + t^2a^2} \right)
\]

\[
b = b_2 \equiv \frac{1}{3t} \left( 2t + 2\alpha + 2ta - \sqrt{4t^2 - 4t^2a - 7t\alpha + t^2a^2 + 8ta\alpha + 4\alpha^2} \right)
\]

\[
b = b_3 \equiv \frac{1}{t} \left( 2t + \sqrt{4t^2 - 3t\alpha - 4t^2a + t^2a^2} \right)
\]

\[
b = b_4 \equiv \frac{1}{3t} \left( 2t + 2\alpha + 2ta + \sqrt{4t^2 - 4t^2a - 7t\alpha + t^2a^2 + 8ta\alpha + 4\alpha^2} \right).
\]

Roots \( b_3 \) and \( b_4 \) are greater than one, so we rule them out. Also note that \( b_1 = \tilde{b} \), where \( \tilde{b} \) is given by (8). As we have mentioned before, we have \( \tilde{b} = b_1 \geq \hat{b} \) if \( a \geq \bar{a} \equiv 1/2 - \alpha/(2t) \) and \( \hat{b} \geq b_1 = \tilde{b} \), if \( a \leq \bar{a} \). This implies the following about the profit function of network \( B \) for any fixed location \( a \) of network \( A \).

\textbf{Case 1}: \( a \leq \bar{a} \equiv 1/2 - \alpha/(2t) \). Figure A3 depicts this case, where we have assumed that \( b_2 \geq \hat{b} \). Whether this holds or not depends, as we explain later, on the magnitude of \( \alpha \).

We have that \( \hat{b} \geq b_1 = \tilde{b} \), so only root \( b_2 \) may be relevant. Given that it is the only relevant root coupled with the result that the profit function of network \( B \) is strictly decreasing at \( b = 1 \) when sharing takes place, the profit function must attain a local minimum at \( b = b_2 \), if \( b_2 \geq \hat{b} \). The two networks share the agents (no tipping) when \( b > \hat{b} \). As network \( B \) moves closer to network \( A \) its prices fall (see (6)) but the market shares after a certain point may increase. This may happen after \( b = 1 - a \) where network \( B \) is closer to the middle point \( 1/2 \) than \( A \). That is why a minimum may be attained at \( b = b_2 \) and after this point the profit function increases. This is not always true as \( b_2 \) may be less than \( \hat{b} \), in which case the profit function of network \( B \) is decreasing until \( b = \hat{b} \).

17. Root \( b_3 \) is clearly greater than one (if it is real). Root \( b_4 \) is greater than one because at \( \alpha = 0, b_4 \) becomes

\[
\frac{2t(a + 1) + \sqrt{(2 - a)^2t^2}}{3t} > 1
\]

and the derivative of \( b_4 \) with respect to \( \alpha \) is

\[
\frac{1}{6t} \left( \frac{-7t + 8ta + 8\alpha}{\sqrt{4t^2 - 4t^2a - 7t\alpha + t^2a^2 + 8ta\alpha + 4\alpha^2}} + 4 \right)
\]

which is always greater than zero for \( a \in [0, 1] \).
Network A is losing market share and at \( b = \hat{b} \) tipping occurs in favor of B. Profits increase for B as it moves closer to \( a \) because its distance to the marginal agents, who are located at 0, decreases. When \( a > b \) the networks reverse identities. Overall, the profit function of network B as a function of its location \( b \) is U-shaped up to \( a = b \) when \( a \leq \hat{a} \).

**Case 2:** \( a \geq \hat{a} \equiv 1/2 - \alpha/(2t) \). Figure A4 depicts this case.

Given that network B’s profit function is strictly decreasing at \( b = 1 \) when the market is shared and that \( \tilde{b} = b_1 \) is now a relevant root, root \( b_2 \) becomes irrelevant. This is because at \( b = \tilde{b} = b_1 \) the market tips in favor of A and the slope of B’s profit function becomes zero. This implies that \( b_2 \) cannot be greater than \( b_1 \), since if that was the case there should be one more root in that range. Hence, in this case it must be that \( b_2 < b_1 \) and therefore \( b_2 \) is irrelevant. The two networks share the market when \( b > \tilde{b} \), unless \( a \geq (2t - \sqrt{3t \alpha + t^2})/t \), in which case network B’s market share is zero. As network B moves closer to \( a \) both prices and market shares fall (because now \( A \) is closer to the middle than in case
and at \( b = \tilde{b} \) the market tips in favor of \( A \). Then, at \( b = 1 - a \) the market tips in favor of \( B \). As in case 1 above, the profit function is U-shaped up to \( a = b \).

The above two cases will be used in the proof of Proposition 1. □

**Proof of Proposition 1**

Without loss of generality, we fix \( a \leq 1/2 \). We examine the optimal location of network \( B \). We showed in Lemma 1 that network \( B \)'s profit function is U-shaped with respect to \( b \) for any \( a \leq b \). This implies that, for any \( a \), network \( B \) (the follower) will either locate right next to network \( A \) and tip the market in \( B \)'s favor, or will locate at \( b = 1 \). Therefore, given network \( B \)'s reaction, it is network \( A \)'s dominated strategy to locate at \( a \in (0, 1/2) \). This can be understood as follows. If network \( B \) finds it profitable to locate at \( b = 1 \), network \( A \)'s profits are maximized at \( a = 0 \). This is because network \( B \) can tip the market in its favor (unless \( a = 1/2 \)), but to choose to locate at \( b = 1 \), it must mean that the externality is not so strong. In this case, \( A \) is better off locating at \( a = 0 \) first. If, on the other hand, network \( B \) finds it profitable to locate right next to \( A \) (and closer to the center \( 1/2 \)), then \( A \) is better off locating first at \( 1/2 \). So, in any subgame perfect equilibrium we have either \( a = 0 \) or \( a = 1/2 \). Also, note that network \( A \) can always secure strictly positive profits for itself because it can always locate at the center and either tip the market in its favor (recall that even if \( B \) locates at the center we have assumed that all agents join network \( A \)) or achieve a sharing equilibrium.

The above discussion suggests that network \( A \) will locate at \( a = 0 \) only if network \( B \) will locate at \( b = 1 \). If, instead, \( B \) locates arbitrarily close to 0, then it attracts all agents and its price (and profit) is \( \alpha \). If it locates at \( b = 1 \), \( B \)'s profit is \((t - \alpha)/2 \). Hence, if \( \alpha \leq t/3 \), network \( B \) has no incentive to deviate from \( b = 1 \) to \( b \approx 0 \). The equilibrium in this case is \( a = 0 \) and \( b = 1 \) (maximum horizontal differentiation).

If \( \alpha > t/3 \), then network \( A \) will locate at the center, \( a = 1/2 \) and \( b = 1 \). Given \( a = 1/2 \), network \( B \) has no profitable deviation. The profits of network \( A \) are

\[
\pi_A(a = 1/2, b = 1) = \frac{(7t - 12\alpha)^2}{144(t - 2\alpha)}
\]

and of network \( B \) are

\[
\pi_B(a = 1/2, b = 1) = \frac{(5t - 12\alpha)^2}{144(t - 2\alpha)}.
\]
The market share of network $B$ (the follower) is positive as long as $\alpha < 5t/12$. If $\alpha \geq 5t/12$, network $B$’s market share is zero. We assume that network $B$ stays at $b = 1$ (its profit is zero regardless of where it locates; so, in that sense the equilibrium is not unique) and hence network $A$’s price (and profit) is $\alpha - t/4$, which is derived as follows. The marginal consumer is at $b = 1$, suggesting that the difference in transportation cost between the two networks—in favor of $B$—is $t/4$ and the difference in the network benefit—in favor of $A$—is $\alpha$. So, the difference in prices should be equal to $\alpha - t/4$. In equilibrium, $B$’s price should be zero and thus $A$’s price is $\alpha - t/4$.

**Proof of Proposition 2**

We differentiate (11) with respect to $a$, $b$, and $x$. There is only one interior solution to the system of first-order conditions

$x = \frac{1}{2}$ and $a = \frac{1}{4}$, $b = \frac{3}{4}$.

The interior solution yields welfare equal to $W = \alpha/4 - t/48$. The corner solution is the one where the social planner has only one network serving all agents (tipping). If we set $x = 1$, for example, then it is easy to verify that welfare is maximized at $a = 1/2$ and is equal to $W = \alpha/2 - t/12$. Finally, it can be easily verified that the interior solution dominates the corner solution if and only if $\alpha \leq t/4$.

**References**


