The Role of Upstream-Downstream Competition on Bundling Decisions: Should Regulators Force Firms to Unbundle?

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We develop an upstream–downstream model to analyze downstream firms’ incentives to bundle. In our framework, the upstream firms are content providers (such as television stations) and the downstream firms are system operators (such as cable/satellite operators). We show that an a la carte regulation (i.e., a regulation that forces downstream firms to unbundle) leads to higher consumer surplus, if the unregulated equilibrium exhibits pure bundling (PB). Hence, our model predicts that in the television industry, which is mainly characterized by PB, an a la carte regulation will be beneficial for the consumers. If, on the other hand, the unregulated equilibrium is characterized by mixed bundling, then an a la carte regulation will increase consumer welfare provided that demand for multiple purchases is strong.

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“Letters have been streaming into my office. I don’t hear as much about highlighted issues, like gay marriage… as I do about rising cable rates. The logical next step is to relieve consumers of the burden of paying for channels they don’t watch.”—Senator Frank Lautenberg (Pittsburgh Tribune Review, “A la Carte Cable Served Up as Solution,” March 27, 2004)

1. Introduction

The purpose of this paper is to examine firms’ incentives to bundle and the welfare implications of bundling in an upstream–downstream model. More specifically, there are two upstream firms, each producing one intermediate good and two downstream firms. Each downstream firm can potentially use both intermediate goods in producing its final product, which is then sold to consumers. The model we develop is particularly useful to study competition in a market where the upstream firms are content providers and the downstream firms are system operators, which offer various contents via their systems to consumers (e.g., wireless communications industry, Internet Service Providers, cable television, satellite television, etc.). In such a context, a downstream system operator may offer each content separately, or the two contents together as a bundle (pure bundling (PB)), or both (mixed bundling (MB)).

In this paper, we focus primarily on the scenario where upstream content providers are television stations (such as ESPN, CNN, etc.) and the downstream operators are cable or satellite systems (such as Comcast or DirecTV), but our set-up can also be thought of in a more general content-system setting. For instance, our model can provide guidance for industries such as wireless communications where wireless providers offer bundles of calling minutes, wireless Internet access, GPS navigation, and so forth. It can also be applied to assess a ban on horizontal tying by downstream firms that bundle a small number of complementary goods, such as hotel rooms and plane tickets, or cable television and cable Internet. Or, we can treat basic cable versus premium channels each as a single good.

There is considerable public interest in regulating the television market. For example, a number of consumer advocacy groups and politicians have expressed dissatisfaction with the current state of cable television pricing. An October 2003 study by the US General Accounting Office (GAO) found that cable television rates had increased approximately 40% in the preceding 5 years. This is in contrast to the 12% increase in general inflation during the same time period.
Consumers and politicians, alike, have been seeking out possible remedies for these rising rates. One of the most oft-cited potential solutions is to allow consumers to purchase cable stations “a la carte.” Under the current system, consumers typically have a choice between two or three different cable packages. Each of these “packages” is essentially a bundle of a number of different cable stations.\(^1\) As Senator McCain’s quotation in the footnote below indicates, under such a system, consumers are forced to purchase products for which they may have no particular interest. For example, a sports fan that purchases Comcast’s basic cable package to receive ESPN is also “purchasing” stations such as PBS, ABC Family, and Nickelodeon. A puzzle, indeed, is why the selection choice offered to viewers is so limited. Proponents of a la carte pricing argue that allowing consumers to purchase stations individually will give them greater control over their cable bill and, presumably, lower the cost of cable for the average viewer. The above conclusion relies upon a rather strong assumption, however: that cable companies would not respond to a la carte regulations by changing the prices charged for each station. If cable providers respond to a la carte regulations, by raising prices, then it is not clear that a la carte pricing will be welfare improving. The model we develop can be used to offer some guidance as to whether downstream firms should be forced to unbundle.

We assume that the two contents and the two systems—as stand alone goods—are horizontally differentiated. Consumers must subscribe to one and only one system, but are allowed to purchase one or both contents. Thus, consumers are one-stop shoppers. In our framework, each consumer has a most preferred content, but also appreciates content variety. In particular, consumers derive an incremental utility from consuming a second content. If the incremental utility parameter is high relative to the cost (price) of purchasing a second content, then some consumers find it profitable to make multiple content purchases. Demand is, therefore, elastic.

In Sections 3 and 4, we present an analysis where we assume that bundling is allowed (or nonlinear prices are permitted). We find (Proposition 1) that if the incremental utility from consuming a second content is low, then the equilibrium is characterized by no bundling (NB). If the incremental utility is in an intermediate range, then the equilibrium is characterized by MB. Finally, if the incremental utility is sufficiently high, then firms offer only the bundle (i.e., PB). In Section 6, we assume that an a la carte regulation is passed. We

\(^1\) “When I go to the grocery store to buy a quart of milk, I do not have to buy a package of celery and a bunch of broccoli. I do not like broccoli.” Senator John McCain (March 26, 2004).
define an a la carte regulation as a regulation where downstream firms must sell the contents separately and charge linear prices. Even in this case, consumers can create the bundle, themselves, by purchasing both contents. The difference with the previous case, where bundling is allowed, is that now consumers who buy a second content do not receive a discount. If the incremental utility parameter exceeds a threshold, then some consumers find it worthwhile to buy both contents (Proposition 2). We, then, compare the welfare properties of the two regimes (see Section 6.3 and in particular Figures 5 and 6 which depict social and consumer welfare under the two regimes).

We find that if, in the unregulated equilibrium, downstream firms offer only the bundle (PB), then an a la carte regulation will increase consumer surplus. Hence, our model predicts that in the cable television industry, which is mainly characterized by PB (e.g., Crawford, 2006), an a la carte regulation will be beneficial for the consumers.

In May 2004, the US House of Representatives Commerce Committee asked the Federal Communications Commission (FCC) to examine the feasibility of a la carte cable pricing. Since this time, the FCC has published two studies on a la carte cable pricing. The first report, released in November 2004, concluded that there would be little benefit to consumers and that a la carte regulation would, therefore, be undesirable, FCC (2004). The second report, released in February 2006, reversed many of the findings of the previous study to conclude that consumers may, in fact, see “substantial benefits” from a la carte offerings, FCC (2006). Our model provides some theoretical support for the latter decision.

Moreover, we show that when the unregulated equilibrium is characterized by MB, an a la carte regulation will benefit consumers if and only if the bundled purchases, in the unregulated equilibrium, are large enough relative to the sales of individual contents. In the wireless industry, for instance, which is characterized with more examples of MB, an a la carte regulation will be beneficial to consumers if and only if the share of bundled purchases for the downstream wireless providers is large (relative to the share of single-purchase consumers).

We identify two effects that are associated with an a la carte regulation: (i) an upstream effect and (ii) a downstream effect (see Section 6.3.1 for more details and an intuition). Both of these effects act in lowering the upstream and the downstream stand-alone prices after an a la carte regulation.

Our analysis also demonstrates that an a la carte regulation will be effective only if downstream firms are forced to unbundle and charge linear prices. Unbundling alone is not enough because firms can offer contents a la carte but through nonlinear pricing (e.g., discounts) can
implement the bundling equilibrium. The PB equilibrium, for example, can be implemented via two-part tariffs where the fixed fee is equal to $t$ (the differentiation parameter) and the stand-alone prices are equal to marginal costs (i.e., the upstream prices).

Furthermore, firms do not become worse off when they are able to offer the bundle (or are allowed to charge nonlinear prices). On the contrary, both the upstream and the downstream firms are better off with the bundle. This may explain why the cable industry is opposed to legislation that would have required the companies to offer a la carte pricing. Crawford (2006), using TV Cable data, finds strong support for the discriminatory theory of bundling. Bundling an average top-15 special interest cable channels is estimated to increase profits and reduce consumer welfare, with an average effect of 4.7 and 4%, respectively. These predictions are qualitatively consistent with our theoretical results. Finally, bundling always (weakly) dominates the a la carte outcome in terms of social welfare.

The remainder of the paper is organized as follows. We offer a literature review and we highlight our main contributions in the next section. The main model is presented in Section 3 and the equilibrium analysis, where bundling is allowed, is presented in Section 4. In Section 5, we derive the first best and the welfare under the noncooperative outcome of Section 4. In Section 6, we derive the equilibrium assuming that downstream firms are not allowed to bundle and we compare the welfare properties of this equilibrium with the one we derived in Section 4. We offer a discussion about the robustness of our predictions in Section 7. We conclude in Section 8.

2. Literature Review

There is a large theoretical literature on bundling. In a monopoly setting, MB will increase the monopolist’s profits (e.g., Adams and Yellen, 1976; McAfee et al., 1989). Fang and Norman (2006) examine a multiproduct monopolist’s incentives to bundle. They compare PB with NB. In an oligopolistic market, bundling may lead to lower firm profits (e.g., Matutes and Regibeau, 1992; Anderson and Leruth, 1993; Economides, 1993; Reisinger, 2006; Thanassouli, 2007). Matutes and Regibeau (1992) investigate the issue of bundling in a duopoly model with complementary goods (software–hardware). They show that for a range of parameters the game is a prisoners’ dilemma. (We explain why we do not obtain a prisoners’ dilemma in Section 6.) Anderson and Leruth (1993) show, in a duopoly model, that MB leads to lower profits. Economides shows that bundling is a dominant strategy and when
the composite goods are not very close substitutes bundling profits are lower than those under a NB commitment on part of the firms. Reisinger (2006) shows that bundling hurts the firms and helps consumers if the correlation of consumer reservation values is highly negative, while profits increase under positive correlation.

Armstrong and Vickers (2006) compare nonlinear with linear prices and find that the effect of nonlinear prices on profits and welfare critically depends on whether consumers are one-stop shoppers or not. In particular, they show that when consumers are one-stop shoppers (as it is the case in our model) linear prices benefit consumers and hurt the firms. This is consistent with our findings.

Thanassoulis (2007) introduces a model with two competing firms each one selling two goods. There is a fraction of consumers who wishes to buy only one good and a fraction who buys both. When differentiation is at the firm level (and not at the product level), MB raises profits and lowers consumer surplus. The result gets reversed under the assumption that differentiation is only at the product level. Gans and King (2006) extend Matutes and Regibeau (1992). Their model involves four firms and two differentiated goods, where each firm produces only one good. Bundling occurs when two firms negotiate and commit to a bundled discount across the two products, prior to any competition for consumers. They show that whether bundling is mutually unprofitable or not critically depends on horizontal integration between firms. In particular, a pair of integrated firms facing a nonintegrated pair finds bundling profitable.

Corts (1995), in a paper which is concerned (as an application) with the effectiveness of a regulation in the TV industry, examines the impact of a price-cap system on a monopolist’s pricing and bundling decisions. He shows, among other things, that such a regulation can lead to unbundling of services.

Bundling may also be used by an incumbent firm as an entry foreclosure device (e.g., Whinston, 1990; Nalebuff, 2004). O’Brien and Shaffer (2005) examine the effects of horizontal mergers in intermediate goods markets, where upstream firms may have the ability to bundle their products. Stole (2007) offers a more systematic review of the bundling literature.

Nevertheless, there is not much work done on bundling (and unbundling) final goods in an upstream–downstream framework.² Such

². Bakos and Brynjolfsson (2000) model the decision to bundle in upstream and downstream competition for information goods, but their model is significantly different from ours. In their model the upstream competition is among firms for the exclusive rights of an information good and downstream competition is among firms who compete for consumers. Moreover, each stage is analyzed separately and there is no link between the upstream and the downstream stages.
an issue may not be particularly interesting in more traditional industries, but we believe that it is becoming increasingly important especially in our digital era where various kinds of contents (e.g., news, movies, games, etc.) are reaching consumers via different types of systems (e.g., Internet, mobile communications, satellite television, etc.). Pricing decisions made at the upstream level determine, to a large extent, the downstream firms’ decisions to bundle, as well as the effect of unbundling on welfare (e.g., the “upstream” effect).

The framework we have developed is rich enough in that it yields different market configurations (e.g., no, mixed, or PB) as equilibrium outcomes. Identifying the conditions under which each kind of an equilibrium will emerge is important, as each type of an outcome may be applicable to different industries. The main innovations of this paper are: (i) the explicit modeling of the vertical channel, (ii) the introduction of competition both at the upstream and downstream levels, and (iii) the examination of the effectiveness of a regulation that forces downstream firms to unbundle (a la carte regulation).

3. Description of the Model

The market consists of two symmetric upstream firms (content providers), denoted by \( U_j (j = a, b) \) and two symmetric downstream firms (system operators), denoted by \( D_i (i = 1, 2) \). Each downstream firm will potentially use both contents as inputs and consumers can subscribe to only one system. This implies that there are, in total, four products, \( 1a, 1b, 2a, \) and \( 2b \) (we have not introduced bundled products yet). For example, \( 1a \) is content \( a \) offered by system 1. We assume that contents and systems, as stand alone products, are horizontally differentiated. We capture this kind of differentiation, by placing the four products equidistantly on the Salop circle (Salop, 1979), of unit circumference (see Figure 1). Consumers are uniformly distributed on the circle and have measure one.

3. Our main focus in this paper is to uncover the basic intuition behind an a la carte regulation when the firms that are involved are symmetric. In Rennhoff and Serfes (2008), where we estimate the parameters of interest and then we perform simulations, TV networks and carriers are asymmetric. An a la carte regulation tends to favor the “bigger” firm more.

4. We assume that the two downstream firms are differentiated, even if they carry the same contents. For instance, cable is in many respects different from satellite, even in the case they both offer exactly the same channels. Also, switching costs and long-term contracts create some degree of differentiation. Both exist and are important in the TV industry, for example (although we do not model them explicitly).

5. In reality, each TV network obtains programming (e.g., movies, sporting events, documentaries, etc.) from various sources. This layer, for simplicity and tractability, is ignored in this paper. We also do not model the advertising side. For models that also consider the advertising side we refer the reader to Anderson and Coate (2005), Anderson and Gabszewicz (2006), and Gabszewicz et al. (2004).
FIGURE 1. LOCATIONS OF PRODUCTS ON THE CIRCLE

To fix ideas, let’s assume that $1a$ is ESPN on Comcast, $1b$ is CNN on Comcast, $2a$ is ESPN on DirecTV and $2b$ is CNN on DirecTV. For a viewer who is located very close to $1a$ (to the right of $1a$) ESPN on Comcast is her “ideal” product. Her “second best” a la carte choice is ESPN on DirecTV (i.e., $2a$). This type cares more about the content than the system that carries the content. For a viewer who is located very close to $1a$ but to the left of that point, CNN on Comcast is her “second best” a la carte choice. This type cares relatively less than the previous type about the content and more about the system. Both types, however, value content variety. As we will prove later the latter type of a consumer is “more likely” to consume the bundle (CNN+ESPN) offered by Comcast than the former consumer type who is more likely to make an a la carte purchase. The main point is that the specification of preferences, although not very general, is flexible enough to allow for the existence of consumers with diverse tastes.

We assume that downstream firm $i$ can offer three products: $a$ and $b$ separately, denoted by $ia$ and $ib$, respectively and $a$ and $b$ together as a bundle, denoted by $iB$. We denote by $p_{ia}$, $p_{ib}$, and $p_{iB}$ the prices of the available products. Moreover, we denote by $ij$ the location of
content \( j \) offered by downstream firm \( i \) and by \( iB \) the bundle offered by downstream firm \( i \). Therefore, there exists a maximum of six products in the market: the four described in a previous paragraph and the two bundles. Consumers purchase only one product and incur a linear disutility from not being able to buy their “ideal” product. The per-unit transportation cost is \( t > 0 \). Each consumer has a most preferred content, but also appreciates content variety. We capture this feature as follows. Consider a consumer who is located at \( x \). Her indirect utility is given by

\[
U = \begin{cases} 
  V(a) - t|x - ia| - p_{ia}, & \text{if she buys product } ia \\
  V(b) - t|x - ib| - p_{ib}, & \text{if she buys product } ib \\
  V(a, b) - t|x - ia| - t|x - ib| - p_{iB}, & \text{if she buys the bundle } iB. 
\end{cases}
\]

We assume that \( V(a, b) \geq V(a) = V(b) = V \) and we set \( V(a, b) = V + \theta \), where \( 0 \leq \theta \leq V \). Furthermore, \( V \) is assumed to be sufficiently high so that the market is covered. The parameter \( \theta \) measures the incremental utility from consuming a second content.\(^6\) By limiting \( \theta \), we are assuming that there is a benefit from receiving two stations (holding price and distance constant), but that consumers experience diminishing marginal utility from consuming the second content (see also Kim and Serfes, 2006). Consumers choose the product that maximizes their indirect utility. A number of the “critical values” in the paper (such as the cut-off that determines when MB is preferred to PB, for example) are measured as ratios between \( \theta \) and \( t \). In the rest of the paper we have set \( t = 1 \), which allows us to present the analysis in terms of the incremental utility parameter \( \theta \). This simplifies the paper, without notable loss of generality. The content prices that the upstream firms charge (in the form of take-it-or-leave-it offers that are also equal across downstream firms) are denoted by \( r_a \) and \( r_b \), respectively.\(^7\) These prices enter into the marginal costs of the downstream firms.

We will analyze a two-stage game, with simultaneous moves in each stage, where upstream firms first choose the prices \( r_a \) and \( r_b \) and then downstream firms choose the product offerings and the prices of their products, \( p_{ia}, p_{ib}, \) and \( p_{iB} \).

In solving for an equilibrium, in Section 4, we assume that bundling (or nonlinear prices) is allowed. In Section 6, we solve for an equilibrium under the assumption that a regulator forces the

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\(^6\) Crampes and Hollander (2005) also allow for multiple purchases.

\(^7\) For tractability reasons we do not allow for more complicated contracts (i.e., bargaining with two-part tariffs) between upstream and downstream firms. Also, we have not considered vertical integration and exclusive dealings. We reserve these very important topics for future research.
downstream firms to sell the contents separately and charge linear prices, in order to assess the welfare implications of bundling.

There are possibly three different types of (symmetric) equilibria: (i) \( NB \) equilibrium, where each downstream firm offers only the two products separately, (ii) \( MB \) equilibrium, where each downstream firm offers the two goods separately and together as a bundle, and (iii) \( PB \) equilibrium, where each downstream firm offers only the bundle. We search for a subgame perfect Nash equilibrium (SPNE) in pure strategies.

**Remark 1** (One Dimensionality Assumption): Due to the one dimensional characteristics space we have assumed, competition is localized (as in standard location models). For example, product 1a does not compete directly with product 2b. Also, as Figure 2 demonstrates, the two bundles do not compete head-on with each other (in the MB equilibrium; in the PB equilibrium they do). A less restrictive modeling framework would be a two-dimensional characteristics space (e.g., Matutes and Regibeau, 1992; Gans and King, 2006). In our context

**FIGURE 2. LOCATIONS OF MARGINAL CONSUMERS ON THE CIRCLE AND MARKET SHARES**
one dimension would capture consumer preference heterogeneity for the contents and the other for the systems. That framework, however, would increase the complexity of the analysis in our set-up dramatically, because, among other things, we explicitly model the vertical channel and competition both at the upstream and downstream levels. Hence, one can view the one-dimensional characteristics space (circle) that we adopt in this paper as a way to obtain clean results and predictions, while still maintaining many of the salient features of differentiation at the upstream and downstream levels. Moreover, we believe that the effects we have identified are robust and our main results will hold qualitatively in a more general model of “global” competition. In Section 7, we offer a discussion on the robustness of our results. In that section, we also consider a logit-based discrete choice model which allows us to relax the one dimensionality assumption. The numerical results we obtain are consistent with the results from our benchmark one-dimensional model.

4. Equilibrium Analysis

4.1 Consumer Demand

As shown in Figure 2, we assume that 1a is located at 0, 2a at \( \frac{1}{4}, 2b \) at \( \frac{1}{3}, \) and 1b at \( \frac{3}{4} \). Figure 2 depicts the MB configuration. There are six indifferent consumers on the circle: (1) consumer \( x_1 \) who is indifferent between 1b and 1b, (2) consumer \( x_2 \) who is indifferent between 1a and 1b, (3) consumer \( x_3 \) who is indifferent between 1b and 2b, (4) consumer \( x_4 \) who is indifferent between 1a and 2a, (5) consumer \( x_5 \) who is indifferent between 2a and 2B, and (6) consumer \( x_6 \) who is indifferent between 2b and 2b (see Figure 2).

The equations of the locations of the marginal consumers are given in Appendix A. Using these equations, the demand functions are given by

\[
\begin{align*}
   d_{1a} &= x_4 + (1 - x_2) = \frac{-8\theta - 12p_{1a} + 4p_{2a} + 8p_{1B} + 3}{8} \\
   d_{1b} &= x_1 - x_3 = \frac{-8\theta - 12p_{1b} + 4p_{2b} + 8p_{1B} + 3}{8} \\
   d_{1B} &= x_2 - x_1 = \frac{8\theta - 8p_{1B} + 4p_{1a} + 4p_{1b} - 1}{4} \\
   d_{2a} &= x_5 - x_4 = \frac{-8\theta - 12p_{2a} + 4p_{1a} + 8p_{2B} + 3}{8}
\end{align*}
\]
\[
d_{2b} = x_3 - x_6 = \frac{-8\theta - 12p_{2b} + 4p_{1b} + 8p_{2B} + 3}{8} \quad (5)
\]
\[
d_{2B} = x_6 - x_5 = \frac{8\theta - 8p_{2B} + 4p_{2a} + 4p_{2b} - 1}{4}. \quad (6)
\]

### 4.2 Equilibrium—Bundling is Allowed

The next proposition summarizes the SPNE of this game.

**Proposition 1** (Bundling is allowed): The SPNE is described as follows,

**NO BUNDLING (NB).** (Low preference for content variety). If \(0 \leq \theta \leq \left(\frac{4\sqrt{66} - 11}{24}\right) \approx 0.896\), then the bundle is not offered. Upstream prices and profits are given by

\[
ra = rb = \frac{5}{6} \quad \text{and} \quad \Pi_{U_a} = \Pi_{U_b} = \frac{5}{12}.
\]

Downstream prices and profits are given by

\[
p_{1a} = p_{2a} = p_{1b} = p_{2b} = \frac{4}{3} \quad \text{and} \quad \Pi_{D_1} = \Pi_{D_2} = \frac{1}{4}.
\]

**MIXED BUNDLING (MB).** (Medium preference for content variety). If \(\left(\frac{4\sqrt{66} - 11}{24}\right) \approx 0.896 < \theta < \frac{135}{88} \approx 1.534\), then firms offer the bundle and the individual goods separately. Upstream prices and profits are given by

\[
r_a = r_b = \frac{10\theta}{21} + \frac{5}{28} \quad \text{and} \quad \Pi_{U_a} = \Pi_{U_b} = \frac{55(3 + 8\theta)^2}{14112}.
\]

Downstream prices and profits are given by

\[
p_{1a} = p_{2a} = p_{1b} = p_{2b} = \frac{10\theta}{21} + \frac{19}{28},
\]
\[
p_{1B} = p_{2B} = \frac{17\theta}{14} + \frac{79}{112} \quad \text{and} \quad \Pi_{D_1} = \Pi_{D_2} = \frac{7744\theta^2 + 16,713 - 8976\theta}{56,448}.
\]

**PURE BUNDLING (PB).** (High preference for content variety). If \(\theta \geq \frac{135}{88} \approx 1.534\), then firms offer only the bundle. Upstream prices and profits are given by

\[
r_a = r_b = \theta - \frac{5}{8} \quad \text{and} \quad \Pi_{U_a} = \Pi_{U_b} = \theta - \frac{5}{8}.
\]
Downstream prices and profits are given by

\[ p_{1B} = p_{2B} = 2\theta - \frac{1}{4} \quad \text{and} \quad \Pi_{D_1} = \Pi_{D_2} = \frac{1}{2}. \]

The proof can be found in Appendix B.

Alternatively, each downstream firm can offer the two contents separately but charge nonlinear prices, allowing consumers to choose whether they will purchase only one content or both. In the PB case, downstream firm \( i \) can charge a cost-based two-part tariff, that is, a fixed fee equal to 1 (which is the per-unit transportation cost parameter) and a price for each content that is equal to firm \( i \)'s marginal costs, \( r_a = \theta - \frac{5}{8} \) and \( r_b = \theta - \frac{5}{8} \), e.g., Armstrong and Vickers (2001, 2006). A consumer who purchases only one content pays \( p_i = 1 + \theta - \frac{5}{8} = \theta + \frac{3}{8} \). For the second content, the price is equal to marginal cost, \( \theta - \frac{5}{8} \). Essentially, the second content is sold at a discount relative to the first one (including the fixed fee). In total, the consumer would pay \( p_{iB} = 2\theta - \frac{1}{4} \) and if \( \theta \) is high (i.e., higher than \( \frac{135}{88} \)) then all consumers, in equilibrium, will buy both contents.

In the MB case, firms charge a fixed fee equal to \( \frac{1}{2} \) (half of the per-unit transportation cost parameter) and a price for the first content equal to marginal cost, \( r_a = \frac{10\theta}{21} + \frac{5}{28} \) or \( r_b = \frac{10\theta}{21} + \frac{5}{28} \). Therefore, a consumer who purchases only one content pays \( p_{ij} = \frac{1}{2} + \frac{10\theta}{21} + \frac{5}{28} = \frac{10\theta}{21} + \frac{19}{28} \). The second content is sold at a price \( \frac{31\theta}{42} + \frac{3}{112} \), which is higher than the marginal cost \( \frac{10\theta}{21} + \frac{5}{28} \). Nevertheless, the second content is sold at a discount relative to the first one (including again the fixed cost).

8. But even when downstream prices equal marginal cost the outcome may not be efficient because there is a pricing distortion at the upstream level (see Section 5 for more details).

9. McAfee et al. (1989, p. 374) argue that MB is always (weakly) better: MB with a bundle price \( P_B \) and single-good prices \( P_1 = P_B - c_2 \) and \( P_2 = P_B - c_1 \) (where \( c_1 \) and \( c_2 \) are the marginal costs; the upstream prices in our model) always does weakly better than PB. This is true in our case. Note that in our framework, and as we demonstrated, when firms charge these prices, no consumer will make an à la carte purchase (when \( \theta \) is high), but the firms are not worse off. Nevertheless, in this case, if there is even an arbitrarily small cost of offering products that have zero demand (i.e., the contents separately) then there is no reason to believe that firms will offer the mixed bundle.

10. In a two-dimensional model, the “candidate” equilibrium prices under MB do not depend on the incremental utility \( \theta \) (more details are available upon request). They only depend on the downstream differentiation parameter and the upstream prices. This has to do with the fact that in our model bundles do not compete head-on in the MB equilibrium and therefore firms are able to appropriate part of the incremental utility. In this respect, the nonlinear tariff we described above will be different in a two-dimensional model. Nevertheless, as we have argued in many places in this paper, the intuition we have derived will very likely hold in a more general model of global competition.
Thus, a consumer who elects to buy both contents (bundle) pays 
\[ p_{i:B} = \left( \frac{10\theta}{21} + \frac{19}{28} \right) + \left( \frac{31\theta}{42} + \frac{3}{112} \right) = \frac{17\theta}{14} + \frac{79}{112} \]. For a medium preference for content variety some consumers (but not all) create the bundle.

### 4.3 Discussion of the Equilibrium

Different types of product offerings (i.e., no, mixed or PB) emerge endogenously in our model. The downstream firms have always incentives to offer the bundle and the two contents a la carte (MB) provided that two constraints are met: (i) the demand for the bundle is positive and (ii) the price of the bundle is less than the sum of the prices of the a la carte products. Upstream firms, through their pricing strategies, influence the above two constraints directly and consequently they influence indirectly the downstream firms’ incentives to bundle the goods.

There are three types of equilibria in this model. If the incremental utility \( \theta \) from consuming a second content (\( a \) or \( b \)) is low (i.e., \( 0 \leq \theta \leq \left( \frac{4\sqrt{66}-11}{24} \right) \approx 0.896 \)), then the upstream firms find it optimal to charge relatively high prices to the downstream firms, see Figure 3. This response is optimal precisely because consumers do not value a second content much and firms focus on single content purchases. As a consequence, the price of the bundle would be high (relative to the incremental benefit \( \theta \)) and no consumer would purchase the bundle, that is, the first constraint mentioned in the above paragraph is not satisfied. This implies that downstream firms do not offer the bundle (NB).

If the incremental utility is in a medium range (i.e., \( \left( \frac{4\sqrt{66}-11}{24} \right) \approx 0.896 < \theta < \frac{135}{88} \approx 1.534 \)), then the upstream firms lower discretely their prices (we explain why the jump at \( \theta = \left( \frac{4\sqrt{66}-11}{24} \right) \) in Figure 3 occurs in the next paragraph). However, as \( \theta \) increases price rises as Figure 3 demonstrates. The downstream firms now offer the bundle and each content separately, MB (see Figure 4).

Finally, if the incremental utility is high (i.e., \( \theta \geq \frac{135}{88} \approx 1.534 \)), then only the bundle is offered (PB). Recall from the discussion in the previous section that in the MB equilibrium the price of the first content (excluding the fixed fee which is \( \frac{1}{2} \)) is equal to marginal cost, \( \frac{10\theta}{21} + \frac{5}{28} \). The price of the second purchase in the MB equilibrium exceeds the marginal cost. When firms switch to the PB equilibrium the price of each content is equal to marginal cost \( \theta - \frac{5}{8} \) and the fixed fee is 1. Moreover, at \( \theta = \frac{135}{88} \), where the switching to the PB equilibrium occurs, \( \theta - \frac{5}{8} = \frac{10\theta}{21} + \frac{5}{28} \). Thus, we see that when \( \theta \) is high enough (and firms switch to the PB equilibrium), the optimal response is to lower the marginal price for the second purchase to marginal cost and increase
the fixed fee, from $\frac{1}{2}$ to 1 (the price of the bundle jumps up; see Figure 4). This induces all consumers to buy the bundle, which is an optimal strategy precisely because consumers value a variety of contents. In this case, the second constraint mentioned at the beginning of this section is not satisfied.

The upstream firms can pursue two distinct types of strategies: (i) set a low price and induce the downstream firms to offer the bundle, or (ii) charge a high price (upward jump) and eliminate the bundle. When $\theta$ is high, the first strategy is more profitable, while for low $\theta$ the second strategy yields higher profits. The threshold of $\theta$, where this switching takes place, is $\left(\frac{4\sqrt{66} - 11}{24}\right)$ (see Figures 3 and 4).\footnote{The profit functions of the upstream firms are not quasi-concave (rather, they exhibit a double hump shape). This is because the demand functions the upstream firms are facing are convex and exhibit a kink. For high upstream prices the bundles are not offered. Below a given threshold for the upstream price downstream firms offer the bundles (consumers start making multiple purchases) and further price reductions increase demand at a higher rate. At that threshold the upstream demand curves exhibit the kink. However, the lack of quasi-concavity of the upstream firms’ profit functions, does not lead to a nonexistence of a pure strategy equilibrium. This is because the upstream firms’ best-response correspondences have only upward jumps (details are omitted). Hence, an equilibrium in pure strategies must exist (see Vives 1999, theorem 2.5, p. 33).} Pricing decisions made by the upstream firms affect downstream firms’ incentives to offer the
bundle. For a range of values of the parameter $\theta$ (see equation (B.17)) there are two equilibria: one with high upstream prices and no bundle at the downstream level and the other with low upstream prices and MB at the downstream level. Upstream firms, as we show in Appendix B, prefer the first one.

5. Welfare Analysis

5.1 First Best

The social surplus is the difference between aggregate benefit and transportation cost and it is given by (details about the derivation of SS are straightforward and are omitted),

$$SS = V - \frac{11}{8} - x_4^2 - \frac{x_2^2}{2} + \frac{3x_4}{4} + \theta (x_6 - x_5) + \theta (x_2 - x_1) + x_1 - \frac{x_1^2}{2} - x_3^2 + \frac{5x_3}{4} + x_4^4 - \frac{x_5^2}{2} + x_5^4 + x_6 - \frac{x_6^2}{2}. \quad (7)$$

The first best locations should maximize (7) and are given by
\begin{align*}
    x_1 &= 1 - \theta, \\
    x_2 &= \frac{4\theta + 3}{4}, \\
    x_3 &= \frac{5}{8}, \\
    x_4 &= \frac{1}{8}, \\
    x_5 &= \frac{1 - 2\theta}{2} \quad \text{and} \quad x_6 = \frac{1 + 4\theta}{4}. \\
\end{align*}

(8)

The size of the group of consumers who buys the bundle is,

\begin{align*}
    x_2 - x_1 &= \frac{8\theta - 1}{4} \quad \text{and} \quad x_6 - x_5 = \frac{8\theta - 1}{4}.
\end{align*}

Note that it is socially optimal to offer the bundle (i.e., MB) if and only if \( \theta \geq \frac{1}{8} = 0.125 \). In the noncooperative equilibrium, firms play the MB equilibrium if and only if \( \theta \geq \left( \frac{4\sqrt{66} - 11}{24} \right) \approx 0.896 \). Moreover, it is socially optimal to offer only the bundle (i.e., PB) if and only if \( \theta \geq \frac{3}{8} = 0.375 \). In the noncooperative equilibrium firms play the PB equilibrium if and only if \( \theta \geq \frac{135}{88} \approx 1.534 \). Not surprisingly, for a range of the \( \theta \) parameter, the noncooperative outcome generates lower output than the socially optimum.

There are two sources of inefficiency. The first one comes from the upstream firms who charge prices that exceed upstream marginal cost (which is assumed to be zero). The second comes from the downstream firms. First, note that there is no downstream inefficiency in the PB case because the prices in the nonlinear schedule are equal to marginal cost. Second, in the MB case, an inefficiency exists (even if the upstream inefficiency is eliminated), because the price of the second content exceeds downstream marginal cost.\(^{12}\)

5.2 Welfare Under the Noncooperative Outcome of Proposition 1

Now we compute the social welfare and the consumer surplus that the noncooperative equilibrium (see Proposition 1) generates. The locations of the marginal consumers, in the MB equilibrium, are given by (after we substitute (B.9) into (B.5)),

\begin{align*}
    x_1 &= \frac{345 - 88\theta}{336}, \\
    x_2 &= \frac{243 + 88\theta}{336}, \\
    x_3 &= \frac{5}{8}, \\
    x_4 &= \frac{1}{8}, \\
    x_5 &= \frac{177 - 88\theta}{336} \quad \text{and} \quad x_6 = \frac{75 + 88\theta}{336}. \\
\end{align*}

(9)

\(^{12}\) Because aggregate demand is not fixed, absolute prices matter (in addition to relative prices) for efficiency.
In the PB equilibrium, the locations of the marginal consumers are
\[ x_2 = x_4 = x_5 = \frac{1}{8} \quad \text{and} \quad x_1 = x_3 = x_6 = \frac{5}{8}. \] (10)

The locations of the marginal consumers, in the NB equilibrium, are given by
\[ x_1 = x_2 = \frac{1}{8}, \quad x_3 = \frac{5}{8}, \quad x_4 = \frac{1}{8} \quad \text{and} \quad x_5 = x_6 = \frac{5}{8}. \] (11)

The social surplus under the MB noncooperative equilibrium, after we substitute (9) into (7), is given by
\[ SS^{MB} = \frac{56448V - 32688\theta + 51392\theta^2 - 1845}{56448}. \] (12)

The social surplus in the PB noncooperative equilibrium, after we substitute (9) into (7), is given by
\[ SS^{PB} = V + \theta - \frac{5}{16}. \] (13)

Under the NB noncooperative equilibrium, after we substitute (10) into (7) the social surplus is,
\[ SS^{NB} = V - \frac{1}{16}. \] (14)

Figure 5 depicts the social surplus (solid line) as a function of the incremental utility parameter \( \theta \).

The upward jump in the social welfare occurs because when \( \theta \) exceeds \((4\sqrt{66} - 11) \approx 0.896\), the upstream firms find it profitable to switch to the second equilibrium (recall, from the discussion in Section 4.3, that for a range of the \( \theta \) parameter there are two equilibria which can be Pareto ranked from the upstream firms’ perspective) by discretely lowering their prices. The upstream price decrease induces the downstream firms to offer the bundle, which in turn increases “total output” and improves efficiency (and consumer surplus—see the first upward jump of the solid line in Figure 6).

Consumer surplus is the difference between social surplus and aggregate profits and it is given, in the MB equilibrium, by
\[ CS^{MB} = \frac{56448V - 35856\theta + 7744\theta^2 - 39231}{56448}. \] (15)
In the PB equilibrium it is given by
\[ CS^{PB} = V - \theta - \frac{1}{16}. \]  
(16)

In the NB equilibrium, consumer surplus is given by
\[ CS^{NB} = V - \frac{67}{48}. \]  
(17)

Figure 6 (solid line) depicts the consumer surplus. The second downward jump of the solid line in Figure 6 occurs because firms after that threshold charge a higher price for the bundle (see Figure 4).

6. A REGULATOR FORCES DOWNSTREAM FIRMS TO UNBUNDLE

In this section, we assume that the downstream firms must sell the contents separately and charge linear prices. This could be due, for
example, to regulation that requires cable/satellite providers to offer a la carte cable pricing. A consumer, if she wishes, she can purchase both contents from a downstream firm (system operator). In other words, consumers can create the bundle themselves.\textsuperscript{13} We will solve for the equilibrium in this modified game and we will compare the welfare properties with those derived in the previous sections where bundling was allowed.

The marginal consumers are again given by (A.1)–(A.6), with the only difference that now $p_{ib} = p_{ia} + p_{ib}$ (i.e., no discounts for the purchase of the second content). $B$ now stands for a bundle that is created by a consumer who purchases both products. Intuitively,

\textsuperscript{13} We, inherently, assume that a consumer creating the bundle does so using products purchased through a single downstream firm. In addition, as in Nalebuff (2004), we assume that consumers can integrate the two contents as well as producers can.
some consumers will purchase both products of a downstream firm if the incremental utility $\theta$ is high. This will make a second purchase worthwhile. The demand functions are given by (1)–(6), with $p_{ib} = p_{ia} + p_{ib}$.

### 6.1 Equilibrium—Bundling is Not Allowed

The next proposition summarizes the SPNE when firms are not allowed to bundle.

**Proposition 2** (Bundling is not allowed): The SPNE is described as follows,

**NO CONSUMER BUNDLES (NCB).** (Low preference for content variety). If \(0 \leq \theta \leq \left(\frac{2\sqrt{20} - 3}{24}\right) \approx 1.053\), then in equilibrium no consumer makes multiple purchases. Upstream prices and profits are given by

\[
\begin{align*}
ra &= r_b = \frac{5}{6} \quad \text{and} \quad \Pi_{Ua} = \Pi_{Ub} = \frac{5}{12}.
\end{align*}
\]

Downstream prices and profits are given by

\[
\begin{align*}
p_{1a} &= p_{2a} = p_{1b} = p_{2b} = \frac{4}{3} \quad \text{and} \quad \Pi_{D1} = \Pi_{D2} = \frac{1}{4}.
\end{align*}
\]

**SOME CONSUMERS BUNDLE (SCB).** (Medium preference for content variety). If \(\left(\frac{2\sqrt{20} - 3}{24}\right) \approx 1.053 < \theta < \left(\frac{37}{24}\right) \approx 1.541\), then some consumers make multiple purchases. Upstream prices and profits are given by

\[
\begin{align*}
ra &= r_b = \frac{\theta}{2} + \frac{1}{16} \quad \text{and} \quad \Pi_{Ua} = \Pi_{Ub} = \frac{3(8\theta + 1)^2}{640}.
\end{align*}
\]

Downstream prices and profits are given by;

\[
\begin{align*}
p_{1a} &= p_{2a} = p_{1b} = p_{2b} = \frac{7\theta}{10} + \frac{7}{80} \quad \text{and} \quad \Pi_{D1} = \Pi_{D2} = \frac{3(8\theta + 1)^2}{1600}.
\end{align*}
\]

**ALL CONSUMERS BUNDLE (ACB).** (High preference for content variety). If \(\theta \geq \left(\frac{37}{24}\right) \approx 1.541\), then all consumers buy both downstream products. Upstream prices and profits are given by

\[
\begin{align*}
ra &= r_b = \theta - \frac{17}{24} \quad \text{and} \quad \Pi_{Ua} = \Pi_{Ub} = \theta - \frac{17}{24}.
\end{align*}
\]

Downstream prices and profits are given by;

\[
\begin{align*}
p_{ia} &= p_{ib} = \theta - \frac{3}{8} \quad \text{and} \quad \Pi_{D1} = \Pi_{D2} = \frac{1}{3}.
\end{align*}
\]
The proof of Proposition 2 is similar to the proof of Proposition 1 and is, therefore, omitted.

### 6.2 Discussion of the Equilibrium

As in the case where bundling is allowed (Proposition 1), there are three types of equilibria. The first one is characterized by no consumer consuming both downstream products (NCB). The incremental utility is relatively low (i.e., $0 \leq \theta \leq (\frac{2\sqrt{20}}{24} - \frac{3}{24}) \approx 1.053$) and firms find it profitable to set high prices so that no consumer finds it worthwhile to purchase a second content. The second type of equilibrium is when $(\frac{2\sqrt{20}}{24} - \frac{3}{24}) \approx 1.053 < \theta < \frac{37}{24} \approx 1.541$. In this case, upstream firms lower their prices and some consumers make multiple content purchases (SCB). However, the threshold above which this switching takes place is higher than the threshold above which firms switch to the MB equilibrium when they are allowed to bundle, that is, $(\frac{2\sqrt{20}}{24} - \frac{3}{24}) \approx 1.053$ versus $(\frac{4\sqrt{66}}{24} - \frac{11}{24}) \approx 0.896$ (see Figures 5 and 6 where both thresholds are depicted). Firms have stronger incentives to induce some consumers to consume both products when they have the ability to offer the bundle. The bundle sorts consumers who have intermediate horizontal tastes versus consumers with stronger brand loyalty and, therefore, firms do not lose much revenue from the consumers with extreme preferences for either one of the two contents. The third type of equilibrium emerges when $\theta \geq \frac{37}{24} \approx 1.541$, in which case all consumers consume both contents (ACB).

### 6.3 Welfare Comparison

Now we look at the welfare properties of the equilibrium, when bundling is prohibited, and compare them with those that emerge when bundling is allowed. When some consumers purchase both downstream products the locations of the marginal consumers—when bundling is not allowed—are given by

\[
\begin{align*}
x_1 &= \frac{3(29 - \theta)}{80}, \\
x_2 &= \frac{24\theta + 53}{80}, \\
x_3 &= \frac{5}{8}, \\
x_4 &= \frac{1}{8}, \\
x_5 &= \frac{47 - 24\theta}{80} \\
\text{and} \quad x_6 &= \frac{13 + 24\theta}{80}.
\end{align*}
\]

We substitute the above locations into (7) to derive the social surplus in the case where SCB. This yields,

\[
SS^{SCB} = \frac{3200V - 149 + 3264\theta^2 - 2384\theta}{3200}.
\]
The locations of the marginal consumers when all consumers bundle (ACB) are the same with (9). Consequently, the social surplus is the same with (13). Finally, when no consumer bundles (NCB) the social surplus is the same with (14).

Figure (5) compares the social surplus between the two regimes: when bundling is allowed (equations (14), (18), and (13)—represented by the solid line) and when it is not (equations (14), (12), and (13)—represented by the dotted line). The social surplus under the former regime is always (i.e., for any $\theta$) higher than that under the latter regime. Total output, in both regimes, is less than the first-best output (see Section 4.1, where the first best was derived), but the output when bundling is allowed is higher than when it is not allowed. The intuition is as follows. First, when the bundle is available firms offer it at a discount which induces more consumers to buy it relative to the case where consumers bundle themselves. Second, firms switch to the equilibrium where consumers buy the bundle sooner (i.e., in terms of the threshold in the $\theta$ dimension) when they can offer the bundle than when they are not allowed to offer the bundle. Both of these effects increase total output (which is below the first best level) and consequently efficiency.

The consumer surplus (social surplus minus aggregate profits) when SCB is given by

$$C_{SCB} = \frac{3200V - 191 + 576\theta^2 - 3056\theta}{3200}.$$  

When all consumers bundle (ACB), it is given by

$$C_{ACB} = V - \theta + \frac{7}{16}.$$  

Finally, when no consumer bundles (NCB) consumer surplus is the same with (17).

Figure (6) compares the consumer surplus between the two regimes: when bundling is allowed (equations (17), (19), and (20)—represented by the solid line) and when it is not (equations (17), (15), and (16)—represented by the dotted line). When $\theta$ is relatively low (i.e., $\theta \leq \frac{(2\sqrt{20} - 3)}{24}$) consumer surplus when bundling is allowed is (weakly) higher than the consumer surplus when bundling is not allowed. For high values of $\theta$ (i.e., $\theta \geq \frac{(2\sqrt{20} - 3)}{24}$) the above ranking of consumer surplus gets reversed. In that case an a la carte regulation will benefit consumers.

Hence, our model predicts that in the television industry, which is mainly characterized by PB, a regulation that would force downstream firms to unbundle is pro-competitive.
Remark 2 (No Upstream Tier): We also solved our model by assuming that the upstream prices are fixed at marginal cost, which is zero. The main results do not change qualitatively (of course, the upstream effect is absent). In particular, what we have termed the downstream effect is still present. For high $\theta$, PB is an equilibrium. An a la carte regulation will lead to lower prices and benefit consumers. Consumers also benefit when the unregulated equilibrium exhibits MB and $\theta$ is high, as in Figure 6. What is different when there is no upstream market is that the $\theta$ thresholds are now lower. In other words, it is easier for the firms to sustain mixed and PB when the upstream tier is absent. This is not surprising, because prices are lower when the upstream prices are equal to marginal cost and therefore more people are willing to make a multiple purchase. In addition, because the upstream effect is absent, the extent of the price reduction due to an a la carte regulation is underestimated when upstream pricing is omitted.

6.3.1. Intuition: Downstream and Upstream Effects

The intuition behind the welfare comparison is as follows. We identify two effects that are associated with an a la carte regulation: (i) a downstream effect and (ii) an upstream effect. Let’s first define these two effects. The downstream effect refers to the incentives of a downstream firm to unilaterally change its prices, following an a la carte regulation, while we hold the upstream prices fixed. The upstream effect refers to the incentives of an upstream firm to unilaterally change its price, following an a la carte regulation, while we allow the downstream tier to respond to the regulation. Below, we explain how each effect works.

**Downstream effect.** When $\theta$ is high PB is an equilibrium. Starting from a PB equilibrium, an a la carte regulation will make the demand of downstream firms more elastic (locally). As a result, stand-alone prices will fall. The intuition is as follows. Fix the upstream prices. In the unregulated PB equilibrium, each firm charges one price: the bundle price. The equilibrium prices balance inframarginal rents and gains from marginal consumers. In equilibrium, firms have no incentive to raise or lower their prices.

Now assume that a regulator forces the firms to unbundle and charge linear prices. Each firm now sets two stand-alone prices. For the sake of comparison, we begin by assuming that each stand-alone price is equal to the preregulation bundle price divided by two (since each firm offers two products). Because consumers have a choice, some of them will choose to purchase only one product and some of them will continue purchasing both. The ones with extreme preferences are those who make the single purchases. As a result, fewer consumers purchase each product.
Let’s now examine the downstream firms’ incentives to raise or lower (locally) their stand-alone prices.

A unilateral decrease of a firm’s stand-alone prices will have three effects: (1) it will attract some marginal consumers from the rival (business stealing), (2) it will lower the inframarginal rents, and (3) it will induce more existing consumers to make multiple purchases, since buying a second product (from the same downstream firm) is now cheaper (demand creation).

Below, we explain in detail how each effect works.

Effect # 1 (business stealing): The first effect is the same as in the unregulated PB equilibrium. The rate at which consumers switch downstream firms in response to a marginal price change depends on the degree of loyalty of the marginal consumers, which in turn depends on how differentiated the two downstream firms are. The degree of differentiation between the two downstream firms does not change, since the a la carte regulation applies uniformly to both downstream firms. So, according to the first effect there is no pressure on the stand-alone prices to change from the preregulation levels.

Two-dimensional model. In a two-dimensional model the bundles consumers create themselves also compete head-on in the a la carte case. Consumers who make single purchases switch downstream firms and consumers who create the bundle also switch firms, in response to marginal changes of the stand-alone prices. In our one-dimensional model the switching comes only from the consumers who make single purchases. But that should not affect the rate at which consumers switch downstream firms, since in any case it is the degree of differentiation across those firms that matters (which has not changed). Therefore, the invariance of the business stealing effect across the two regimes (i.e., PB vs. a la carte) should also hold in a two-dimensional model. (This will be used in the discussion on nonlocalized competition that we offer later).

Effect # 2 (inframarginal rents): The second effect is weaker now because downstream firms make fewer sales, that is, the revenue loss due to price cuts is less than in the unregulated equilibrium. So price cuts in the a la carte scenario are more profitable.

Effect # 3 (demand creation): The third effect is absent in the unregulated equilibrium and creates incentives for price cuts in the a la carte scenario. This induces more consumers to make multiple purchases (from the same downstream firm).

The last two effects are key for our intuition and as we argue later will also be present in a more general model of nonlocalized
competition. By combining all three effects, we can see that price reductions are profitable.

The same three effects are present when a firm unilaterally raises its stand-alone prices. The first effect is the same as in the unregulated equilibrium. The second effect (inframarginal rents) is weaker because downstream firms make fewer sales, that is, the revenue gain due to the price increase is less than in the unregulated equilibrium. The third effect (demand creation) implies that fewer consumers make multiple purchases. By combining all three effects, we can see that price increases are unprofitable.

Hence, the first order conditions, evaluated at the point where the stand-alone prices are equal to the preregulation bundling price divided by two, are negative. Put differently, downstream demand in the a la carte case becomes more elastic (locally). In the new regulated (a la carte) equilibrium, therefore, the stand-alone prices will be lower. (Second-order conditions are satisfied).

When the unregulated equilibrium is characterized by MB, an a la carte regulation will benefit consumers if and only if the bundled purchases, in the unregulated equilibrium, are large enough relative to the sales of individual contents, that is, relatively high $\theta$. The downstream effect indicates that the stand-alone prices after a regulation will be lower than the corresponding prices in the unregulated equilibrium. However, the bundle is offered at a discount in the unregulated equilibrium and therefore we cannot say in general whether all consumers will benefit from an a la carte regulation. This is because some consumers (i.e., the ones who purchase the bundle) may pay a higher price for both contents in the regulated equilibrium. Nevertheless, in our set-up, all consumers are better off following an a la carte regulation. In general, when $\theta$ is high, demand for multiple purchases is strong and downstream firms have strong incentives to lower their stand-alone prices significantly. This effect is likely to outweigh the absence of discounts and make consumers better off in an a la carte equilibrium (even when upstream prices are fixed).\footnote{Armstrong and Vickers (2006) offer a similar intuition, about why linear prices benefit consumers over nonlinear prices, in a model where consumers have elastic demands and make all their purchases from one firm (as it is the case in our model). Thanassoulis (2007) notes that the magnitude of the consumer surplus effect of MB with one stop shopping (and unit demands) depends on how many people want one good (light users) versus both (heavy users). With discounts (i.e., MB) the price for the heavy users drops, while more is extracted from the light users. Without discounts, prices are kept low to attract the heavy users. This is precisely the driving force behind our downstream effect.}

\textbf{Uncovered market.} An assumption that we have made is that there is no outside option (covered market). Consumers must buy at least
one product. However, it is conceivable that in the PB unregulated equilibrium some consumers choose not to buy the bundle, if their only choice is the bundle, but they may buy one product if products are offered a la carte. In this case the number of consumers who purchase a product may increase in the a la carte scenario, if there are sufficiently many consumers who had chosen the outside option in the unregulated equilibrium. This will affect the second effect that we mentioned above (inframarginal rents), which will make price cuts more costly for the firms. The net effect becomes ambiguous and depends on how attractive the outside option is. In the TV industry, where approximately 90% of households subscribe to either cable or satellite television services, we do not believe that it will be enough to overturn the price decrease prediction.15 Not allowing for an outside option is one limitation of our model, that we partially address with the numerical example of Section 7.2.

**Upstream effect.** An a la carte regulation makes the demand of each upstream firm *more elastic*. The intuition is as follows. We fix the upstream prices at the level of the unregulated equilibrium and we allow the downstream tier, after an a la carte regulation, to respond. Then, we examine the incentive upstream firms have to change their prices (locally) in response to an a la carte regulation. Each upstream firm sells its product through both downstream firms.

When the unregulated equilibrium is characterized by PB upstream firms create no demand by unilaterally lowering their prices, since all consumers buy the bundle. Consider now an a la carte regulation. Such a regulation will decrease the number of consumers who purchase each upstream firm’s product and it will generate a group of consumers who make single purchases (as in the downstream effect, see above). Price reductions are now more profitable, because they: (i) are less costly, relative to the preregulation case (inframarginal rents) and (ii) will induce downstream firms to lower their prices which in turn will induce more consumers to make multiple purchases (demand creation). Demand has become more elastic for each upstream firm.

Now suppose that the unregulated equilibrium exhibits MB. The demand of each upstream firm consists of those who buy a single product (from either downstream firm) and those who buy the bundle (also from either downstream firm). The consumers who buy the bundle are the ones with intermediate horizontal preferences. Furthermore, the marginal consumers for each upstream firm are the ones who are indifferent between buying the bundle from that firm

and buying a single product from the rival firm. (This would be true even in a two-dimensional model, since the dimension which captures downstream differentiation—the second dimension—plays no critical role in this discussion). An a la carte regulation will result in fewer (and on average more loyal) consumers purchasing each upstream firm’s product.\textsuperscript{16} The reason is the presence of discounts in the MB equilibrium. In other words, a second purchase costs less when downstream firms can bundle than when they are forced to charge linear prices and that is why more consumers prefer to consume the bundle in the unregulated regime than those who create the bundle themselves in the a la carte regime. Moreover, a unilateral upstream price reduction attracts more marginal consumers in the a la carte regime than under MB. This has to do with how downstream firms react to a unilateral upstream price reduction. In the MB unregulated regime, the upstream price reduction of, say, content $a$ is passed on to the downstream price of content $a$, the price of the bundle as well as on to the downstream price of rival content $b$, see equation (B.3). In contrast, under a la carte, only the downstream price of content $a$ decreases.\textsuperscript{17} The implication is that upstream firm $a$ can induce a bigger relative downstream price change (and hence can attract more marginal consumers) for its product in the a la carte regime relative to MB, for the same upstream price reductions. This implies a more elastic upstream demand under an a la carte regulation, which leads to lower upstream prices.

More formally, this can be seen as follows. The demand function of upstream firm $a$, for example, in the unregulated MB equilibrium is,

$$D_a = \frac{4r_b + 15 - 44r_a + 40\theta}{40}$$

\textsuperscript{16} As an example suppose that $\theta = 1.2$ and $t = 1$. Then, it follows from our model that 65% of the consumers buy the bundle (from either firm) in the MB equilibrium and 59% create the bundle themselves in the a la carte equilibrium. Thus, each upstream firm experiences a demand reduction after an a la carte regulation. Moreover, those who buy an upstream firm’s product are more loyal, on average, in the sense that they have more extreme horizontal tastes.

\textsuperscript{17} We do not present in this paper the derivations under the a la carte regime. For the sake of comparison, below we present the a la carte downstream prices when SCB,

$$p_{1a} = p_{2a} = \frac{2\theta}{5} + \frac{3r_a}{5} + \frac{t}{20} \quad \text{and} \quad p_{1b} = p_{2b} = \frac{2\theta}{5} + \frac{3r_b}{5} + \frac{t}{20}.$$  

As it can be immediately observed, $r_a$ only affects $p_a$. This happens because the two contents, under a la carte and when SCB, do not compete head-on. Each upstream firm’s goal is to induce consumers (through the downstream firm) to make a second purchase (create the bundle), rather than to steal consumers from the rival. As a result, only the own upstream (input) price matters in determining a downstream price, as the above expressions indicate. Effectively, each content becomes a local monopoly. This is not the case in the MB unregulated regime, where an upstream price change affects all downstream prices via the price of the bundle, as in (B.3).
and in the a la carte equilibrium is,

\[ D_a = \frac{3 + 24\theta - 24r_a}{20}. \]

For the sake of comparison, fix the upstream prices at the preregulation symmetric MB equilibrium. First, observe that the demand function under a la carte has a steeper slope with respect to \( r_a \) (\(-\frac{12}{10}\) vs. \(-\frac{11}{10}\)). As we argued above, in the a la carte regime, a unilateral upstream price reduction results in a lower downstream price for that content only, while in the MB case rival contents benefit as well (at the partial expense of the upstream firm who initiated the price decrease). Second, fewer consumers purchase \( a \) under a la carte. This can be easily computed using the above upstream demand functions evaluated at the preregulation MB equilibrium upstream prices. Steeper slope and fewer consumers imply a more price elastic upstream demand under an a la carte regulation.

Hence, both the upstream and the downstream effects imply that the final prices will be lower if, starting from a PB equilibrium, a regulator forces firms to unbundle. When the unregulated equilibrium exhibits MB, then an a la carte regulation will benefit consumers provided that demand for multiple purchases is strong.\(^{18}\)

The comparison so far has assumed that regulation does not affect the type of the equilibrium, that is, it has assumed that some consumers make multiple purchases irrespective of whether the bundle is allowed or not. However, it is possible that a regulation will bring an outcome where no consumer purchases both contents. This happens when the preference for variety is weak, in which case upstream firms find it profitable to switch to a “high price” equilibrium, (when \( \theta < (\frac{2\sqrt{20}}{24} - \frac{3}{24}) \)), Figure 6). The high upstream prices are passed on to consumers who have no incentive to make multiple purchases. In this case, a regulation makes consumers worse off. It happens when preference for variety is weak, which in turn implies that in the unregulated MB equilibrium very few consumers demand the bundle.

Finally, the profits of the upstream and the downstream firms are always (i.e., for any \( \theta \)) higher when bundling is allowed than when it is not. This can be easily derived by comparing the equilibrium profits that are stated in Propositions 1 and 2. Therefore, bundling in our model does not lead to a prisoners’ dilemma, as in Matutes and Regibeau (1992) and Economides (1993), for example. This may explain why the cable

\(^{18}\) In the TV industry, for example, a proxy for this could be the fraction of households with more than a certain number of members, with the presumption that a larger household will have a stronger demand for multiple channels (heavy users, Thanassoulis, 2007).
industry is opposed to an a la carte regulation. A key difference between our model and those models is that consumers in our model do not have to buy both downstream products. The downstream products in our model (e.g., CNN and ESPN) are not two indispensable components of a system, as it is the case in those papers. Clearly, our assumption fits certain industries better, as for example the TV industry. An implication of this assumption is that price competition, after an a la carte regulation, intensifies in our model, precisely because downstream firms try to induce more consumers to make multiple purchases. This effect is absent in Matutes and Regibeau and Economides because consumers must buy both products. In addition, the upstream effect is not present in those papers and other similar papers. As we have argued, the upstream effect reduces the prices even further.

In the next section, we explore which modeling assumptions are most likely to be responsible for this result.

7. Discussion about the Robustness of our Results

7.1 The Effects of Nonlocalized Competition

The most restrictive implication of our model is that competition is localized. In this section, we conjecture whether our main predictions will hold and how they might be affected in a more general model of nonlocalized competition.

Pure bundling. In the discussion on the two-dimensional model that we offer in Section 6.3.1 when we discuss the downstream effect, we argue that the business stealing effect should be the same across the two regimes (i.e., PB vs. a la carte). The other two effects that we identified (inframarginal rents and demand creation) should work the same way in a two-dimensional model as they do in the one-dimensional model. Thus, we expect our intuition and results to hold in a model of nonlocalized competition. The results from the logit model that we consider next, where competition is not localized, do not contradict this assertion.

Mixed bundling. The assumption of localized competition is less innocuous when the unregulated equilibrium is characterized by MB.

19. We have confirmed this intuition using a two-dimensional model. In particular, we solved for the downstream prices under the assumption of PB. Then, we imposed an a la carte regulation and we evaluated the first order conditions at the point where the stand-alone prices are equal to the preregulation prices divided by two. The derivatives were negative, implying that firms would have incentives to lower (locally) their prices.
In this case the bundle is offered at a discount in the unregulated equilibrium and that discount is likely to be deeper if the bundles are competing directly with each other. Therefore, on the one hand an à la carte regulation lowers the upstream and the stand-alone downstream prices, but on the other hand in the unregulated equilibrium the price of the bundle may be lower than the sum of the stand-alone prices in the regulated equilibrium. Consumer welfare will depend on which effect dominates the other. As we argued in Section 6.3.1, when $\theta$ is high demand for multiple purchases is strong and downstream firms have strong incentives to lower their stand-alone prices significantly. When this effect is strong an à la carte regulation is likely to benefit consumers.\(^{20}\)

### 7.2 Relaxing the One Dimensionality Assumption

To see how our results hold up under different assumptions, we consider a numerical simulation based on the multinomial logit. The simulation allows us to relax the one dimensionality assumption by incorporating unobserved heterogeneity. Here, we briefly describe the simulation model and present some of the results. The indirect utilities from purchase are given as follows,

$$U_k = \begin{cases} 
(V + \epsilon_k^a) + \epsilon^i_k - p_{ia} + \epsilon_{kia} & \text{if } k \text{ buys product } ia \\
(V + \epsilon_k^b) + \epsilon^i_k - p_{ib} + \epsilon_{kib} & \text{if } k \text{ buys product } ib \\
(V + \theta + \epsilon_k^a + \epsilon_k^b) + \epsilon^i_k - p_{iB} + \epsilon_{kiB} & \text{if } k \text{ buys the bundle } iB
\end{cases}$$

where $i$ indexes the carriers and $k$ the consumers. We also use $j$ ($j = a, b$) to index the contents.

Consumer $k$'s valuation of content $j$, which is independent of the identity of the content provider, is $V + \epsilon^i_k$. $V$ is the value of the content common to all consumers and $\epsilon^i_k$ is assumed to be distributed $N(0, \Omega)$, where $\Omega = \begin{bmatrix} \rho_j & 1 \\ 1 & \rho_i \end{bmatrix}$. The $\epsilon^i_k$ term represents $k$’s preference for carrier $i$, which is equal across all products offered by carrier $i$. We assume $\epsilon^i_k$ is distributed $N(0, \Omega)$, where $\Omega = \begin{bmatrix} \rho_i & 1 \\ 1 & \rho_i \end{bmatrix}$. The final unobserved

20. The assumption of localized competition does not affect the NB equilibrium. In this case products compete head-on, regardless of whether competition takes place in one or two dimensions. Nonlocalized competition will also affect which type of bundling is used in equilibrium. Our conjecture is that nonlocalized competition will make PB easier. When bundles compete head-on, firms have incentives to offer deeper discounts to those who purchase the bundle, in the MB equilibrium. This results in more consumers consuming the bundle as opposed to making single purchases. In turn, this will lower the threshold for $\theta$ above which firms switch to PB equilibrium.
Table I.
Numerical Pricing Simulation with the Multinomial Logit

<table>
<thead>
<tr>
<th>Equilibrium Type</th>
<th>( r_i )</th>
<th>( p_{ij} )</th>
<th>( p_{iB} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mixed Bundling</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta = 10 )</td>
<td>7.92</td>
<td>9.91</td>
<td>17.83</td>
</tr>
<tr>
<td>( \theta = 25 )</td>
<td>21.58</td>
<td>23.58</td>
<td>45.16</td>
</tr>
<tr>
<td>Pure Bundling</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta = 30 )</td>
<td>26.36</td>
<td>54.71</td>
<td></td>
</tr>
<tr>
<td>( \theta = 45 )</td>
<td>40.87</td>
<td>83.73</td>
<td></td>
</tr>
<tr>
<td>A La Carte</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta = 10 )</td>
<td>7.07</td>
<td>8.09</td>
<td></td>
</tr>
<tr>
<td>( \theta = 25 )</td>
<td>20.65</td>
<td>21.64</td>
<td></td>
</tr>
<tr>
<td>A La Carte</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta = 30 )</td>
<td>25.40</td>
<td>26.40</td>
<td></td>
</tr>
<tr>
<td>( \theta = 45 )</td>
<td>39.88</td>
<td>40.89</td>
<td></td>
</tr>
</tbody>
</table>

The term \( \varepsilon_{kij} \) is \( k \)’s idiosyncratic preference for the specific combination \( ij \) (perhaps because of advertising, previous experience, or information from friends). We assume that \( \varepsilon_{kij} \) is distributed Type I extreme value (logit).

We allow for correlation in the distributions of \( \varepsilon_k^i \) and \( \varepsilon_k^{-j} \) in order to allow for differences in the degree of differentiation between choices. To illustrate how this works intuitively, consider \( \varepsilon_k^i \) with positive correlation. A positive correlation implies that an individual’s valuation of content \( a \) will be positively correlated with her valuation of content \( b \). The end result is that the positive correlation diminishes the differentiation between contents. A negative correlation implies the opposite.

In order to conduct our pricing simulation, we assume that \( \rho_j = -0.20 \) (negative correlation between the content valuations), \( \rho_i = 0 \) (zero correlation between the carrier valuations), and \( V = 50 \). We solved the model numerically and the table below presents some of the results (\( r \) denotes the upstream prices and \( p \) the downstream). The top portion of Table I compares the MB equilibrium with the a la carte equilibrium, while the lower portion compares the PB equilibrium with the a la carte equilibrium.

The results are qualitatively consistent with our benchmark theoretical model. An a la carte regulation lowers both the upstream and
Table II.
Bundling Example with Discrete Valuations

<table>
<thead>
<tr>
<th>Consumer</th>
<th>Valuations of Consumers</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Product A</td>
<td>Product B</td>
<td>Bundle</td>
<td></td>
</tr>
<tr>
<td>Group 1</td>
<td>3</td>
<td>10</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Group 2</td>
<td>8</td>
<td>3</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

the downstream prices, whether the unregulated equilibrium exhibits mixed or PB. Consumer surplus increases. We also re-ran the simulation using $\rho_i = 0.80$, which allows the two carriers to be less differentiated. The results, which we omit, are similar to those in the tables above. We believe that these numerical simulations indicate that the intuitions we have extracted from our model hold in more general environments.

7.3 Continuity of Preferences

Our main results and intuition rely on the assumption that valuations are continuous (or, at least not so discrete). If, instead, valuations are discrete, then in some cases our main predictions may change. To see this, consider the example given in Table II.21

Under PB, the optimal price is 10 and consumer surplus is 2. In the a la carte regime the price of product A is 8 and the price for product B, yielding zero consumer surplus. Thus, an a la carte regulation will make consumers worse off.

8. Conclusion

We develop a parsimonious model of upstream downstream competition to investigate downstream firms’ incentives to bundle. The upstream firms are content providers and the downstream firms are system operators. We use this model to shed light on the question about whether cable and satellite television companies should be forced to offer stations a la carte (i.e., to unbundle). The nature of the equilibrium depends on the strength of consumer preference for content variety. When such a preference is: (i) low, the firms do not bundle (NB), (ii) medium, each firm offers the bundle and the two products separately (MB), and (iii) high, each firm offers only the bundle (PB). A regulation that forces downstream firms to unbundle and charge linear prices

21. We thank an anonymous referee for bringing this point up and for offering us this example.
leads to higher consumer surplus if and only if the share of the bundled purchases for any downstream firm is large enough (relative to the shares of the single purchases). One of our model’s prediction is that when downstream firms offer only the bundle (which is arguably the case in the television industry), then an a la carte regulation will increase consumer surplus (see Figure 6).

In this paper we have assumed that both contents are of equal quality. A concern is whether offering a la carte programming would hinder the ability of lesser-watched channels (i.e., channels of lower “quality” ) to exist when not bundled with popular channels. This will have an adverse effect on program variety. We intend to address this issue in future work.

**Appendix A: Locations of Marginal Consumers**

Consumer $x_1$ is located at,

$$V - \left( x_1 - \frac{3}{4} \right) - p_{1b} = V + \theta - \frac{1}{4} - p_{1B} \Rightarrow x_1 = -\theta - p_{1b} + p_{1B}. \quad (A.1)$$

Consumer $x_2$ is located at,

$$V - (1 - x_2) - p_{1a} = V + \theta - \frac{1}{4} - p_{1B} \Rightarrow x_2 = \frac{4\theta - 4p_{1B} + 4p_{1a} + 3}{4}. \quad (A.2)$$

Consumer $x_3$ is located at,

$$V - \left( \frac{3}{4} - x_3 \right) - p_{1b} = V - \left( x_3 - \frac{1}{2} \right) - p_{2b} \Rightarrow x_3 = \frac{-4p_{2b} + 4p_{1b} + 5}{8}. \quad (A.3)$$

Consumer $x_4$ is located at,

$$V - x_4 - p_{1a} = V - \left( \frac{1}{4} - x_4 \right) - p_{2a} \Rightarrow x_4 = \frac{4p_{2a} - 4p_{1a} + 1}{8}. \quad (A.4)$$

Consumer $x_5$ is located at,

$$V - \left( x_5 - \frac{1}{4} \right) - p_{2a} = V + \theta - \frac{1}{4} - p_{2B} \Rightarrow x_5 = \frac{-2\theta + 2p_{2B} - 2p_{2a} + 1}{2}. \quad (A.5)$$

Finally, consumer $x_6$ is located at,

$$V - \left( \frac{1}{2} - x_6 \right) - p_{2b} = V + \theta - \frac{1}{4} - p_{2B} \Rightarrow x_6 = \frac{4\theta - 4p_{2B} + 4p_{2b} + 1}{4}. \quad (A.6)$$
The consumers who are located in the interval \([x_2, x_1]\) prefer, by construction, the bundle 1B to either 1a or 1B. Similarly, the consumers who are located in the interval \([x_5, x_6]\) prefer, by construction, the bundle 2B to either 2a or 2B. In addition, \(x_3\) and \(x_4\) must be located, by construction, between 1B and 2B and between 1a and 2a, respectively.

**Appendix B: Proof of Proposition 1**

The downstream firms have always incentives to offer the bundle and the two contents a la carte (MB) provided that two constraints are met: (i) the demand for the bundle is positive and (ii) the price of the bundle is less than the sum of the prices of the a la carte products. Upstream firms, through their pricing strategies, influence the above two constraints directly and consequently they influence indirectly the downstream firms’ incentives to bundle the goods.

**B.1 Proof—Mixed Bundling Equilibrium (MB)**

We first search for an equilibrium where firms offer the bundle and the individual goods separately. We solve the game backwards.

**Second stage (downstream).** The demand functions are given by (1)–(6). The downstream firms’ profit functions are given by

\[
\pi_{D_1} = (p_1a - r_a)d_{1a} + (p_{1b} - r_b)d_{1b} + (p_{1B} - r_a - r_b)d_{1B}, \tag{B.1}
\]

\[
\pi_{D_2} = (p_2a - r_a)d_{2a} + (p_{2b} - r_b)d_{2b} + (p_{2B} - r_a - r_b)d_{2B}. \tag{B.2}
\]

Each firm’s profit function is strictly concave with respect to the firm’s strategic variables (second-order condition is satisfied). The unique and symmetric solution to the system of first order conditions is given by

\[
\begin{align*}
p_{1a} &= p_{2a} = \frac{4r_a}{5} + \frac{r_b}{5} + \frac{1}{2}, \\
p_{1b} &= p_{2b} = \frac{4r_b}{5} + \frac{r_a}{5} + \frac{1}{2} \quad \text{and} \\
p_{iB} &= \frac{\theta}{2} + \frac{3r_a}{4} + \frac{3r_b}{4} + \frac{7}{16}. \tag{B.3}
\end{align*}
\]

Note that the implicit requirement \(p_{ia} + p_{ib} \geq p_{iB}\) (i.e., the price of the bundle is lower than the sum of the individual prices of the goods in the bundle) holds if and only if, \(2\theta \leq r_a + r_b + \frac{9}{4}\). If we substitute (B.3) into (1 and 4), (2 and 5) and (3 and 6) we obtain the stage 2-equilibrium demand functions,
\[ d_{1a} = d_{2a} = \frac{-40\theta - 4r_a + 44r_b + 25}{80}, \]
\[ d_{1b} = d_{2b} = \frac{-40\theta - 4r_b + 44r_a + 25}{80} \] and
\[ d_{1B} = d_{2B} = \frac{8\theta - 4r_a - 4r_b - 1}{8}. \] (B.4)

By substituting (B.3) into (A.1), (A.2), and (A.3) we obtain the stage 2-equilibrium locations of the marginal consumers,
\[ x_1 = \frac{-40\theta + 44r_a - 4r_b + 75}{80}, \quad x_2 = \frac{40\theta - 44r_b + 4r_a + 65}{80}, \quad x_3 = \frac{5}{8}, \]
\[ x_6 = \frac{40\theta - 44r_a + 4r_b + 25}{80}, \quad x_5 = \frac{-40\theta + 44r_b - 4r_a + 35}{80}, \quad x_4 = \frac{1}{8}. \] (B.5)

For \( d_{iB} \geq 0 \) we must have \( \theta \geq \frac{r_a}{2} + \frac{r_b}{2} + \frac{1}{8} \). Therefore, for bundling to arise in equilibrium, it must be that: (i) \( d_{iB} \geq 0 \) and (ii) \( p_{iB} \leq p_{ia} + p_{ib} \). The following inequalities ensure that these two constraints are satisfied,
\[ \frac{r_a}{2} + \frac{r_b}{2} + \frac{1}{8} \leq \theta \leq \frac{r_a}{2} + \frac{r_b}{2} + \frac{9}{8}. \] (B.6)

By substituting (B.3) into (B.1) we derive the equilibrium profit functions of both downstream firms,
\[ \Pi_{Di} = \frac{200r_a - 1600\theta (r_a + r_b) + 200r_b + 825 + 32r_a r_b + 784 (r_a^2 + r_b^2) + 1600\theta^2 - 400\theta}{3200}. \] (B.7)

**First stage (upstream).** The profit functions of the upstream firms are given by (we assume a zero marginal cost),\(^{22}\)
\[ \pi_{Ua} = (d_{1a} + d_{2a} + d_{1B} + d_{2B})r_a \quad \text{and} \quad \pi_{Ub} = (d_{1b} + d_{2b} + d_{1B} + d_{2B})r_b. \] (B.8)

\(^{22}\) The demand functions of the upstream firms are given by
\[ D_a = \frac{4r_b + 15t - 44r_a + 40\theta}{40t} \quad \text{and} \quad D_b = \frac{4r_a + 15t - 44r_b + 40\theta}{40t}. \]
The solution to the first order conditions is,

\[ r_a = r_b = \frac{10\theta}{21} + \frac{5}{28}. \] (B.9)

By substituting (B.9) into the constraints, given by (B.6), we derive the region of \( \theta \) so that our assumptions hold,

\[ \frac{51}{88} \leq \theta \leq \frac{219}{88} \quad \text{or approximately} \quad 0.58 \leq \theta \leq 2.49. \] (B.10)

If \( \theta < \frac{51}{88} \), then \( d_{iB} = 0 \) and if \( \theta > \frac{219}{88} \) then the price of the bundle is greater than the sum of the prices of the individual goods. By substituting (B.9) into (B.7) we derive the profits of the upstream firms,

\[ \Pi_{U_a} = \Pi_{U_b} = \frac{55(3 + 8\theta)^2}{14112}. \]

The stage 2 (downstream) equilibrium prices can be obtained by substituting (B.9) into (B.3) and are given by

\[ p_{ia} = p_{ib} = \frac{10\theta}{21} + \frac{19}{28} \quad \text{and} \quad p_{iB} = \frac{17\theta}{14} + \frac{79}{112}. \]

The equilibrium bundle demand is,

\[ d_{1B} = d_{2B} = \frac{88\theta - 51}{168}. \]

The bundle demand is less than or equal to \( \frac{1}{2} \) provided that \( \theta \leq \frac{135}{88} \approx 1.534 \). Otherwise, we move to the PB case where all consumers purchase the bundle (this case will be analyzed later).

The downstream equilibrium profits are given by

\[ \Pi_{D_1} = \Pi_{D_2} = \frac{7744\theta^2 + 16713 - 8976\theta}{56448}. \]

Next, we to examine a “global” deviation on part of the upstream firms.

**Checking for Profitable Deviations from the Proposed Equilibrium**

**Deviation by firm** \( U_a \). Due to symmetry, we consider only one upstream firm’s deviation. Firm \( U_a \) can deviate by increasing its price \( r_a \)

23. The second order conditions are satisfied with strict inequality. Therefore, the first order conditions are sufficient for a “local” maximum. Still, an upstream firm can deviate globally, by increasing its price drastically, so that downstream firms do not find it profitable to offer the bundle. We examine this kind of deviation next.
beyond the point where \( d_{ib} = 0 \). We solve the stage 2 game assuming that no consumer purchases the bundle. It is important here to emphasize that we allow consumers to create their own bundle by purchasing both products even when a firm does not offer the bundle. Nevertheless, in this deviation, this option will not be exercised. This is because, in what follows, the bundle demand is zero when the bundle price is less than the sum of the prices of the individual goods. Therefore, it must be the case that no consumer would want to create her own bundle by paying \( p_{ia} + p_{ib} \).

The stage 2 equilibrium prices, assuming that downstream firms do not offer the bundle, are,

\[
p_{ia} = \frac{4r_{a}^{dev}}{5} + \frac{r_{b}}{5} + \frac{1}{2} \quad \text{and} \quad p_{ib} = \frac{4r_{b}}{5} + \frac{r_{a}^{dev}}{5} + \frac{1}{2},
\]

where \( r_{b} \) is given by (B.9). For \( d_{ib} \) to be zero it must be that (from (B.5)),

\[
\frac{r_{a}^{dev}}{2} + \frac{r_{b}}{2} + \frac{1}{8} > \theta \Rightarrow r_{a}^{dev} > \frac{32\theta}{21} - \frac{12}{28}.
\]

The optimal deviation price is, \( r_{a}^{dev} = \frac{5\theta}{21} + \frac{85}{168} > \frac{32\theta}{21} - \frac{12}{28} \), provided that \( \theta < \frac{32\theta}{4536} \approx 0.723 \). The maximum profits after deviation are given by

\[
\Pi_{Ua}^{dev} = \frac{5(17 + 8\theta)^{2}}{9408}.
\]

It turns out that this type of deviation is unprofitable (i.e., \( \Pi_{Ua}^{dev} \geq \Pi_{Ua}^{dev} \)), if and only if \( \theta \geq \frac{1}{8}(\frac{-15 + 14\sqrt{66}}{19}) \approx 0.65 \).

Therefore, if

\[
\theta \geq \frac{1}{8}\left(\frac{-15 + 14\sqrt{66}}{19}\right) \approx 0.65
\]

then neither upstream firm has an incentive to deviate. Note that

\[
\frac{1}{8}(\frac{-15 + 14\sqrt{66}}{19}) \approx 0.65 > \frac{51}{88} \approx 0.58,
\]

which has to hold (see the previous section) in equilibrium for the bundle demand to be positive. Therefore, if

\[
\frac{1}{8}\left(\frac{-15 + 14\sqrt{66}}{19}\right) \approx 0.65 \leq \theta \leq \frac{135}{88} \approx 1.534
\]

then a SPNE is described by (B.9) and (B.11).

Downstream firms in this case have no incentive to deviate from (B.11) as long as (B.10) is satisfied.
**B.2 Proof—Pure Bundling Equilibrium (PB)**

If $\theta > \frac{135}{88} \approx 1.534$, then all consumers buy the bundle, half from one firm and half from the other. To find the equilibrium prices we assume that both downstream firms offer the bundle. The optimal downstream prices (after we solve the system of first order conditions) are given by

$$p_{1B} = 1 + r_a + r_b \quad \text{and} \quad p_{2B} = 1 + r_a + r_b. \quad (B.12)$$

The demand function that each upstream firms faces for its product is inelastic, since all consumers buy the bundle. Hence, the upstream firms will set their prices to ensure that the downstream firms have no incentives to deviate from the PB equilibrium. To this end, assume that firm 1 deviates and offers the bundle and the two contents a la carte (i.e., MB), while firm 2 offers only the bundle. It can be shown that the optimal prices for the deviating firm are,

$$p_{1a}^{\text{dev}} = r_a + \frac{r_b - \theta + 17}{2} \quad p_{1b}^{\text{dev}} = r_b + \frac{r_a - \theta + 17}{2} \quad \text{and} \quad p_{1B}^{\text{dev}} = 1 + r_a + r_b.$$  

The profits after deviation are given by

$$\Pi_{D_i}^{\text{dev}} = \frac{64\theta^2 - 80\theta + 73 - 64\theta r_a + 40r_a + 32r_a^2 - 64\theta r_b + 40r_b + 32r_b^2}{96}.$$  

The profits before deviation are $\frac{1}{2}$. It turns out that a deviation is profitable, provided that (in a symmetric equilibrium where $r_a = r_b = r$), $r > \bar{r} = \theta - \frac{5}{8}$. Also, at $r = \bar{r} = \theta - \frac{5}{8}$ the a la carte demands, that is, $d_{1a}^{\text{dev}} = d_{1b}^{\text{dev}} = \frac{-80+5+8\theta}{12}$, are zero. When $r < \bar{r} = \theta - \frac{5}{8}$, the a la carte demands are strictly negative and the best a downstream deviating firm can do is to offer only the bundle. But in this case such a deviation does not yield strictly higher profits. Therefore, to sustain the PB outcome as an equilibrium, upstream firms must charge $r_a = r_b = \theta - \frac{5}{8}$.

Because all consumers buy the bundle, it is not profitable for the upstream firms to charge prices that are strictly less than $\bar{r}$ (an upstream deviation to a price strictly greater than $\bar{r}$ is examined in the next section). So, given a PB equilibrium, the upstream firms will charge the following prices,

$$r_a = r_b = \theta - \frac{5}{8}. \quad (B.13)$$
The downstream prices for the bundle are obtained by substituting (B.13) into (B.12) and are given by

\[ p_{1B} = p_{2B} = 2\theta - \frac{1}{4}. \] (B.14)

The profits of the upstream firms are,

\[ \Pi_{Ua} = \Pi_{Ub} = \theta - \frac{5}{8} \] (B.15)

and those of the downstream firms are,

\[ \Pi_{D1} = \Pi_{D2} = \frac{1}{2}. \] (B.16)

**Checking for Profitable Deviations from the Proposed Equilibrium**

**Deviation by firm \( U_a \).**

Given (B.13) no downstream firm wishes to deviate from (B.14) assuming that the deviating firm offers only the bundle and also no firm wants to deviate by offering the bundle and the contents a la carte (MB). What remains to be shown is whether upstream firms have an incentive to deviate. Fix \( r_b \) at the equilibrium value (B.13) and allow firm \( U_a \) to deviate. A deviation is meaningful if it induces the downstream firms to play an equilibrium that is different from the PB one. If \( r_{a\text{dev}} \leq \bar{r} = \theta - \frac{5}{8} \), then, as we demonstrated above, firms play the PB equilibrium, and such a deviation would only reduce profits. If \( r_{a\text{dev}} > \bar{r} = \theta - \frac{5}{8} \), then the PB outcome is no longer an equilibrium. As we showed in the above section, a downstream firm has a profitable deviation. We then assume that \( r_{a\text{dev}} \) is strictly greater than \( \bar{r} = \theta - \frac{5}{8} \) and downstream firms play the MB equilibrium. The demand functions for the downstream firms are given by (1–6). The downstream prices are given by (B.3). It can be easily calculated that the optimal deviation price is \( r_{a\text{dev}} = \frac{\theta}{2} + \frac{25}{176} \). But \( r_{a\text{dev}} > \bar{r} = \theta - \frac{5}{8} \) if and only if \( \theta < \frac{135}{88} \). The PB case we are examining is valid when \( \theta > \frac{135}{88} \) and consequently such a deviation is not profitable.

**B.3 Proof—No Bundling Equilibrium (NB)**

The proof is similar to the proofs of the MB and PB equilibria and it is omitted.
**Equilibrium selection.** Based on the analysis so far we can conclude that there are two equilibria when,

\[
\frac{1}{8} \left( -15 + 14\sqrt{66} \right) \approx 0.65 \leq \theta \leq \left( -11 + 4\sqrt{66} \right) \approx 0.896, \tag{B.17}
\]

one with MB and the other with NB. Moreover, the NB one is the Pareto optimal equilibrium (from the perspective of the two upstream firms) provided that \( \theta \leq \frac{1}{8} (-3 + \frac{14\sqrt{66}}{11}) \approx 0.917 \) and consequently it will be the one preferred by these firms. This is a property of supermodular games, when the payoff to a player is increasing in the strategy of the other player, as it is the case at the upstream stage in our model (see Vives, 1999, remark 14, p. 34). We assume that upstream firms are able to coordinate on the better equilibrium. Therefore, the NB equilibrium emerges if and only if \( \theta \leq \left( -11 + 4\sqrt{66} \right) \approx 0.896 \).

**REFERENCES**


