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Abstract

We perform a quantitative analysis of the nuclear modification factor in deuteron–gold collisions $R_{dAu}$ within the Color Glass Condensate approach, and compare our results with the recent data from RHIC experiments. Our model leads to Cronin enhancement at mid-rapidity, while at forward rapidities it predicts strong suppression of $R_{dAu}$ at all $p_T$ due to low-$x$ evolution. We demonstrate that our results are consistent with the data for $dAu$ charged hadron spectra, $R_{dAu}$ and $R_{CP}$ recently reported for rapidities in the interval $\eta = 0–3.2$ by the BRAHMS experiment at RHIC. We also make a prediction for $R_{pA}$ at mid-rapidity in $pA$ collisions at the LHC.

Recent observations [1–5] of the suppression of high $p_T$ hadron yields at forward rapidities at RHIC have attracted considerable interest. The observed suppression is in sharp contradiction with the naive multiple scattering picture, in which the magnitude of Cronin enhancement observed at mid-rapidity is expected to increase further at forward rapidities, reflecting the growth of the number of scattering centers (partons) at small Bjorken $x$. On the other hand, the observed effect has been predicted [6–9] as a signature of quantum evolution in the Color Glass Condensate (CGC) [10–16]. Very recently, the first exploratory experimental results [17] on the back-to-back azimuthal correlations of high $p_T$ particles separated by several units of rapidity in $dAu$ collisions indicated the possible onset of the “mono-jet” behavior expected in the quantum CGC picture [18] (the azimuthal correlations in the classical approach to the CGC were studied in [19]). Nevertheless, the origin of the observed effects is certainly not beyond a reasonable doubt at present; to
clarify it, one needs to perform dedicated and careful experimental and theoretical studies. While the data is qualitatively consistent with the predictions based on the CGC picture, a detailed comparison to the data requires a quantitative analysis taking into account, for example, the contributions of both valence quarks and gluons, and the influence of realistic fragmentation functions. Such an analysis is the goal of this note. Recently, related work in the more traditional multiple scattering picture supplemented by shadowing has been done in Refs. [20,21], and in Ref. [22] where the contribution of valence quarks scattering off the CGC has been addressed.

In this Letter we use a simple model for the dipole–nucleus forward scattering amplitude which describes the onset of the gluon anomalous dimension in the color glass condensate regime. Since the inclusive gluon and quark production cross sections in \( p(d)A \) collisions can be expressed in terms of the adjoint dipole–nucleus scattering amplitude, our model allows us to describe inclusive hadron production in deuteron–gold collisions at \( \sqrt{s} = 200 \text{ GeV} \) at RHIC. Our model is based on a detailed analytical analysis performed in our previous publication [7] stemming from the idea put forward in [6].

Inclusive cross section for production of a gluon in \( dA \) collisions was calculated in [23–25] and is given by

\[
\frac{d\sigma_{G}}{d^2kdy} = \frac{C_FS_A S_d}{\alpha_s(2\pi)^2} \int d^2x \nabla^2 z G(z, y) \times e^{-ikz} N_G(z, y),
\]

where \( S_A \) and \( S_d \) are cross sectional areas of the gold and deuteron nuclei correspondingly and \( y \) is the total rapidity interval. We assume a simple form of the scattering amplitude of the gluon dipole of transverse size \( zT = |z| \) on the deuteron inspired by the two-gluon exchange [7]

\[
N_G(z, y) = \ln 1/x_p \rightarrow (1 - x_p)^4 x_p^{-\lambda} \alpha_s^2 z^2 \ln(1/zT \mu) \frac{1}{S_d}
\]

with \( \lambda \) to be fixed later and \( x_p \) the gluon’s Bjorken \( x \) in the deuteron’s (or proton’s) wave function. Integrating over directions of \( z \) we rewrite (1) as

\[
\frac{d\sigma_{G}}{d^2kdy} = \frac{\alpha_s C_F S_A}{k_T^2} \int_0^{\infty} dzT J_0(k_T, zT) 
\times \ln \frac{1}{zT \mu} \frac{1}{zT} \partial_{zT} \left[ zT \partial_{zT} N_G(zT, y) \right],
\]

where \( \mu \) is a scale associated with deuteron and is fixed at \( \mu = 1 \text{ GeV} \) thereof. The gluon dipole scattering amplitude on a gold nucleus \( N_G(zT, y) \) should be determined from the non-linear evolution equation [15]. Since an exact solution of the non-linear evolution equation [15] is a very difficult task we are going to construct a model for \( N_G(zT, y) \) satisfying its asymptotic behavior: at \( zT \ll 1/Q_s(y) \) one should have \( N_G(zT, y) \sim z^2T \), while at \( zT \gg 1/Q_s(y) \) we should get \( N_G(zT, y) \sim 1 \) [15,26,27]. \( Q_s(y) \) is the nuclear saturation scale at rapidity \( y \) ) This behavior can be modeled by a simple Glauber-like formula

\[
N_G(zT, y) = 1 - \exp\left[ -\frac{1}{4} (z^2T_0)^2 \gamma(y, z^2T) \right],
\]

where \( \gamma(y, z^2T) \) will be given by (7). Note, that when \( \gamma = 1 \) Eqs. (3) and (4) reproduce the results of McLerran–Venugopalan model [13,14,23] (for similar results see [28]).

At forward rapidities, in the deuteron fragmentation region, the Bjorken \( x \) of the nucleus acquires its lowest possible value for a given \( \sqrt{s} \), while the Bjorken \( x \) of the proton is close to unity. In that region rescatterings of valence quarks of the proton in a nucleus can give a substantial contribution to the hadron production cross section. This problem was discussed in a series of papers listed in [29,30] leading to the following expression for inclusive valence quark production cross section [30]

\[
\frac{d\sigma_{Q}}{d^2k} = \frac{S_A}{2\pi} \int_0^{\infty} d\zeta T \zeta T J_0(k_T, zT) [2 - N_Q(zT, y)],
\]

where \( N_Q(zT, y) \) is the quark dipole–nucleus forward scattering amplitude. In the quasi-classical approximation (\( \gamma = 1 \)) \( N_Q(zT, y) \) is given by the same quasi-classical formula (4) with \( Q_s^2(y) \) replaced by \( \frac{x_p}{S_d} Q_s^2(y) = \frac{x_p}{S_d} Q_s^2(y) \). Therefore, by analogy with (4), we model the quark dipole scattering ampli-
tude \( N_Q(z_T, y) \) as

\[
N_Q(z_T, y) = 1 - \exp \left[ -\frac{1}{4} \left( \frac{z_T^2}{N_c} Q_s^2 \right) \gamma(y, z_T^2) \right].
\] (6)

To model the anomalous dimension \( \gamma(y, z_T^2) \) we use the following interpolating formula

\[
\gamma(y, z_T^2) = \frac{1}{2} \left( 1 + \frac{\xi(y, z_T^2)}{\xi(y, z_T^2) + 2\xi(y, z_T^2) + 7\xi(3)\epsilon} \right),
\] (7)

where

\[
\xi(y, z_T^2) = \frac{\ln[1/(z_T^2 Q_s^2)]}{(\lambda/2)(y - y_0)},
\] (8)

and \( \epsilon \) is a constant to be fitted. This form of the anomalous dimension is inspired by the analytical solutions to the BFKL equation [31]. Namely, in the limit \( z_T \to 0 \) with \( y \) fixed we recover the anomalous dimension in the double logarithmic approximation \( \gamma \approx 1 - \sqrt{1/(2z_T^2)} \). In another limit of large \( y \) with \( z_T \) fixed, Eq. (7) reduces to the expression of the anomalous dimension near the saddle point in the leading logarithmic approximation \( \gamma \approx 1 + \frac{\xi}{1 + \xi} \). Therefore, Eq. (7) mimics the onset of the geometric scaling region [27,33]. A characteristic value of \( z_T \) is \( z_T \approx 1/(2k_T) \), so we will put \( \gamma(y, z_T^2) \approx \gamma(y, 1/(4k_T^2)) \).

The saturation scale \( Q_s(y) \) that we use is the same as the one used in [34] to fit the low-\( x \) DIS data and in [35] to describe the hadron multiplicities at RHIC. It is given by

\[
Q_s^2(y) = \Lambda^2 A^{1/3} e^{\gamma} = 0.13 \text{ GeV}^2 e^{-y} N_{\text{coll}}.
\] (9)

Here \( N_{\text{coll}} \) is the number of binary collisions at a given centrality in a \( dAu \) collision. Parameters \( \Lambda = 0.6 \text{ GeV} \) and \( \gamma = 0.3 \) are fixed by DIS data [34]. The initial saturation scale used in (8) is defined by \( Q_s^2(y_0) = Q_s^2(30) \) with \( y_0 \) the lowest value of rapidity at which the low-\( x \) quantum evolution effects are essential.

The Cronin effect [36] is usually attributed to multiple rescatterings of partons in the nucleus [7,9,28,37]. However, it is also present in the low energy data, i.e., at energies where saturation is unlikely to play a significant role for the production of high \( p_T \) particles. For example, at \( \sqrt{s} = 20 \text{ GeV} \) the nuclear enhancement for \( \pi^\pm \) produced in proton–nucleus collisions peaks at \( k_T \approx 4 \text{ GeV} \) [36]. This implies that the typical non-perturbative scale \( \kappa \) associated with such low energy hadronic rescatterings may be rather large. It becomes much smaller than \( Q_s(y) \) at high energies/rapidities as one can see from (9). However, at the central rapidity region at RHIC the influence of this non-perturbative scale cannot yet be neglected. To take it into account in describing the nuclear modification at RHIC we shift the saturation scale in the Glauber exponents (4) and (6) as follows \( Q_s^2 \to Q_s^2 + \kappa^2 A^{1/3} \). This shift is also performed for \( Q_{\text{frag}}^2 \) in (8).

In our numerical calculation we chose two values of \( \kappa: \kappa = 1 \text{ GeV} \) takes into account additional momentum broadening due to a non-perturbative effects and \( \kappa = 0 \) neglects such effects. The nuclear modification factor is usually defined as

\[
R_{dAu}(k_T, y) = \frac{dN_{dAu}^{dN_{pp}}}{d^{2}k_{y}d^2y}.
\] (10)

where \( dN_{dAu}/d^{2}k_{y}d^2y \) are multiplicities of hadrons per unit of phase space in \( dAu \) and \( pp \) collisions. Both expressions for gluon (3) and quark (5) production contribute to the hadron production cross section in \( dAu \) collisions. The cross section of hadron production is given by

\[
\frac{d\sigma_{h}^{dA}}{d^{2}k_{y}dy} = \int \frac{dz}{z^2} \frac{d\sigma_{G}^{dA}}{d^{2}k_{y}dy} (k_T/z) D_{\text{frag}}^G(z, k_T) F(k_T/z, y) + \int \frac{dz}{z^2} \frac{d\sigma_{Q}^{dA}}{d^{2}k_{y}dy} (k_T/z) x q_{V}(y, k_T/z) \times D_{\text{frag}}^Q(z, k_T) F(k_T/z, y).
\] (11)

We use the LO fragmentation functions from Ref. [38]. We choose the renormalization scale of the fragmentation functions to be \( k_T \). Eq. (5) is derived for production of a valence quark in the deueteron fragmentation region. To generalize it to smaller values of Bjorken \( x \) one has to convolute it with the deueteron’s valence quark distribution, which is fixed by quark counting rules at high \( x \) and by the leading Regge trajectory at low \( x \)

\[
x q_{V}(x) = 1.09(1 - x_p)^{3} x^{0.5},
\] (12)

where \( x_p = (k_T/\sqrt{s}) e^\gamma \). Eq. (12) is normalized to give the distribution of a single valence quark in the deueteron to keep normalization the same as in (1). Valence quarks are increasingly less important at low \( x \) [39], where the quark production is dominated.
Fig. 1. Charged particle spectra in deuteron–gold collisions at $\sqrt{s} = 200$ GeV at RHIC. For the plots with $\eta = 0$, 1 the solid line corresponds to $(h^- + h^+)/2$ contribution calculated in the isospin-independent approximation for the fragmentation functions with $\kappa = 0$, while the dashed line gives the same $(h^- + h^+)/2$ contribution for $\kappa = 1$ GeV. In the plots for $\eta = 2.2, 3.2$ the solid line denotes the $h^-$ contribution calculated in the constituent quark approximation with $\kappa = 0$, the dashed line gives the same $h^-$ contribution for $\kappa = 1$ GeV, while the dotted line at $\eta = 2.2, 3.2$ gives the $(h^+ + h^-)/2$ isospin-independent contribution calculated for $\kappa = 0$. Data is from [2].

by gluons splitting in $q\bar{q}$ pairs. The factor of $x^{0.5}$ insures that this is indeed the case here [39]. Analogously, the high $x$ behavior of the nuclear gluon distribution is taken into account by introducing the function $F(k_T, y)$

$$F(k_T, y) = (1 - x_A)^4 \left( \frac{A^2}{k_T^2 + A^2} \right)^{1.3\alpha_s} ,$$

(13)

where the Bjorken $x$ of a gluon in the nuclear wave function is given by $x_A = (k_T/\sqrt{s})e^{-y}$ and $\alpha_s = 0.3$. The last factor in Eq. (13) arises when we impose momentum conservation constraint on the anomalous dimension of the distribution functions. Namely, we use the following phenomenological parametrization of the anomalous dimension in the Mellin momentum variable $\omega$ [32]

$$\gamma(\omega) = \alpha_s \left( \frac{1}{\omega} - 1 \right).$$

(14)

This parametrization takes into account high $x$ corrections to the QCD splitting functions.

The differential hadron multiplicity can be calculated by dividing (11) by the total inelastic cross section $\sigma_{dAu}$ for a given centrality selection. The baseline $pp$ multiplicity is calculated by expanding the Glauber exponent (4) to the leading term at $zT \ll 1/Q_s$. The free parameters of our model are $y_0$ in (8), which sets the initial value of $y$ at which the quantum evolution sets in, $c$ in (7), which describes the onset of quantum regime, the momentum scale $\kappa$, which specifies the typical hadronic rescatterings momentum, and $\mu$ in (2), which is the infrared cutoff. The value of $\mu = 1$ GeV and the range of values for $\kappa = 0$–1 GeV
are fixed by lower energy data. Parameters $y_0$ and $c$ are fitted to RHIC $d$Au data reported by BRAHMS Collaboration [1,2]. The parameter $\Lambda$ from (9) is fixed by the DIS data and is not a free parameter in our model.

The data reported in Ref. [2] is for charged particles at pseudo-rapidities $\eta = 0, 1$ and for negative ones at pseudo-rapidities $\eta = 2.2, 3.2$. At forward rapidities (in the deuteron fragmentation region) the valence quarks begin to dominate over gluons in the production of hadrons with high transverse momenta. In particular, in $pp$ collisions this leads to an asymmetry between positive and negative hadrons—an effect which is well-established (see [40] and references therein). Since the nuclear modification factor $R_{dA}$ has been experimentally defined as the ratio of $d$Au and $pp$ cross sections, this factor is modified by the isospin asymmetry effects. Unfortunately, it is difficult to evaluate quantitatively the magnitude of these effects—the isospin dependence of fragmentation functions is poorly known, and the relative importance of valence quarks and gluons in various kinematical regions heavily depends on the choice of the structure functions. Nevertheless, to account for the influence of this effect we performed calculations for two limiting cases: (i) assuming no isospin dependence for the valence quark fragmentation and (ii) in the opposite limit of the constituent quark model, with $u$-quarks fragmenting only into positive hadrons and $d$-quarks fragmenting only into negative ones.

The results of our calculations are presented in Figs. 1–3 along with the data collected by BRAHMS Collaboration [1,2]. In these figures we use $c = 4$ with $y_0 = 0.6$ for both $\kappa = 0$ and $\kappa = 1$ GeV. We would like to emphasize that the ratios $R_{dA}$ and $R_{CP}$ are al-
most insensitive to the values of $\kappa$ and $\mu$ at $\eta \geq 1$ and $p_T \geq 2$ GeV. Their dependence on $y_0$ is also weak at forward rapidities.

In Fig. 1 we present our calculation of the charged particle transverse momentum spectra in dAu collisions at several different rapidities compared to BRAHMS data [2]. We find a reasonable agreement with experimental data [2]. To evaluate the degree of agreement with the data one should also keep in mind that BRAHMS data at $\eta = 0$ and 1 are for the average of positive and negative hadrons, and so contain the baryons which production dynamics still remains puzzling at present (the shown data at $\eta = 2.2$ and 3.2 are for the negative hadrons only). In Fig. 2 we show the nuclear modification factor $R^{dAu}$ as a function of $p_T$ at different rapidities calculated in our model and compared to the data from [2]. At rapidities $\eta < y_0$ we observe Cronin enhancement of the nuclear modification factor at $p_T \sim 2$–3 GeV due to the multiple rescatterings of the deuteron in the gold nucleus [7,9,28,37]. At $\eta > y_0$ the low-$x$ quantum evolution effects modify the anomalous dimension $\gamma$ leading to suppression in $R^{dAu}$ at forward rapidities and disappearance of the Cronin maximum in accordance with our qualitative predictions in Ref. [7] (see also [8]). Fig. 3 demonstrates the centrality dependence of the hadronic spectra by plotting the central-to peripheral ratio $R^{CP}$ for the same rapidities as in Figs. 1 and 2. It is important to emphasize that $R^{CP}$ is much less sensitive than $R^{dAu}$ to isospin-dependent effects. At mid-rapidity the Cronin maximum increases and moves to the right as centrality increases. Conversely, at forward rapidities, $\eta \gg y_0$ the suppression gets stronger with centrality since, ap-
proximately, $R^{dAu} \sim \sqrt{1/N_{Au}^{part}}$ [6,7]. This behavior at forward rapidities is in agreement with the BRAHMS data [1,2]. Nuclear modification factor obtained by numerical solution of the BK equation [8] as well as other approaches [9,30] qualitatively agree with our conclusions. Further research in the area included an analysis of running coupling corrections [41] and a study of similar suppression in di-lepton production [42].

It will be interesting to check the predictions of our approach at the LHC energy of $\sqrt{s} = 5.5$ TeV. In Fig. 4 we show our result for the nuclear modification factor $R^{pA}$ for $pA$ collisions at LHC at mid-rapidity compared to $R^{dA}$ for RHIC $dAu$ collisions at $\eta = 3.2$. Our model predicts that the nuclear modification factor will be quite similar for both cases. This conclusion seems natural to us since the effective values of nuclear Bjorken $x$ for mid-rapidity LHC collisions will be similar to the effective $x$ achieved in the forward rapidity RHIC collisions. If observed, the mid-rapidity suppression predicted in Fig. 4 for $pA$ collisions at LHC would indicate that an even stronger suppression due to the CGC initial state dynamics should be present in $AA$ collisions at LHC. The overall high-$p_T$ suppression in mid-rapidity $R^{AA}$ at the LHC would then be due to both the initial state saturation/CGC dynamics and jet quenching in quark–gluon plasma [43–46].

In summary, we presented a simple but quantitative model which incorporates the main features of the small-$x$ evolution in the color glass condensate for particle production in ultra relativistic proton (deuteron)-heavy ion collisions. We find that at $\sqrt{s} = 200$ GeV the evolution sets in at rapidity $y_0 \simeq 0.2$. As a result, at central rapidities the nuclear modification factor $R^{dAu}$ exhibits Cronin enhancement at $k_T \simeq 2–3$ GeV, whereas at forward rapidities it is strongly suppressed at all $p_T$.

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References
