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Inferring impulsive-analytic disposition from written responses

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INFERRING IMPULSIVE-ANALYTIC DISPOSITION FROM WRITTEN RESPONSES

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Impulsive disposition refers to one's proclivity to spontaneously proceed with an action that comes to mind without checking its relevance. Analytic disposition refers to one's proclivity to analyze a problem situation and establishes a goal to guide one's actions. An instrument, called the likelihood-to-act survey, was developed to measure students' impulsive-analytic disposition. In this study, we sought to test and refine this instrument by analyzing 92 participants' written responses to open-ended questions that were adapted from items in the likelihood-to-act survey. We found relatively strong correlations between participants' disposition scores for written responses and those from the likelihood-to-act survey.

Introduction

In solving mathematics problems, "doing whatever first comes to mind … or diving into the first approach that comes to mind" (Watson & Mason, 2007, p. 307) is commonly observed among students. To demonstrate, consider the following problem: *Sue and Julie were running equally fast around a track. Sue started first. When she had run 9 laps, Julie had run 3 laps. When Julie completed 15 laps, how many laps had Sue run?* Cramer, Post and Currier (1993) observed that 32 out of 33 pre-service teachers solved the problem by setting up a proportion such as $9/3 = x/15$. Instead of analyzing and reasoning through the problem situation, these students had "blindly" applied the proportional strategy familiar to them. Lim, Morera, and Tchoshanov (2009) use the term *impulsive disposition* to refer to students' proclivity to spontaneously proceed with an action that comes to mind without checking its relevance.

A contrast to impulsive disposition is *analytic disposition*. When one approaches a problem with analytic disposition, one "analyzes the problem situation and establishes a goal … to guide one's actions" (Lim, 2008, p. 45). We have developed a survey instrument that seeks to measure one's problem-solving disposition in the impulsive-analytic dimension. This instrument, called the *Likelihood-to-Act* (LtA) survey, consists of six-point Likert items where participants are asked to indicate how likely they are to respond to a given mathematical problem in the described manner (Lim, Morera, & Tchoshanov, 2009). In order to increase the reliability of the instrument, we have increased the number of LtA items from 18 to 32. A study was recently conducted to investigate the validity and reliability of the 32-item version of the LtA survey. Three other instruments—an open-ended questionnaire, a classification test, and a multiple-choice mathematics test—were developed and administered in conjunction with the LtA survey. This paper focuses on participants' written responses to questions in the open-ended questionnaire and a question that asked participants what they thought the LtA survey was trying to measure.

Theoretical Background

Students are impulsive because they tend to approach a problem with conceptual tools that are familiar to them. In the famous water jar experiment (Luchins, cited in NRC, 2000), subjects, after solving many problems using one approach, spontaneously used the same approach to solve other problems that could have been easily solved using a different approach. This phenomenon

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of solving a given problem in a fixated manner even when a better approach exists is called the *Einstellung* effect. In the context of mathematical problem solving, the Einstellung effect is described by Ben-Zeev and Star (2001) as a *spurious correlation*, which involves a two-phase process: A person first conceives an association between a problem feature and an algorithm for solving the problem, and then uses the algorithm upon perceiving the feature in another problem. In their study, Ben-zeev and Star found that students not only relied on surface-level features in solving problems but “also generate[d] and use[d] correlations between irrelevant surface-level features and solution strategies” (p. 272). Even experienced students were found to be susceptible to this tendency. We call the tendency of making spurious correlations impulsive disposition.

Einstellung effect, spurious correlation, and impulsive disposition refer to the same phenomenon but emphasize different aspects. Whereas the Einstellung effect refers to a mental fixation and spurious correlation refers to a process, impulsive disposition refers to a proclivity. We chose to focus on impulsive disposition because it can be conceived as a negative *habit of mind* (Cuoco, Goldenberg, & Mark, 1996) or an undesirable *way of thinking* (Harel & Sowder, 2005) that can be changed to analytic disposition. Our research is motivated by the intent to create an awareness of impulsive disposition among mathematics teachers so that they can be mindful in teaching their students and avoid propagating a culture where “doing mathematics means following rules laid down by the teacher, knowing mathematics means remembering and applying the correct rule when the teacher asks a question, and mathematical truth is determined when the answer is ratified by the teacher” (Lampert, 1990, p. 31).

Spurious correlations tend to lead to errors, which Radatz (1979) would classify as “errors due to incorrect associations or rigidity of thinking” (p. 167). Impulsive disposition is generally inferred from such errors, examples of which include overgeneralizing proportionality in solving non-proportional missing-value problems (Lim, 2009) and misapplying a procedure for solving linear equations to solve linear inequalities (Tsamir & Almog, 2001). Examples of the latter include $x^2 < 16$ implies $x < \pm 4$ and $\frac{2x-2}{x+1} < 1$ implies $2x - 2 < x + 1$. We also infer impulsive disposition when students use an inefficient strategy to solve a problem that could be solved using a simpler method (i.e., when the Einstellung effect occurs).

We regard an instrument that can assess one’s impulsive-analytic disposition to be a viable way to motivate teachers and students to progress from impulsive disposition to analytic disposition. The LtA survey was developed with this purpose in mind. To investigate the validity of the LtA items and to improve them, we developed an open-ended questionnaire to find out the initial steps participants would take to solve a mathematics problem. Our objectives were two-fold: (a) to investigate the validity of the 32-item version of the LtA survey, and (b) to improve the LtA items.

In this paper we address the following research questions: (a) What solution strategies were mentioned in participants’ responses to the items in the open-ended questionnaire? (b) How well did the impulsive-analytic disposition scores for the open-ended questionnaire correlate with their impulsive-analytic scores in the LtA survey? And (c) What did the respondents think the LtA survey was trying to measure?

Method

Data Collection

Two groups of participants completed the open-ended questionnaire. The first group consisted of 27 in-service teachers and 10 pre-service teachers who were enrolled in a program for improving mathematics and science education in El Paso. The second group consisted of two mathematics classes for pre-service EC-8 (Early Childhood to Grade 8) teachers: 33 and 22 students. A convenience sample of pre-service and in-service teachers was used because the purpose of this round of data collection was to investigate the validity of the instruments rather than to test hypotheses. Since the LtA survey was the prime instrument that we sought to investigate, we administered it prior to the open-ended questionnaire.

Likelihood-to-Act Survey. The 32-item survey that was administered is comprised of the following categories: algebra, word problem, fraction, and non-mathematically-specific description. Each of the four categories includes four impulsive and four analytic items.

A1i $(x – 5)(x – 8) = 0$. When asked to solve for x, how likely are you to multiply out the terms (i.e., FOIL) and then solve $x^2 – 13x + 40 = 0$ using the quadratic formula? [impulsive]

A1a $(x – 7)(x – 4) = 0$. When asked to solve for x, how likely are you to study the equation and predict the solution? [analytic]

A2i 30 workers took 8 hours to complete Project P whereas 20 workers took 3 hours to complete Project Q. When asked to determine which project was bigger in size, how likely are you to compare rates (e.g., comparing 30/8 to 20/3)? [impulsive]

A2a 26 workers took 10 hours to complete Project ABC whereas 18 workers took 7 hours to complete Project XYZ. To determine which project was bigger in size, how likely are you to visualize the two scenarios and predict the answer without doing any computation? [analytic]

B6i $\frac{55}{95} \div \frac{11}{95}$. When asked to find the answer without using a calculator, how likely are you to use the invert-and-multiply rule, obtain $\frac{55}{95} \times \frac{95}{11}$, and then simplify the answer? [impulsive]

B6a $\frac{44}{82} \div \frac{11}{82}$. When asked to find the answer without using a calculator, how likely are you to study the two fractions and predict the answer? [analytic]

Three measures can be derived from the LtA survey: (a) the *analytic subscale* is based on the 16 analytic LtA items, (b) the *impulsive subscale* is based on the 16 impulsive items, and (c) the *analytic-impulsive difference* is computed based on the difference between the analytic score and the impulsive score for each pair of items.

Opinion-seeking Question. Appended to the end of the LtA survey is a page with the following question: “In your opinion, which aspect(s) of problem-solving disposition do you think the 32-item survey is trying to quantify (i.e., measure)?” The intent of this question was for us to get a sense of what the participants thought the survey was about.

Open-ended Questionnaire. In this questionnaire, six open-ended questions were posed to uncover participants’ initial approaches for solving selected impulsive items in the LtA survey. Two versions were created to cover the 12 mathematically-specific impulsive items (i.e., the four non-mathematically specific impulsive items were excluded). Of the 92 participants, 47 took Version A and 45 took Version B. Below are three of the 12 open-ended questions.

A1 What are the first few actions that you would take when asked to solve $(x – 5)(x – 8) = 0$ for x?

A2 30 workers took 8 hours to complete Project P whereas 20 workers took 3 hours to complete Project Q. What are the first few actions that you would take when asked to determine which project was bigger in size?

B6 What are the first few actions that you would take when asked to find the answer for $\frac{55}{95} \div \frac{11}{95}$ without using a calculator?

Data Analysis

The authors and a graduate student coded all the responses in Version A. Another team of a full-time research assistant and a final-year doctoral student coded all the responses in Version B. Members from both teams met to analyze the responses in a training set. Five responses per item were selected for training. They included typical responses as well as challenging responses. The purpose was for both teams to establish consistency in coding and to agree on a set of guidelines such as analyzing a response in its entirety instead of automatically assigning a code based on the presence of a particular strategy or keyword.

Each written response was first analyzed in terms of solution strategies. Strategies that were similar were grouped together to form a category. The response was then assigned a code according to whether the response had a strong or a weak indication of analytic disposition (A+ or A-) or impulsive disposition (I+ or I-). In situations where a response was incomplete or irrelevant and we could not decide one way or the other, we coded the response as unsure (U). The inter-rater reliabilities for the two teams were 0.89 and 0.96 respectively. The codes, I+, I-, U, A-, A+, were later quantified using a five-point scale for statistical analysis.

We also attempted to quantify participants' depth of mathematical understanding. However, because the participants were only asked to describe the initial steps that they would take instead of actually solving the problem, we found it difficult to infer depth of understanding. Hence, we used a two-point scale: "0" for a response that contained major error(s) and "1" for a response without any major error. The inter-rater reliabilities were 0.93 and 0.98 for the two teams.

Participants' comments on the LtA survey were analyzed in two rounds to find common themes. In round one, the first author noted specific descriptions within each response in order to generate categories which were then collapsed. In round two, the second author used these categories to code the data. Since the purpose was to get a sense of how participants perceived the survey, we did not proceed to establish inter-rater reliability.

Results and Discussion

Solution Strategies

Table 1 shows the distribution of strategies from most common to least common for the 12 open-ended items. Strategies were not the same across items. For example, the top four strategies for Item A1 were: (a) using the FOIL method; (b) setting the two factors, $x – 5$ and $x – 8$, equal to 0; (c) using a guess-and-check approach; and (d) analyzing the problem or following a proper procedure; whereas the top three strategies for Item A2 were: (a) dividing 30 by 8 and 20 by 3 or comparing two ratios; (b) setting up a proportion and/or using cross-multiplication; and (c) commenting that more information was needed in order to solve the problem.

The most common strategy (Strategy 1) for all the items, except B6, in the open-ended questionnaire was consistent with the act described in the corresponding impulsive LtA item. For example, the most common strategy among respondents for Item A1 was using the FOIL method and the act described in the LtA Item A1i was "multiply out the terms (i.e., FOIL) and then solve $x^2 – 13x + 40 = 0$ using the quadratic formula." The top two strategies for Item A5 (using a proportion and finding unit cost) were equally common and both of them were depicted in its corresponding impulsive LtA item. The correspondence between the most common strategy and the act described in the impulsive LtA item suggests that the act described in the impulsive LtA item was appropriate for assessing impulsive disposition.

Table 1. Percent Distribution of Solution Strategies for Each Open-ended Item

	A1	A2	A3	A4	A5	A6	B1	B2	B3	B4	B5	B6
Strategy 1	57%	60%	79%	45%	30%	68%	73%	38%	58%	78%	33%	30%
Strategy 2	23%	21%	11%	9%	30%	13%	24%	38%	20%	13%	27%	25%
Strategy 3	6%	6%	8%	6%	26%	9%	2%	9%	7%	4%	24%	18%
Strategy 4	6%	-	2%	6%	4%	4%	-	4%	4%	-	7%	7%
Others	6%	13%	-	34%	11%	6%	-	11%	11%	4%	9%	23%

Note: The percentages for each item may not add up to 100% because of rounding.

The top three strategies for Item B6 were: (a) immediately cancelling out the common number 95; (b) inverting and then simplifying; and (c) inverting and then multiplying across. In terms of impulsive-analytic disposition, the difference between the first two strategies is subtle. For several students who “cancelled” out the 95s, we found it difficult to determine whether or not they spontaneously applied the invert-multiply strategy (i.e. invert the second fraction prior to canceling the 95s). We ended up replacing items B6i and B6a by a different pair of items in the revised LtA survey.

Coding for Impulsive-Analytic Disposition

Table 2 shows the distribution of disposition codes assigned to responses for all 12 items in the open-ended questionnaire. The frequency of I+ is highest for all the 12 items. This result might be a consequence of participants’ working on the LtA survey before working on the open-ended questionnaire. Participants’ initial exposure to the LtA items might have influenced their subsequent written responses for the open-ended items.

In general, the disposition code (I+ = strong indicator of being impulsive, A+ = strong indicator of being analytic) was dependent on the solution strategy associated with the response. Take responses to Item A1, for example. Out of 31 responses that were categorized as impulsive (I+ or I-), 27 mentioned using the FOIL method to arrive at a solution. These participants said that they would use the FOIL method probably because they could do something to those factors rather than analyze the form of the equation. Some participants commented that they would combine like terms and then solve for *x*.

Table 2. Percent Distribution of Disposition Code for Each Open-ended Item

	A1	A2	A3	A4	A5	A6	B1	B2	B3	B4	B5	B6
I+	62%	75%	81%	66%	60%	94%	76%	58%	78%	87%	76%	61%
I-	4%	11%	-	15%	2%	-	-	-	-	-	-	32%
U	6%	6%	-	11%	4%	-	-	2%	2%	-	-	-
A-	6%	6%	13%	2%	6%	-	-	2%	-	-	-	-
A+	21%	2%	6%	6%	28%	6%	24%	38%	20%	13%	24%	7%

For example, the response in Figure 1 contains a minor computational error when adding -8*x* and -5*x* and a major error in assuming that the quadratic equation, with three terms, could be solved by isolating *x*. We inferred impulsive disposition when there was evidence of spurious correlations such as associating the FOIL method with the factored form and associating isolating *x* with solving an equation for *x*. We are aware of the inaccuracy of inferring students' problem-solving disposition based on written responses alone, especially given the lack of detail in the descriptions. A more reliable way to infer impulsive-analytic disposition is to conduct task-based interviews (see Lim, 2008).

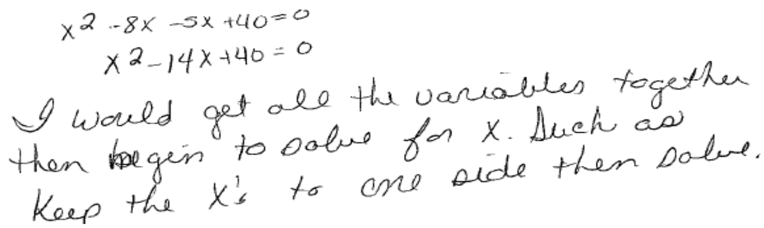

Figure 1. Using the FOIL method and combining like terms

Correlation between Coded-Disposition Score and LtA Subscales

Table 3 shows for each open-ended item, the mean value (1 = strong indicator of being impulsive, 5 = strong indicator of being analytic), its correlation with the Impulsive LtA subscale, the Analytic LtA subscale, and the difference between the two subscales. All the mean values are below three which is the midpoint on a five-point scale. This result suggests that the solution strategies were more indicative of being impulsive than being analytic. The mean value for each open-ended item was negatively correlated with the Impulsive LtA subscale and positively correlated with the Analytic LtA subscale. Seven of the 12 open-ended items were significantly correlated with the impulsive LtA subscale, but only three items were significantly correlated with the analytic LtA subscale. These results might be due to the items in the open-ended questionnaire being constructed based on the impulsive items in the LtA survey.

Table 3. Relating Coded-disposition Score and the LtA Subscales

	Mean Value (five-point scale)	Correlation with Impulsive LtA Subscale	Correlation with Analytic LtA Subscale	Correlation with Analytic-Impulsive Difference
Item A1	2.23	-0.21	0.25	0.31*
Item A2	1.51	-0.11	0.20	0.22
Item A3	1.64	-0.40**	0.45**	0.58**
Item A4	1.68	-0.41**	0.21	0.41**
Item A5	2.40	-0.16	0.17	0.22
Item A6	1.26	-0.43**	0.50**	0.63**
Item B1	1.98	-0.48**	0.44**	0.62**
Item B2	2.62	-0.39**	0.17	0.39**
Item B3	1.89	-0.47**	0.21	0.47**
Item B4	1.53	-0.48**	0.19	0.46**
Item B5	1.98	-0.13	0.14	0.18
Item B6	1.56	-0.17	0.08	0.17

*$p < .05$, **$p < .01$.

Eight of the 12 items have significant correlations with the analytic-impulsive difference score. Items that did not have strong correlations (A2, A5, B5, and B6) were considered for refinement or replacement. For example, after studying items A2i and A2a critically, we modified them to highlight the number of hours worked by *each* worker. The revised version for item A2i now reads "Project P took 30 workers, each working 8 hours, to complete. Project Q took 20 workers, each working 3 hours, to complete. When asked …?"

Item B6 was the least correlated with the analytic LtA subscale score. The supposedly-analytic act described in B6a might be considered *likely-to-act* for someone who would spontaneously invert the second fraction, cancel the 82s, and predict 4 as the answer.

Hence, in the revised version items B6i and B6a were replaced with the following pair of items:

- 3875.4 + 367.9 – 875.4. When asked to find the answer without using a calculator, how likely are you to begin by adding 3875.4 and 367.9 and then subtract 875.4?
- 1545.9 + 694.8 – 545.9. When asked to find the answer without using a calculator, how likely are you to study the decimals and obtain the answer almost instantly?

Table 4. Correlations between Coded Scores and LtA Subscales

	Analytic Subscale	Impulsive Subscale
Coded-disposition Score	0.373^{**}	-0.488^{**}
Coded-correctness Score	0.256^{*}	-0.205^{*}

$^{*}p < .05$, $^{**}p < .01$.

Each participant took either Version A or Version B of the open-ended questionnaire. We added the disposition scores for all six items in each version to produce the Coded-disposition score for each participant. We added the six correctness scores to produce the Coded-correctness score. Table 4 shows the correlations between the two coded scores and the two LtA subscales. The strong correlations between the coded-disposition score and both the LtA subscales strengthen our confidence in the validity of the LtA survey.

Opinion on Purpose of LtA Survey

Of the 92 participants who were asked to write what the survey was designed to assess, three participants did not respond. From the remaining 89 responses, 48 categories were initially generated and subsequently collapsed into 13 categories. Because more than one category could be assigned to a response, we ended up with a total of 200 counts. Table 5 shows the number of responses for each category and its rank based on frequency count.

The first three categories are related to analytic disposition and the next three categories are related to impulsive disposition. The other categories are not directly related to impulsive or analytic disposition, including the highest-count category (e.g., "find out the way you would answer a given problem situation") and the third-highest category (e.g., "measure critical thinking skills in problem solving").

Predicting was the fourth-highest category and this might be a consequence of the word "predict" being used in 11 of the 16 analytic LtA items. We intended "predict" to be interpreted as an analytic act, but only 9 of the 17 respondents who thought that the survey was about predicting viewed predicting in the manner we intended (e.g., "predict and think about a problem rather than solving it immediately). Five respondents viewed predicting in opposition to analytic disposition (e.g., “automatically begin to predict a solution before analyzing a problem”). The remaining three respondents viewed predicting in a neutral manner (e.g., “what problems they would predict rather than solve”). Knowing that “predict” might be misinterpreted as *guessing without analyzing*, we revised several items. For example, the original version of an analytic LtA item reads: “Given that 6 bottles of mineral water cost $2.10. When asked to find the cost of 30 bottles, how likely are you to notice a relationship and predict that the answer is $10.50?” The revised version reads “… how likely are you to notice that 5 times $2.10 will give you the answer?” In addition, we reduced the number of times “predict” was used from 11 in the original version to 6 in the revised version.

Table 5. Number of Responses and Rank for Each Category about the LtA Survey

	Count	Rank
Analyzing or identifying relationships	30	2
Interpreting or understanding the problem	13	6
Finding an easier way, a shortcut, or alternative ways	7	10
Following procedures	14	5
Acting quickly without thinking	8	9
Assessing fastness in solving problems	7	10
Finding out how one approaches a problem	43	1
Assessing knowledge/skill/problem-solving	26	3
Predicting	17	4
Commenting about teaching and learning	13	6
Visualizing or mentally computing	11	8
Assessing competence in specific math topics	6	12
Others	5	13

Note: Total count = 200 > 89 because the 13 categories are not mutually exclusive.

Conclusions

In this paper, we reported our analysis of students’ written responses to open-ended questions with the intent to investigate the validity of the items in the likelihood-to-act survey, which was designed to measure students’ problem-solving disposition along the impulsive-analytic dimension. We found that in each question, except Item B6, the most-mentioned strategy was consistent with the act described in its corresponding impulsive LtA item. We also found significant correlations, for 8 of the 12 open-ended questions, between the assigned disposition code for written responses to these questions and the analytic-impulsive difference LtA score. In addition, the Coded-disposition score for the open-ended questionnaire was positively correlated to the analytic subscale and negatively correlated to the impulsive LtA subscale, both with $p < .01$. Less promising results were taken as opportunities to improve the LtA survey items. We are currently administering and testing the revised LtA survey to more than 400 pre-service teachers.

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