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Stochastic Resource-Constrained Project Scheduling and Its Military Applications

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Contribution

This research aims at developing computationally tractable algorithms to obtain near-optimal closed-loop policy

for the stochastic resource-constrained project scheduling problem (SRCPSP). It has a wide range of military applications such as mission planning, path planning, and logistics network configuration.

Introduction

Project management has a long and colorful history in the military setting, dating back to the well-known program evaluation and review technique (PERT [Malcolm et al. 1959]) in the late 1950s to aid the U.S. Navy's nuclear submarine development program. PERT extends the deterministic critical path method (CPM [Kelly 1961]) by explicitly considering the uncertainty of task durations. Although they are widely used today,

CPM and PERT's modeling and solving capability have been surpassed by the growing dynamics and complexity of modern warfare.

The following are a few examples. Planning an onboard mission requires accomplishing mission-specific tasks on time not only in the right order, but also with a fixed and multiskilled crew. Further challenges arise when task durations or outcomes are uncertain. Routing an unmanned aerial vehicle (UAV) often involves finding a shortest path from base to destination subject to various operational, physical, and logistical constraints. Several factors in this setting can be uncertain, e.g., the traversal time over an edge due to weather condition, and the estimated survivability. A military logistics network is often dynamic and adaptive in nature with random demand, lead time, and potential supply disruption. Designing and

configuring military logistics networks calls for a systematic supply chain approach to optimize the system-wide performance.

The above examples have the following in common. First, their underlying structure can be described by a general network with complex interactions and dependencies among nodes. Second, they are all combinatorial or nonlinear in nature, which is in general more difficult to handle than a linear program (LP). Last but not the least, several varieties of uncertainty and randomness must be modeled to obtain quality, reliable, and robust solutions. They all relate to a general category of optimization problems known as the stochastic resource-constrained project scheduling problem (SRCPSP [Herroelen and Leus 2005, Ballestin 2007]) in the operations research (OR) literature.

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Stochastic Resource-Constrained Project Scheduling

As shown in Figure 1, since the introduction of CPM and PERT in the early 1960s, the research field of project scheduling has evolved in two directions: the one studying more general and complex network structures and randomness called the *graphical evaluation and review technique* (GERT [Moore and Clayton 1976, Elmaghraby 1977]), and the one focusing on modeling explicit resource constraints known as the *resource-constrained project scheduling problem* (RCPSP [Demeulemeester and Herroelen 2002, Brucker et al. 1999]). That is, uncertainty and resource constraints have been studied separately under the topics of PERT/GERT and RCPSP for almost 40 years. The stochastic RCPSP (SRCPSP) has emerged recently to simultaneously address both resource constraints and uncertainty. We refer to Herroelen and Leus (2005) and Demeulemeester and Herroelen (2002) for an introduction to this topic.

An SRCPSP may involve either *non-structural randomness*, such as stochastic activity duration and resource capacity, or GERT-type *structural randomness*, such as uncertain activity outcome, which may potentially alter the underlying network structure.

An example of SRCPSP with only structural randomness, e.g., stochastic activity durations, is provided in Figure 2. The example project consists of six real tasks and two dummy tasks representing the start and end of the project. The tasks and precedence constraints among them can be depicted by an activity-on-node (AON) network. For instance, Task-6 cannot start before Task-1 is finished. Two types of resources R1 and R2 are required for the project to execute. R1 has a capacity of three units available, and R2 has five units available. The bracket $[a, b]$ above each task specifies a units of R1 and b units of R2 are simultaneously required for the corresponding task to execute. For instance, Task-1 requires two units of R1 and three units of R2. Dummy tasks do not require any resource. Unlike a deterministic scheduling problem, here the exact task duration is not known a priori, but is assumed to be stochastic following a certain known probability distribution. In our example, each task

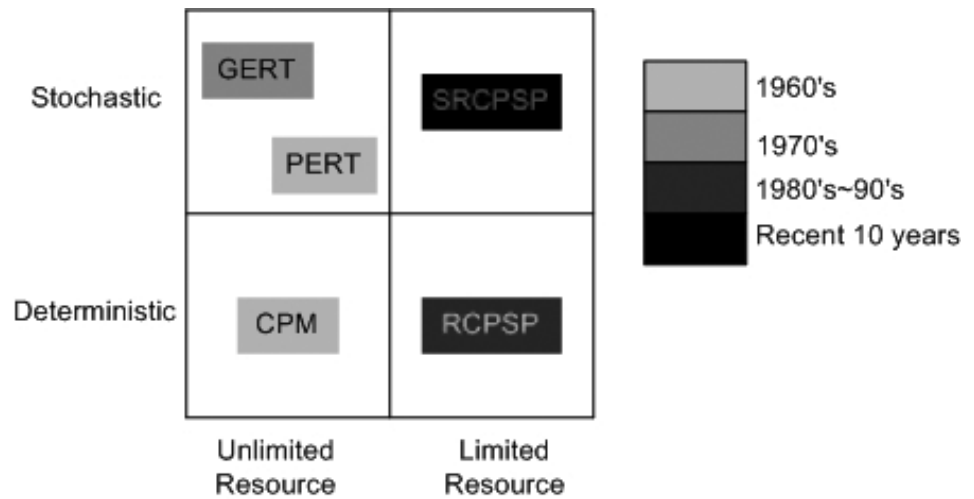


Figure 1. Evolution of the research field of project scheduling.

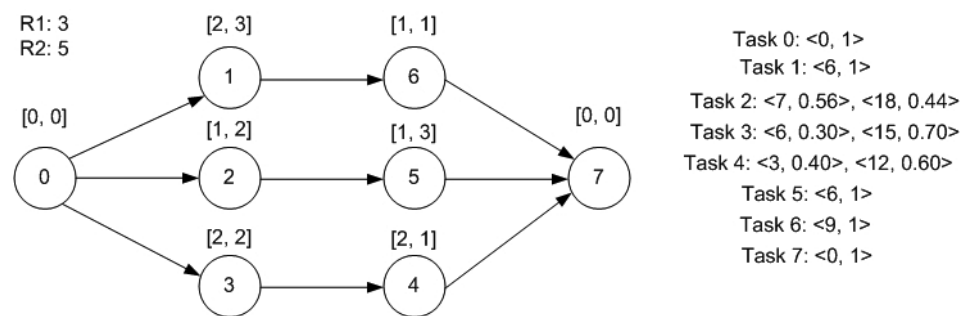


Figure 2. An example of SRCPSP with stochastic activity durations.

follows a discrete probability distribution with at most two outcomes. For instance, the duration of Task-1 is 6 with probability 100%; the duration of Task-2 may be 7 (with 56% probability) or 18 (with 44% probability). The goal is to find a time- and resource-feasible schedule that minimizes the expected makespan (project completion time).

Figure 3 illustrates a simple production process with structural randomness. Part A goes through Process-1, which has a reliability of 50%, i.e., 50% probability of success and 50% probability of failure. A state of success allows the production to continue to Process-2 and subsequently Process-4; a state of failure, however, requires going through Process-11 (a repair or restore operation), which has two possible outcomes: satisfying and unsatisfying. A satisfying outcome (80% chance) allows repeating Process-1, whereas an unsatisfying outcome (20% chance) activates Process-3 (a backup plan, e.g., alternative plant/production line, outsourcing, or alternative supplier/vendor). The nodes and paths, which are uncertain until the realization of the process, have

been indicated by dashed rectangles and lines. In a more general setting, the probabilities may be dependent. For instance, the probability of success of an activity may depend on the outcome of its predecessors: highly successful or satisfying.

Military Applications

Military applications of SRCPSP can take many forms. Below we expand on the three problem settings mentioned earlier.

Mission Planning

A military mission can be modeled as a project consisting of a hierarchy of tasks, as those in the work-break-down (WBS) structure in project management. Various resources such as manpower, material, and equipment may be crucial for accomplishing a mission.

Consider a mission planning problem in the context of a naval vessel as in Li and Womer (2009a). Depending on the mission, the list of tasks to be accomplished on board varies from period to period and, in emergency situations, survival may depend on scheduling tasks in

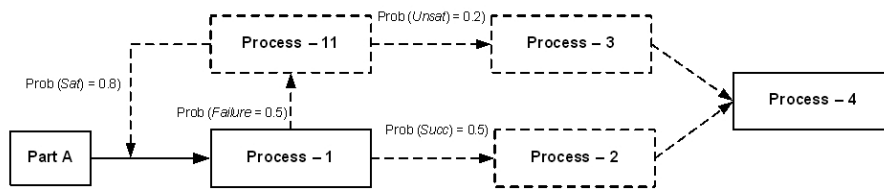


Figure 3. Structural randomness due to uncertain outcome of an activity.

the correct order. Furthermore, the jobs must be accomplished with limited resources, i.e., a fixed and multiskilled crew. The deterministic version of the problem has been modeled as a project scheduling problem with multiple-purpose resources (cf. Li and Womer 2006, 2009a, 2009b), which is a variant of RCPSP. Both non-structural and structural randomness may arise in this setting, which results in an SRCPSP. For instance, task duration may be stochastic; a task may succeed or fail.

Path Planning

The simplest path planning problem is perhaps the well-known shortest path problem (SSP [Ahuja et al. 1993]), which attempts to find a minimum-cost path from the source to destination. Mathematically, SSP is equivalent to the problem of finding a longest path (by the critical-path method or CPM) in project scheduling.

Military path planning involves guiding a UAV to fly to a specified destination through a shortest path while maintaining certain survivability (Washburn and Kress 2009). The inclusion of survivability and other physical and operational constraints makes the problem an instance of the resource-constrained shortest path problem (RCSP [Beasley and Christofides 1989]). Additional complexity arises when routing with uncertain information about positioning of threat regions (Jun and D'Andrea 2003).

The relevance of path planning to project scheduling is established through the insight that both can be modeled as a *sequential decision problem*, in which decisions are made sequentially in stages, often through the methodology of dynamic programming (DP [Bertsekas 2007]).

Configuring Logistics Networks

In 2005, the Government Accountability Office (GAO) identified the Department of Defense (DoD) "Supply Chain Management" as one of 25 activities across the entire federal government that is in need of "broad-based transformation" (GAO 2005). The U.S. Army's logistics transforming initiative aims at employing a supply chain approach to minimize the system-wide spare parts inventory while achieving certain readiness (Parlier 2005). A similar problem is faced by the Navy to manage thousands of depot level repairable (DLR) line items in its wholesale inventory (Reich 1999, Burton 2005).

Military logistics network configuration problems involve not only the traditional network design issues on site location (Daskin 1995), but also issues concerning the *inventory location and level* throughout the supply chain, which have been studied by the topic of inventory positioning or safety stock placement in the research literature (cf. Simpson 1958, Gallego and Zipkin 1999, Magnanti et al. 2006). An inventory positioning problem attempts to find optimal inbound service time (the time at which a process in the chain may start) and outbound service time (the delivery time promised to a successor of the process), so as to minimize the system-wide safety stock cost while achieving certain service level. The scheduling nature of the problem and its relevance to project scheduling has been discussed and exploited in Li and Womer (2008, 2010) and Li and Jiang (2009).

Solution Methodologies

What constitutes a solution to SRCPSP? A deterministic schedule of task starting times will not provide an implementable solution in the stochastic setting because the schedule can easily become time- or resource-infeasible in an uncertain environment. An *implementable*

solution to SRCPSP requires a *policy* determining which task(s) to start at each decision point.

There are two distinct approaches in the literature for obtaining policy-type solutions to SRCPSP. The first approach attempts to find a sequence for all tasks at time 0, without waiting to see subsequent realization of task durations. The predetermined task sequence is *static* in nature and not updated during real-time executions. Using the terminology in optimal control theory, it corresponds to an open-loop policy. Various approaches on finding optimal or near-optimal *open-loop policy* include those of Radermacher (1985), Stork (2001), Golenko-Ginzburg and Gonik (1997), Tsai and Gemmill (1998), and Ballestin and Leus (2009).

The second approach consists of finding a *dynamic* or *closed-loop* policy, in which scheduling decisions are made in a sequential fashion through the methodology of dynamic programming (DP [Bertsekas 2007]). Instead of being interested in finding an optimal task sequence at one time, a closed-loop policy seeks to find optimal rule for selecting the task(s) to start at each decision-point, given the current state of the system. The closed-loop policy is adaptive in nature and more flexible than the open-loop policy. In general, an optimal closed-loop policy dominates an open-loop policy as the latter is a special case of the former when the sequence of tasks is fixed (Dreyfus 1977). Although theoretically attractive, obtaining closed-loop policy has been so far regarded being computationally intractable for SRCPSP.

Current Research

Our current research models the SRCPSP as a Markov decision process (MDP [Puterman 2005]). MDP is general enough to model both nonstructural and structural randomness in SRCPSP. Because of the large number of states and decisions in the resulting MDP model, the classical DP algorithm based on the exact Bellman's recursion (Bellman 1957) suffers the well-known "curses of dimensionality" and is computationally intractable.

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Our main research effort will focus on developing computationally tractable heuristics to obtain a near-optimal closed-loop policy for SRCPSP. This is achieved by developing approximate dynamic programming (ADP [Si et al. 2004, Powell 2007]) algorithms, which replace the optimal cost-to-go function in DP by some form of approximation. ADP has its roots in reinforcement learning (RL [Sutton and Barto 1998]) and neuro-dynamic programming (NDP [Bertsekas and Tsitsiklis 1996]).

The success of ADP relies on effectively and efficiently solving a deterministic subproblem in each iteration. When the problem involves only continuous decision variables and linear relationships, the resulting linear program can be routinely solved by the simplex method or network optimization algorithms where applicable. The SRCPSP, however, is combinatorial in nature and requires discrete sequencing decisions to be made. In fact, even the deterministic RCPSP is NP-complete (Bartusch et al. 1988), so the classical integer linear programming (ILP [Nemhauser and Wolsey 1988]) based methods are not able to handle it efficiently (Hooker 2002). Therefore, we are motivated to build constraint programming (CP [Mariotti and Stuckey 1998]) into the ADP framework. CP originated in the artificial intelligence (AI) area and has proven to be an effective approach for dealing with complex scheduling problems (cf. Baptiste et al. 2001, Brucker 2002, and Dorndorf et al. 2000).

The preliminary results along this direction are encouraging. We have developed an ADP that embeds CP into the rollout framework (Bertsekas et al. 1997, Bertsekas and Castanon 1999) for the deterministic RCPSP. Computational results on the standard Project Scheduling Program Library (PSLIB) benchmark instances show that our well-configured hybrid CP-ADP algorithm is competitive with the state-of-the-art algorithms for deterministic RCPSP. Our ongoing research effort focuses on developing ADP with various approximation architectures for the more challenging SRCPSP.

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References

- Ahuja, R.K., T. L. Magnanti and J. B. Orlin, *Network flows*, Prentice Hall, 1993.
- Ballestin, F. When it is worthwhile to work with the stochastic rcpsp, *Journal of Scheduling* 10 (2007), 153-166.
- Ballestin, F., and R. Leus. Resource-constrained project scheduling for timely project completion with stochastic activity durations, *Production and Operations Management* 18 (2009), no. 4, 459-474.
- Baptiste, P., C. Le Pape and W. Nuijten. *Constraint-based scheduling: Applying constraint programming to scheduling problems*. Springer, 2001.
- Bartusch, M., R. H. Mohring and F. J. Randermacher. Scheduling project networks with resource constraints and time windows, *Annals of Operations Research* 16 (1988), 201-240.
- Beasley, J.E., and N. Christofides. An algorithm for the resource constrained shortest path problem, *Networks* 19 (1989), 379-394.
- Bellman, R. *Dynamic programming*, Princeton University Press, 1957.
- Bertsekas, D. *Dynamic programming and optimal control*, Athena Scientific, 2007.
- Bertsekas, D., and D. A. Castanon. *Rollout algorithms for stochastic scheduling problems*, *Journal of Heuristics* 5 (1999), no. 1, 89-108.
- Bertsekas, D., and J. N. Tsitsiklis. *Neurodynamic programming*, Athena Scientific, 1996.
- Bertsekas, D., J. N. Tsitsiklis and C. Wu. Rollout algorithms for combinatorial optimization, *Journal of Heuristics* 3 (1997), no. 3, 245-262.
- Brucker, P., A. Drexl, R.H. Mohring, K. Neumann, and E. Pesch. Resource-constrained project scheduling: Notation, classification, models and methods, *European Journal of Operational Research* 112 (1999), no. 1, 3-41.

- Brucker, P. Scheduling and constraint propagation. *Discrete Applied Mathematics* 123 (2002), no. 1-3, 227-256.
- Burton, L. Strategic inventory positioning of navy depot level repairables, *Naval Postgraduate School, vol. Master Thesis*, Monterey, CA, 2005.
- Daskin, M.S. *Network and discrete location: Models, algorithms, and applications*, John Wiley & Sons, Inc. 1995.
- Demeulemeester, E.L., and W.S. Herroelen. *Project scheduling: A research handbook*, Kluwer Academic Publishers, 2002.
- Dorndorf, U., E. Pesch and T. Phan-Huy. A time-oriented branch-and-bound algorithm for resource-constrained project scheduling with generalized precedence constraints, *Management Science* 46 (2000), no. 10, 1365-1384.
- Dreyfus, and A. M. Law. *The art and theory of dynamic programming*, vol. 130, Academic Press Inc., 1977.
- Elmaghraby, S.E. *Activity networks: Project planning and control by network models*, John Wiley & Sons Inc., 1977.
- Gallego, G., and P. Zipkin. Stock positioning and performance estimation in serial production-transportation systems, *Manufacturing & Service Operations Management* 1 (1999), no. 1, 77-88.
- GAO. High risk series: An update, *U.S. Government Accountability Office (GAO), Government Accountability Office*, Washington, D.C., 2005.
- Golenko-Ginzburg, D., and A. Gonik. Stochastic network project scheduling with non-consumable limited resource, *International Journal of Production Economics* 48 (1997), no. 1, 29-37.
- Herroelen, W., and R. Leus. Project scheduling under uncertainty: Survey and research potentials, *European Journal of Operational Research* 165 (2005), 289-306.
- Hooker, J. Logic, optimization and constraint programming, *INFORMS Journal on Computing* 14 (2002), no. 4, 285-321.

Jun, M., and R. D'Andrea. Path planning for unmanned aerial vehicles in uncertain and adversarial environments, *Cooperative control: Models, applications and algorithms*, R. Butenko, R. Murphey, and P.M. Pardalos (editors), Kluwer Academic, 2003.

Kelly, J. Critical-path planning and scheduling: Mathematical basis, *Operations Research* 9 (1961), no. 3, 296-320.

Li, H., and D. Jiang. New model and heuristics for safety stock placement in general acyclic supply chain networks, *Working Paper, University of Missouri - St. Louis* (2009).

Li, H., and K. Womer. Project scheduling with multi-purpose resources: A combined milp/cp decomposition approach, *International Journal of Operations and Quantitative Management* 12 (2006), no. 4, 305-325.

Li, H., and K. Womer. Modeling the supply chain configuration problem under resource constraints, *International Journal of Project Management* 26 (2008), no. 6, 646-654.

Li, H., and K. Womer. A decomposition approach for shipboard manpower scheduling, *Military Operations Research* 14 (2009a), no. 3, 1-24.

Li, H., and K. Womer. Scheduling projects with multi-skilled personnel by a hybrid milp/cp benders decomposition algorithm, *Journal of Scheduling* 12 (2009b), no. 3, 281-298.

Li, H., and K. Womer. Optimizing the supply chain configuration for make-to-order (MTO) manufacturing, *Working Paper, University of Missouri - St. Louis* (2010).

Magnanti, T.L., Z.-J. Shen, J. Shu, D. Simchi-Levi and C.-P. Teo. Inventory placement in acyclic supply chain networks, *Operations Research Letters* 34 (2006), 228-238.

Malcolm, D.G., J.H. Roseboom, C.E. Clark, and W. Fazar. Applications of a technique for research and development program evaluation, *Operations Research* 7 (1959), no. 5, 646-669.

Marriott, K., and P. J. Stuckey. *Programming with constraints*, The MIT Press, 1998.

Moore, L.J., and E.R. Clayton. *Gert modeling and simulation: Fundamentals and applications*, Petrocelli/Charter, 1976.

Nemhauser, G., and L. Wolsey. *Integer and combinatorial optimization*, John Wiley, 1988.

Parlier, G.H. Transforming U.S. Army logistics: A strategic "supply chain" approach for inventory management, *The Land Warfare Papers, The Institute of Land Warfare*, 2005.

Powell, W.B. *Approximate dynamic programming: Solving the curse of dimensionality*, John Wiley & Sons Inc., 2007.

Puterman, M.L. *Markov decision processes: Discrete stochastic dynamic programming*, John Wiley & Sons Inc., 2005.

Radermacher, F.J. *Scheduling of project networks*, *Annals of Operations Research* 4 (1985), 227-252.

Reich, W.F. Optimizing navy wholesale inventory positioning, *Naval Postgraduate School*, vol. Master Thesis, Monterey, CA, 1999.

Si, J., A. G. Barto, W. B. Powell and D. Wunsch. *Handbook of learning and approximate dynamic programming*, Wiley, 2004.

Simpson, K.F. In-process inventories, *Operations Research* 6 (1958), no. 6, 863-873.

Stork, F. Stochastic resource-constrained project scheduling, vol. Ph.D., Technische Universitat, Berlin, 2001.

Sutton, R.S., and A. G. Barto. *Reinforcement learning: An introduction*, The MIT Press, 1998.

Tsai, Y.-W., and D. D. Gemmill. Using tabu search to schedule activities of stochastic resource-constrained projects, *European Journal of Operational Research* 111 (1998), 129-141.

Washburn, A., and M. Kress. *Combat modeling*, Springer, 2009.

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