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# Environmental Policy Choice with Learning

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**Abstract**

Uncertainty and learning are key components to many environmental externalities. Often the true costs of pollution, be they from greenhouse gases or deforestation are unknown at the time the pollution is created, and policy makers need to decide on mitigation before they know the full extent of the damage. In this paper, we ask two related questions: (1) What is the effect of the potential for learning on the timing and amount of investment and (2) In which environmental policy situations will the potential for learning lead to an increase in initial mitigation? By explicitly modeling the structure of information, and treating learning as a continuous variable, we derive a simple condition that dictates when the prospect of learning will increase initial mitigation, namely, when the curvature elasticity of the marginal cost of mitigation is at least twice as large as the curvature elasticity of marginal benefit. The lower the amount of anticipated learning, the higher the ratio of curvature elasticity of marginal cost to benefit required for this 'precautionary' result. Facing a positive discount rate increases the required ratio of curvature elasticities, while the introduction of a small stock externality makes it more likely that learning will increase the initial optimal level of mitigation.

# 1 Introduction

Uncertainty and learning are integral to many environmental policy decisions. Most environmental externalities take time before their effects are felt, and their full impact is unknown at the time of their production. Often we learn more about the magnitude of the future environmental damage after the creation of the pollution itself, whether it be greenhouse gases, heavy metals, loss in genetic diversity, or chemical spills with long plumes. During this time, policy-makers can mitigate against the future damage, for example by reducing emissions or building dykes to ward off rising ocean levels. This paper asks what is the effect of the potential for learning on the timing and magnitude of investment in mitigation. Specifically, the paper addresses the question: under what circumstances does the potential for learning increase, or decrease, the optimal level of initial mitigation?

In the case of the effect of greenhouse gases, it takes years before we learn the true level of damage associated with the pollutants. For example, predictions of temperature increases due to climate change range all the way from 2 to 11 °C (Nature 2005). Those arguing for more immediate mitigation note that some aspects of climate change may be highly damaging if they are allowed to continue unchecked; for example, if the polar ice cap melts, global warming will accelerate as sunlight is no longer reflected by the white ice mass (Stern Report 2006). On the other hand, critics of tighter emissions regulation and expenditure on mitigation often argue that the investment should wait until better information is obtained about the magnitude of damage and the benefit of mitigation (Office of the President 2001). The tension between whether the potential for learning should induce more or less initial mitigation has been brought forward in other policy debates, such as the regulation of biotechnology, pesticides, and the preservation of biodiversity. Key to the question is what to do in the presence of scientific uncertainty, and how does a government best respond now to anticipated future information.

Much prior literature looks at the effect of learning in a world with uncertainty, irreversible investments, and/or irreversible outcomes. Instead, we approach the problem without imposing strict irreversibility, and we use curvature of cost and of the benefit of investment. In this respect, we follow Epstein (1980), but to get an explicit rule about the effect of learning on the optimal initial mitigation, we add structure to the problem. In particular, we identify the importance of the curvature of the marginal cost of mitigation separately from the curvature of its marginal benefit. Explicitly modelling learning allows us to explore the effect of a stock externality and of a discount rate on the effect of learning.

In our model, having convex mitigation costs (a positive second derivative) gives the social planner the incentive to smooth mitigation over time, and having convex benefits of mitigation means that she wants to avoid high damage. On the other hand, having convex *marginal* costs or benefits (a positive third derivative) means that the potential for learning may induce the social planner to change the timing of her investment. We find that unless the marginal cost of mitigation is increasing at a higher rate than marginal damage, the potential for learning tends to decrease the initial amount of mitigation. The intuition behind this result is that since the social planner has the incentive to try to balance her expected expenditure over the two periods, and since she can react more precisely with better information, the expected future expenditure will be lower with better potential for learning. Therefore, her optimal level of initial expenditure will also be lower with better learning. This result is exacerbated by a higher discount rate, while it is reduced with the introduction of a small stock externality. Further, overall expected social welfare is higher with anticipated future learning. We achieve this preference for flexibility without imposing irreversibility of investments, irreversibility of environmental damage, or risk aversion. Yet, these components can be added when applicable.

Because pollution problems differ in terms of how much we will learn about the damages before they occur, and in terms of the curvature of marginal costs and benefits of mitigation, these results provide some conceptual guidance for policy makers. Given two environmental mitigation proposals, each with steeply increasing slopes of marginal cost, the one with the greater potential for learning optimally receives a larger amount of initial mitigation. Further, if a third case has a marginal cost of environmental mitigation that increases at a relatively constant rate, the potential for learning decreases the optimal amount of initial mitigation.

We see four contributions of this paper. (1) By using a simple Bayesian information structure, we develop a sufficiency condition for when potential learning increases optimal initial mitigation based on the curvature of marginal cost and benefit of mitigation. (2) This simple model allows us to walk through our results graphically and give intuition behind the conditions. (3) Like Epstein and Kolstad (1996a,b), we take the meaningful view that learning is marginal. However, by treating the learning as a continuous variable, we can consider the effect not only of learning, but of the degree of learning, on mitigation. (4) We explore the impact of the discount rate and the stock externality on how learning affects the optimal level of initial mitigation.

Our approach applies generally to many environmental decisions. The three primary constraints imposed by the model are (a) the effect of mitigation is additive, (b) government knows the range of possible damage,[1] and (c) society does not see the true level of damage (or benefit of mitigation) until after the decisions on mitigation have been made. This scenario can cover many stock externalities (e.g. bioaccumulants, airborne emissions, biodiversity), but it could also cover the situation of a one-time activity like a chemical spill where the severity is unknown for years. For example, consider the mercury mines in Marin county, California, where the pollution created by the tailings are largely being felt 100 years later. Also, in Santa Tecla, El Salvador, deforestation allowed for a subsequent earthquake to result in a devastating landslide (BBC 2001). Thus, although we use climate change as an example throughout the paper, the model applies quite broadly. The paper proceeds as follows: In the next section, we provide background, and in section 3, we develop the model. In section 4, we present our results on the effect of future information on the timing and amount of current mitigation. We then walk through the model and the results to develop graphical intuition. In section 6, we discuss the effect of the stock externality and discount rate, and we end with conclusions.

## 2 Background

Imagine a country facing damage from global warming. The government or "social planner" optimally pursues two policy measures: one to reduce the amount of GHGs and one to address future droughts caused by climate change. To reduce the amount

[1] This paper models risk, not Knightian uncertainty (Knight 1921). Expanding this simple model of learning to incorporate Knightian uncertainty, although interesting, is beyond the scope of this paper.

of GHGs, the country considers switching the production of energy from coal to a carbon-neutral source of energy such as wind, solar, or geothermal. The cost of this mitigation is $C > 0$. Converting the first 10 percent of energy production in any one year has costs ($C' > 0$) that are relatively low. Converting the next 10 percent is more expensive per kwh ($C'' > 0$), but still possible with current technology. The last 10 percent of energy is very expensive to convert, and it may require technology that is currently unavailable, such as broad-spectrum photovoltaic cells. Thus, for any single time-period, marginal cost is increasing at an increasing rate ($C''' > 0$).

On the other hand, avoiding environmental damage is the benefit of mitigation, ($B > 0$). Marginal benefits are positive ($B' > 0$) and falling ($B'' < 0$). Assume that the social planner is uncertain about the severity of climate change, and that reducing emissions from energy production is more valuable if climate change is severe than if it is mild, but that the incremental decrease in marginal benefit is reasonably steady ($B''' > 0$ but small). In other words, the decrease in marginal benefit from reducing GHG flow from 201 to 200 tons in any one year is about the same as the decrease in marginal benefit from reducing the flow from 1001 to 1000 tons. This set-up means that the curvature elasticity of cost is much larger than that of benefits. Assume further that the social planner expects to learn something more about the severity of climate change in the future. Detailed below, we find that the protential for learning increases the optional amount to spend to cut GHG emissions today.

Our second primary policy example is one where the social planner wants to mitigate against future droughts in an area where climate change is expected to make water supply more variable, for example, in the Pacific Northwest of the

United States, due to reduced snow-pack. The social planner considers building above-ground water storage to make water available during dry spells. The cost of building a dam increases with the amount of water storage capacity. Assume for the moment that the marginal cost of building water storage within a single period increases ($C'' > 0$), but at a relatively constant rate. In other words, the increase in marginal cost of storing an extra acre-foot by adding to a 200,000 acre-feet dam is not much more than the increase in marginal cost of adding to a 10,000 acre-feet dam. In this case, $C'''$ is not very large.

On the other hand, the first thousand acre-feet of stored water would have a huge marginal benefit during a drought. The highest value uses of water, such as human consumption, could benefit from this storage. As soon as those highest-value uses are satisfied, the marginal benefit of an extra acre-foot of stored water decreases substantially ($B'' < 0$). As more water is available from storage, the marginal benefit of an extra acre-foot of water decreases, as it goes to satisfy other needs, such as irrigation of low-value crops. Thus, we can think of the marginal benefit curve as convex ($B''' > 0$). The social planner is debating how much storage to build now. If the social planner expects to learn something about the severity of future droughts, we find below that expected learning can reduce the amount of storage to build today.

If the investment in either the green energy or the water storage is irreversible, and learning is anticipated, the standard approach is to use option value to calculate the benefits from retaining flexibility by reducing the initial investment (Dixit and Pindyk 1994). On the other hand, Epstein (1980) develops a general formulation of investment under uncertainty and learning. He shows that the effect of learning on investment depends on the curvature of the value function of investment with respect to information. Specifically, he shows that if the marginal value function of the investment is convex in the vector of probabilities, then the initial level of investment is no larger than investment without learning. Similarly, if the marginal value function of the investment is concave in probabilities, the optimal initial investment is no smaller than with learning. One of our contributions is to decompose Epstein's marginal value function, to look separately at curvature of costs and benefits, which aids in interpetation in our policy examples.

Like investments in mitigation, the environmental outcomes such as the melting of the polar ice cap can also be irreversible. Arrow and Fisher (1974) and Henry (1974) develop the notion of a quasi-option value, where society is willing to pay a premium for environmental policy that enables flexibility in the future. More recently, Ulph and Ulph (1997), Kolstad (1996a, b) and Fisher and Narain (2003) explore the effect of learning on mitigation with environmental irreversibility, modeled as a constraint that emissions cannot become negative. In their model of climate change, Ulph and Ulph show that the Epstein result cannot be applied to determine the effect of learning on initial levels of mitigation. They derive a sufficient condition for learning to increase the demand for flexibility in the first period, which states that the non-negative restriction on emissions is binding in the no-learning case. In other words, for learning to induce the social planner to cut emissions today, the unconstrained optimal emissions must be negative tomorrow. Next, Ulph and Ulph simulate climate change and show that for most parameter values, the potential for learning actually decreases mitigation efforts today. Kolstad (1996a) looks at the effect of learning in the case with both a sunk investment and a stock externality of pollution, where the stock externality acts as the sunk costs of not abating now. Thus, the stock effect is a form of environmental irreversibility. Whether the environmental or investment irreversibility dominates depends on the relative magnitudes of the decay and depreciation rates and on expectations about damages.

Simulating greenhouse gas emissions, Kolstad (1996b) finds that the optimal level of investment is affected by the capital stock irreversibility, while emissions irreversibility has no impact. Too little investment in emission control in the early periods can be compensated by a bit more investment in later periods, but it is never optimal to emit negatively in the future to correct for over-emission today. Using a similar framework, Fisher and Narain (2003) consider how the level of first-period investment varies with the irreversibility of the investment, and with the degradability of the stock of greenhouse gases. Intuitively, a larger sunk cost of investment means a lower investment in the first period. The higher the irreversibility of greenhouse gases, the higher the investment in the first period. Quantitatively, like Kolstad (1996b), they find that the effect of capital irreversibility is much stronger than either the effect of emissions irreversibility or of endogenous risk.[2]

In this previous literature, the effect of learning depends on whether the investment irreversibility dominates the environmental irreversibility, or vice-versa. For a different approach, we return to Epstein's earlier model to compare the nature of

[2] In a related literature on the precautionary principle, Gollier et al (2000) and Gollier and Treich (2003) develop a two-period model of decision-making under uncertainty with learning. They assume a degree of irreversibility and a risk-averse decision-maker, and find that an increase in future information decreases consumption or increases investment in mitigation in the first period. With a stock externality, they find two contrasting effects: by clarifying the risk in the second period, an increase in information widens the difference in perceived outcomes, producing an incentive to remain flexible in the first period. However, because the agent will be able to react more efficiently in the second period, her perceived future income has increased, allowing her initially to consume more (or mitigate less).

the cost and benefit functions, and we develop results based on the nature of the environmental problem and the mitigation technique. We allow for curvature in the marginal cost and marginal benefit of mitigation. Thus, we assume that it may be increasingly expensive to mitigate all at once, and/or that the marginal benefit from abatement is decreasing at a decreasing rate. These types of curvature yield an effect of learning on the optimal level of initial mitigation. We then explore how the effect of learning on the initial level of mitigation changes with a discount rate or with a stock externality from pollution.

## 3 Model

We use a three-period model to consider the effect of the degree of learning on the optimal level and timing of mitigation, as shown in the timeline presented in Figure 1. Assume that in period 1 a social planner can invest in damage mitigation, $m_1$. In period 2, the social planner learns something about the probability of high versus low damage, and then can invest again in mitigation, $m_2$. In period 3, the economy experiences the damage.

Environmental damage is a function of a stochastic variable $K$, and the amount spent on mitigation ($m_1$ and $m_2$). Damage is either "high" (when $K = H$), or "low" ($K = L$), assuming $H > L > 0$. One can think of $K$ as the future stock of GHGs without mitigation, where that level is uncertain (due, for example, to the potential for methane to be released from melting permafrost). Damage is reduced by mitigation, $m$, which can represent the reduction in greenhouse gas emissions, or, in case of drought, investment in water storage. Further, assume

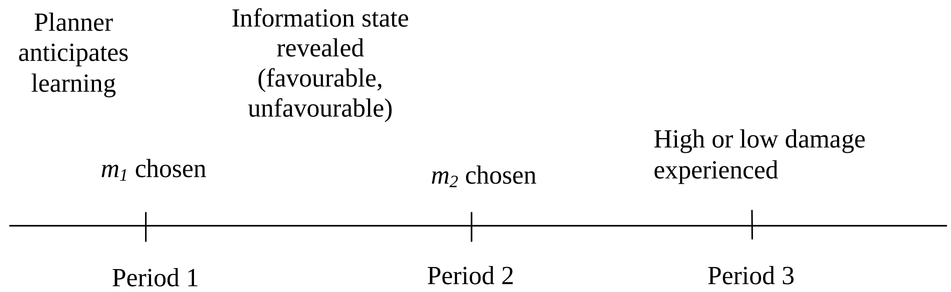


Figure 1: **Time Line**

that mitigation tomorrow may not be as useful as mitigation today. For a specific example, converting to green electricity generation next period ($m_2$) rather than in this period ($m_1$) means that the stock of GHGs has increased by the additional emissions in the first period. This stock effect is represented by $\delta$, where $0 < \delta \leq 1$, so mitigation in the second period is assumed to be only $\delta$-times as effective as mitigation in the first period.

***Cost and Benefit Functions:*** In our model, the mitigation cost and environmental damage functions are endowed with curvature, which gives the social planner an incentive to mitigate similar amounts across the two periods and to limit the spread between the damages across the two possible outcomes. The curvature in cost reflects the idea that altering the economy is assumed to be expensive, and more alteration in a single period increases the marginal cost as in our first scenario where electricity generation is converted from coal to various forms of green energy. Thus, we assume marginal cost is positive and increasing ($C' > 0$ and $C'' > 0$). For our model, we do not make assumptions on curvature of marginal cost ($C''' \lesseqgtr 0$).[3]

We model environmental damage as a function $D = D(K - m_1 - \delta m_2)$, where we assume $D' > 0$. Note that our model differs slightly from that of Ulph and Ulph (1997) in that we have an additive, rather than multiplicative, stochastic variable ($K$). Also, we assume that an increase in GHG stocks could result in a more than proportional increase in damage ($D'' > 0$). This assumption means that a little mitigation may have a larger impact when damages are high than when damages are low. We assume that the benefit of mitigation is simply the damage avoided. That is, if $E$ represents the benefits of an undamaged environment, then we define benefits as $B = E - D(K - m_1 - \delta m_2)$, which we simplify to: $B = B(m_1 + \delta m_2 - K)$. It follows from the damage function that benefits are increasing in mitigation at a decreasing rate (i.e. $B' > 0, B'' < 0$). We make no assumptions on $B''' \lesseqgtr 0$.

Given that some degree of climate change is generally accepted, we make the assumption that even a 'good' outcome implies some damage. Thus, even with low damage ($K = L$), we assume that $L > m_1 + \delta m_2$. Therefore, we do not need to place irreversibility constraints on the level of mitigation. For example, once a portion of electricity generation has switched to solar energy, we assume that the social planner does not want to revert that generating capacity back to coal. Similarly, we do not need to limit the amount of mitigation in any one period. Thus, we assume no physical limits on mitigation, only financial ones. Last, we do not impose risk-aversion, instead assuming the social planner is risk-neutral.

[3]We assume that the cost 're-sets' itself each period. This way of modeling cost applies well to situations where one might face annual capacity constraints (such as in the case of converting electricity generation). However, it does not apply to situations that face land-constraints, such as carbon sequestration, where the cost in period two would be a function of the amount chosen in period 1.

Imposing irreversibility or risk-aversion would capture some of the features of option value, quasi-option value, and the precautionary principle, without using curvature. However, these attributes could be explicitly modeled within our framework if desired.[4] Here, we focus on the implications of curvature.

***Learning:*** In period 1, the social planner believes the damage outcome could be high ($K = H$) or low ($K = L$) with equal probability, and she chooses mitigation level $m_1$. However, the social planner anticipates learning more about the damage in period 2, at which point additional levels of mitigation can be chosen. Specifically, the social planner believes that the information forthcoming in period 2 will be either favorable ($F$) or unfavorable ($U$), and that those two outcomes are equally likely.[5] Conditional on period 2 information being favorable, the social planner will revise beliefs about period 3 damages as follows: outcome $H$ with probability $\frac{1}{2} - 2\lambda$, and outcome $L$ with probability $\frac{1}{2} + 2\lambda$. Similarly, if period 2 information is unfavorable, then beliefs become outcome $H$ with probability $\frac{1}{2} + 2\lambda$, and outcome $L$ with probability $\frac{1}{2} - 2\lambda$.

The parameter $\lambda$ plays a central role in the analysis because it describes the degree or quality of learning. In the extreme case of no learning, ($\lambda = 0$), the social

[4] Traditional option value requires irreversibility. Irreversibility of mitigation requires a situation where one would want negative mitigation in the second period. If the government initially invested in mitigation, for example, with no damage in the low-damage state, the government would like to reverse that mitigation in the following period, i.e. $m_2 < 0$. One simple way of modeling option value is to constrain mitigation in period 2 to be non-negative, i.e. $m_2 \geq 0$.

On the other hand, envionmental irreversibility can be modeled as a limit on the quantity of mitigation in any one period. If climate change results in irreversible damage, then no level of mitigation can address that damage, and the marginal cost of mitigation at that point is infinite, i.e. $C'(x) = \infty$, where $x$ is some upper limit on the feasible level of mitigation.

Last, to model 'precaution' explicitly, one could assume that the decision-maker is risk averse, and therefore maximises the expected utility of benefit less cost.

[5] Although we take the timing and degree of learning as exogenous, Kelly and Kolstad (1999) develop a model where learning is endogenous, and they solve for the timing of the resolution of uncertainty. Hennessy and Moshini (2006) also assume uncertainy about when information is revealed, and they allow the decision-maker to invest in research that affects the timing of learning.

planner continues to believe that high and low damages are equally likely. At the other extreme, with perfect learning ($\lambda = \frac{1}{4}$), favorable information means that the social planner is certain that damages will be low (and if information is unfavorable, the social planner knows damages will be high). The period 1 unconditional probabilities of all four possible outcomes can be summarized as follows:

**Unconditional probabilities**

| | | Degree of Damage | |
|---|---|---|---|
| | | Low | High |
| Information | Favorable | $\frac{1}{4}+\lambda$ | $\frac{1}{4}-\lambda$ |
| | Unfavorable | $\frac{1}{4}-\lambda$ | $\frac{1}{4}+\lambda$ |

In other words, an increase in the degree of learning, $\lambda$, raises the probability that favorable information is associated with low damage and that unfavorable information is associated with high damage.

***Optimization problem:*** The social planner's objective function is to maximize expected welfare. In her initial plan, she can set three choice variables: the level of mitigation in the first period $(m_1)$, the level of mitigation to be undertaken in the second period conditional on favorable information $\left(m_2^F\right)$, and the level of mitigation in the second period conditional on unfavorable information $\left(m_2^U\right)$. From the vantage of period 1, expected welfare $E_1(W)$ is the sum of the benefit of each outcome times its respective probability, less the expected cost of mitigation. Assume that the social planner has a discount factor, $\beta$, where $0 < \beta \leq 1$. Then the objective function is:

$$(3) \underset{m_1,m_2^F,m_2^U}{Max} E_1(W) = (\tfrac{1}{4}+\lambda)\beta^2 B\left(m_1+\delta m_2^F-L\right)+(\tfrac{1}{4}-\lambda)\beta^2 B\left(m_1+\delta m_2^F-H\right)$$
$$+(\tfrac{1}{4}-\lambda)\beta^2 B\left(m_1+\delta m_2^U-L\right)+(\tfrac{1}{4}+\lambda)\beta^2 B\left(m_1+\delta m_2^U-H\right)$$

$-\frac{1}{2}\beta C\left(m_2^F\right) - \frac{1}{2}\beta C\left(m_2^U\right) - C\left(m_1\right).$

The first order conditions are as follows:[6]

(4) $\frac{\partial E(W)}{\partial m_2^F} : \left(\frac{1}{4}+\lambda\right) B'_{LF} + \left(\frac{1}{4}-\lambda\right) B'_{HF} - \frac{1}{2\beta\delta} C'_F = 0$

(5) $\frac{\partial E(W)}{\partial m_2^U} : \left(\frac{1}{4}-\lambda\right) B'_{LU} + \left(\frac{1}{4}+\lambda\right) B'_{HU} - \frac{1}{2\beta\delta} C'_U = 0$

(6) $\frac{\partial E(W)}{\partial m_1} : \left(\frac{1}{4}+\lambda\right) B'_{LF} + \left(\frac{1}{4}-\lambda\right) B'_{HF} + \left(\frac{1}{4}-\lambda\right) B'_{LU} + \left(\frac{1}{4}+\lambda\right) B'_{HU}$

$-\frac{1}{\beta^2} C'_1 = 0$

where $B_{ij}$ is the benefit with damage $i = \{H, L\}$ and information $j = \{F, U\}$.

As long as our above assumptions on the curvature of cost and benefit hold, the second order conditions yield a maximum (for proof, see appendix).

# 4 Results

To determine how the social planner's plan for mitigation changes with the degree of learning, we totally differentiate the first order conditions *w.r.t.* the three mitigation levels, $m_1, m_2^F, m_2^U$, and the exogenously-determined degree of learning, ($\lambda$). To keep the terms non-negative, we multiply both sides by -2 and write them in matrix form as:

(7) $$\begin{bmatrix} \delta A_F + \frac{1}{\beta\delta} C''_F & 0 & A_F \\ 0 & \delta A_U + \frac{1}{\beta\delta} C''_U & A_U \\ \delta A_F & \delta A_U & A_F + A_U + \frac{2}{\beta^2} C''_1 \end{bmatrix} \begin{bmatrix} \frac{dm_2^F}{d\lambda} \\ \frac{dm_2^U}{d\lambda} \\ \frac{dm_1}{d\lambda} \end{bmatrix}$$

[6] In a dynamic programing context, the planner's choice of $m_2^F$ and $m_2^U$ are conditioned on $m_1$. However, the envelope theorem allows $m_1$, $m_2^F$ and $m_2^U$ to be solved simultaneously, yielding the same results.

$$= \begin{bmatrix} -2N_F \\ \\ 2N_U \\ \\ -2\left(N_F - N_U\right) \end{bmatrix}$$

where $A_F \equiv -E_2(B''_F) = -2\left[\left(\frac{1}{4}+\lambda\right)B''_{LF} + \left(\frac{1}{4}-\lambda\right)B''_{HF}\right] \geq 0$,

$A_U \equiv -E_2(B''_F) = -2\left[\left(\frac{1}{4}-\lambda\right)B''_{LU} + \left(\frac{1}{4}+\lambda\right)B''_{HU}\right] \geq 0$

and $N_F \equiv B'_{HF} - B'_{LF}$, $N_U \equiv B'_{HU} - B'_{LU} \geq 0$.

In (7), $A_F$ or $A_U$ is the absolute value of the expected slope of the marginal benefit function, given that the learned information is favorable or unfavorable, respectively. Thus, if the social planner receives favorable information, her expectation in period 2 ($E_2$) of the slope of marginal benefit ($B''_F$) is $A_F$, and if she sees unfavorable information, she will expect marginal benefit to have a slope of $A_U$. In contrast, $N_F$ and $N_U$ are the differences in the marginal benefit at high and low damages when the information is favorable and unfavorable, respectively. A large $N_i$ implies that for a given outcome of learning, and the resulting level of mitigation, the social planner faces a large difference between the marginal benefit from experiencing high damage versus low. On the other hand, a small $N_i$ means little difference in marginal benefit between high and low damage. Both $N_i$ and $A_i$ are functions of the total level of effective mitigation ($m_1 + \delta m_2$).

Using Cramer's rule, we calculate how the first period chosen mitigation varies with the degree of anticipated learning. Solving for $\frac{dm_1}{d\lambda}$ :

(8) $\frac{dm_1}{d\lambda} = \frac{1}{\beta}C''_F N_F \left(A_F + \frac{1}{\beta\delta^2}C''_F\right)\left[\frac{\left(\delta^2 A_U + \frac{1}{\beta}C''_U\right)}{\left(\delta^2 A_F + \frac{1}{\beta}C''_F\right)} - \frac{C''_U N_U}{C''_F N_F}\right]\frac{1}{SOC}$

where the SOC is the determinant of the matrix of second order conditions of the objective function with respect to mitigation. Note that the expression before the square brackets is positive, and SOC must be negative. Therefore, the sign of $\frac{dm_1}{d\lambda}$ depends on the sign of the term inside the square brackets. The sign of this term depends on the size of the ratio of the slope of the marginal costs times the ratio of the difference in the marginal benefit $\left(\frac{C''_U N_U}{C''_F N_F}\right)$, compared to the size of the ratio of the second derivatives of expected welfare with respect to second period mitigation $\left(\frac{\delta^2 A_U + \frac{1}{\beta} C''_U}{\delta^2 A_F + \frac{1}{\beta} C''_F}, \text{ which equals } \frac{E(W)''_U}{E(W)''_F}\right)$. We will show this ratio depends on the relative curvature of marginal cost and marginal benefit functions ($C'''$ vs $B'''$).

Consider our first example of conversion to carbon-neutral sources of energy, where we explained why $C'''$ is high relative to $B'''$. In the simple case where the slope of the marginal benefit function is constant ($B''' = 0$), and the slope of marginal cost is increasing ($C''' > 0$), the sign of equation (8) depends on $(C''_F - C''_U) \leq 0$, implying $\frac{dm_1}{d\lambda} \geq 0$. In contrast, consider our second primary example of water storage, where $C'''$ was small compared to $B'''$, and take the extreme case where the slope of the marginal cost curve is constant ($C''' = 0$), while the marginal benefit curve is convex ($B''' > 0$). Then $C''_U = C''_F$, while $\frac{N_U}{N_F} \leq 1$ and $\frac{E(W)''_U}{E(W)''_F} \geq 1$ from above. In this case, the term inside the square brackets on the right-hand side of equation (8) is positive. This term is multiplied by a positive term times the negative SOC, and so $\frac{dm_1}{d\lambda} \leq 0$ for any initial learning $\left(0 \leq \lambda \leq \frac{1}{4}\right)$. More learning does not increase initial mitigation. Finally, if $B'''$ and $C'''$ are both zero, then the degree of learning ($\lambda$) has no effect on the initial level of mitigation.[7] This last differs from Ulph and Ulph (1997) who find that even with quadratic costs and benefits, the degree of learning has an effect. This difference arises from our use of an additive stochastic damage term, while they use multiplicative risk.

[7] Although we focus on the case when marginal benefits are convex, if $B'''$ is $< 0$ and $C''' \geq 0$, then $\frac{dm_1}{d\lambda} \leq 0$. If $C'''$ is $< 0$ and$B''' \geq 0$, then $\frac{dm_1}{d\lambda} \geq 0$.

If both marginal cost and marginal benefit are convex ($C''' > 0$ and $B''' > 0$), however, the interpretation of equation (8) becomes a bit more tricky. To compare the relative influence of marginal cost and marginal benefit on how $\lambda$ changes the optimal initial level of mitigation, define the curvature elasticity of marginal cost as the third derivative of the cost of mitigation over the second ($\frac{C'''}{C''} = \varepsilon$), and define the curvature elasticity of marginal benefit as the third derivative of the marginal benefit curve over the second ($\left|\frac{B'''}{B''}\right| = \eta$). Further, define $\omega$ as the ratio of the curvature elasticity of cost to benefit, i.e. $\varepsilon/\eta \equiv \omega$. We find the following necessary and sufficient conditions for $\frac{dm_1}{d\lambda} \geq 0$.

**Proposition 1** ***(necessary)*** *If both marginal cost and marginal benefit are convex ($B''' > 0$ and $C''' > 0$), then optimal mitigation in the first period increases with a marginal increase in the degree of learning only if the curvature elasticity of marginal cost of mitigation is at least as large as the curvature elasticity of marginal damage.*

**Proposition 2** ***(sufficient)*** *With no stock externality or discount rate, optimal mitigation in the first period increases with the prospect of learning if the curvature elasticity of marginal cost of mitigation is more than twice as large as the curvature elasticity of marginal benefit.*

The proof of these propositions is in the appendix. If both marginal benefit and marginal cost are convex (i.e. $B''' > 0$ and $C''' > 0$), then no sufficiency condition holds for all levels of discount rate or stock externality. As we show later, a higher discount rate or stock externality can always offset a larger ratio of the elasticity of marginal cost to that of marginal benefit, so that an increase in the degree of learning has either no effect or induces a smaller optimal level of initial investment,

$\left(\text{making } \frac{\partial m_1}{\partial \lambda} \leq 0\right)$. Without a stock externality or discount rate, however, we find that if the curvature elasticity of cost is more than twice that of marginal benefit, the potential for learning always increases the optimal level of investment in mitigation.

How the initial level of mitigation varies with learning depends in general on the relative size of the curvature elasticities of marginal cost and marginal damage. Specifically, for potential learning to increase current mitigation $\left(\frac{\partial m_1}{\partial \lambda} > 0\right)$, the curvature of the marginal cost function must be positive and greater than the curvature of the marginal damage function. Essentially, marginal cost must be increasing at a highly-increasing rate, making it very expensive to increase mitigation in the second period. The prospect of information means that in the second period, the social planner will want to increase or decrease mitigation, and a high curvature elasticity of marginal cost means that the marginal cost of increasing expenditure in response to unfavorable information outweighs the savings from decreasing mitigation in response to favorable information. Therefore, the social planner chooses an initial level of mitigation that is higher than she would without learning, in anticipation of this possible future expense.

On the other hand, if the curvature elasticity of marginal benefit of mitigation is very high, the social planner is able to forestall a great deal of damage with information about the chances of a bad outcome. Because she has the potential to benefit from learning in the second period, the social planner is not obliged to mitigate as much in the initial period. Otherwise, with no learning, the social planner is stuck mitigating the same amount in both periods, and therefore she has to compensate for this lack of ability to respond to downside risk by mitigating more today.

The critical level of the ratio of curvature elasticities is not constant over all levels of learning. Specifically, we find that the critical ratio of curvature elasticities needed for $\frac{\partial m_1}{\partial \lambda} > 0$ is decreasing in $\lambda$.

**Proposition 3** *Starting at no information* $(\lambda = 0)$*, for an increase in the degree of learning to result in an increase in optimal initial mitigation* $\left(\frac{\partial m_1}{\partial \lambda} > 0\right)$*, the minimum ratio of cost to benefit curvature elasticity must be at least two,* $(\omega \geq 2)$*. Further, the* $\omega$ *needed for an increase in the degree of learning to increase the optimal initial level of mitigation is decreasing with the degree of information.*

In other words, if the social planner initially expects not to learn anything in the second period, the curvature elasticity of marginal cost must be twice as large as the curvature elasticity of benefit for the prospect of some learning to increase the optimal level of initial mitigation. The more the potential for learning, the smaller the ratio of curvature elasticities of cost to benefit needed for a marginal increase in learning to increase optimal initial level of mitigation. In the limit, if the social planner expects to learn almost fully in the second period, the curvature elasticity of cost to benefit needs only be greater than one for a marginal increase in learning to increase the optimal level of initial mitigation.

Next, we shift focus from initial mitgation to the discussion of total mitigation, $m_1 + m_2$ (as opposed to 'effective' mitigation, $m_1 + \delta m_2$). As one might expect, in our model, increasing the degree of learning always results in lower levels of total mitigation and a larger expected welfare.

**Proposition 4** *Total mitigation* $(m_1 + m_2)$ *generally decreases with an increase in the degree of learning.*

The proof of this proposition is in the appendix. The intuition is as follows: learning allows the social planner to respond more efficiently in the second period, reducing the total expected second-period mitigation level. These efficiency gains, both in terms of reduced cost of mitigation and reduced damage, outweigh the potential that information might induce the social planner to increase initial mitigation ($m_1$) by more than that reduction in $m_2$. The greater the curvature of either the marginal cost or marginal benefit curves, the more information is valuable in terms of avoiding high damage and/or high adjustment costs, and therefore the more it reduces total mitigation.

## 5 Graphical Intuition

In our general results above, we show the importance of the relative size of $B'''$ and $C'''$. We next describe the model graphically. For simplicity, figure 2 considers the extreme case where $C''' = 0$ and $B''' > 0$. It could represent the case where the social planner considers building water storage in the case of drought, with linear marginal cost and convex marginal benefits. Later, figure 3 illustrates the case where $C''' > 0$ and $B''' = 0$, which could represent the energy-switching example, with linear marginal benefits and convex marginal costs. For simplicity, we ignore the stock externality and discount rate in these graphs, assuming $\beta = \delta = 1$.

Within figure 2, we illustrate the choice of mitigation under the two extremes: a situation with perfect learning (solid lines) compared to that with no learning (dashed lines).

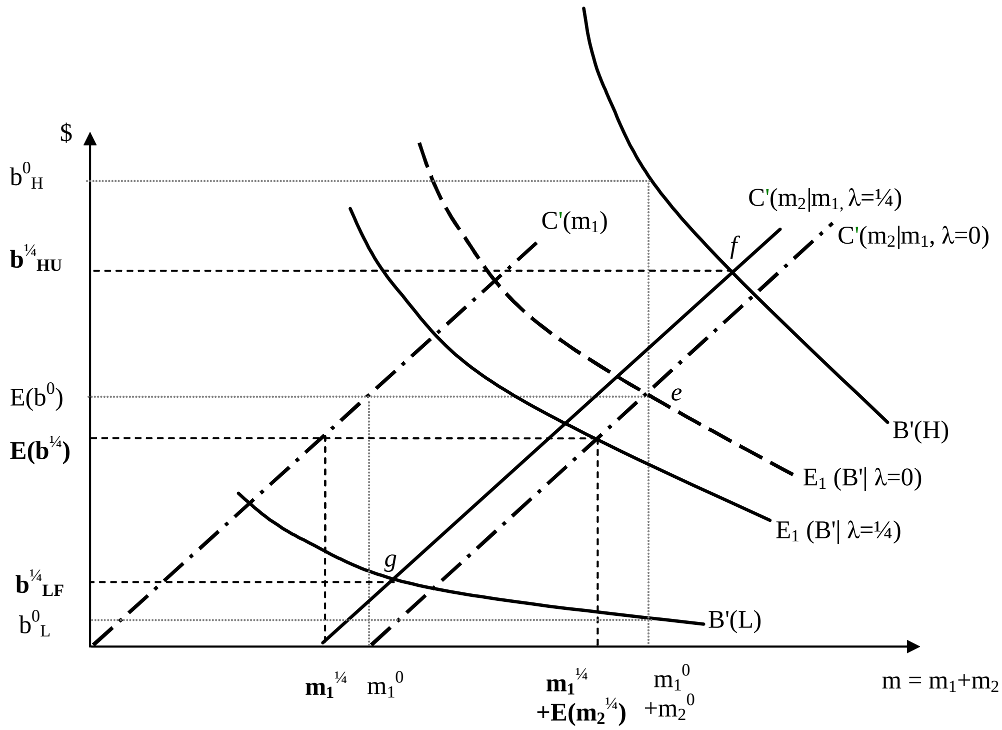


Figure 2: Mitigation with convex marginal benefits ($B''' > 0$) and linear marginal costs ($C''' = 0$) with no stock externality or discount rate ($\beta = \delta = 1$).

Because of the incentive for smoothing coming from the rising marginal cost and falling marginal benefit schedules, the social planner wants to equate expected marginal benefit and expected marginal cost across the two periods. With no information, either outcome is equally likely. Therefore, the curve for *expected* marginal benefit in the second period is always half the vertical distance between the curves for marginal benefit with high damage $B'(H)$ and with low damage $B'(L)$: $E_1\left(B'|\lambda = 0\right) = \frac{1}{2}B'(L) + \frac{1}{2}B'(H)$.[8] This curve is illustrated as the dashed line labelled as: $E_1\left(B'|\lambda = 0\right)$. Since in this case the social planner receives no new information in the second period, she sets mitigation equal across the two

[8]The graph uses $B'(H)$ to represent $B'(m_1 + m_2 - H)$, and $B'(L)$ to represent $B'(m_1 + m_2 - L)$.

periods, $m_1 = m_2$ (assuming $\delta = 1$). Thus, marginal cost is also equal across both periods, and is equal to expected marginal damage $C'(m_1) = C'(m_2|m_1, \lambda = 0) = E_1\left(B'|\lambda = 0\right)$. In our example, with no information, the social planner would build the same amount of water storage in each period, and would set the marginal cost of water storage equal to the average of the marginal benefit of the water in a severe and a light drought.

Graphically, to find the optimal quantity of total mitigation, start with the marginal cost $C'(m_1)$ in figure 2, and imagine a curve (not drawn) also starting at the origin but with with half the slope, representing the marginal cost of total mitigation, $m_1 + m_2$, given $m_1 = m_2$. That total marginal cost curve would intersect expected marginal benefit $E_1\left(B'|\lambda = 0\right)$ at point $e$. Then divide this level of total mitigation by 2 to obtain $m_1^0$. Starting at $m_1^0$ on the horizontal axis from zero marginal cost , we can plot out the marginal cost for mitigation in the second period, $C'(m_2|m_1, \lambda = 0)$. Total mitigation occurs where this marginal cost curve intersects the marginal benefit curve, labelled as $m_1^0 + m_2^0$. Total mitigation with no information will then result in an expected marginal benefit level of $E(b^0)$.[9]

With perfect learning (represented by solid lines and points in bold), the social planner in the second period knows whether the damage will be high or low with certainty, and can mitigate accordingly. If the information is unfavorable, indicating high damage, the social planner invests to the point where the marginal cost of second period mitigation equals the marginal benefit with high damage. Thus, she sets $m_1 + m_2^U$ where the $B'(H)$ curve intersects the $C'(m_2|m_1, \lambda = \frac{1}{4})$ curve, at point

[9] Whereas an upper case $B'$ is used to label a marginal benefit curve, we use a lower case $b$ to denote the specific marginal benefit at a certain level of mitigation. In other words, the lower case $b$ represents a height.

$f$, giving a marginal benefit of $b_{HU}^{\frac{1}{4}}$. Similarly, with favorable information indicating a good outcome, she sets $m_1 + m_2^F$ where the $B'(L)$ curve intersects $C'(m_2|m_1, \lambda = \frac{1}{4})$, at point $g$, giving a marginal benefit of $b_{LF}^{\frac{1}{4}}$. Expected marginal benefit $E(b^{\frac{1}{4}})$ lies vertically half-way between these two points, $E(b^{\frac{1}{4}}) = \frac{1}{2}b_{HU}^{\frac{1}{4}} + \frac{1}{2}b_{LF}^{\frac{1}{4}}$. The social planner will chose $m_1$ where the expected marginal benefit equals marginal cost, and where the marginal costs are equal in both periods, so where $E(b^{\frac{1}{4}}) = C'(m_1)$. With full information, these levels of mitigation yield a total expected amount of mitigation $m_1^{\frac{1}{4}} + E(m_2^{\frac{1}{4}})$.

With perfect learning, the expected marginal benefit is lower than with no learning. In our example, with full information, the social planner knows she will learn whether droughts will be severe or light next year, and can therefore wait to build the extra storage needed for a severe drought, ensuring the population has enough to drink, and enough water for industrial uses. If the social planner has no information, and only mitigates at the expected marginal benefit, the population may still have enough to drink in a severe drought, but industry might shut down. Thus, in the case of a severe drought, an extra unit of water is less valuable with full information, where it might go to irrigation, than with no information, where that water might go to industrial uses, or human consumption. On the other hand, if the social planner has no information and the drought turns out to be light, the extra stored water would go to a low-value use, such as watering lawns. So the difference in marginal benefit of water between full and no information is higher in a severe drought, than in a light drought. Thus, in figure 2, $b_H^0 - b_{HU}^{\frac{1}{4}} > b_{LF}^{\frac{1}{4}} - b_L^0$ (because of the curvature of marginal benefits, $B''' > 0$). The average of the marginal benefits with high and low damage is therefore greater with no learning than with learning, shown as $E(b^0) > E(b^{\frac{1}{4}})$, $\forall m$. Because of this reduction in expected marginal benefit with learning, the social planner has less incentive to build water storage capacity now.

Note that in this situation, where $C''' < D'''$, the optimal level of mitigation in the initial period is smaller with full learning than with no learning.

In figure 3, we illustrate the case where marginal benefit is linear and marginal costs are increasing and convex. Here, return to our a story where mitigation comes in the form of switching to non-fossil fuel sources of electricity generation. Switching from 10 to 20 percent 'green power' is relatively cheap, while moving from 80 to 90 percent green electricity is very costly. That is, we assume the marginal cost is increasing at an increasing rate. On the other hand, we assume the marginal benefits are decreasing at a relatively constant rate.

With no learning, the marginal cost curves for the two periods are illustrated as the dashed lines; the one for the second period starts at the choice of $m_1^0$, and the expected marginal benefit $E(B')$ is vertically half-way between the marginal benefit curves with high and low damage ($B'(H)$ and $B'(L)$). The scenario is the same as above: the social planner has an incentive to mitigate the same amount in the two periods. She sets $m_1^0 = m_2^0$ and chooses the level of total mitigation where marginal cost of $m_2$ is equal to the expected marginal benefit, i.e. where the height of $C'(m_1)$ equals the height of $C'(m_2|m_1, \lambda = 0)$, which in turn equals the height of $E\left(B'\right) = E(b^0)$. The resulting level of total mitigation, $m_1^0 + m_2^0$, gives a marginal benefit of $b_H^0$ if damage is high, and $b_L^0$ if damage is low.

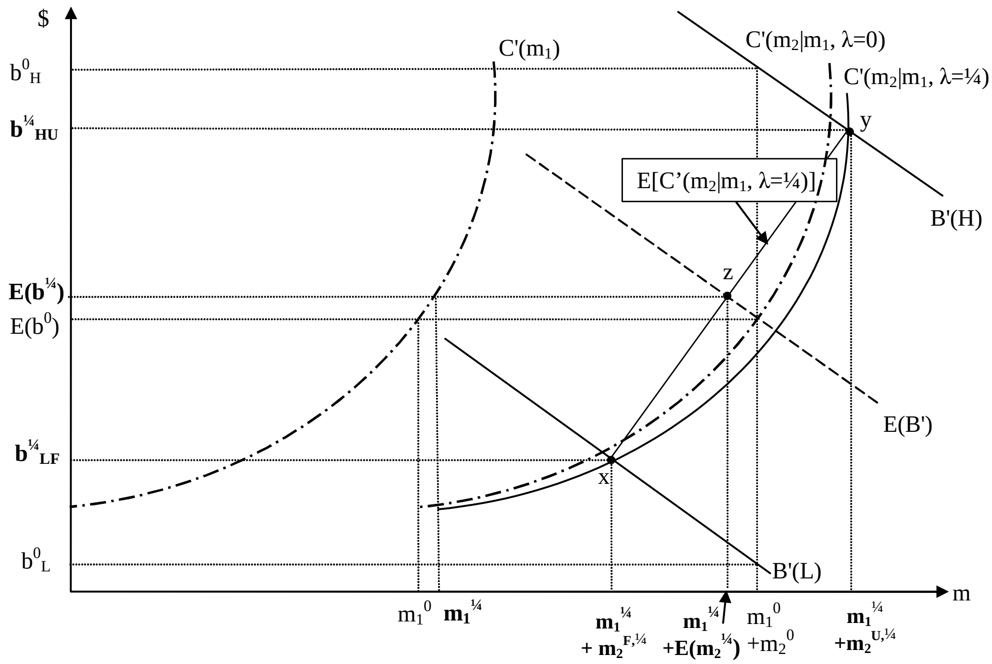


Figure 3: Mitigation with linear marginal benefit ($B''' = 0$) and convex marginal cost ($C''' > 0$) with no stock externality and no discount rate ($\beta = \delta = 1$).

With perfect learning (illustrated with solid lines), the social planner chooses $m_1^{\frac{1}{4}}$ such that the marginal cost of mitigation in period 1 is equal to the expected marginal cost of mitigation in period 2, setting both equal to the expected marginal benefit: $C'(m_1) = E\left[C'(m_2|m_1, \lambda = \frac{1}{4})\right] = E\left(b^{\frac{1}{4}}\right)$. In figure 3, this intersection is labelled as point $z$, where the expected marginal cost curve, $E\left[C'(m_2|m_1, \lambda = \frac{1}{4})\right]$ intersects the expected marginal benefit curve, $E(B')$, resulting in a total mitigation equal to $m_1^{\frac{1}{4}} + E(m_2^{\frac{1}{4}})$. The expected marginal cost of mitigation in period 2 is the average of the marginal cost of mitigation if information is favorable (point $x$) and unfavorable (point $y$), which are respectively equal to the marginal benefit with low and high damage, or $E\left[C'(m_2|m_1, \lambda = \frac{1}{4})\right] = \frac{1}{2}b_{HU}^{\frac{1}{4}} + \frac{1}{2}b_{LF}^{\frac{1}{4}}$ in figure 3. Note that, unlike the situation where $B''' > 0$, the linear marginal benefit curves here mean that the expected marginal benefit, $E(B')$ remains the same for the situations with both perfect and no learning. Thus, the social planner chooses $m_1$ so that the expected marginal benefit, $E\left(b^{\frac{1}{4}}\right)$, is equal to half the vertical distance between the marginal benefit with favorable and unfavorable information ($b^{\frac{1}{4}}_{HU}$ and $b^{\frac{1}{4}}_{LF}$).

The intuition for the change in the sign of $\frac{dm_1}{d\lambda}$ from figure 2 is based on the fact that with convex marginal cost ($C''' > 0$) in figure 3, it will cost the social planner more to react at the margin to unfavorable information than to favorable information. Therefore, in period 1, the social planner optimally mitigates slightly more initially, knowing she has a 50% chance of a costly upward adjustment in mitigation in reaction to learning in the following period. Thus, $m_1^{\frac{1}{4}} > m_1^0$ in figure 3. In a sense, the incentive to smooth expenditure outweighs the savings in initial mitigation provided by the social planner's ability to react.

Returning to our examples, we find that in the case illustrated in figure 2 where the slope of marginal benefits is increasing rapidly, such as in the case of water storage for a drought, an increase in the degree of learning leads to a decrease in the optimal level of initial investment. Conversely, where the slope of marginal costs are increasing rapidly, as illustrated in figure 3, such as the case of switching to green energy, an increase in the degree of learning leads to an increase in the optimal level of initial investment. Later we show that the latter result does not necessarily hold with a high stock externality of pollution ($\delta < 1$), as is the case for greenhouse gases, nor does it necessarily hold with a high discount rate ($\beta < 1$).

As noted in proposition 3, an increase in the potential for learning tends to decrease the total amount of mitigation. This result can be seen in the above figures.

In both figures 2 and 3, total mitigation is higher with no learning than with perfect learning. As one might expect, in figure 2, expected marginal damage is lower with perfect learning than with no learning, implying that learning not only allows the social planner to save on total mitigation, but also on expected damage. In figure 3, we see that learning does not necessarily reduce marginal damage. When marginal cost is more convex than marginal benefit, expected marginal damage increases with more learning.

## 6 Stock Externality and Discount Rate

Until this point, we have ignored the effect of the stock externality ($\delta$) and discount rate ($\beta$) on the level and timing of mitigation. Since these two important characteristics have driven many of the models of climate change mitigation, we would be remiss not to consider their influence. First, consider the effect of the stock externality on total mitigation. Figure 4 illustrates the effect of a stock externality such that it takes twice as much mitigation in the second period to have the same effect as mitigation in the first (i.e. $\delta = \frac{1}{2}$). Effective mitigation, or $m_1 + \delta m_2$, is shown on the horizontal axis. Thus, the illustrated marginal benefit function remains unchanged from figure 2, but the marginal cost of mitigation doubles its slope in the second period. As before, the case with no learning is shown as the dashed marginal cost, and the case with full learning is illustrated with solid lines.

As one would expect, the stock externality increases the initial level of mitigation in both the case with no learning and with perfect learning.[10] This result is

[10] This relationship can be proven mathematically by extending the comparative statics in equations (4'), (5') and (6') to include $\delta$ and then using Cramer's rule to solve for $\frac{dm_1}{d\delta}$. We find that

intuitive, since the stock externality makes it more expensive for the social planner to mitigate in the second period regardless of learning, and therefore, she spends more initially. Further, note that the total level of mitigation $(m_1 + m_2)$ is larger with the stock externality. The social planner still has the incentive to set marginal cost of (effective) mitigation equal across the two periods. Since the initial level of mitigation increases with a stock externality, the amount of mitigation in the second period increases as well. However, the total amount of effective mitigation decreases $(m_1 + \delta m_2)$, implying that the expected marginal benefit is higher with a stock externality than without.

all terms in the numerator are negative, implying $\frac{dm_1}{d\delta} < 0$.

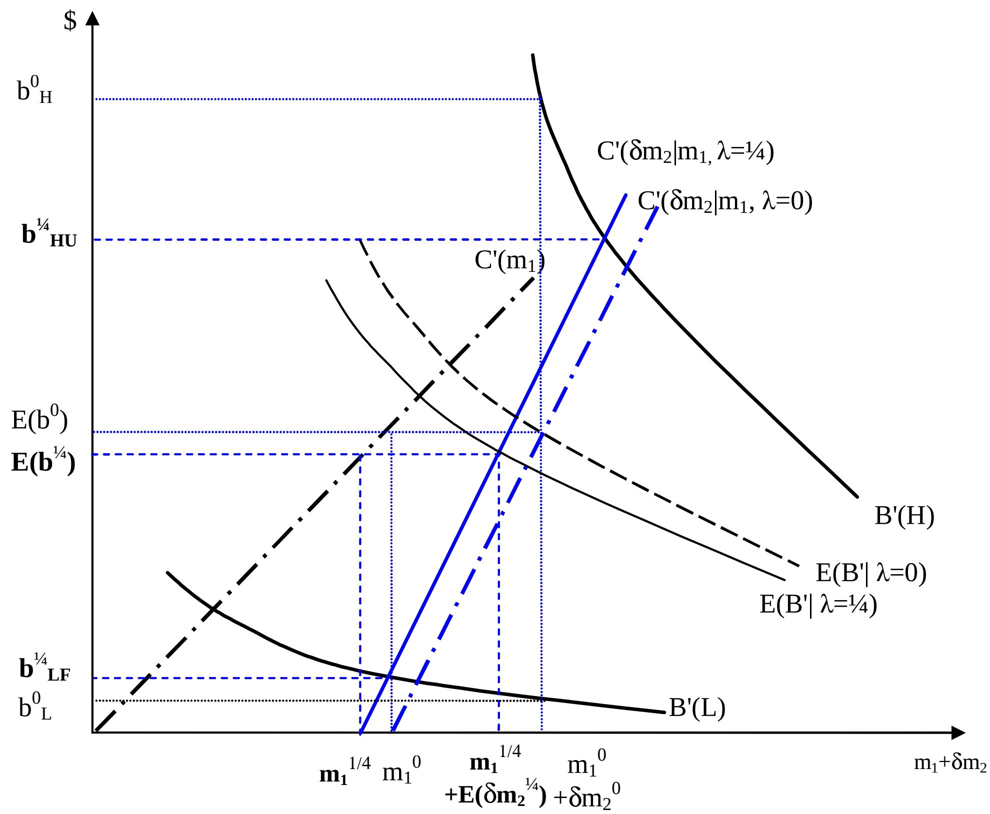


Figure 4: Mitigation with convex marginal benefit, linear marginal cost, and stock externality ($\delta = \frac{1}{2}$).

In figure 4, we can find the optimal level of total effective mitigation with no information where marginal cost in the second period $C'(\delta m_2|m_1, \lambda = 0)$ crosses expected marginal benefit $E(B'|\lambda = 0)$, giving total effective mitigation of $m_1^0 + \delta m_2^0$. This level of mitigation implies that if damages are high, marginal benefit is given by the height of the marginal benefit curve, $B'(H)$ at $m_1^0 + \delta m_2^0$, yielding $b_{H.}^0$ Similarly, if damages are low, the marginal benefit occurs at the height of the marginal benefit curve, $B'(L)$ at $m_1^0 + \delta m_2^0$, that is, $b_{L.}^0$ The resulting expected marginal benefit is vertically half-way between these two points at $E(b^0)$.

With full information, the social planner sets $C'(\delta m_2|m_1, \lambda = \frac{1}{4}) = E(B'|\lambda = \frac{1}{4})$, giving an expected total effective mitigation of $m_1^{\frac{1}{4}} + E(\delta m_2^{\frac{1}{4}})$. If information is favorable, implying damages will be low, the social planner reduces second-period effective mitigation, yielding a marginal benefit of $b_{LF}^{\frac{1}{4}}$. If information is unfavorable, implying that damage will be high, the social planner increases the effective mitigation in the second period, resulting in a marginal benefit of $b_{HU}^{\frac{1}{4}}$.

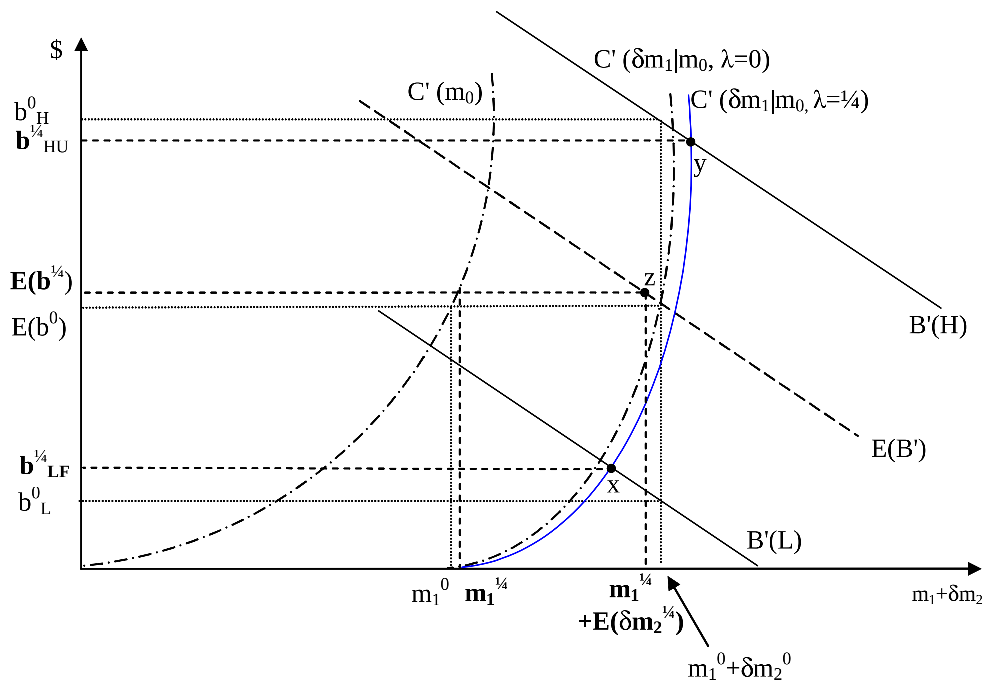


Figure 5: Mitigation with linear marginal benefit, convex marginal cost and stock externality ($\delta = \frac{1}{2}$)

In figure 5 we illustrate the situation where the curvature of marginal cost is greater than that of marginal benefit. As before, the social planner chooses mitigation so that the marginal cost of effective mitigation is equal across the two periods, and the marginal cost of effective mitigation in the second period equals the expected marginal benefit. Graphically, with no information, the optimal level of total effective mitigation $m_1^0 + \delta m_2^0$, occurs where the marginal cost of effective mitigation in the second period given mitigation in the first $C'(\delta m_2 | m_1, \lambda = 0)$ crosses expected marginal benefit $E(B')$. If damages are high, the resulting level of marginal benefit $b_H^0$ is given by the height of the marginal benefit curve, $B'(H)$ at the chosen level of total effective mitigation $m_1^0 + \delta m_2^0$. If damages are low, the level of resulting marginal benefit is found at the height of the marginal benefit curve, $B'(L)$ at $m_1^0 + \delta m_2^0$, giving $b_{L.}^0$ As before, the resulting expected marginal benefit is vertically half-way between these two points at $E(b^0)$.

In the case where marginal cost is convex, the expected marginal cost is equal to the average of the marginal cost of effective mitigation given favorable information (at point $x$ in figure 5) and the marginal cost of effective mitigation given unfavorable information (at point $y$). Expected marginal cost and expected marginal benefit then cross at point $z$, giving an expected total effective mitigation of $m_1^{\frac{1}{4}} + E(\delta m_2^{\frac{1}{4}})$. If information is favorable, and the social planner knows that damages will be low, she responds with a low level of effective second-period mitigation, and the marginal benefit becomes $b_{LF}^{\frac{1}{4}}$. If information is unfavorable and the social planner anticipates high damages, she increases her effective mitigation in the second period, resulting in an expected marginal benefit of $b_{HU}^{\frac{1}{4}}$.

Although we do not illustrate it graphically, the effect of the discount rate is intuitive. As the discount rate increases, or the discount factor $\beta$ decreases, the social planner mitigates much less initially; she perceives it to be cheaper to mitigate later, and the expected marginal benefits, not felt until period 3, are much less valuable. The social planner also mitigates less in period 2, again because of the decrease in the present value of the expected marginal benefits.

Next, let us consider how the effect of learning changes with the discount factor and the stock externality.

**Proposition 5** *The minimum ratio of cost curvature elasticity to benefit curvature elasticity necessary for learning to increase the optimal initial level of mitigation* ($\widehat{\omega}$) *is increasing with the discount rate.*

From the vantage of the first period, a positive discount rate implies that mitigation in the second period is less expensive. Thus, the social planner is less concerned about the potential increase in second-period mitigation associated with learning of a possible high damage. Therefore, the dominant factor for the social planner is that learning decreases the expected future damage, thereby allowing her to mitigate less initially. The formal proof is in the appendix.

**Proposition 6** *Given some degree of learning greater than zero* ($\lambda > 0$)*, the minimum ratio of cost curvature elasticity to benefit curvature elasticity needed for learning to increase the optimal initial level of mitigation* ($\widehat{\omega}$) *decreases with the introduction of a stock externality (i.e. for* $\delta < 1$*). At high levels of stock externality* ($\delta \ll 1$)*, a marginal increase in the stock externality increases the minimum ratio of curvature elasticities needed for learning to increase the optimal level of mitigation* ($\widehat{\omega}$)*.*

The effect of the stock externality is more complex than the effect of the discount rate. The introduction of a stock externality makes information less beneficial than with no stock externality, since it is more expensive to respond to that information by adjusting effective mitigation. If this were the only effect, it would imply that the change in marginal benefit from learning would be smaller with a stock externality.

Given that the shift in expected marginal benefit drives the optimal level of initial mitigation down, one therefore would anticipate that a stock externality would make it more likely for learning to increase initial mitigation. However, the fact that the social planner mitigates more initially means that she faces a smaller increase in marginal costs in the second period if she learns that damage will be high. Because she has already mitigated more, the extra cost of the added second-period mitigation induced by learning is smaller. Therefore, the potential for learning does not concern the social planner as much in terms of potential increased costs. Depending on the ratio of cost curvature elasticity to benefit curvature elasticity, this effect on the additional marginal cost may outweigh the smaller decrease in marginal benefit generated with learning, making the social planner less likely to increase initial mitigation with learning when a stock externality applies.

Last, let us turn to the value of learning. Both a stock externality and a discount rate diminish the value of learning. Because the stock externality makes it more expensive to react to learning in the second period, information yields fewer benefits in terms of avoided downside risk. On the other hand, a discount rate implies that the social planner cares less about future damage, which means she takes less pains to avoid high damage, implying information is less valuable.

# 7 Conclusions

In this paper, we ask how the potential for learning affects the timing and quantity of mitigation. Following work by Epstein (1980), Kolstad (1996a) and Ulph and Ulph (1997), we develop a model where the social planner can choose how much to invest in mitigation before and after learning about future benefits of mitigation. Using a simple Bayesian approach, we model the degree of learning as a continuous parameter, which allows us to do comparative statics on the optimal level of initial mitigation in response to an increase in anticipated learning. We generate the conditions necessary for a marginal increase in learning to increase the initial level of mitigation, conditions that depend on the curvature elasticities of the curves for marginal cost and benefit of mitigation. Even if these curvature elasticities may be difficult to measure, policymakers may find it relatively easier to determine if their ratio is above of below some threshold level.

First, we find that unless the marginal cost of mitigation is increasing at least as rapidly as marginal benefit, an improvement in anticipated information tends to decrease the initial amount of mitigation. Our result differs from that of Ulph and Ulph (1997). With irreversibility, they find that for learning to induce the social planner to mitigate more initially, the irreversibility constraint must bind. In contrast, we find circumstances where initial mitigation increases with an increase in the degree of learning even without irreversibility. Thus, we find situations that justify precaution, without resorting to irreversibility and without risk aversion. This scenario of rapidly rising marginal cost of mitigation (or investment) may well be plausible in light of mitigation that involves large social and capital costs, such as mass dislocation or wholesale transformation from a car-based transportation system.

Because of the convex marginal cost of mitigation ($C''' > 0$), the social planner has an incentive to try to balance its expected expenditure over the two periods. Increasing second-period mitigation by one unit in reaction to unfavorable information costs more at the margin than the social planner saves by reducing second-period mitigation by one unit with favorable information. Thus, the fact that the marginal cost curve is convex induces the social planner to exhibit 'precaution' and mitigate more initially. However, a convex marginal benefit curve ($B''' > 0$) implies that the social planner benefits more from information, in that she is able to avoid the very damaging outcomes, while saving on mitigation if damage is light. Only when the curvature elasticity of the marginal cost of mitigation is large relative to the curvature elasticity of benefit will the potential for learning increase the optimal level of initial mitigation.

Second, we find that starting with a lower degree of learning makes it less likely for learning to increase the optimal level of initial expenditure. Specifically, if a social planner initially does not anticipate any learning and then hears of a study coming out a year from now that will begin to clarify the probabilities of future environmental damage, the curvature elasticity of marginal cost of mitigation has to be at least twice that of the curvature elasticity of marginal benefit for her to optimally increase mitigation today.

The third finding is perhaps more predictable: total expected expenditure on mitigation is lower with a greater potential for learning. Because the social planner can respond to the learned information, she can decrease her expected expenditure in the second stage.

Since it is often the stock of pollution, such as GHGs, that damage the environment, it makes sense to include a stock externality in the model. As one might anticipate, we find that less effective second-period mitigation makes the social planner spend more on mitigation in the first period, and spend more overall. However, the increase in total mitigation is not enough to offset the decrease in its effectiveness. The stock externality also affects how the timing of mitigation responds to information. Since the stock externality makes it more expensive to respond to unfavorable information, the social planner increases the initial level of mitigation with the prospect of better learning. This result only holds for a small stock externality, however. If the stock externality is large enough such that mitigation in the second period has very little effect on expected damage, then a marginal change in the stock externality has either no impact on the effect of learning, or may imply that learning decreases the optimal initial level of mitigation a small amount. The intuition here is simply that if the social planner can do very little in the second period, she mitigates more fully in the first period. Learning primarily decreases the expected marginal benefit, inducing the social planner initially to mitigate slightly less.

Discount rates are staples in policy decisions with long horizons such as climate change. When the social planner discounts the future, she understandably spends less now to mitigate against future damage. Further, she is also less likely to increase the initial level of mitigation in response to an increase in the degree of learning, since the information affects the expected future marginal benefits that are discounted. The discount rate acts as one would expect: it decreases mitigation in both periods, particularly in the first.

The model provides insight to debates beyond climate change. Public policy makers face a number of scenarios where they have to determine the level of investment when facing unknown environmental threats. For example, governments debate how much to invest in parks and other means of preserving endangered species before knowing their exact rate of decline. By developing a relatively simple, yet, we hope, broadly-applicable model, this paper helps develop economic intuition for when mitigation today should be increased or decreased, for causes that have different potentials for learning, based solely on knowledge of the shape of the marginal cost and benefit curves.

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# 9 Appendix

## 9.1 Second order conditions

From equation (7), we can solve for the second order conditions:

$$\text{(A.1) } SOC = -\frac{1}{2}\left[\begin{array}{c} \left(\delta A_F + \frac{1}{\beta\delta}C_F''\right)\left(\delta A_U + \frac{1}{\beta\delta}C_U''\right)\left(A_F + A_U + \frac{2}{\beta^2}C_1''\right) \\ -\delta A_F^2\left(\delta A_U + \frac{1}{\beta\delta}C_U''\right) - \delta A_U^2\left(\delta A_F + \frac{1}{\beta\delta}C_F''\right) \end{array}\right]$$

Simplifying, one obtains:

$$\text{(A.2) } SOC = -\frac{1}{2}\left[\begin{array}{c} \frac{1}{\beta^2\delta^2}C_F''C_U''\left(A_F + A_U + \frac{2}{\beta^2}C_1''\right) + \frac{1}{\beta}A_F A_U\left(C_F'' + C_U''\right) \\ +\frac{2}{\beta^2}C_1''\left(\frac{1}{\beta}\left(A_F C_U'' + A_U C_F''\right) + \delta^2 A_F A_U\right) \end{array}\right].$$

If assumptions (1) and (2) hold, $\mathrm{A}_F, A_u, C_F''$ and $C_U''$, $\geq 0$. As long as not all of these terms equal 0, then $SOC < 0$, implying a maximum exists.

## 9.2 Derivation of $\frac{dm_1}{d\lambda}$

To find the ratio of the two curvature elasticities, $\omega$, that allows for an increase in the degree of learning to increase the optimal initial level of mitigation, we need to be able to compare the terms, $A'', N$ and $C''$. Therefore, we take a Taylor series expansion of each of these terms, show that $\frac{dm_1}{d\lambda}$ is monotonic in $\omega$, and solve for the minimum level of $\omega = \widehat{\omega}$ where $\frac{dm_1}{d\lambda} = 0$.

Note from equation (8) that for $\frac{dm_1}{d\lambda} > 0$, the following must be true:

(2A.1) $\frac{A_u}{C''_u} - \frac{A_F}{C''_F}\frac{N_u}{N_F} < \frac{1}{\beta\delta^2}\left(\frac{N_u}{N_F} - 1\right) < 0$

First, let us re-define these terms. From its definition, note that

$A_U|_{\lambda=0} = -\left\{\frac{1}{2}B''(\overline{m_1} + \delta\overline{m_2} - H) + \frac{1}{2}B''(\overline{m_1} + \delta\overline{m_2} - L)\right\} = A_F|_{\lambda=0} = |\overline{B}''|$

while using a Taylor series expansion, we can write

$A_U|_\lambda = -\left\{\left(\frac{1}{2} + 2\lambda\right)\left(\overline{B}''_H - B'''\frac{\partial m^u_T}{\partial\lambda}\lambda\right) + \left(\frac{1}{2} - 2\lambda\right)\left(\overline{B}''_L - B'''\frac{\partial m^u_T}{\partial\lambda}\lambda\right)\right\}$

(2A.1) $A_U|_\lambda = |\overline{B}''| + 2\lambda\left(\overline{B}''_H - \overline{B}''_L\right) - B'''\frac{\partial m^u_T}{\partial\lambda}\lambda$, where $\overline{B}''_H - \overline{B}''_L = B'''(H-L)$.

Dividing through by $B'''$, we get

(2A.3) $\frac{A_U}{B'''} = \frac{1}{\varepsilon} + 2\lambda(H-L) - \frac{\partial m^u_T}{\partial\lambda}\lambda$.Similarly,

(2A.4) $\frac{A_F}{B'''} = \frac{1}{\varepsilon} - 2\lambda(H-L) - \frac{\partial m^f_T}{\partial\lambda}\lambda$

We can also use a Taylor series expansion to re-write cost:

(2A.5) $\frac{C_U}{C'''} = \frac{1}{\eta} + \frac{\partial m^u_2}{\partial\lambda}\lambda$

(2A.6) $\frac{C_F}{C'''} = \frac{1}{\eta} + \frac{\partial m^f_2}{\partial\lambda}\lambda$

Last, we can also re-write $N$ as a function of the elasticity of marginal benefit.

(2A.7) $N_U = \overline{N} - B'''(H-L)\frac{\partial m^U_T}{\partial\lambda}\lambda$, and

(2A.8) $N_F = \overline{N} + B'''(H-L)\frac{\partial m^F_T}{\partial\lambda}\lambda$

where $\frac{\overline{N}}{H-L} = \overline{B}''$.

First, we need to show that $\frac{\partial^2 m_1}{\partial\lambda\partial\omega} < 0$ at $\frac{\partial m_1}{\partial\lambda} = 0$. Setting $\frac{\partial m_1}{\partial\lambda} = 0$, we notice from the third first order condition (equation 6) that at the optimal level of mitigation, $\frac{1}{\eta}\left(\alpha^F - \alpha^u\right) = \left(\alpha^{F2} + \alpha^{u2}\right)\frac{\lambda}{2}$.

Define $\delta\left(\alpha^F + \alpha^u\right) = 4\theta(H-L)$, where $\theta < 1$. Further, define $\delta^2\left(\alpha^{F2} + \alpha^{u2}\right) = 16\phi(H-L)^2$ and $\sigma = \frac{\phi}{\theta}$.

Essentially, $\theta$ can be thought of as the proportion of the difference between good and bad outcomes to which the government reacts in the second period with full learning. The function, $\theta$, is increasing in the ratio of the curvature elasticity of cost over benefit, $\omega$, increasing in information, $\lambda$, and increasing in $\delta$ and $\beta$. Note that the smaller the curvature elasticity of cost, the more likely is the left side of equation (9) to be positive. Further, note that the larger the difference between high and low damage, the more likely is the left hand side of equation (9) to be negative.

Substituting the above definitions back into (A.3), rearranging and we obtain:

(2A.9) $\Delta = \delta^2 \left(\frac{1}{\eta}\right)^2 (\varphi + \frac{\omega}{\theta} - \frac{\omega^2}{\delta}) - 4\lambda^2(H-L)^2 \left[2\theta^2(\varphi - \delta) + \sigma(2\theta+1)(\omega+\delta)\right]$

where $\varphi = \frac{C'''}{\beta\delta^2 B'''}$. Note that $\Delta$ is decreasing in $\theta$ and $\sigma$. The first term in brackets in equation (2A.9), is positive for all values of the stock externality and discount factor $(\delta, \beta)$ as long as $\omega \leq 2$. The term in square brackets, $2\theta^2(\frac{\rho}{\beta\delta^2} - \delta) + \sigma(2\theta+1)(\omega+\delta)$, is positive for all values of $\omega$, and is increasing in $\theta$. Therefore, if we solve for the critical level of omega, $\widehat{\omega}$, where, for any $\omega \leq \widehat{\omega}$, $\Delta < 0$ at the maximum level of $\theta$, $\Delta < 0$ will hold for all levels of $\theta$.

### 9.2.1 Solving for maximum $\theta$ and $\sigma$

Now, let us turn to $\theta \equiv \frac{\delta(\alpha^F + \alpha^u)}{4(H-L)}$. Note that $\alpha^u = \frac{\partial m_2^U}{\partial \lambda}$ and $\alpha^F = -\frac{\partial m_2^F}{\partial \lambda}$ are themselves a function of $\omega$. Using the first order conditions for $\frac{\partial m_2^U}{\partial \lambda}$ and using the Taylor series expansion for $C_U'$ and $A_U'$, we can solve for $\frac{\partial m_2^U}{\partial \lambda}$.

(2A.10) $\frac{1}{\beta\delta}\left(\overline{C}' + \overline{C}''\alpha\lambda + \frac{C'''}{2}\alpha^2\lambda^2\right)$

$= \overline{B}' + 2\lambda(B_H' - B_L') + \overline{B}''\frac{\partial m_T^U}{\partial \lambda}\lambda + \frac{B'''}{2}\left(\frac{\partial m_T^U}{\partial \lambda}\lambda\right)^2 - 2\lambda^2 B'''(H-L)\frac{\partial m_T^U}{\partial \lambda}$

Note that if $\frac{\partial m_1}{\partial \lambda} = 0$, $\frac{\partial m_T^U}{\partial \lambda} = \delta\frac{\partial m_2^U}{\partial \lambda} \equiv \delta\alpha$. Also, note that in equilibrium,

$\frac{1}{\beta\delta}\overline{C}' = \overline{B}'$. Thus,

(2A.11) $\delta\alpha^u = \frac{2\omega\eta(H-L)}{\frac{1}{\eta}(\varphi+\omega)+\frac{\alpha^u\lambda}{2}(\varphi-\delta)+2\lambda(H-L)}$

and, using the same trick,

(2A.12) $\delta\alpha^F = \frac{2\omega\eta(H-L)}{\frac{1}{\eta}(\varphi+\omega)-\frac{\alpha^F\lambda}{2}(\varphi-\delta)-2\lambda(H-L)}$.

Define $a = \left(\frac{1}{\eta}(\varphi+\omega) + \frac{\alpha^u\lambda}{2}(\varphi-\delta) + 2\lambda(H-L)\right) > 0$

and $b = \left(\frac{1}{\eta}(\varphi+\omega) - \frac{\alpha^F\lambda}{2}(\varphi-\delta) - 2\lambda(H-L)\right) > 0$. Thus,

(2A.13) $\delta\left(\alpha^F + \alpha^u\right) = 4(H-L)\left[\frac{\omega(a+b)}{2\eta ab}\right]$

where the term in square brackets is defined as $\theta$. Since $\theta$ contains endogenous terms $\alpha^U$ and $\alpha^F$, we calculate a maximum level of $\theta$, which we denote as $\overline{\theta}$, and use it to solve for $\omega$. To get $\overline{\theta}$, we need to make some assumptions about the maximum level of $\alpha^U$ and $\alpha^F$. First, note that by the assumption of $C'' \geq 0$,

(2A.14) $\alpha^F\lambda \leq \frac{1}{\eta}$.

Second, note that by the assumption that $B'' \leq 0$ for all possible outcomes, then $B''$ is still greater or equal to zero with a good outcome, but where the social planner assumed a bad outcome, and mitigated accordingly (setting $m_2 = m_2^u$). In that case, $\frac{\omega}{\eta} \geq \delta\alpha^U\lambda + 2\lambda(H-L)$ for all possible levels of $\alpha^U$ and $\delta$. The maximum level of $\alpha^U$ from equation 2A.11 above, is:

(2A.15) $\frac{\omega}{\eta} - 2\lambda(H-L) \geq \frac{2\omega(H-L)}{(\varphi+\omega)}$

Further, note that $\alpha^U \geq 0$. Substituting both both (A.16) and (A.17) into (A.15), and solving for $\overline{\theta}$, we obtain the following, admittedly ugly, expression:

(2A.16) $\overline{\theta} = \frac{\omega(3\varphi+4\omega+\delta)(\varphi+2\omega)^2}{2(\varphi+\omega)(\varphi+3\omega)(2\omega\varphi+2\omega^2+\varphi^2+\delta(\varphi+2\omega))}$.

### 9.2.2 Showing that $\frac{\partial\Delta}{\partial\omega} > 0$

To show that $\frac{\partial\Delta}{\partial\omega} > 0$, we first need to show that $\frac{\partial\theta}{\partial\omega} \geq 0$.

(2A.17) $\frac{\partial\theta}{\partial\omega}=\frac{\frac{1}{\eta}\left[a^2(b-\omega\eta)+b^2(a-\omega\eta)\right]+\omega\left[a^2\xi_b-b^2\xi_a\right]}{2a^2b^2}$,

where $\xi_a=\frac{\lambda(\varphi-\delta)}{2}\frac{\partial\alpha^U}{\partial\omega}, \xi_b=\frac{\lambda(\varphi-\delta)}{2}\frac{\partial\alpha^F}{\partial\omega}$, and $\xi_a<\xi_b$.

The sign of (2A.17) depends on $b-\frac{\omega}{\eta}$, since $a-\frac{\omega}{\eta}>0$. Note that $\frac{\varphi}{\eta}-\frac{\alpha^F\lambda}{2}(\varphi-\delta)-2\lambda(H-L)>(\frac{\varphi+\delta}{\eta 2})-2\lambda(H-L)$. Further, note that from (2A.15), $2\lambda(H-L)\leq\frac{\omega(\varphi+\omega)}{\eta(\varphi+2\omega)}$. Therefore, as long as $\omega\leq 1.6$, the term $b\geq\frac{\omega}{\eta}$, and $\frac{\partial\theta}{\partial\omega}>0$. Last, note that at the maximum level of $\theta$, $\frac{\partial\overline{\theta}}{\partial\omega}>0\forall\omega$.

Next, consider $\frac{\partial\sigma}{\partial\omega}$. We can write $\sigma=\frac{\delta^2\left(\alpha^{F2}+\alpha^{u2}\right)}{\delta(\alpha^F+\alpha^u)4(H-L)}=\theta-\frac{\delta^2\left(\alpha^F\alpha^U\right)}{\theta 8(H-L)^2}=\theta-\frac{\omega}{\eta(a+b)}$. Solving for $\frac{\partial\sigma}{\partial\omega}$ we find:

(2A.18) $\frac{\partial\sigma}{\partial\omega}=\frac{a^4(b-\frac{\omega}{\eta})+b^4(a-\frac{\omega}{\eta})+2ab(a+b)\frac{\omega}{\eta}}{2a^2b^2(a+b)^2}$

By the same reasoning above, if $\omega\leq 1.6$, $b>\frac{\omega}{\eta}$, which implies that $\frac{\partial\sigma}{\partial\omega}>0$. Note as well that the maximum $\sigma$, $\overline{\sigma}=\theta-\frac{\omega}{2(\omega+\varphi)}$, is increasing in $\omega$, $\frac{\partial\overline{\sigma}}{\partial\omega}>0\ \forall\omega$.

To show that $\frac{\partial\Delta}{\partial\omega}<0$, note that the first term in 2A.9 will decrease with $\omega$ if $\frac{1}{\theta}-\frac{\partial\theta}{\partial\omega}\frac{\omega}{\theta^2}-2\frac{\omega}{\delta}<0$. Note that $\frac{1}{\theta}-\frac{\partial\theta}{\partial\omega}\frac{\omega}{\theta^2}=2\left[1-\frac{ab}{(a+b)^2}\right]<2$, while $2\frac{\omega}{\delta}\geq 2$ if $\omega\geq 1$. Therefore, if $\omega\geq 1$, $\frac{\partial\Delta}{\partial\omega}<0\forall\beta,\delta,\lambda$.

Now that we've shown that for the range of $\omega$ that can generate $\frac{dm_1}{d\lambda}=0$, $\frac{\partial\Delta}{\partial\omega}<0$, we know a critical level of omega exists, $\widehat{\omega}$, such that , $\frac{dm_1}{d\lambda}\leq 0\forall\omega\leq\widehat{\omega}$.

### 9.2.3 Showing that for $\omega\leq 1, \frac{dm_1}{d\lambda}<0$

Returning to equation (A.11), note that the maximum level of $\alpha^U=\frac{2\omega(H-L)}{(\varphi+\omega)}$ (from equation 2A.15 above), therefore $\frac{\omega\eta}{2\lambda(H-L)}\geq\frac{\varphi+2\omega}{\varphi+\omega}$. Substituting into (2A.9), we get

$$\text{(2A.19)}\quad \Delta\geq\underline{\Delta}\equiv 2\delta^2\frac{(\varphi+2\omega)^2}{(\varphi+\omega)^2}\left(\frac{\varphi}{\omega^2}+\frac{1}{\omega\overline{\theta}}-\frac{1}{\delta}\right) \\ -4\overline{\theta}^2(\varphi+\omega)-\frac{(\omega+\delta)}{(\varphi+\omega)}\left(2\overline{\theta}\varphi-\omega\right)$$

Next, we need to show that at $\omega=1$, $\underline{\Delta}\geq 0$. Note from A.18, $\underline{\Delta}$ is no longer a function of $\eta$. Because $\frac{\partial\overline{\theta}}{\partial\varphi}<0$, $\frac{\partial\overline{\theta}^2\varphi}{\partial\varphi}<0$ and $\partial\frac{\frac{2\overline{\theta}\varphi}{(\varphi+\omega)}}{\partial\varphi}<0$, $\frac{\partial\underline{\Delta}}{\partial\varphi}>0$. Since the discount factor only affects the value of $\varphi$, where $\frac{\partial\varphi}{\partial\beta} < 0$, we know that the minimum value of $\underline{\Delta}$ occurs at $\beta = 1$. The stock externality, however, effects both the value of $\varphi$ as well as other components of $\underline{\Delta}$. By simulating (2A.19), we find that at $\omega = 1$, the minimum $\Delta$ occurs at $\delta = .523$. At these values, $\overline{\theta} = 0.318$ and $\overline{\sigma} = 0.21$. Plugging these values into equation (2A.19), we find that at $\omega = 1$, $\underline{\Delta} = 0.88, \forall\beta, \delta$.

We want to know, for some initial level of learning, $\lambda > 0$, at what $\omega$ is $\frac{\partial m_1}{\partial\lambda} = 0$? Because $\frac{\partial\varphi}{\partial\delta} < 0, \frac{\partial\theta}{\partial\delta} > 0$ and $\frac{\partial\Delta}{\partial\varphi} > 0$. Since $\varphi$ is defined as $\frac{C'''}{\beta\delta^2 B'''}$, as long as $\Delta$ is near zero, $\frac{\partial\Delta}{\partial\delta} < 0$.

Thus, for a marginal increase in learning to increase the optimal initial level of mitigation, the curvature elasticity of marginal cost has to be greater than 1.14 times as large as the curvature elasticity of marginal benefit, regardless of stock externality, discount rate or the initial degree of learning expected.

## 9.3 3. Derivation of $\frac{\partial\widehat{\omega}}{\partial\lambda}$.

To consider how the critical level of $\omega$ changes with learning, we need to consider how $\Delta$ changes with learning. Given that $\frac{\partial\Delta}{\partial\theta} < 0$ and $\frac{\partial\Delta}{\partial\sigma} < 0$, if we can show $\frac{\partial\theta}{\partial\lambda} > 0$, then it follows that $\frac{\partial\Delta}{\partial\lambda} > 0$. Note that $\frac{\partial\theta}{\partial\lambda} = \frac{\delta}{4(H-L)}\left(\frac{\partial\alpha^F}{\partial\lambda} + \frac{\partial\alpha^U}{\partial\lambda}\right)$, where $\frac{\partial\alpha^F}{\partial\lambda} = \frac{\left[\left(\frac{\alpha^F}{2} + \frac{\partial\alpha^F}{\partial\lambda}\frac{\lambda}{2}\right)(\varphi-\delta)+2(H-L)\right]}{b} \equiv \frac{b'}{b}$ and $\frac{\partial\alpha^U}{\partial\lambda} = -\frac{\left[\left(\frac{\alpha^U}{2} + \frac{\partial\alpha^F}{\partial\lambda}\frac{\lambda}{2}\right)(\varphi-\delta)+2(H-L)\right]}{a} \equiv \frac{a'}{a}$. Further, note that $\frac{\partial\alpha^F}{\partial\lambda} > -\frac{\partial\alpha^U}{\partial\lambda}$. Thus, $\frac{\partial\theta}{\partial\lambda} > 0$. Now, $\sigma = \theta - \frac{\omega\eta}{a+b}$, where $\frac{\partial\sigma}{\partial\lambda} = \frac{\frac{\omega}{\eta}(a+b)^3(b'a-a'b)-(b'-a')ab\left[(a+b)^2+2\frac{\omega}{\eta}ab\right]}{2a^2b^2(a+b)^2}$. Note that $(b'a - a'b) > (b' - a')a$. Therefore $\frac{\partial\sigma}{\partial\lambda} > \frac{(b'-a')\left[(a+b)^2 a\left(\frac{\omega}{\eta}(a+b)-b\right)-2\frac{\omega}{\eta}a^2b^2\right]}{2a^2b^2(a+b)^2}$. Note that $2\frac{\omega}{\eta}a^3b > 2\frac{\omega}{\eta}a^2b^2$ and assume $H - L > 1$, so that $\frac{\omega}{\eta} > 2\lambda(H-L) > 1$. Thus $\frac{\partial\sigma}{\partial\lambda} > 0$.

To see the sign of $\frac{\partial\Delta}{\partial\lambda}$return to equation A.11. Note that as long as $\omega < 2$, the first term is positive. For all values of $\omega, \beta, \delta$ and $\lambda > 0$,the second term is positive, and is increasing in $\lambda$. Thus, the first term minus the second is decreasing in learning, or $\frac{\partial\Delta}{\partial\lambda} < 0$. Further, this result implies that the critical level of $\omega$ needed to yield a $\Delta < 0$, is decreasing in information. Thus, $\frac{\partial\widehat{\omega}}{\partial\lambda} > 0$.

### 9.4 4. Derivation of $\frac{\partial^2 m_1}{\partial\lambda^2}$ at $\lambda = 0$

From equation (8) above, we know:

$$(4A.1)\, \frac{dm_1}{d\lambda} = \frac{1}{\beta\delta^2}\begin{bmatrix} C_F''\left(\delta^2 A_U + \frac{1}{\beta}C_U''\right)N_F \\ -C_U''\left(\delta^2 A_F + \frac{1}{\beta}C_F''\right)N_U \end{bmatrix}\frac{1}{SOC}$$

Therefore, the sign of $\frac{dm_1}{d\lambda}$ depends on the ratio of how marginal welfare changes with mitigation in the second period under unfavorable and favorable information: $\frac{\delta^2 A_F + \frac{1}{\beta}C_F}{\delta^2 A_U + \frac{1}{\beta}C_U} = \frac{E(W)_U''}{E(W)_F''}$ compared to the ratio of the slope of marginal cost times the average slope of marginal damage at the two states of information: $\frac{C_U'' N_U}{C_F'' N_F}$. If $\frac{E(W)_U''}{E(W)_F''} > \frac{C_U N_U}{C_F N_F}$ then $\frac{dm_1}{d\lambda} < 0$. Note that with no information ($\lambda = 0$), $\frac{C_U}{C_F} = \frac{N_U}{N_F} = \frac{E(W)_U''}{E(W)_F''} = 1$, implying $\frac{dm_1}{d\lambda} = 0$. To determine what happens as the initial information increases, consider how $\frac{dm_1}{d\lambda}$ evaluated at no information changes with $\lambda$. In general we know that $\frac{\partial\frac{C_U}{C_F}}{\partial\lambda} > 0$ $\frac{\partial\frac{N_U}{N_F}}{\partial\lambda} < 0$ and $\frac{\partial\frac{E(W)_U''}{E(W)_F''}}{\partial\lambda} > 0$. For simplicity, assume $C'''' = B'''' = 0$.

First, let us consider the derivative of each of these terms with respect to a marginal decrease in the quality of information. To start, consider the ratio of cost $\frac{\partial C_U''}{\partial\lambda} = C'''\frac{\partial m_2^U}{\partial\lambda} > 0$. Similarly, $\frac{\partial C_F''}{\partial\lambda} = C'''\frac{\partial m_2^F}{\partial\lambda} < 0$. At no information ($\lambda = 0$), assuming small changes, $\frac{\partial\left(m_2^U\right)}{\partial\lambda} = -\frac{\partial\left(m_2^F\right)}{\partial\lambda}$ the change in the ratio of the two costs simplifies to:

$$(4A.2)\ \frac{\partial\frac{C_U}{C_F}}{\partial\lambda} = 2\frac{C'''}{C''}\left|\frac{\partial(m_2)}{\partial\lambda}\right|$$

Next, consider the change in average marginal damage with a change in information quality.

$$\frac{\partial N_u}{\partial \lambda} = \frac{\partial\left(B'_{HU} - B'_{LU}\right)}{\partial \lambda} = B''(m_0 + \delta m_1^u - H)\frac{\partial\left(m_1 + \delta m_2^U\right)}{\partial \lambda} - B''(m_0 + \delta m_1^u + L)\frac{\partial\left(m_1 + \delta m_2^U\right)}{\partial \lambda}$$

$$= \left(B''_{HU} - B''_{LU}\right)\frac{\partial\left(m_1 + \delta m_2^U\right)}{\partial \lambda} > 0 \text{ since } \frac{\partial\left(m_1 + \delta m_2^U\right)}{\partial \lambda} < 0 \text{ and } B''_{HU} < B''_{LU} < 0.$$

$$\frac{\partial N_F}{\partial \lambda} = \frac{\partial\left(B'_{HF} - B'_{LF}\right)}{\partial \lambda} = B''(m_0 + \delta m_1^F - H)\frac{\partial\left(m_1 + \delta m_2^F\right)}{\partial \lambda} - B''(m_0 + \delta m_1^F - L)\frac{\partial\left(m_1 + \delta m_2^F\right)}{\partial \lambda}$$

$$= \left(B''_{HF} - B''_{LF}\right)\frac{\partial\left(m_1 + \delta m_2^F\right)}{\partial \lambda} > 0 \;\text{ since }\; \frac{\partial\left(m_1 + \delta m_2^F\right)}{\partial \lambda} < 0 \text{ and } B''_{HF} < B''_{LF} < 0.$$

Using the above, we can derive the change in the ratio of average marginal damage with a decrease in information quality:

$$\text{(4A.3)} \quad \frac{\partial \frac{N_U}{N_F}}{\partial \lambda} = \frac{1}{N_F^2}\left(\begin{array}{c} N_F\left(B''_{HU} - B''_{LU}\right)\frac{\partial\left(m_1 + \delta m_2^U\right)}{\partial \lambda} \\ -N_U\left(B''_{HF} - B''_{LF}\right)\frac{\partial\left(m_1 + \delta m_2^F\right)}{\partial \lambda} \end{array}\right) > 0.$$

At no information, an increase in unfavorable information will increase mitigation in the second period by the same amount that favorable information will decrease second-period mitigation, $\frac{\partial\left(m_2^U\right)}{\partial \lambda} = -\frac{\partial\left(m_2^F\right)}{\partial \lambda}$, and A.3 simplifies to:

$2\delta\left(\frac{B''_H - B''_L}{N}\right)\left|\frac{\partial(m_2)}{\partial \lambda}\right|$. Further, note that $\frac{B''_H - B''_L}{H - L} = -B'''$ and $\frac{N}{H-L} = -B''(\overline{m_2})$. Thus, (A.3) simplifies to:

$$\text{(4A.4)} \quad \frac{\partial \frac{N_U}{N_F}}{\partial \lambda} = 2\delta\frac{B'''}{B''}\left|\frac{\partial(m_2)}{\partial \lambda}\right| < 0.$$

Combining the two derivatives, we can solve for $\frac{\partial \frac{C''_U N_U}{C''_F N_F}}{\partial \lambda}$ at $\lambda = 0$.

$$\text{(4A.5)} \quad \frac{\partial \frac{C''_U N_U}{C''_F N_F}}{\partial \lambda} = \frac{C''_U}{C''_F}\left[2\delta\frac{B'''}{B''}\left|\frac{\partial(m_2)}{\partial \lambda}\right|\right] - \frac{N_U}{N_F}\left[2\frac{C'''}{C''}\left|\frac{\partial(m_2)}{\partial \lambda}\right|\right].$$

Since at no information, the slope of marginal cost and average marginal damage at favorable and unfavorable information are the same, $\frac{C''_U}{C''_F} = \frac{N_U}{N_F} = 1$, the above can be simplified to:

(4A.6) $\frac{\partial \frac{C''_U N_U}{C''_F N_F}}{\partial \lambda} = 2 \left| \frac{\partial (m_1)}{\partial \lambda} \right| \left( \delta \frac{B'''}{B''} + \frac{C'''}{C''} \right)$

Next, we need to solve how the second derivatives of expected welfare with respect to mitigation in the second period change with a decrease in information quality.

$$\frac{\partial E(W)''_U}{\partial \lambda} = \delta^2 \frac{\partial A_U}{\partial \lambda} + \frac{1}{\beta} \frac{\partial C''_U}{\partial \lambda} \text{ where } A''_U \equiv -2 \left[ \left( \tfrac{1}{2} - \lambda \right) B''_{LU} + \lambda B''_{HU} \right]$$
$$= 2\delta^2 \left( B''_{HU} - B''_{LU} - \frac{1}{2} B''' \frac{\partial \left( m_1 + \delta m_2^U \right)}{\partial \lambda} \right) + \frac{1}{\beta} C''' \frac{\partial \left( m_1^U \right)}{\partial \lambda}$$

$$\frac{\partial E(W)''_F}{\partial \lambda} = \delta^2 \frac{\partial A_F}{\partial \lambda} + \frac{1}{\beta} \frac{\partial C''_F}{\partial \lambda} \text{ where } A''_F \equiv -2 \left[ \lambda B''_{LF} + \left( \tfrac{1}{2} - \lambda \right) B''_{HF} \right]$$
$$= -2\delta^2 \left( B''_{HF} - B''_{LF} + \frac{1}{2} B''' \frac{\partial \left( m_1 + \delta m_2^F \right)}{\partial \lambda} \right) + \frac{1}{\beta} C''' \frac{\partial \left( m_1^F \right)}{\partial \lambda}. \text{ Thus,}$$

$$\frac{\partial \frac{E(W)''_U}{E(W)''_F}}{\partial \lambda} = \left( \begin{array}{c} \frac{1}{E(W)''_F} \left[ \begin{array}{c} 2\delta^2 \left( B''_{HU} - B''_{LU} - \frac{1}{2} B''' \frac{\partial \left( m_1 + \delta m_2^U \right)}{\partial \lambda} \right) \\ + \frac{1}{\beta} C''' \frac{\partial \left( m_1^U \right)}{\partial \alpha} \end{array} \right] \\ + \frac{E(W)''_U}{\left[ E(W)''_F \right]^2} \left[ \begin{array}{c} 2\delta^2 \left( B''_{HF} - B''_{LF} + \frac{1}{2} B''' \frac{\partial \left( m_1 + \delta m_2^F \right)}{\partial \lambda} \right) \\ - \frac{1}{\beta} C''' \frac{\partial \left( m_1^F \right)}{\partial \lambda} \end{array} \right] \end{array} \right)$$

At no information, $E(W)''_F = E(W)''_U$ giving:

(4A.7) $\frac{\partial \frac{E(W)''_U}{E(W)''_F}}{\partial \lambda} = \frac{\left[ 2\delta^2 \left( 2 \left( B''_H - B''_F \right) + \delta B''' \left| \frac{\partial (m_2)}{\partial \lambda} \right| \right) - \frac{2}{\beta} C''' \left| \frac{\partial (m_2)}{\partial \lambda} \right| \right]}{E(W)''}$

Now we have all partial derivatives as a function of $\frac{dm_2}{d\lambda}$. Solving $\frac{dm_2}{d\lambda}$ at no information using Cramer's rule from equation (7), we obtain:

(4A.8) $\frac{dm_2}{d\lambda} = \frac{2\delta N}{\left( \delta^2 A + \frac{1}{\beta} C'' \right)}$

We are interested in how $\frac{dm_1}{d\lambda}$ changes with a marginal increase in information, thus, with an increase in $\lambda$. We can address this by combining equation (8) above with (A.6) and (A.7). Given $\frac{E(W)''_U}{E(W)''_F} - \frac{C''_U N_U}{C''_F N_F} = 0$ at $\lambda = 0$, (4A.9) $\frac{\partial \frac{dm_1}{d\lambda}|_{\lambda=0}}{\partial\lambda} = \left[\frac{\partial \frac{E(W)''_U}{E(W)''_F}}{\partial\lambda} - \frac{\partial \frac{C''_U N_U}{C''_F N_F}}{\partial\lambda}\right]\Psi + \left(\frac{E(W)''_U}{E(W)''_F} - \frac{C''_U N_U}{C''_F N_F}\right)\frac{\partial\Psi}{\partial\lambda}$

$$= \left[-\frac{4\delta^2\left(B''_H - B''_L\right)}{\left(\delta^2 A'' + \frac{1}{\beta}C''\right)} + 2\frac{\partial(m_2)}{\partial\lambda}\left(\delta\frac{B'''}{B''} - \frac{C'''}{C''} - \frac{\delta^3 B''' - \frac{1}{\beta}C'''}{\left(\delta^2 A'' + \frac{1}{\beta}C''\right)}\right)\right]\Psi$$

where $\Psi$ is $\frac{2}{\beta}C''_F N_F\left(A_F + \frac{1}{\beta\delta^2}C''_F\right)\frac{1}{-2SOC}>0$. Substituting equation (A.8) for $\frac{dm_2}{d\lambda}$ , we obtain:

(4A.10) $\frac{\partial \frac{dm_1}{d\alpha}|_{\lambda=0}}{\partial\lambda} = . - \left[\begin{array}{c} \frac{4\delta^2\left(B''_H - B''_L\right)}{\left(\delta^2 A'' + \frac{1}{\beta}C''\right)} \\ + \frac{4\delta N}{\left(\delta^2 A + \frac{1}{\beta}C''\right)}\left(\delta\frac{B'''}{B''} - \frac{C'''}{C''} - \frac{\delta^3 B''' - \frac{1}{\beta}C'''}{\delta^2 A'' + \frac{1}{\beta}C''}\right)\end{array}\right]\Psi$

Note that $\frac{\left(B''_H - B''_L\right)/(H-L)}{N/(H-L)} = \frac{B'''}{B''}$. Further, notice that the last term in brackets is just the elasticity of the second order conditions with respect to mitigation in the second period, since $\delta^3 B''' - \frac{1}{\delta}C''' = \frac{\partial\left(\delta^2 B'' - \frac{1}{\beta}C''\right)}{\partial m_2} = 2\frac{\partial^3 E(W)}{\partial m_2^3}$ and $\delta^2 A'' + \frac{1}{\beta}C'' = -2\frac{\partial^2 E(W)}{\partial m_2^2}$. So we can rewrite (4A.10) as follows:

(4A.11) $\frac{\partial \frac{dm_1}{d\lambda}|_{\lambda=\frac{1}{4}}}{\partial\lambda} = \left[-2\delta\frac{B'''}{B''} - \frac{C'''}{C''} + \left(\frac{E(W)'''}{E(W)''}\right)\right]\left|\frac{dm_2}{d\lambda}\right|\Psi$

If equation (4A.11) is positive, the introduction of some information will decrease the initial level of mitigation. With no stock externality or discount rate, the elasticity of the slope of the marginal cost curve has to be twice the absolute value of the elasticity of the slope of marginal benefit curve plus the elasticity of the second derivative of expected welfare with respect to $m_2$ for an increase in information quality to have no effect on the initial level of mitigation.

For (4A.11) to be negative, $-2\delta\frac{B'''}{B''} - \frac{C'''}{C''} + \left(\frac{E(W)'''}{E(W)''}\right) < 0$. Multiplying through by $E(W)'' < 0$. We can then obtain the following condition.

(4A.12) $\left|\frac{C'''/C''}{B'''/B''}\right| - \frac{2}{\beta\delta}\left|\frac{C''}{B''}\right| < \delta.$

If $C''' = B'''$, (A.12) then requires the slope of the marginal cost curve to be less than half the slope of the marginal benefit curve ($C'' < \frac{1}{2}|B''|$). Even before considering their effect on the slope of the marginal cost or benefit curve, the introduction of a stock externality or discount rate increases the required difference between $C''$ and $|B''|$. Further, because it decreases the amount of effective mitigation, an increase in the stock externality (or a decrease in $\delta$) specifically increases $B''$; and because it decreases the amount of mitigation in the second period, it decreases $C''$. Discounting the future clearly decreases mitigation in both periods, and therefore a decrease in the discount factor $\beta$ increases $|B''|$ and decreases $C''$, also further increasing the difference.

If the absolute value of the slopes of the marginal cost and marginal benefit are the same, $C'' = |B''|$, the change in the slope of marginal cost has to be at least three times as large as the change in the slope of marginal benefit for (A.12) to hold. As above, the introduction of a discount rate or a stock externality increases the magnitude of this requirement. Thus, for the amount of initial mitigation to increase with an increase in quantity, i.e. for $\frac{dm_1}{d\lambda} > 0$, the curvature elasticity of cost must be more than twice the curvature elasticity of benefit, $\frac{C'''}{C''} > 2\frac{B'''}{B''}$.

### 9.4.1 5. Proof of proposition 2

To solve for the effect of the quality of information on total mitigation, we first need to solve for $\frac{dm_2^F}{d\lambda}$ and $\frac{dm_2^U}{d\lambda}$. To determine $\frac{dm_2^F}{d\lambda}$ and $\frac{dm_2^U}{d\lambda}$ we can once again use Cramer's rule from equation (7) above to obtain:

$$\text{(5A.1)}\quad \frac{dm_2^F}{d\lambda} = -\frac{1}{\beta\delta}\left[\begin{array}{c} \frac{2}{\beta}N_F C_1''\left(\delta^2 A_U + \frac{1}{\beta}C_U''\right) \\ +C_U''\left(N_F A_U + N_U A_F\right) \end{array}\right]\frac{1}{-SOC} < 0$$

Similarly, solving for $\frac{dm_2^U}{d\lambda}$ we obtain:

$$(5A.2)\ \frac{dm_2^U}{d\lambda} = \frac{1}{\beta\delta}\left[\begin{array}{c} \frac{2}{\beta}N_U C_1''\left(\delta^2 A_F + \frac{1}{\beta}C_F''\right) \\ +C_F''\left(N_F A_U + N_U A_F\right) \end{array}\right]\frac{1}{-SOC} > 0$$

To determine how total expected mitigation changes with information, we combine the above expressions: $\frac{dm_T}{d\lambda} = \frac{dm_1}{d\lambda} + \frac{1}{2}\left(\frac{dm_2^U}{d\lambda} + \frac{dm_2^F}{d\lambda}\right)$ obtaining (A.14)

$$(5A.3)\ \ \frac{dm_T}{d\lambda} = \left\{\begin{array}{c} 2\delta\left(C_F''N_F A_U - C_U''N_U A_F\right) \\ +\frac{2}{\beta\delta}C_F''C_U''\left(N_F - N_U\right) \\ +\left(C_U'' - C_F''\right)\left(N_F A_U + N_U A_F\right) \\ +\frac{2}{\beta}C_1''\left[\begin{array}{c} N_F\left(\delta^2 A_U + \frac{1}{\beta}C_U''\right) \\ -N_U\left(\delta^2 A_F + \frac{1}{\beta}C_F''\right) \end{array}\right] \end{array}\right\}\frac{1}{-2\beta\delta SOC}$$

Note that $N_F \geq N_U$, $C_U'' \geq C_F''$ and $A_U \geq A_F$, so that the only term with a sign in question is the first one: $C_F''N_F A_U - C_U''N_U A_F$.

If $C_F''N_F A_U - C_U''N_U A_F \geq 0$, then equation (A.14) is clearly positive or zero. If $C_F''N_F A_U - C_U''N_U A_F < 0$, note that the third line in (A.14) can be written as $\left[\left(C_U''N_U A_F - C_F N_F A_U\right) + \left(C_U''N_F A_U - C_F''N_U A_F\right)\right]$. The first difference in the brackets is positive by the assumption that $C_F''N_F A_U - C_U''N_U A_F < 0$. As for the second difference, since $N_F A_U > N_U A_F$, note that $C_U''N_F A_U > C_U''N_U A_F''$ and $C_F N_F A_U > C_F''N_U A_F$ implying the second difference is positive and larger than the first. Thus, $\left(C_U''N_F A_U - C_F''N_U A_F\right) > \left(C_U''N_U A_F - C_F N_F A_U\right) > 0$. Therefore, the magnitude of the third line has to be larger than the magnitude of the first, so $\left[\begin{array}{c} \left(C_U''N_U A_F - C_F N_F A_U\right) \\ +\left(C_U''N_F A_U - C_F''N_U A_F\right) \end{array}\right] > 2\delta\left(C_F N_F A_U - C_U N_U A_F\right)$. Therefore $\frac{dm_T}{d\lambda} \leq 0$, regardless of stock externality or discount rate.