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Statistics Based Modeling of Wind Speed and Wind Direction in Real Time Optimal Guidance Strategies via Ornstein-Uhlenbeck Stochastic Processes

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4.5 STATISTICS BASED MODELING OF WIND SPEED AND WIND DIRECTION IN REAL-TIME GUIDANCE STRATEGIES VIA ORNSTEIN-UHLENBECK STOCHASTIC PROCESSES

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1. INTRODUCTION

In real-time guidance strategies, the characteristics of the wind and wind modeling plays an important role in obtaining reasonable results from the devised algorithm (especially if the wind is the main driving mechanism of the trajectory optimization routine). It is desirable to have an as realistic as possible wind model to reflect the real nature and the true outcome of the proposed guidance strategies. Therefore, in this paper, it is aimed to come up with a statistics-based stochastic wind model that will reflect observed, true nature of the wind characteristics (namely wind magnitude and wind direction). This will provide a tool that could be used in trajectory optimization studies to achieve meaningful and realistic results in trajectory optimization calculations.

There are many valuable works in existing literature attacking the problem in hand from different perspectives. Komen and Hasselmann (1984) have investigated a fully developed wind seas spectrum approach for modeling wind behavior in sea dynamics. Janssen (1991) has studied quasi-linear theory of wind-wave generation applied to wave forecasting. Hellerman and Rosenstein (1983) concentrated on the wind stress analysis over the world ocean with uncertain analysis. Hasselmann, Allender, and Barnett (1985) worked on parametrization of the nonlinear energy transfer in a gravity wave spectrum. Kirk and Cox (1968) established the foundations of nonlinear modeling of wind dynamics in their thorough analysis of three dimensional velocity and density fields. Thomson and Cardone (1996) took a practical approach towards modeling the hurricane surface wind fields. Banner and Young (1994) analyzed spectral dissipation characteristics in evolution of wind waves. Bougeault (1991) provided useful information on modeling the specific trade-wind cumulus boundary layer. Holzworth (1967) demonstrated the fundamental principles of mixing depths, wind speeds and air pollution potential in United States. Crespo, Hernandez and Frandsen (1999) surveyed the modeling methods for wind turbine wakes and wind farms. Chapman (1985) provided a barotropic coastal ocean wind model for cross-shelf open boundaries.

In these studies, the general approach taken was to concentrate on a specific portion of the wind magnitude and wind modeling, rather trying to provide a generalist approach. But for the sake of trajectory optimization studies and to observe the impact of wind behavior in such methodologies, we are in need of a simple, yet valid and data driven stochastic wind model. To accomplish this, in this study, existing/recorded meteorological data (from weather stations located at airports in whole continental US) and publicly accessible NOAA (and IGRA) archives have been used. Based on these real data sets, it desired to devise an approximate wind model that will signify the behavior of wind magnitude and direction.

In this specific study, (to reflect the global behavior of wind characteristics), wind data sets from two East coast locations (namely from Albany, NY and Pittsburg, PA), two Mid-West locations (Green Bay, WI and Minneapolis, MN) and two West coast locations (Tucson, AZ and San Diego, CA) have been considered. With this, the intent is to demonstrate the global, long term and low-frequency wind behavior over different locations in continental US (varying from East Coast, to Mid-West and to West Coast). Moreover, the expectation with deriving such statistics based stochastic wind model is that it can be used with simplicity in not only aerospace engineering, but also in several other areas and applications. Some of these possible application areas may be wind turbines (wind load models), environmental engineering (atmospheric dispersion models) and so on. Thus, the main emphasis is on obtaining an approximate wind model, which will assist to demonstrate the overall outcome of the proposed guidance strategies in general wind conditions.

Obtaining an analytical wind model is still an ongoing research effort in Meteorological and Wind Engineering society and is a very complicated, highly challenging task. Derivation of a purely analytical

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expression for wind behavior as a function of pressure (gradients), temperature, humidity, surface roughness and density is an elaborate procedure and relates itself to Navier-Stokes, Conservation of Mass and Conservation of Momentum equations as well as Heat Equation and Diffusion Equations. To avoid such complex nature of airflow, to simplify the evaluation procedures and to increase the applicability of the proposed wind model in the field of engineering, a statistical study has been conducted to obtain a wind model expression. For this purpose, monthly data (for the year of 2010) from each (above mentioned) location has been considered and compared. In this process, not only monthly wind magnitude characteristics are evaluated, but also wind direction characteristics (for each location) have been investigated to devise a meaningful pattern for the analysis of wind behavior.

Here, the analysis has been broken into two parts: First, the change of wind magnitude and wind direction characteristics with respect to altitude has been analyzed. Second, horizontal wind magnitude and wind direction characteristics are examined. The following sections discuss the insights of these procedures in further detail.

2. VERTICAL WIND PROFILE

2.1 Wind Magnitude

The analysis starts with investigating recorded wind data sets extracted from weather stations. For the above mentioned stations, obtained wind data plots are given in Fig-1, which characterizes the change in wind magnitude with respect to altitude up to 20km (~ 65,600 [ft]).

In Fig-1, colored dots are observations from locations as specified in legends. With careful investigation of Fig-1, it is possible to see that for each month and altitude, the maximum and minimum wind magnitude values are varying from location to location, as well as from altitude to altitude. This brings the concept of uncertainty into the picture. This also introduces the benefit of describing the statistical wind model as a function of uncertainties around the mean value.

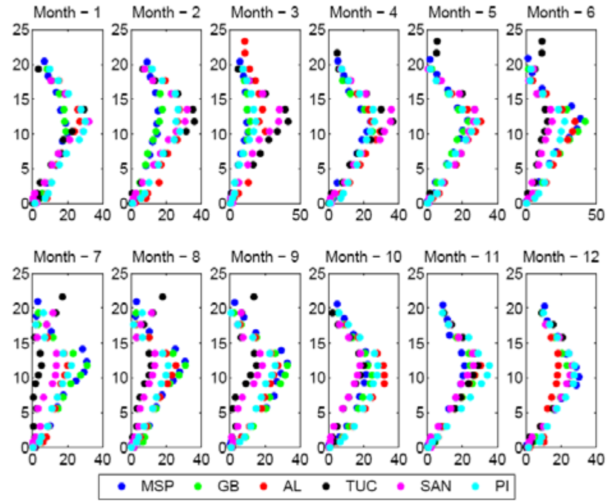

Figure – 1 12months statistical wind data obtained from 6 different states for year 2010. X-label: Wind magnitude [m/sec] and Y-label: Altitude [km]

To address this, next, mean value and uncertainty characteristics for vertical wind profile has been obtained. Associated uncertainties can also be defined with respect to an upper and lower bound as a function of maximum and minimum observed values, at each specific altitude. With this, it is appropriate to plot mean value characteristics together with uncertainty bounds (error bars) for each specific month (during the year of 2010).

Using all this information, in aid to obtain an approximate model of observed wind magnitude data, it is possible to fit several linear/nonlinear curves to match the observed wind characteristics. In our case, for its simplicity and use of algebraic manipulations, a polynomial fit is attained to specify changes in wind magnitude with respect to altitude. Finally, if all the measured data sets are combined, average vertical wind profile (with given uncertainty bounds) is obtained. Each uncertainty region consists of an upper bound which defines the maximum overall observed wind magnitude at that altitude, while the lower bound defines the overall minimum observed wind magnitude at that specific altitude. For clarification, the boundaries of uncertainty regions specify ``the bandwidth of the change in wind magnitude with respect to altitude''. As a result, obtained fit for the mean of the observed vertical wind magnitude is a 5th order polynomial and is given in Eq.(1).

$$P_{W_{avg}} = 0.0003h^5 - 0.0127\,\mathrm{h}^4 + 0.1814\,\mathrm{h}^3 - 1.0250\,\mathrm{h}^2 + 4.0995\,\mathrm{h} + 0.6336 \quad (1)$$

This model will be used to generate wind magnitude data for a specific flight altitude. As a further step in this study, it is also possible to obtain data sets for a collection of several (for example, 10) years of statistical data and associated uncertainties at each location with respect to different altitudes. With this extended approach, it will also be possible to define the vertical wind model for change magnitude as a function of altitude, which is based on statistically obtained wind data. This could be extended to the 52 states all around the continental US. This is an ongoing research effort, and will be presented in future research efforts, and is not pursued in this study.

2.2 *Wind Direction*

Following the analysis of wind magnitude for vertical profiles, the same analysis is conducted for the wind direction. The corresponding data for the above selected locations is plotted and obtained results are as given in Fig-2.

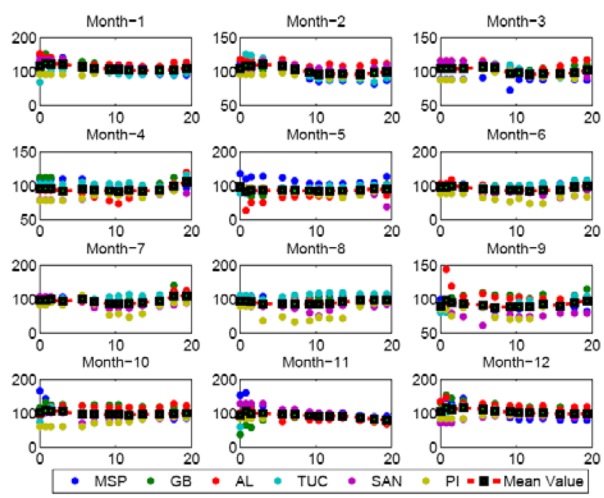

Figure – 2 Vertical wind profile characteristics for selected locations over 2010. X-label: Altitude [km] and Y-label: Wind direction (w.r.t. true north) [deg].

In order to derive sensible wind models, the average of the observed wind direction characteristics over different altitudes has been taken into consideration. As it is justifiable from Fig-2, change in wind direction is significant and noticeable, up to ~9[km]. Starting from ~9[km] up to ~16 [km], wind magnitude tends to remain constant and mostly deviates around a constant value of 90[deg] (with slight variations of ~1-4[deg]).

This natural observation gives the opportunity to the designer that, if the specific operation region (of aerial vehicle in hand) is within the range of (9-16km), it is fair to assume that the wind direction is approximately constant and deviation around the mean is within certain negligible amount (~ [1-4[deg]]). If it is favorable, this condition could be used in derivations and may also aid to further simplify the proposed wind model, increase the efficiency of algebraic manipulations and reduce the computational cost of real-time calculations. In the following sections, it will be observed that this approach has not been used for this study, but has been provided here as an intuitive detail about the study.

$$\psi_{W_{avg}} = 0.0007\,\mathrm{h}^5 - 0.0195\,\mathrm{h}^4 + 0.2772\,\mathrm{h}^3 - 1.9711\,\mathrm{h}^2 + 5.1323\,\mathrm{h} + 102.1122 \quad (2)$$

Once the mean value of observed wind direction data is obtained, the corresponding polynomial fits results in a 5^{th} -order polynomial and is as given in Eq.(2). In addition, it is also possible to analyze the uncertainties around the mean value, which will be investigated in further detail in the following sections.

With the definition of uncertainties around the mean values of both wind magnitude and wind direction obtained, the next step is to investigate the core of the wind model, which is the analytical modeling of wind uncertainty around the mean value. This analysis has been conducted in the following section and detailed explanations are provided.

3. WIND UNCERTAINTY MODELING

3.1 *Uncertainty Modeling for Wind Magnitude*

Through the analysis conducted in previous section, it is possible to deduct that uncertainty characteristics (i.e. deviation around the mean) can be defined based on observed upper and lower limits, for both wind magnitude and wind direction.

At this point, it is important to obtain a stochastic uncertainty model which will help to generate distributed stochastic data in simulations. For this purpose, it is assumed that, the wind magnitude and wind direction are consisted of two main parts: the mean value and the uncertainty (deviation) around the mean. This has been formulated in an efficient manner as shown in Eq.(3),

$$\begin{aligned} W_m &= W_{mean}(h) + w_m \\ \Psi_m &= \Psi_{mean}(h) + \psi_m \end{aligned} \tag{3}$$

where W_{mean} and ψ_{mean} depicts the mean values of wind magnitude and wind direction, respectively. In addition, w_m and ψ_m stands for variations (uncertainties) around the mean value, for wind magnitude and wind direction, all at once. Following to this formulation, the next step will be to derive models that will aid to define wind uncertainties (deviations) around the mean value, as a stochastic process.

For simplicity in design and modeling, without the loss of generality, variations (uncertainties) around the mean value will be modeled as a first order stochastic differential equation system driven by white noise. Here, the stochastic differential equation defines a first order differential equation with a stochastic nature, which it has to be dealt under the umbrella of Stochastic Differential Equations (SDEs). This will constitute the next step of derivation.

3.1.1. <u>Ornstein - Uhlenbeck Process:</u>

In literature, stochastic differential equations have been studied extensively since the 1960ies and still are an ongoing research topic. Throughout the years, stochastic differential equations has found ground for many applications varying from statistical physics to finance, from molecular biology to aerospace applications, and are still extensively used in numerous areas and purposes.

In this context, one of the vastly studied stochastic differential equation is Langevin equation. In compact form, it could be written as given in Eq.(4).

$$\begin{aligned} \frac{dX(t)}{dx} &= -\mu X(t) + \sigma \frac{dW_t}{dt} \\ &= -\mu X(t) + \sigma v \end{aligned} \tag{4}$$

Here, μ and σ are positive constants peculiar to the system itself, and υ is the white noise that drives the stochastic evolution of given dynamics. Since, the main interest in this section is to model vertical wind (magnitude and/or direction) behavior with respect to the altitude, it is clear that for the case of modeling of wind uncertainties, the Langevin equation becomes a function of altitude.

But unfortunately, with this form of Langevin equation, it is not possible to come up with an exact analytical solution in terms of a simple stochastic process. Instead, as stated in Sauer (2010), the solution to the Langevin equation is provided numerically. This specific numerical solution is named as Ornstein-Uhlenbeck process in mathematics literature, after Leonard Ornstein and George Eugene Uhlenbeck. Once the stochastic system is converted to an Ornstein-Uhlenbeck process, such a system can be solved using several numerical techniques.

Arithmetic Ornstein-Uhlenbeck process is an extension to Ornstein-Uhlenbeck process and is provided in literature as a numerical recipe for the original process. In arithmetic Ornstein-Uhlenbeck process, the fundamental formulation has been slightly changed, so that there exists a driving force/mechanism which enforces the system dynamics to go back to a steady-state (mean) value with time. And therefore, it is also called as Mean-Reverting Ornstein-Uhlenbeck process. Furthermore, the Arithmetic (Mean-Reverting) Ornstein-Uhlenbeck process is considered as the continuous branch of the discreet-time AR(1) process. In this study, the emphasis is given on the implementation of Arithmetic Ornstein-Uhlenbeck process, and therefore, the main concentration will be focused on obtaining numerical recipes to implement the problem in computer (thus, in real-time on-board instruments). In order to be able to implement the Arithmetic Ornstein-Uhlenbeck process in a computer, it is desired to discretize existing formulation with respect to the dependent variable (in our case altitude). The solution itself is given in Kloeden and Platen (1992) as presented in Eq.(5),

$$\begin{aligned} X(H) = X(0)exp(-H) &+ (1 - \exp(-\theta H))\mu \\ &+ \sigma \int_0^H \exp(\theta h)\, dz(h) \end{aligned} \tag{5}$$

And then, it is possible to apply the result in computer simulations. The process, as elaborated in further detail in Dixit and Pindyck (1994), is defined as given in Eq.(6)

$$X_h = X_{h-1}\exp(-\theta\Delta h) + \mu(1-\exp(-\theta\Delta h)) + \sigma\sqrt{\frac{1-\exp(-2\theta\Delta h)}{2\theta}}N(0,1) \quad (6)$$

This provides a numerical guideline that can be applied with ease in any computational platform. After obtaining such an exact numerical solution for a desired uncertainty model (with the help of Mean-Reverting Ornstein-Uhlenbeck process), the intention is to come up with mean and standard deviation characteristics (μ and σ, respectively), which will complete the stochastic uncertainty model analysis. Here, the characterization of the nature of the uncertainty distribution in wind magnitude (namely the characteristics of the variation around the mean value) is portrayed by the Gaussian distribution, which explains the behavior of the uncertainty characteristics associated with the wind magnitude and/or direction. Here, using given data sets, it is possible to obtain (and define) the Gaussian distribution of the uncertainties around the mean value for each specific altitude. Having derived all the necessary components (μ and σ characteristics) for the stochastic wind model, 100 Monte Carlo simulations were conducted leading to the stochastic wind model results as given in Fig-3. It is possible to see from Fig-3 that the obtained stochastic wind model is a very good fit and approximation of the given data set. It is not only able to cover the general stochastic nature of the wind behavior, but also able to maintain the observed upper and lower bounds.

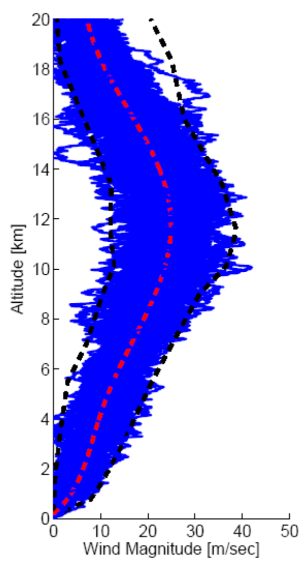

Figure – 3 100 Monte Carlo simulations of obtained stochastic wind model for vertical wind magnitude with θ = 1

Here, it is advisable to note that ``the nature of the given 100 Monte Carlo simulations'' and ``how well it maintains the observed upper and lower bounds'', heavily depends on the intensity parameter-θ.

3.2 Uncertainty Modeling for Wind Direction

It is possible to follow the same procedure for the derivation of mean and uncertainty values in wind direction, as well. If the data points are plotted together with the corresponding wind direction values, obtained results are as given in Fig-5.

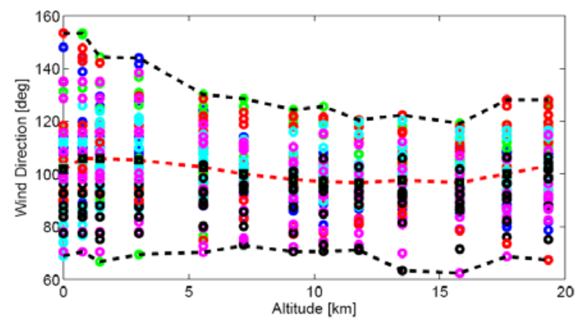

Figure – 4 Data points for selected 6 locations among continental US.

If Fig-4 is investigated in further detail, it is possible to see that using the uncertainty bandwidth, not only the mean value (red dashed line) can be defined, but also the characteristic deviation around the mean can also be derived. Both the 5^{th} order polynomial fit and characterized mean wind direction behavior were already presented in Eq.(2) and corresponding normalized distributions could be easily derived using the analogy presented in previous section.

Using given data points, it is also possible to come up with Gaussian distribution fits, which will help to characterize the uncertainty in wind direction around the mean value. Once all the necessary components for a were derived, 100 Monte Carlo simulations were ran and the obtained results of stochastic wind direction model are as given in Fig-5.

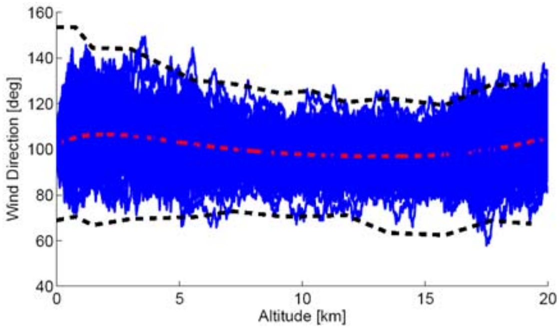

Figure – 5 100 Monte Carlo simulations of obtained stochastic wind model for horizontal wind direction with θ = 1.

It is possible to witness in Fig-5 that obtained stochastic wind model for wind direction is a very good fit and obtained results are able to mimic the nature of the stochastic wind direction characteristics very well. Through this analysis, it is not only possible to reflect the general stochastic nature of the wind direction but also is able to maintain the observed upper and lower bounds in wind direction profile. As it was noted beforehand, for the case of wind magnitude, again, it is also good to note that ``the nature of the given 100 Monte Carlo simulations'' and ``how well it maintains the observed upper and lower bounds'', heavily depends on the intensity parameter-θ.

With all the aforementioned derivations and analysis, modeling of vertical wind profile characteristics is complete.

4. CONCLUSIONS

Analysis of real-time guidance strategies in the presence of wind is a concept that heavily relies on the suggested wind formulation. Especially when the outcomes of the proposed strategies are of significant importance, the wind model formulation becomes critical. For this reason, and to be able to obtain realistic outcomes from real-time guidance strategies based on wind utilization methodologies, a statistics based wind model is derived in paper. With the nature of statistics based derivation, it is aimed to not only represent true stochastic nature of the wind, but also to assess the true outcomes of the proposed guidance strategies in realistic wind conditions. For this purpose, real measurement data collected from different NOAA weather stations are utilized for the analysis. For the mathematical formulation, Ornstein-Uhlenbeck process is deployed which is a stochastic differential equation driven by the white noise and obtained data/measurement statistics. At the end of the analysis, it has been shown that both wind magnitude and wind direction modeling are successfully accomplished and a simple (yet effective) engineering model of the wind behavior has been obtained to be utilized in trajectory optimization problems.

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