2010

The Ontological Status of Systems at the Juncture of Quantum Mechanics and Relativity

Juan Ferret, *University of Texas at El Paso*

The Ontological Status of Systems at the Juncture of Quantum Mechanics and Relativity

Juan Ferret

March 1, 2007

Contents

> We need to go back to the insights behind general relativity and quantum field theory, learn to hold them together in our minds, and dare to imagine a world more strange, more beautiful, but ultimately more reasonable than our current theories of it. For this daunting task, philosophical reflection is bound to be of help. – John C. Baez

1 Introduction

Even a brief analysis of the foundations of mechanics from Galileo to current approaches to quantum gravity reveal the evolution of the concepts of relativity of motion and the relational nature of systems. Galileo showed that velocities are relative properties of systems. Assuming inertial frames of reference, an entity's velocity depends on the frame used and no preferred frame is needed. Einstein expanded the relativity of motion of inertial frames to non-inertial frames in general relativity (GR). This expansion of the relativity of all types of motion dragged along a deeper and more fundamental relational concept. Quantum mechanics (QM) makes similar demands on the relations between systems and a sufficiently general account of QM proves consistent with the conceptual developments of the concept of motion in GR. Bits and pieces of these positions have been proposed and some even accepted in large by the community of scientists and philosophers of physics, but the relativity of reference frames and relational nature of systems as fundamental notions in QM, GR, and quantum gravity has lacked a consistent and dedicated treatment.

In the first part of this paper, an introductory example from classical mechanics shows how the concepts of relativity and relations of systems are already part of the conceptual development of frames of reference in physics. For this analysis, I introduce a distinction between the concepts of relative and relational frames of reference with the aim to clarify the state of systems, first in classical mechanics, then in QM, and in Relativity. I use the term relative to mean primarily the epistemic idea that measurement or knowledge of a system's properties of motion depends on a reference frame of choice. The term relative is often used for the relativity of motion, but as Einstein introduced in Special Relativity (SR), it can also be of other properties such as extension or mass. By relational I mean a deeper, more ontological idea about the necessity of boundaries or interactions for the existence of a system. In a relational account a system can only become a system in interaction with at least one other system that will provide the original system with boundaries. A consequence of this relational account will be that the dynamical interactions will be crucial for accounting for the state of the system.

The paper also examines Carlo Rovelli's account of relational quantum mechanics, his general views on GR, and, briefly, his approach to quantum gravity, Loop Quantum Gravity (LQG), to show that his account is in line with the evolution of the ideas of relative and relational frames of reference that emerge from a conceptual examination of some elementary phenomena in classical mechanics, QM, and relativity. Along the way, I offer some criticism of Rovelli's treatment to clarify the status of systems in mechanics.

The main aim of this paper is to examine the ontological status of entities as systems or parts of systems in classical mechanics, quantum physics, general relativity to show that there is a clear line of development that LQG attempts to fulfill. This analysis will also serve to offer an ontological foundation for LQG and similar approaches to quantum gravity. From this ontological foundation, which is part of a larger programme to develop a dynamic ontology of relations for mechanics, some recommendations are put forth for understanding entities in LQG.[1]

2 Status of Systems in Classical Mechanics

The development of Galilean relativity of motion and Newton's laws of dynamics have been two early cornerstones of the development of classical mechanics. Galilean relativity shows that the motion of an entity is dependent on (relative to) the reference frame chosen. So if we ask what is the motion of an entity from the reference frame of that same entity, then the motion is always null. Instead, if we inquiry as to the motion of the same entity from the point of view of another entity, the motion or velocity may have a non-zero value. To speak meaningfully about velocity in the Galilean perspective one needs to specify a reference frame and any reference frame will do. The choice of reference frame is arbitrary. Since there is no preferred reference frame, we also know that motion is a meaningful property of a system only when a reference frame is selected. In short, motion of a system is relative to a chosen reference frame.

The laws of dynamics of Newton, and in particular his law of gravity, instructed us into realizing that the motion of objects depend on the relationship with other objects that affect its trajectory. An entity's motion is, through an action-at-a-distance, affected by the gravitational pull of all other entities. The entities were often assumed to be particle-like or corpuscles and the laws of motion explained the means of interaction among these corpuscles.

The development of Laplacian and Hamiltonian dynamics moved mechanics toward the recognition of the entity or the aggregation of entities and their present and future behavior. The present behavior is accounted for in terms of kinetic energy, T, the amount of work that a system exhibits thanks to its speed, and the future behavior is accounted as part of the system in terms of potential energy, V, the amount of work that can be exhibited in relation to a chosen reference frame. The potential energy is the mechanical energy that the system has in store but has not

[1]This main sketch and history of this programme is in development in conjunction with Alexis Saint-Ours in "Dynamic Relational Ontology" (forthcoming).

yet exhibited and an object under the pull of gravity has the potential to exhibit displacement if some constraints are removed. In the Hamiltonian picture, for instance, the energy (in terms Hamiltonian $H = T + V$) becomes the central notion that defines the state and the evolution of the system. With this move, mechanics became the account of the dynamics of a system, where the Hamiltonian encapsulates the full description and evolution of the system in a particular scenario. Since the Hamiltonian is defined in terms of kinetic and potential energy, the system is then basically defined by its energy. So we can conceive of the Hamiltonian as the energy formulation of the system and therefore the entity.

2.1 An Example from Elementary Classical Mechanics

Consider the case of a stationary car hanging at the very edge of a cliff h meters deep from the reference frame of the ground. The car has zero kinetic energy, since it is not moving, and a potential energy mgh, where m is the mass of the car, h the height of the cliff, and g the gravitational constant, which is the gravitational pull of the earth on all objects near the surface. In general, the energy of the system is:

$$E = T + V$$

and the kinetic energy is

$$T = \frac{1}{2} m v_{h'}^2$$

where m is the mass of the object and v the velocity.[2] Before the car starts falling the energy of the system is:

$$E_h = 0 + mgh$$

Once the car starts falling, the car gains a small velocity, the kinetic energy becomes non-zero and the potential energy decreases because the car is no longer at h, but at shorter distance from the ground, h'.

$$E_{h'} = \frac{1}{2} m v_{h'}^2 + mgh'$$

Right before the car crashes to the ground, the kinetic energy is large because the velocity is large, but the potential energy of the car is almost zero since h' is now

[2] Where $h' = h(y)$ and $h = max(h(y))$.

very small. Once the car hits the ground, the kinetic energy is at its maximum (v_0 is the speed right before impact) and the potential energy is zero since h' is zero.

$$E_0 = \frac{1}{2}mv_0^2 + 0$$

If we treat this system as a closed system, the energy is conserved, thus $E_h = E_{h'} = E_0$ and

$$\frac{1}{2}mv_0^2 = mgh$$

This simple exercise from introductory physics illustrates the crucial role of reference frames. Consider the property of position for a moment. Imagine that in the example we remove the cliff and everything else in the universe (including its boundaries) except by the car; would it make sense to say that the car has a position? The only way we can respond positively to this question would be by assuming the existence of a background space frame of reference from which we could determine the position of the object. This background would offer a fixed frame of reference from which to measure the motion of entities and would thus be absolute. Claiming that position is given by the existence of a background space forces a demand for strong physical reasons (like Newton thought he had with the rotating bucket experiment) or evidence for a preferred absolute reference frame. If this background space is not found or shown to exist via solid theoretical exposition, then the property of position needs to be thought in purely relative terms, where it would make sense to speak of the position of an object only relative to a reference frame. Thus, position is a relative property of a classical system if we do not assume the existence of a space container. In our example, the choice of the ground as a reference frame is as valid as any other (the top of the cliff, Alpha Centauri, etc.). Yet some choices are more natural and more effective for calculating and considering an event than others. Since the position of the bottom of the cliff will become the likely resting place of the entity (or what is left of it), then it is a natural choice for a reference frame. In sum, it is meaningless to speak of the position of the entity without reference to a frame from which to assert that the entity has a position relative to it. Unless, of course, we were to assume the existence of an absolute reference frame or background from which we could absolutely determine the state of the entity.

2.2 Relative vs. Relational in Classical Mechanics

The relative state of properties of motion and the system are natural consequences of classical mechanics when the notion of a fixed background of space and time, as an absolute frame of reference, is not assumed. A relational system entails that the properties of the system express a relation between two interacting systems in addition to the relative state view that we need a separate frame of reference to speak meaningfully of properties of motion. The difference between relative and relational is a conceptual distinction I introduce to clarify the role of interactions between systems and the role of reference frames. A main difference between the relative and the relational is that the relative is primarily epistemological while the relational is primarily ontological. By relational we mean that the reference frame is not solely an epistemic tool for being able to talk meaningfully of or measure position, velocity, or energy, but rather that the reference frame has some ontological significance for the account of the properties of the system and, of course, the system itself. This means that the system, to be a system, needs another system to interact or relate. A system needs boundaries, and the boundaries of a system are provided by other system(s). The relational account takes the boundaries to imply an interaction between the original entity with another system or entity. In our example, the car's energy is in relation (potentially albeit) with the ground and the car's energy (and thus the system) is thus defined as being in that relation. The system's energy depends on the possible interaction (which may become real) with the ground.

To become clear on this difference an examination ensues to show that, like position, velocity is a relative property of bodies, but it is not relational. In the example, the velocity of the car depends on the reference frame of the ground, but any other reference frame would serve. We could have chosen the reference frame of Alpha Centauri or the Eiffel Tower. The property of velocity is relative to the choice of reference frame and that choice is arbitrary. Yet, clearly, some choices of reference frames may be preferred over others in some circumstances, since we often want to know the velocity of bodies with respect with frames of reference that are in or may potentially be in interaction.[3] The choice of the ground for our reference frame is made because of the anticipated interaction of the car with the ground. We presume

[3] This potential of interaction is what curtails the limitless choices of reference frames into a smaller subgroup of reference frames. This smaller subgroup are local reference frames that may interact with the body. This is what I will describe shortly as relational. What is local, of course, varies on the scale of our system, so Alpha Centauri may turn out to be a local reference frame in some circumstances. The idea of potential interaction harbors a notion of time-space scale, since anything within the range of present and future light-cones can become potential interactions. Wait long enough and the car may be in interaction with a body now far away.

that someone in the car would care to know that information about the velocity of the car with respect to the ground rather than with respect to Alpha Centauri. This preference is due to the relation or potential relation between the body and the ground, however, one can still meaningfully ask about the velocity of the car from the reference frame of Alpha Centauri. There is no reference frame for velocity that is absolute, truer, or ontologically more fundamental so the choice of reference frame is arbitrary.

Since kinetic energy is proportional to the product of mass and velocity squared, $T = \frac{1}{2}mv^2$, it is a property of a system relative to the reference frame chosen. From the reference frame of the body, the kinetic energy is zero at all times, since no object moves with reference to itself. There is no intrinsic velocity and, therefore, no intrinsic kinetic energy.[4] In the example, the kinetic energy of the car from the car itself is as valid as the kinetic energy consideration from the frame of reference of the ground. The difference lies in that the former tell us little of the coming interaction while the latter is setup with the interaction in mind. We choose the ground as our reference frame since we were particularly interested in finding the state of the energy upon impact. Hence, the choice of reference frame for determining the kinetic energy is modulated by the fact that we anticipate an interaction, in this case, with the boundary which hosts the chosen reference frame. Although we could chose any reference frame we wanted to determine the kinetic energy of a system, often our choice is dependent on the information we want to draw from a given interaction. In this case, we want to know the dynamics of interaction between the car and the ground boundary.

Since the potential energy is dependent on the position and the position is relative, then the potential energy is also relative to the chosen reference frame. From the reference frame of the car, the potential energy would be zero at all times as well since the distance the body travels with reference to itself is always null. Thus the total mechanical energy of a body in reference to itself is zero. The energy can become non-zero only when assessed from an outside reference frame, i.e., the reference frame of the ground as per our example. Obviously, there is no dynamics of an entity measured from its own reference frame, so to do dynamics we must determine the state of the system from a reference frame outside of it.[5] Since the

[4] Jeremy Butterfield, in a recent article "Against *Pointillisme* about Mechanics", argues against the idea that velocity can be intrinsic and points to the fundamental physical fact of the necessity of an outside reference frame to speak meaningfully of motion. It is true that we can speak of a body's velocity from its own reference frame, but clearly this carries little explanatory power or physical significance since it will always be null.

[5] Often, by the system a theorist entails the body being measured, the reference frame outside of the body from which we measure its dynamics, and the boundaries of the system, which often,

potential energy exists by virtue of the reference frame-boundary outside of the entity, the potential energy is also relational. The preference of treating the ground as a 'preferred' reference frame lies in the interaction that will occur between car and ground. Consider now the following situation so we can understand the meaning of potential energy of the system in terms of a future interaction. Imagine that as the car begins to fall we remove a fake ground and reveal the true ground level 50 meters below. What happens to the potential energy? The potential energy was measured to be mgh and now is measured at $mg(h+50)$. Changing the system (by moving the point of impact, the frame of reference) changes the potential energy. The potential energy will vary depending on the reference frame we choose to measure the system. We can still speak of the potential energy at a distance h, but that would constitute a partial account of the mechanical potential of the system. The full potential occurs how at $h + 50$.

Imagine further that we make the cliff to be excessively large, $h \to \infty$. The potential energy of the system becomes infinite large and, hence, the total energy becomes infinitely large. In the absence of physical boundaries, or when the reference frame is infinitely far away, the potential energy becomes infinite and the system is then ill defined. Thus, the potential energy of a system is relational since it only makes sense to speak of a system's energy, or to speak of a system altogether, in the presence of some boundary. Changing the boundaries of the system also affect the final kinetic energy of the system, and, obviously, the final energy of the system. The energy of a system is relative to the reference frame chosen (like velocity), but is also dependent on the boundaries chosen for the system. The energy is relational to the choice of boundary of the system, that is, the energy of an entity is relative to the frame of reference from which we consider the physical situation. In the first scenario, the system was the car-entity as referenced by the cliff and the ground at a distance h. In the second scenario, the system was the car-entity, the cliff and the ground at a distance $h + 50$. The energy of a system is relative to the choice of reference frame but some reference frames will be preferred over than others because of the possible interaction. This interaction is a relation between entities (the car and the ground). Although the choice for a reference frame for position and velocity is purely relative, the relational nature of energy ensures that some reference frames, those that contain a present or future interaction, will be preferred. In the example, the choice of car and cliff as the system is altered when the ground is removed to reveal the new ground 50 meters below. It turns out that our system was slightly different than anticipated, and the energy of the system varies according to its boundaries.

---

but not always, host the reference frame. In our example, the system was the car, the reference frame of the ground, and the boundaries of the system which include the ground.

Since Hamiltonian and Lagrangian formulations tell us that we can speak of the system as the Hamiltonian-Energy and the laws of motion, then since the energy of a system is relative to the choice of reference frame, the state of the system is obviously also relative. The state-energy of the system is then relational to the choice of the system and its boundaries from which it will experience an interaction. The relational aspect of classical mechanics is subdued since we can treat the interaction of the systems boundaries to be minimal or negligible. That is, that we can accurately enough treat the system as closed. Saying that the entity has energy is meaningless if a reference frame is not specified.[6] A system is made by the interaction of the entity and its boundaries. So, when we speak of an entity, we necessarily pick a reference frame or boundary of interaction from which we can then meaningfully describe it as a system. The system does not exist prior to the assignment of a boundary. Energy is relational.

If we can describe the mechanical state of the system solely in terms of its energy, then we are also making the claim that state of the system is relational. The state of the system depends on the frame of reference chosen (an obvious point) and on the boundaries of choice. A system has energy (both kinetic and potential) only in reference to a boundary. In our scenario, the boundary of the ground is an important reference frame to consider since it points to an interaction between the entity and the ground. The ground prevents any further motion and thus constitutes a natural frame of reference because of the significant interaction. So, although the energy of a system is relative to whatever reference frame we chose, the system is really formed by the interactions of entities (the car) with other entities that form some sort of boundary (the ground). Thus the system is relational in that without a boundary (or ground to fall to) the concept of energy is arbitrary (relative) to any reference frame.

It is important to note that this preferred reference frame (the ground in our example) to relate and define the system does not imply it is an absolute reference frame. Rather, the suggestion is only that the notion of energy in the original system becomes meaningful only when an interaction occurs at the system's boundaries. That is what creates a preferred system (the boundaries of the electron box in quantum mechanics or the gravitational field of general relativity as we will see shortly). There is no absolutely preferred reference frame, but rather the preferred frame of reference to constitute the relation between systems is given by the interactions (potentially or actually). In sum, we can see that the energy of the system is relative and relational. Velocity is relative to the choice of reference frame and so is posi-

[6]We ignore for the moment the more complex question of the internal energy, but the quantum mechanics examination should help elucidate this.

tion. Relative in that we use the Galilean reference frame as the ground to measure position, velocity, and energy, but we could choose any reference frame we wanted.

3 Status of Systems in Quantum Mechanics

It is well known that the development of quantum mechanics forced us to rethink the nature of the universe. What is not so clear is the way in which we should interpret the results of the discipline. I will show that the notions of relative and relational are already part of the foundations of quantum mechanics, if we pay close attention to the status of systems and entities. In addition, I propose that Carlo Rovelli's introduction of relational quantum mechanics is a robust step in this direction.

3.1 The state of the System in Quantum Mechanics

Let's examine the relatively elementary case of the unbounded free electron that is unconfined by any other system or boundary. The easiest way to picture this system is an electron as the sole entity in an unbounded universe.

The state of the system can be designated by the eigenstate ψ and the momentum $\hat{p}$. From the formalism we can state the momentum of the system as

$$\hat{p}|\psi\rangle = p_\psi|\psi\rangle$$

In terms of position, the momentum operator can be written as $\hat{p} = -i\hbar\frac{\partial}{\partial r}$ and the above eigenvalue equation becomes

$$-i\hbar\frac{\partial}{\partial r}|\psi\rangle = p_\psi|\psi\rangle$$

Solving for ψ we obtain

$$|\psi\rangle = Ae^{ikr}$$

where $p_\psi = \hbar k$.

The energy of a free electron is purely kinetic since there is no potential interaction with any other system. From the previous section we should understand now that we are assuming that there are reference frames from which to measure the velocity and thus the momentum to be non-zero. We can do this because we do not necessarily assign existence to a reference frame; it is just an epistemic tool to speak of the momentum of the electron. To speak of the kinetic energy, however, goes beyond this assumption of the existence of epistemic frames of reference (relative to the frame of reference of choice) and demands, at least, the hope of the existence of a boundary for the system to interact. Clearly, we set ourselves out for failure with the example of an electron system without boundaries of interaction for we cannot speak of energy unless a potential interaction is set. Let's however, examine the example further as it is treated in elementary quantum mechanics.

The state of the electron given by the momentum operator also represents an eigenstate of the Hamiltonian operator, since

$$\hat{H}|\psi\rangle = E_\psi|\psi\rangle$$

$$\frac{\hat{p}^2}{2m}|\psi\rangle = E_\psi|\psi\rangle$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2}|\psi\rangle = E_\psi|\psi\rangle$$

also has the same solution

$$|\psi\rangle = Ae^{ikr}$$

We know that since the position of the free electron is in full dispersion, the indeterminacy of its location is maximized and we could say that the probability of the position of the electron is infinite. Since the momentum is given by $p_\psi = \hbar k$ in full determinacy, we notice that it fulfills the Heisenberg inequality of position and momentum:

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

To normalize this eigenstate, however, we need to invent some boundaries (a box of sides L) to be able to solve for the constant A. We can consider this boundary, in order to keep the notion that it is a free particle, to be at $L \to \infty$. Doing so results in $A = \frac{1}{L^{\frac{3}{2}}}$

$$|\psi\rangle = \frac{1}{L^{\frac{3}{2}}}e^{ikr}$$

This mathematical trick commonly used to determine the normalized eigenstate of the free electron reveals a nice insight about the state of the system in quantum mechanics. To speak about an unambiguous state of a system (in this case the electron), we need to introduce a boundary (even though it goes to infinity, thus emulating the state of the free unbounded electron) to specify the normalized eigenstate. This implies, if taken seriously, that the state of the system is ill-defined until a boundary is specified. That is, we need a separate system to present a boundary to speak of a defined normalized eigenstate of a system in quantum mechanics. This should resonate with the examination of the state of system in classical mechanics. To have the energy of a system, to have a system for that matter, boundaries are required. Notice what happens to the solution for $|\psi\rangle$ when $L \to \infty$.

Consider the well-know example of the electron in a box with sides of length L. The introduction of the boundaries, allows for the system to have a normalized eigenstate that is quantized. If we move the boundaries far away, we return to the previous situation of the free unbounded electron. Consider instead the situation where we make L to be very small. The electron is more and more localized. From Heisenberg's principle we know that the momentum of the system becomes fully dispersed. The now localized electron becomes so when the boundaries of the box creates an interaction which, in fact, defines the state of the electron. This state has a fairly well defined position, but ill-defined momentum.

From these two elementary examples, we notice that the electron needs to be considered and electron-in-interaction with its boundaries. Since the boundaries are provided by other systems, the electron is a relational entity, coming to being by interaction with other entities or systems. Hence, quantum mechanics is relational.

Another argument for the relational nature of quantum mechanics is advanced by Carlo Rovelli.

3.2 Relational Quantum Mechanics

Since its early years, quantum mechanics has struggled to account for how systems come into interaction. The mathematical formalism describes the state of a system as the addition or superposition of its possible states, yet the interaction of measuring the system reveals it as being in only one of these states. This problem, often referred as the problem of measurement or the problem of the collapse of the wave function, has received much attention and many possible solutions, or dissolutions in some cases, exist.

The traditional or Copenhagen interpretation, which is so widespread that practioners often just equate it to quantum mechanics, indicate that somehow some type of interactions, measurements with macroscopic devices, will collapse or select one of the possible values of the state of the system. (Bohr 1935) Other interpretations have included the attempt to claim that quantum mechanics is incomplete (Einstein 1932, Bohm 1952) since the formalism gives a state of the system that is not accurately representing what we obtain when we measure. The Many-Worlds interpretation, which postulates that there is no collapse of the wave function or state of the system, and rather, at the moment of measurement, a new universe is opened for each of the possible states of the system (Everett 1957, Dewitt 1973). There is also Decoherence (Zeh 1970, Omnès 1994), Transactional interpretation (Cramer 1986), Modal Interpretations (Shimony 1969, van Fraassen1991), and more.

In 1996 Carlo Rovelli published Relational Quantum Mechanics (Rovelli 1996) where he proposed a new account of quantum mechanics and a novel way to examine the interactions between systems. Rovelli argues that there is a seeming contradiction on how quantum mechanics treats the interactions of systems. Although Rovelli was not the first to propose such a relational view of the state of the system, he was the first one to carry out the physical, conceptual, and initial philosophical consequences of treating systems relationally. As Rovelli acknowledges, Zurek, in 1982, and Kuchan, in 1979, proposed similar accounts of the state of the system and its relation to other measuring systems. Although a discussion of the differences in the accounts would worthwhile from a historical and philosophical angle, it is a discussion better left for another project.

Rovelli begins his argument for the need of a relational account of the problem of measurement in quantum mechanics by considering a simple general system with two discrete possible values or eigenstates. Here I follow the spirit of the illustration of Rovelli, but expanded to be able to examine it critically later in the paper. Consider the following example from elementary quantum mechanics.

Imagine the state of a system ψ , with two possible eigenstates $\uparrow$ or $\downarrow$.

$$|\psi\rangle = a|\uparrow\rangle \; + b|\downarrow\rangle \tag{1}$$

When the state ψ is measured, it will be found in only one of the two possible eigenstates $\uparrow$ or $\downarrow$. The formalism of quantum mechanics predicts this result and experiments confirm it. Associated with the possible states are the eigenvalues, a and b, of $\uparrow$ or $\downarrow$, respectively. The square of the absolute value of the eigenvalues gives the probability of finding the system in such a state upon measurement. The probability of finding the system in the state $\uparrow$ is given by $P(\psi,\uparrow)$ is

$$P(\psi,\uparrow) = |a|^2$$

and similarly

$$P(\psi,\downarrow) = |b|^2$$

where

$$|a|^2 + |b|^2 = 1$$

if we have the eigenstates normalized. This indicates that the square of the absolute value of these eigenvalues values, assuming that the eigenstates are normalized, can range in value from 0 to 1 and their sum must add to 1.

A specific example of such a system that would fit the above mathematical description would be the spin state of one electron. The formalism of quantum mechanics tells us that the spin state ψ can be written as the sum of the projection states that span the Hilbert space, $\uparrow$ and $\downarrow$.

For instance, if there was a 50% chance to find the state of the electron upon measurement in the state $\uparrow$ or $\downarrow$, then

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \ + |\downarrow\rangle)$$

where

$$a = b = \frac{1}{\sqrt{2}}$$

and therefore

$$|a|^2 + |b|^2 = \frac{1}{2} + \frac{1}{2} = 1$$

Consider now the situation where a system ϕ comes into interaction with system ψ at time t_1. We can consider that this system measures and reports at a short interval later at t_1 the value found for the spin of ψ. It reports one of the two possible eigenstates of the spin of the electron. For instance, $\uparrow$. Prior to measurement, at t_0, ϕ has not come to interaction with system ψ, but it can report on the state of ψ

At t_0, system ϕ reports:

$$|\psi\rangle = a|\uparrow\rangle \ + b|\downarrow\rangle \tag{2}$$

At t_1, system ϕ reports:

$$|\psi\rangle = |\uparrow\rangle \tag{3}$$

Imagine that a third system ξ was nearby at t_1 but not interacting with either ψ or ϕ. Yet at t_2 it interacts with the combination state ψ and ϕ.

At t_0, system ξ reports:

$$|\psi\rangle = a|\uparrow\rangle + b|\downarrow\rangle \tag{4}$$

and

$$|\phi\rangle = |\phi_{initial}\rangle \tag{5}$$

At t_1, system ξ reports:

$$|\psi\rangle \otimes |\phi\rangle = (a|\uparrow\rangle + b|\downarrow\rangle) \otimes (|\phi_\uparrow\rangle + |\phi_\downarrow\rangle) = a|\uparrow\rangle \otimes |\phi_\uparrow\rangle + b|\downarrow\rangle \otimes |\phi_\downarrow\rangle \tag{6}$$

At t_2, system ξ reports:

$$|\psi\rangle \otimes |\phi\rangle = a|\uparrow\rangle \otimes |\phi_\uparrow\rangle \tag{7}$$

Where $|\phi_\uparrow\rangle$ is the eigenstate of the interacting system ϕ when it correlates to the state $|\uparrow\rangle$ and $|\phi_\downarrow\rangle$ is the eigenstate of the interacting system ϕ when it correlates to the state $|\downarrow\rangle$. Notice that the non-correlated terms vanish.

Rovelli points out that here we have a situation where, at t_1, measuring system ϕ reports that the state of ψ is $\uparrow$, but for system ξ (although it has yet to measure the combined system ψ and ϕ) the state of ψ is entangled in a superposition with state ϕ. Rovelli claims "[i]n quantum mechanics different observers may give different accounts of the same sequence of events." (Rovelli 1996, 4) In short, one system accounts for a collapse of the state of possibilities of ψ as being $\uparrow$, while another system ξ reports no collapse and the state of the combined system ψ and ϕ is in a state of superposition. It follows then that a quantum mechanical description of the state of a system (state and/or values of physical quantities) cannot be taken as an absolute (observer independent) description of reality, but rather as a formalization, or codification, of properties of a system relative to a given observer. (Rovelli 1996, 6)

Rovelli claims that his motivation for treating the state as relative comes from a similar move by Einstein in SR of abandoning the notion of absolute simultaneity of events. This move should not be confused, Rovelli warns us, as a loss of descriptive power about the system or systems. If the notion of observer-independent description of the world is unphysical, a complete description of the world is exhausted by the relevant information that systems have about each other. (Rovelli 1996, 7) The relation between the accounts of different measuring systems must be treated as a quantum mechanical interaction as well.

A critique of Rovelli's account may center on questioning, first, how (3) and (6) can be both the state of the system ψ and second, how (7) will be the case and not

$$|\psi\rangle \otimes |\phi\rangle = a|\downarrow\rangle \otimes |\phi_{\downarrow}\rangle \tag{8}$$

In the first problem we need to clarify that the difference between equation (3) and (6) could be taken to be solely an epistemic difference, meaning that (3) is the state of ψ from the reference frame of ϕ and (6) is the state of ψ from the reference frame of ξ. From this epistemic consideration both accounts of the state of ψ are valid in that they illustrate the knowledge that ϕ and ξ have of ψ and nothing more. If this was the case, however, then quantum mechanics would be a theory about a system's knowledge of the quantum world and not of the quantum world itself or simply an incomplete theory of phenomena. Besides, it appears that we would have two incompatible accounts of the system ψ from two different reference frames. If we take into consideration the analysis of classical mechanics, the energy state of the system is relative to the reference frame and in relation to the boundaries of interactions. From the reference frame of ϕ, ψ is in a state of possibility represented by (2) and upon interaction in state (3). The state ϕ is now entangled with ψ, but it does not enter explicitly into its account of ψ in (3). It is no surprise, then, to have another system, ξ report that entanglement in (6). The surprising bit comes from the report of (6) that there is no collapse, but it is consistent with traditional quantum mechanical accounts of the postulate of measurement in that the actualization does not occur unless there is an interaction. From the reference frame of ξ, ψ and ϕ constitute a single system $\psi \otimes \phi$ which is in a superposition of states until measurement. However, this is only problem if we insist to hold on to a non-relative view of a system or state. This view conflicts with the way we have been doing mechanics since Galileo. The challenge here is to understand that relativity of states does not imply loss of objectivity. Rather objectivity needs to be disassociated from the notion that an absolute reference frame exists. The relativity of motion is objective in that a particular reference frame will report a unique value in classical mechanics and a set of possibilities in the more fundamental account of QM. I am afraid that a fuller examination of the topic of objectivity would be needed in the future.[7]

An ontological reading would claim that the state at t_1 of ψ is both (3) and (6). As in the epistemic case the state of the system differs from system to system and

[7] This is an unfinished forthcoming article "What is relational about relational quantum mechanics?"

there is no absolute state of the system. But notice what happens when ξ finally interacts with ψ (which is entangled with ϕ) as depicted in (7), which brings us to the second problem. It reveals the state collapsed into one of its possible values as in (3). But how come we expect to find it $\uparrow$ and not $\downarrow$? One response would be to appeal to Luder's rule and claim that if t_2 occurs right after t_1 we will expect the state of ψ and its entanglement with ϕ to remain $\uparrow$ when interacted by ξ. But this response would seem to avoid the real problem raised in that (8) is as good an answer as (7) unless we assume that there is an absolute, already given, state for ψ ($\uparrow$ in our example). From the simple lesson of classical mechanics we know that the state of the system depends on the anticipated interaction. In quantum mechanics this still holds true and the state of ψ depends on boundaries given by other systems. In (3) the boundary of ψ is ϕ while in (6) the boundary of ψ (entangled with ϕ) is ξ. Since the boundaries of the systems change, we should expect a different account, in the same way we expect (3) (at t_2 albeit) and (7) to be distinct. The state of the system ψ depends on the interactions it experiences and those interactions will be different for different systems.

The further problem in QM, unlike classical mechanics, is the existence of the state of superposition, as in (4) and (6). How can it be that ψ is both an actual state and a superposition of states, at the same time? This seemingly contradiction at an ontological level is similar to the epistemic question we raised in the previous paragraph. In the ontological account it appears that the collapsed state of ψ can only occur when interacted by a system. Since the system can be chosen to have any boundaries, the entangled stated of ψ and ϕ is not collapsed until a external system, ξ, interacts with the entangled state. Rovelli does not expose the nature of interaction and how it produces such a change in the state.[8] In classical mechanics the interaction seems clear, as when the car hits the ground. But in the quantum realm, what constitutes and interaction or measurement is still subject to debate and seems crucial for understanding precisely how Rovelli's relational account would work. His proposal to understand interaction as information has possibilities, but leaves some of the conceptual questions still open.

Despite the difficulty of ascertaining what constitutes measurement or interaction the ontological account, the state of a system ψ depends on the system or reference frame and that the system requires another system to create its boundaries. When a

[8]Rovelli is aware of the main problem in QM, the measurement problem. He mentions that the solution probably lies in a combination of the interpretations, but he does not specify what the solution is in full, but that it may disappear given his relational account. Rovelli recognizes that more work would need to be done in RQM to show this is the case, possibly with the complicity of other interpretations

system interacts with another system, then the possibilities become actualized, but only from the reference frame of the external system, in our example ϕ and later ξ. What a system is depends on the system that is interacting with it. In Rovelli's example, we assumed that there was nothing interacting with ψ and it was in a state of superposition. Once ϕ interacted, then the state of ψ changed into one of its possibilities, but for ξ, ψ was entangled with ϕ in a state of possibilities.

3.3 Conceptual Picture that Emerges from Quantum Mechanics

David Bohm and others have already hinted at the role of external systems in determining the nature of a quantum mechanical system. Bohm declares:

> Thus, under all circumstances, we picture the electron as something that is itself not very definite in nature but that is continually producing effects which, whether they are actually observed by any human observers or not, call for the interpretation that the electron has a nature that varies in response to the environment." (Bohm 1951, 610)

This variation of the state of the system in relation to the environment could often be ignored for practical purposes in classical mechanics, but it creates a conceptual vacuum in QM if ignored. QM is relational and the entities described by the theory can only be understood in relation to other entities. Without boundaries or other systems to interact, the electron vanishes. With set boundaries the electron develops its properties and certain types of interactions can render this properties of the entity actualized. The sacrifice is that other possible properties of the entity are lost. Rovelli's argument for a relative and relational quantum physics adds another aspect to the understanding of an entity in QM. A system's state will vary depending on what system is reporting (relative) and that difference depends on the type of interaction (relational) the systems have.

Taking QM to be relative and relational this way fits well with the lessons we can draw from the status of an entity in Relativity.

4 Status of Systems in Relativity

In Relativity we find that the space coordinates are now linked inexorably to the time coordinates. In the absence of gravitation, spacetime is rendered as a Minkwoski manifold which is often conceived as independent of the states or entities inhabiting it, much like Newton's notion of space and time can be conceived as containers independent of the states of the system and of each other. This background dependence begins to fade away if we look closely at the consequences of giving up absolute simultaneity in SR. An observer may report that a different system experiences length contraction and time dilation as well as different simultaneous events. Thus, the spatial and temporal aspects of a system are not inherently part of the system, but only meaningful when measured by another system. Of course, the system itself will report a length and a time lapse, referred to as proper length and proper time, but this hardly gives the true state of the system by proper properties considerations. The state of a system, and thus and entity is given only in relation to another system. In a historical context, the concept of relativity of velocities of Galileo trumped the challenge of the lack of relativity in Maxwell's theory of electromagnetism.

From this, the time lapsed in a system can never be defined at a particular absolute value, since some other system will likely report a different time lapse. So the description of a system (by another system) will entail that there is a time lapse of some sort. Any system is, therefore, inherently dynamical since an absolute zero-time lapse or instant does not exist.[9]

Hence, in SR, even though the Minkowski spacetime can be understood as separate from the system, in truth, it makes little sense to treat it as such conceptually, since the system is defined in terms of its relation to another part (system) within the manifold. The mathematical formalism of SR makes no such demand, but from the consideration of the role of simultaneity we can begin to see the move toward a more general account of a systems in relation where the relativity of motion is pushed further toward ontology. Clearly, however, even in the absence gravitational field considerations, SR begins the conceptual move toward a relational account of systems and entities that demand a full relativity of motion. The problem in SR is that the relation between the spacetime manifold and the entities or systems is not clear. This problem is what GR clarifies.

Some, like Rovelli, view SR as holding on to the mathematical and physical notion of Minkowski spacetime as the background for the dynamics of systems and thus SR fails to capture the conceptual power of Relativity, which demands a dynamics devoid of the background spacetime. GR offers such a theory, becoming a legitimate heir of the account of relative motion that began with Galileo. In GR the addition of the gravitational field makes the system ill defined in reference to a background of any sort and only becomes meaningful in reference to another system in the gravitational field. For instance, the notion of a space point where a system is located is no longer

[9] Unless, of course there was only one distinguishable system or all systems were not in motion with respect to each other

sensible in general relativity. Only in reference of the field itself. Rovelli expresses this idea succinctly

> The space and time of Newton and Minkowski are re-interpreted as a configuration of one of the fields, the gravitational field. This implies that physical entities–particles and fields–are not immersed in space, and moving in time. They do not live on spacetime. They live, so to say, on one another. (Rovelli 2004, 9)

Historically, Einstein's achievement in GR was to find that the laws of motion remain the same in all frames of reference and not just ones in the absence of gravitation. In this achievement Einstein furthered the concept of relativity of motion to a general case. Systems will obey the same laws of motion regardless of what reference frames is chosen; this is a consequence of the diffeomorphism invariance of the field equations. This makes the reference frame necessary for considering the dynamics of a system.

4.1 The Significance of Background Independence

In Relativity the system is a system in relation to other systems, both epistemically and ontologically. Epistemically, to know the status of a system or entity, one must refer to the reference frame of another system (it can be the system itself, as we saw in the case in classical mechanics, but that offers little valuable or meaningful information). A system is then known in reference to another system. Ontologically, a system does not have already made boundaries (even though some may be more obvious than others), but rather the boundaries of the system are given via the interactions with other systems. [10] Thus, in relativity a system has boundaries given by other systems and not by relation to a background (thus relativity is background independent, but I remind the reader that some still try to interpret relativity as background dependent, spacetime substantivalism, yet the debate has severely eschewed against this position.

[10] A variation on this point has received much attention in the last few years with the introduction of the Holographic principle, where a physical state of a system can be fully described by the boundary that limits the system (t'Hooft and Susskind). This boundary occurs in the interaction with other system, which indicates that a meaningful description of a physical system can only occur when another system interacts with it, creating then a boundary between the two systems. This boundary, of course, is arbitrary (as we saw in the case of the falling car in classical mechanics) since any interaction can set a boundary, but some interactions and thus boundaries will make more practical sense to pursue than others.

In sum, Relativity, and particularly GR, is background independent. The implications of background independence are that the systems and entities are defined, both epistemically and ontologically, relative to and in relation to other systems. GR gives us a relative as well as a relational view of reality. GR may not prove to be the final theory, but it is worthwhile, given its theoretical and empirical success to treat it ontologically seriously. Doing this implies the adoption of an ontology that is dynamic and relational. The dynamical element in the ontology implies that systems are inherently dynamic and the relational element implies that the boundaries of systems, and thus the ability to speak of systems, are given by other systems (other systems, of course may include the fields, EM or gravitational or else, that interacts and forms the system).

Herman Weyl offered an insightful account of the dynamical implications of relativity soon after Einstein's papers on GR appeared in print:

> We have thus attained a new, purely dynamical view of matter. Just as the theory of relativity has taught us to reject the belief that we can recognize one and the same point in space at different times, so now we see that there is no longer meaning in speaking of the same position of matter at different times. (Weyl 1952, 202)

Furthermore, regarding the relational nature of matter, Weyl states

> The electron, which was formerly regarded as a body of foreign substance in the non-material electromagnetic field, now no longer seems to us a very small region marked off distinctly from the field, but to be such that, for it, the field quantities and the electrical densities assume enormously high values. An energy-knot of this type propagates itself in empty space in a manner no different from that in which a water-wave advances over the surfaces of the sea; there is no "one and the same substance" of which the electron is composed at all times. (Weyl 1952, 202-3)

For Weyl, matter or entities cannot be understood as a prepackaged bundle. Matter is whatever has localized energy and what it is needs to be re-conceptualized.

> Since there is no sharp line of demarcation between diffuse field-energy and that of electrons and atoms, we must broaden our conception of matter, if it is still to retain an exact meaning. (Weyl 1952, 203)

The move to show that a system or entity needs to be understood dynamically and in relation to the boundaries given by other system is precisely this type of broadening. In addition, the more fundamental entities of classical mechanics, the point particles that trace out specific paths have been replaced by relativistic world lines and fields, since points in space independent of their evolution lose meaning. In its evolution the entity will be part of many interactions that make it what it is. Those interactions are often given by fields. Weyl declares that

> It is not the field that requires matter as its carrier in order to be able to exist itself, but matter is, on the contrary, an offspring of the field. (Weyl 1952, 203)

The distinction between particle and field depends on the type of system in consideration. Weyl adds

> There is only a potential; and no kinetic energy-momentum-tensor becomes added to it. The resolution into these two, which occurs in mechanics, is only the separation of the thinly distributed energy in the field from that concentrated in the energy-knots, electrons and atoms; the boundary between the two is quite indeterminate. (Weyl 1952, 203)

With this lesson from Relativity in mind, we can look back at classical mechanics and realize that some of these ideas about the nature of entities and systems were already available if we avoided unnecessary assumptions about their constitution. Jean Marc Levy-Leblond and Franoise Balibar reminds us that:

> The notion of particle acquires its full (classical) meaning only with reference to that of an interaction. The physics of the classical period (that of Newton) entertained the ambition of explaining the world through the combined interplay of the reciprocal actions between particles– the interactions. It is for this reason that the notion of a particle includes, besides its spatio-temporal properties, its dynamical properties–i.e., properties related to the movement of the particle under the effect of the interaction. (Levy-Leblond 1990, 38)

However, this is not the most common way that practitioners or many philosophers have understood particles to be, and they have emphasized the non-interacting and non-dynamical aspects of particleness as the essential elements. Levy-Leblond and Balibar accurately point out a simple but often neglected element of the ontology of entities in classical mechanics; that the full sense of them being particles entails interaction of their boundaries with other particles or systems. A particle has an associated momentum from a particular reference frame (Galilean transformations). Thus, even in classical mechanics a particle can only be considered fully a particle if it is in interaction with another particle (ontological) and can only be known relative to another system (epistemic).

The lack of a background spacetime to host and frame the entities and systems force us to reconceptualize what entities and systems are. They are dynamic relations in interaction with other systems that help them form their boundaries. Along with the lesson from classical mechanics and quantum mechanics, Relativity shows us that systems cannot be considered absolute entities. Rather, entities are constituted by the interaction of the system with its boundaries which are given by other systems in the environment.

4.2 Rovelli's Vision of Relativity

For Rovelli, the greatest conceptual lesson of relativity is that there is no background spacetime. Hence, he argues, we need to look for theories that are background independent. This background independence implies that spacetime is not an entity in its own right, but rather that the entities that contribute to the gravitational field rely solely on each other (relational view) to constitute the connected gravitational field.

Further, since space is what constitutes the relation between systems, space is relational.

> *"Space is a relation"* means that the world is made up of physical objects, or physical entities. These objects have the property that they can be in touch with one another, or not. Space is this "touch", or "contiguity", or "adjacency" relation between objects. Aristotle, for instance, defines the spatial location of an object as the set of the objects that surround it. This is relational space. (Rovelli 2004, 53)

In addition:

> The space and time of Newton and Minkowski are re-interpreted as a configuration of one of the fields, the gravitational field. This implies that physical entities–particles and fields–are not immersed in space, and moving in time. They do not live on spacetime. They live, so to say, on one another. (Rovelli 2004, 9)

That is, if there are no entities in the universe, there is no spacetime. The entities can only be made sense if they are in relation to other entities. The gravitational field is what determines the basis of interaction of these entities and it is also what determines inertial motion of these entitites in relation to one another. But the gravitational field does not become a substitute for substantival spacetime, since the gravitational field is made by the entities that compose it. Along these lines, the notion of distance does not need a physical metric already there to meaningfully speak of length or distance. Rather, the length or distance of an entity or event is given in relation to the gravitational field experienced by the entity.

From this relativity of motion, not just of inertial frames, but all frames of reference, Einstein was able to craft the generally covariant field equations, and also invariant under active diffeomorphisms, that form the basis of general relativity. Hence, in GR the status of the reference frame goes from being merely relative, that is the measure of the properties of another system, to relational, that is, the necessary condition for the entity being an entity. Einstein declares:

> If, for example, events consisted merely in the motion of material points, then ultimately nothing would be observable but the meeting of two or more of these points...The introduction of a system of reference serves no other purpose than to facilitate the description of the totality of such coincidences. (Einstein 1921, in Rovelli 2004, 70)

These coincidences form the system and thus the reference frames are necessary. From these considerations of the foundations of GR, Rovelli concludes:

> It follows that localization on the manifold has no physical meaning...In GR, general covariance is compatible with determinism only assuming that individual spacetime points have no physical meaning by themselves...What disappears in this step is precisely the background spacetime that Newton believed to have been able to detect with great effort beyond the apparent relative motions...Einstein's step toward a profoundly novel understanding of nature is achieved. Background space and spacetime are effaced from this new understanding of the world. Motion is entirely relative. Active diffeomorphism invariance is the key to implement this complete relativization...Because of background independence–that is, since there are no nondynamical objects that break this invariance in the theory–diffeomorphism invariance is formally equivalent to general covariance, namely the invariance of the field equations under arbitrary changes of the spacetime coordinates $\vec{x}$ and t. (Rovelli 2004, 71-74)

For Rovelli, however, the difference between Galilean relativity and relationism lies in that relationism refers to the relativity of any kind of motion and not just inertial motions. This distinction is a good start for generalizing the relativity of motion and towards a more robust ontological account of GR, yet the problem with it is that relationism taken in this sense would simply be a more general epistemological form of relativity like the one we obtain in SR and GR. It is the case the Galilean relativity is superseded by the more comprehensive account of motion given the special relativistic treatment of frames of reference and in turn, GR gives an even more general account of the relativity of frames of reference with respect to the gravitational field. But if this was just the only improvement that relativity offered the Galilean-Newtonian account, that we could conceive and describe it as a more general type of relativity of motion, then we would miss the deeper lesson. The deeper lesson occurs at an ontological level in that the theory demands a reference frames to have ontological significance in that they carry the possibility, at least, of interaction of their gravitational fields. To speak of the motion or being of an entity demands that we specify a reference frame with a specific gravitational potential. This is what should properly be referred as relationism.[11] Without the interaction, and not just a mere epistemic reference frame, the entity in GR does not have a status.

5 Status of Systems in Loop Quantum Gravity

The conceptual picture of spacetime that arises from considerations of the foundations of quantum mechanics and relativity is one that demands a background-independent theory where the entities form and are influenced by the gravitational field. In Loop Quantum Gravity (LQG) Rovelli claims that the relativity and relationism of QM and GR are preserved and used to direct the inquiry of the search for quantum gravity. The states of entities are described by loop states $|\alpha\rangle$ that form spin networks and in their evolution, spinfoams (the sum over the spacetimes). These spin networks are sets of relations that define space and since these loop states are discrete, space is discrete.

> Notice that this is not imposed on the theory, or assumed. It is the result of a completely conventional quantum mechanical calculation of the spectrum of the physical quantitites that describe the geometry of space...Space is effectively granular at the Planck scale, and there is no infinite ultraviolet limit. (Rovelli 2004, 21)

[11]In fact, that is how Rovelli uses the term in his account of relationism in quantum mechanics.

It is not clear in Rovelli's account, however, whether the discreteness of the loop states (the entities in LQG) is an inherent property of the loops or one that emerges. From our previous considerations, the discreteness of a state comes from the boundaries of interaction. So, the discreteness of the loop states and of "space" arises from the interaction with its environment. Discreteness emerges from interaction. But since we always need boundaries to have entities, we can expect the loop states to always be discrete.

The goal of LQG is to "merge the conceptual insight of GR into QM." (Rovelli 2004, 14) This conceptual insight is the lack of a background spacetime. This allows spacetime to be made of loop states and where "physical systems reveal themselves by interacting with other systems." (Rovelli 2004, 17) This is the relational insight of GR that is also available in QM and Classical Mechanics. LQG is then a conceptual extension of mechanics and not just a forced marriage between QM and GR. Nevertheless, in joining QM and GR, LQG allows the eigenstates of the gravitational field to be represented by loop states and the time variable does not appear separately from the space variables, but is embedded in the physical variables. Time appears intrinsically in the equations of motion.

> The quantum dynamics is governed by the corresponding quantum operator H. In quantum gravity, H is defined on the space of the spin networks. There is no external time t in the theory, and the quantum dynamical equation which replaces the Schrödinger equation is the equation $H\Psi = 0$, called the Wheeler-DeWitt equation." (Rovelli 2004, 24)

The dynamics of LQG lack a preferred time variable that can be treated distinctly from the system. In the same way that we lose the notion of trajectory of a particle in QM, LQG gives up on the notion of spacetime. It is replaced by a sum over spinfoams which will represent the dynamical version of spacetime.

> Thus, there is no background "spacetime", forming the stage on which things move. There is no "time" along which everything flows. The world in which we happen to live can be understood without using the notion of time.

6 Concluding Remarks

An analysis of the epistemic and ontological role of reference frames reveals that motion is relative and systems are relational. These systems exist in relation to each other and not in relation to a background absolute frame of reference.

7 References

Bohm, David. (1952). A Suggested Interpretation of the Quantum Theory in Terms of "Hidden Variables", I, Physical Review, 84, pp 166-179

Bohm, David. (1952). A Suggested Interpretation of the Quantum Theory in Terms of "Hidden Variables", II, Physical Review, 85, pp 180-193

Bohm, David. (1979) Quantum Theory. New York: Dover Publications.

Bohr, Niels. (1935) Nature 12, 65.

Cramer, John . (1986). The Transactional Interpretation of Quantum Mechanics Reviews of Modern Physics 58, pp 647-688.

DeWitt BS, Graham N. (1973). The Many World Interpretation of Quantum Mechanics, Princeton: Princeton University Press.

Everett, Hugh. (1957). Relative State Formulation of Quantum Mechanics, Reviews of Modern Physics vol 29, pp 454-462

Hughes, RIG. (1989). The structure and interpretation of Quantum Mechanics, Cambridge: Harvard University Press.

Kochen, S. (1979). The Interpretation of Quantum Mechanics. Unpublished.

Lévy-Leblond, Jean-Marc and Balibal, Françoise. (1990) Quantics: Rudiments of Quantum Physics. Amsterdam: North Holland Elsevier Publications.

Nottale, Laurent. (1992). The Theory of Scale Relativity. Int. J. Mod. Phys. A7, 4899-4936

Omnès, Roland. (1994). The Interpretation of Quantum Mechanics. Princeton: Princeton University Press.

Rovelli, Carlo. (1996). Relational Quantum Mechanics, International Journal of Theoretical Physics 35 pp. 1637-78, Revised: arXiv:quant-ph/9609002v2 24 Feb 1997.

Rovelli, Carlo. (2004). Quantum Gravity. Cambridge: Cambridge University Press.

Shimony, Abner. (1969). in Quantum Concepts and Spacetime, Eds. R. Penrose and C. Isham. Oxford: Clarendon Press.

Van Fraassen, Bass. (1991). Quantum Mechanics: an Empiricist View, Oxford: Oxford University Press.

Weyl, Hermann. (1952). Time, Space, Matter. New York: Dover Publications.

Zeh, H. Dieter . (1970). On the Interpretation of Measurement in Quantum Theory, Foundation of Physics, vol. 1, pp. 69-76.

Zurek, W.H. (1982). Environment-induced superselection rules Phys. Rev. D 26, 1862 - 1880