Can the Threat of Entry Reduce Competition?

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Abstract

This paper is the first to provide a general context whereby potential entry can permanently reduce the intensity of competition in a market. All previous results found that potential entry would lead to lower prices and greater competition. Examining markets where entry occurs by the acquisition of access rights from existing incumbents, we demonstrate that, when competitive choices are strategic complements, a more efficient entrant may be unable to acquire those rights from a less efficient incumbent due to the accommodating behavior of efficient incumbents. Similarly, such accommodating behavior may deter efficient investment by an incumbent or mergers that would generate social welfare improvements. These results have implications as to how economists view potential entry and its benefits.

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Economic models of the impact of potential entry on competition give rise to two distinct predictions. First, theories of contestable markets and limit pricing predict that credible entry threats will induce incumbents to price lower so as to deter that entry. Second, originating with Selton’s chain store paradox, potential entry will not induce a prior change in incumbent behavior as it either does not commit them to maintaining that behavior in the post-entry environment or their own forecasted behavior in that environment is sufficient to deter entry in of itself. Resolving the theoretical tension between the two predictions has relied on considerations of the actions incumbents can take to commit themselves to tough post-entry competition or the possibility that pre-entry behavior might signal relevant information regarding forecasted behavior post-entry. Regardless, together, all of these theories predict that potential entry will not lead to less competition in a market.\(^1\)

The existing models all have in common one feature: that entry, if it occurs, will be de novo; that is, the entrant adds to the pool of competitors in a market. In contrast, there are many situations where entry arises by acquisition of an incumbent. While, in some situations, this may be a choice aimed at eliminating incumbent competitors, in others, there may be market constraints on de novo entry. For instance, in a market with high sunk entry costs, a natural oligopoly might arise with a ceiling on the number of suppliers (Gilbert and Newbery, 1992). In this case, entry may only be credible if it is by acquisition.

Here we consider this by examining a situation where an environment where a firm may only participate in the market if it owns one of a strictly limited number of access rights. These rights could be in the form of complementary assets that are uneconomic to replicate but critical for competition. Or, alternatively, these rights could arise as a result of government licensing arrangements. Examples of this include the need for broadcasting licenses for television and radio or the rights to spectrum.

\(^1\)See Davis, Murphy and Topel (2004) for a recent treatment and Wilson (1992) for a review.
used by wireless carriers. It could also apply to situations in which firms need to access a constrained distribution system (Dana and Spier, 2007) or limitations on physical capital restricts access. For example, sea and air ports require significant amounts of land that satisfy the very specific requirements of offering sea and air port services. Likewise, an asset such as “eyeballs” is necessary to compete in the the market for internet advertising; particularly targeted advertising connected to web searches and internet portals. In these markets, entry to a market can only occur by acquisition of access rights.

Simple intuition would suggest that if an entrant emerged that was more efficient than one of incumbents in utilising those rights, there would be an incentive for that incumbent to sell the right to the entrant. Hence, potential entry of this kind would (a) not result in any change in pre-entry behavior of incumbents and (b) would result in eventual entry.

However, this simple intuition neglects the role that rival incumbents might play in influencing the terms of trade in the acquisition market for the access rights. Using a dynamic model of competition, we show that, in a Markov perfect equilibrium, *the threat of entry may result in reduced competition*. In the model, an inefficient firm competes with an efficient firm in a restricted access market. Potential entry creates an incentive for the inefficient firm to sell its rights. Nonetheless, we demonstrate that, where the actions of firms are strategic complements, there may exist a Markov perfect equilibrium in which the efficient incumbent relaxes competition, directing a stream of profits to the inefficient firm sufficient to eliminate the gains from trade in the acquisition market and preventing the firm from selling its access rights. Because the threat of entry allows the efficient firm to commit to this less aggressive stance, even in the absence of collusion, prices are higher. The higher prices, in turn, *deter exit*. Interestingly, compared to the situation where no potential entrant existed, both incumbent firms enjoy higher profits. Hence, incumbents may be in favour of policies
(e.g., spectrum or broadcast re-sale rights) that free-up entry into such markets.

The key to this result is the fact that potential entry creates the possibility of trade in access rights and that this possibility drives an incumbent to adopt a softer competitive stance to deter such trade. This depends critically on the assumption of strategic complements in key competitive actions. If those actions are strategic substitutes (as they are for price and quantity in Cournot competition), pre-emptive action to accommodate the inefficient incumbent is too costly for the efficient incumbent and it is better off allowing the access right to be acquired by the entrant.

While our baseline model focusses on entry, we demonstrate that a similar logic can arise for potential threats to an incumbent in other environments. For instance, consider an incumbent firm facing a less efficient rival who, perhaps because a technology is coming off patent, has the option to invest in greater efficiency. If their actions are strategic complements, the efficient incumbent has an incentive to soften competition to deter that investment. Similarly, in a situation where there are three firms in a market and any two have the option to merge and generate synergies, all three firms have an incentive to soften competition so as to prevent a merger from taking place. Consequently, it is possible that socially beneficial mergers might be detered in markets where competitive actions are strategic complements. This was something that was thought might only occur when those actions were strategic substitutes (see Salant, Switzer and Reynolds, 1983).

This result is significant because, to our knowledge, it is the first paper that provides a situation whereby potential entry could reduce competition in a market. There are numerous examples of papers that show that increasing actual competition in a market can lead to less competition and increased prices (Satterthwaite, 1979; Perloff and Salop, 1985; Stiglitz, 1987; Schulz and Stahl, 1996; Janssen and Moraga-González, 2004; Gabaix, Laibson, and Li. 2005; Perloff, Suslow, and Seguin, 2006; and Chen and Riordan, 2008). These results are based on the interplay of product
differentiation and/or consumer search. In contrast, in the model developed in this paper competition is greater post-entry than the pre-entry benchmark. Stiglitz (1981) develops a model whereby potential entry might temporarily cause prices to increase for a monopolist. He considers a situation whereby a monopolist chooses to extract a resource at a slower rate so as to forstall the entry of a competing substitute. Strictly speaking, in his model, it is the availability of the resource (akin to the incumbent holding additional capacity) that is entry deterring and the ability to sustain higher prices is, at best, temporary.

The paper proceeds as follows: Section 1 sets out the definition of a restricted access market. The general form of the model and the central results are set out in section 2. Section 3 presents two simple examples covering both the cases of strategic complements and substitutes. The model is extended to the case of an investment in a cost reducing technology in section 4, efficient mergers in section 5, and accommodation with multiple efficient firms in section 6. The paper concludes with a discussion of the results.

1 Entry in a Restricted Access Market

We consider a market where there exists an asset that is required in order for a firm to gain access to the market.

**Definition 1.** An *access right* is defined as an asset that is (a) necessary for a firm to participate in a market, (b) transferable and (c) rival.

In other words, an owner of such an asset may sell, but can never share, its access to a market. The assumption of transferability is not as strict as it may at first appear. Even where a firm is prevented from selling its access right, the right may still be transferred to a second firm through a merger, acquisition or management buyout.\(^2\)

\(^2\)The rivalry assumption may seem questionable in a world in which regulators frequently require
Definition 2. A *restricted access market* is defined as a market in which there is a strictly limited number of *access rights*.

Entry into a restricted access market differs from unrestricted entry insofar as entry is only possible if the entrant displaces an incumbent firm. As each incumbent has control rights over access, the replacement of an incumbent by an entrant must occur on terms that create benefits for both the entrant and the displaced incumbent. That is to say, in order to purchase an incumbent’s access rights, the entrant must pay a fee to the incumbent that is at least as great as the earnings the incumbent foregoes by exiting the market. At the same time, the fee cannot be greater than the profit that the entrant expects to earn following entry.

2 The Model

Here we present a dynamic model of competition in a *restricted access* market. At any time there can be at most two firms competing in the market. Initially, the access rights are owned by firms 1 and 2, with firm 1 assumed to be more efficient than firm 2. We term this the *duopoly phase*. At some point a potential entrant, firm 3, arrives; modelled here as the result of an exogenous policy change. For instance, that policy change may be to allow broadcast or spectrum license re-sale after some earlier allocation to firms 1 and 2. Firm 3 is assumed to be as efficient as firm 1 and hence, from a social perspective there would be a motive for allowing such re-sale.\(^3\) All firms discount the future at the common rate \(\delta \in (0, 1)\).

After the entrant arrives, the game consists of two distinct phases. Each phase is itself a repeated game with the number of periods in each phase endogenously determined.

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\(^5\)See Cramton (2000) on the effect of the FCC’s "unjust enrichment" requirements on the ability to resell spectrum.
1. In the entry phase firms 1 and 2 compete under the threat of entry by firm 3. Each period begins with firms selecting actions and realising instantaneous profits. Firm 2 then has the opportunity to sell its access rights to firm 3. If firm 2 sells out, the game proceeds to the post-entry phase, otherwise the game repeats. It is assumed that the decision to exit the market is irreversible.\footnote{The critical element here — as demonstrated by the investment problem considered in section 4 — is that the technical advancement facilitated by entry is irreversible.}

2. In the post-entry phase, firms 1 and 3 compete. In each period of the post-entry phase firms 1 and 3 simultaneously select actions before realising instantaneous profits. This game is repeated infinitely.

Because the transition from the entry to post-entry phase only occurs with firm 2’s consent, the game may never reach the post-entry phase. Figures 1 and 2 illustrate the intra- and inter-period timing of the game.

It will be useful to benchmark the outcomes here with a baseline duopoly phase involving competition between firms 1 and 2. Each period of the duopoly phase sees firms 1 and 2 simultaneously selecting actions and realising the consequent instan-
taneous profits. As will be demonstrated below, the duopoly and post-entry phases share the characteristic that in a Markov perfect equilibrium market outcomes involve firms considering only instantaneous profits. In contrast, as behavior in the entry phase might trigger a transition the post-entry phase, firms active in the entry phase consider outcomes beyond the current period.

2.1 Primitives

The common instantaneous profit function of firms 1 and 3 is written $\pi(a_i, a_j)$ where $a_i \in A$ is the firm’s own action, $a_j \in A$ is the rival firm’s action, and $A$ is a closed bounded interval on $R$. It is assumed that $\pi$ is twice continuously differentiable in both its arguments. Moreover, $\pi$ is assumed to be strictly concave in the firm’s own action ($\partial^2 \pi / \partial a_i^2 < 0$), while for all $a_j$ there exists $R(a_j) \in A$ such that $\partial \pi(R(a_j), a_j) / \partial a_i = 0$. It follows from the continuity and differentiability of $\pi$ that $R$ is likewise continuously differentiable. Moreover, if $\partial^2 \pi / \partial a_i \partial a_j < 0$ ($> 0$) the actions of firms are strategic substitutes (compliments) and $R' < 0$ ($> 0$). It is assumed that $R'$ is bounded such that $R' \in (-1, 1)$.

For the purposes of this paper, aggressive behaviour by a firm is characterised by that firm selecting a high value of $a_i$, while a low value of $a_i$ is interpreted as the firm adopting a passive position. Formally, this characterisation requires the firm’s profit to be strictly decreasing in its rival’s action ($\partial \pi / \partial a_j < 0$).

Firm 2 is less efficient than firms 1 and 3 (who have equal efficiency). Firm 2’s instantaneous profit is given by the function $\pi_c(a_2, a_1)$ where the subscript $c > 0$ represents the degree of firm 2’s inefficiency or cost disadvantage. Specifically, it is assumed that $\lim_{c \to 0} \pi_c = \pi$ and $\partial \pi_c / \partial c < 0$. As with the profit function of the efficient firms, it is assumed that $\pi_c$ is twice continuously differentiable in $a_1, a_2$ as well as the additional argument $c$, strictly concave in $a_2$ and there exists $R_c(a_1) \in A$. 

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5 The bounds $R'$ ensure the existence and uniqueness of an equilibrium in the static game.

6 Note that firm 2 is never in competition with firm 3.
such that \( \partial \pi_c(R_c(a_1), a_1)/\partial a_2 = 0 \) for all \( a_1 \) and \( c \). Finally, it is assumed that \( \partial^2 \pi_c/\partial a_2 \partial c < 0 \). This last condition implies that the marginal return to aggressive behaviour declines as inefficiency increases. It is also a necessary and sufficient condition for \( \partial R_c/\partial c < 0 \). In words, this condition causes the best response function to shift inward as inefficiency increases.

### 2.2 Markov Strategies

Firms are restricted to employing Markov strategies; strategies that depend only on payoff relevant information. This dependance can be captured by describing the value of the payoff relevant variables as a state and defining each firm’s action a function of the state. In the competition stage of each period the payoff relevant variables are the identity of the firms in the market, and the existence, or otherwise, of a potential entrant. This information is entirely captured by the phase of the game. Let the superscripts \( D \), \( E \) and \( P \) denote actions and instantaneous profits in the duopoly, entry and post-entry phases respectively. It follows that firm 1’s strategy is a triplet \((a_1^D, a_1^E, a_1^P)\), while firm 3’s strategy is a singleton \( a_3^P \).

In the exit stage of the entry phase the payoff relevant variable is the magnitude of the profit firm 2 will receive as a result of remaining in the market. Given that each firm’s strategy in the competition stage is invariant within a phase, and that firm 2 declining to sell its access rights causes the game to remain in the entry phase infinitely, the profit that firm 2 receives going forward is,

\[
\Pi_2 = \frac{\delta}{1 - \delta} \pi_c(a_2^E, a_1^E).
\]

Thus, firm 2’s strategy is a triplet \((a_2^D, a_2^E, \pi_c)\) where the set \( \pi_c \) defines the range of instantaneous profits for which firm 2 will sell out to firm 3 in the exit stage.
2.3 Equilibrium Duopoly and Post-Entry Phase Profits

In a Markov perfect equilibrium, actions, and therefore instantaneous profits, in the duopoly and post-entry phases are independent of firm behaviour in any other phase.

Lemma 1. In a Markov perfect equilibrium firms play their static Nash equilibrium actions in both the duopoly phase and post-entry phase.

Proof. Once entered the post-entry phase is repeated indefinitely. It follows that in a Markov perfect equilibrium firms 1 and 3 select actions to maximise instantaneous profits, thus $a^p_1 = a^p_3 = R(R(a^p_1))$.

While the game will eventually proceed beyond the duopoly phase, actions in the duopoly phase neither effect instantaneous payoffs in later phases nor the probability of entering the entry phase. It follows that in a Markov perfect equilibrium firms 1 and 2 select actions to maximise instantaneous profits, thus $a^d_1 = R(R_c(a^d_1))$ and $a^d_2 = R_c(R(a^d_2))$.

Lemma 1 establishes that both prior to the emergence of a potential entrant, and following the displacement of the inefficient incumbent, the market behaves as in a one shot game. It is only during the entry phase that the strategic effect of the option to exit comes into play.

Lemma 1 also defines the instantaneous profits for the duopoly and post-entry phases, and therefore both the viability of the potential entrant and the incentive for the efficient incumbent to resist entry. Let $\pi^D = \pi(a^D_1, a^D_2)$ and $\pi^d_c = \pi_c(a^D_2, a^D_1)$ represent the equilibrium duopoly phase instantaneous profits of firms 1 and 2 respectively; and let $\pi^p = \pi(a^p_1, a^p_j)$ represent the equilibrium instantaneous profits of firms 1 and 3 in the post-entry phase.

Lemma 2. In a Markov perfect equilibrium $\pi^D > \pi^p$.

Proof. First note that as $R_c$ lies strictly inside $R$, $a^D_2 < a^p_3$. It follows that,

$$\pi^D = \pi(a^D_1, a^D_2) > \pi(a^D_1, a^D_2) > \pi(a^D_1, a^D_3) = \pi^P,$$

9
where the first inequality results from the strict concavity of $\pi$, while the second is implied by assumption that $\pi$ is strictly decreasing in a rival’s action.

Lemma 2 unambiguously states that entry has a negative impact on the profits of firm 1, and therefore that firm 1 has an incentive to prevent entry.

**Assumption 1.** $\pi_c^D < \pi^P$.

Assumption 1 is necessary for entry by firm 3 to be viable. Intuitively, firms 2 and 3 will never be able to agree on a price for firm 2’s access rights if the access rights are more valuable to firm 2. Note that assumption 1 is an assumption over the form of the profit function. It always holds where actions are strategic substitutes as $a_1^D > a_1^P$ and therefore,

$$\pi_c^D = \pi(a_2^D, a_1^D) < \pi(a_3^P, a_1^D) < \pi(a_3^P, a_1^P) = \pi^P.$$

### 2.4 Entry Phase

Firm 3 can only enter the market if there exists a fee, acceptable to both firms 2 and 3, at which firm 2 is willing to exit the market, trading its access rights to firm 3. For the fee to be acceptable to firm 2 it must be greater than the stream of profits that firm 2 receives for staying in the market.

The emergence of a potential entrant causes the game to move to the entry phase. The return to firm 2 of remaining in the market indefinitely, thereby preventing the game from proceeding to the post-entry phase, is,

$$\Pi_2 = \frac{\delta}{1 - \delta} \pi_c^E.$$

Here $\pi_c^E = \pi_c(a_2^E, a_1^E)$ is firm 2’s instantaneous profit in each period of the entry phase.

If firm 2 sells out to firm 3 the game transitions to the post-entry phase. Going forward firm 3 expects a profit of,

$$\Pi_3 = \frac{\delta}{1 - \delta} \pi^P.$$
It is assumed that firm 2 weakly prefers to stay in the market thus a fee must exceed $\Pi_2$ in order to be preferred. Moreover, the fee can be no greater than $\Pi_3$ and still be acceptable to firm 3.

Let $F(E_c)$ represent the fee that firm 3 pays to firm 2 in exchange for firm 2’s access right such that,

$$F(\pi_c^E) \in \left( \frac{\delta}{1 - \delta} \pi_c^E, \frac{\delta}{1 - \delta} \pi_p \right),$$

and $F' \geq 0$, for all $\pi_c^E < \pi_p$. In words, $F$ is in the range that is acceptable to both firms, and is weakly increasing in the current profitability of firm 2. Given firm 2’s weak preference to remain in the market, any fee that would induce firm 2 to exit where $\pi_c^E \geq \pi_p$ would not be acceptable to firm 3. The following lemma summarises firm 2’s strategy in the exit stage of the entry phase.

**Lemma 3.** In a Markov perfect equilibrium firm 2 will exit in the entry phase if, and only if, its instantaneous profit is less than firm 3’s equilibrium instantaneous profit in the post-entry phase; $\pi_c = (-\infty, \pi_p)$.

Firm 2’s response to firm 3’s offer of $F$ is purely mechanical; an optimal response to the offer at the point in time that the offer is made. This must be the case in a Markov perfect equilibrium as Markov perfect equilibria are sequential. It turns out that this mechanical behavior extends to the competition stage of the entry phase. In other words firm 2’s strategy in the entry phase, does not encompass any strategic objective beyond maximising instantaneous profits.

**Lemma 4.** In a Markov perfect equilibrium firm 2 always plays its static best response in the competition stage of the entry phase; $a_2^F = R_c(a_1^F)$.

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7 A weak preference is necessary to avoid an openness problem.

8 This formulation for $F$ is satisfied by both the generalised Nash bargaining solution — a reasonable assumption where firms 2 and 3 are engaged in bilateral bargaining over the price of the access rights — and the case $F = \Pi_3$ which would be expected to arise where two or more equally efficient potential entrants are in competition for the access rights.
Proof. There two possibilities to consider. If, given firm 1’s choice of \( a_1^E \), firm 2 can select \( a_2^E \) such that \( \pi_c(a_1^E, a_2^E) \geq \pi^P \) then selecting \( a_1^E \) to maximise \( \pi_c \) delivers firm 2 an outcome that is strictly preferred to any fee that firm 3 is willing to offer.

Where firm 2’s choice of \( a_2^E \) cannot result in \( \pi_c^E \geq \pi^P \) firm 2 will exit at the end of the first period of the entry phase. Nevertheless, maximising \( \pi_c \) remains in firm 2’s best interests as it maximises firm 2’s instantaneous payoff in the single period of the entry phase as well as weakly increasing the fee firm 2 receives from firm 3. \( \square \)

2.5 Markov Perfect Equilibria

It follows from lemma 4 that there exist at most two Markov perfect equilibria to this game: A competitive equilibrium in which firm 2 exits in the first period of the entry phase, and an accommodating equilibrium in which firm 2 remains in the market indefinitely. The following lemmas characterise firm 1’s strategies in each of these equilibria.

Lemma 5. In a Markov perfect equilibrium in which firm 2 remains in the market indefinitely — an accommodating equilibrium — \( a_1^E \) solves,

\[
\pi_c^E(R_c(a_1^E), a_1^E) = \pi^P.
\]

Proof. From lemmas 3 and 4 it is straightforward to see that \( a_1^E \) must satisfy,

\[
\pi_c^E(R_c(a_1^E), a_1^E) \geq \pi^P. \tag{2.1}
\]

To exclude the inequality note that \( \pi_c^E(R_c(a_1^E), a_1^E) \) is strictly decreasing in \( a_1^E \). Given that \( \pi_c^D < \pi^P \) it follows that in order to satisfy (2.1) \( a_1^E < a_1^D \) and thus from the single crossing property of the best response functions \( a_1^E < R(R_c(a_1^E)) \).

Suppose to the contrary that \( \pi_c^E(R_c(a_1^E), a_1^E) > \pi^P \) in a Markov perfect equilibrium. Given the continuity and concavity of \( \pi \), firm 1 can increase its instantaneous profit in each period of the exit phase by marginally increasing \( a_1^E \). For a small
enough increase (2.1) will continue to hold and firm 2 will remain in the market. A Contradiction.

Lemma 6. In a Markov perfect equilibrium in which firm 2 exits in the first period of the exit phase — a competitive equilibrium — firm actions and instantaneous profits in the entry phase are identical to their actions and instantaneous profits in the duopoly phase.

Proof. It follows from lemmas 2, 3 and 4 that it is only necessary to show that $a_1^E = a_1^D$. To see that this must be the case note that firm 1’s only concern in the exit phase is to maximise its instantaneous profits.

It follows from lemma 5 that a necessary condition for the existence of an accommodating equilibrium is,

$$\frac{1}{1-\delta} \pi(a_1^E, R_c(a_1^E)) \geq \pi \left( R(R_c(a_1^E)), R_c(a_1^E) \right) + \frac{\delta}{1-\delta} \pi^P. \quad (2.2)$$

In words, the stream of profits firm 1 receives accommodating firm 2 must be at least as great as the profit firm 1 receives from deviating to its static best response in the first period of the entry phase, followed by infinite repetition of the post-entry game. Note that there exists a $\delta \in (0, 1)$ such that (2.2) holds if, and only if,

$$\pi(a_1^E, R_c(a_1^E)) > \pi^P. \quad (2.3)$$

Similarly, from lemma 6 it follows that a necessary condition for the existence of a competitive equilibrium,

$$\pi^D + \frac{\delta}{1-\delta} \pi^P \geq \frac{1}{1-\delta} \pi(a_1, a_2^D), \quad (2.4)$$

where $a_1$ solves $\pi_c(a_2^D, a_1) = \pi^P$. This condition is the converse of (2.2). Firm 1 will not attempt to accommodate firm 2 so long as the return from accommodating firm 2 is weakly less than the return from playing static best responses in every phase of the game. As $\pi^D > \pi(a_1, a_2^D)$, condition (2.4) is always satisfied for $\delta$ sufficiently close to zero.
Proposition 1. (a) Where the actions of firms are strategic complements there exists a $\bar{c} > 0$ and $\bar{\delta} \in (0, 1)$ such that for all $c \in (0, \bar{c})$ and $\delta \in (\bar{\delta}, 1)$ there exists an accommodating equilibrium. (b) Where the actions of firms are strategic substitutes there does not exist an accommodating equilibrium.

Proof. Define $a(c)$ as the action that solves,

$$\pi_c[R_c(a(c)), a(c)] = \pi^P,$$

and note that $a(c)$ is continuous, strictly decreasing in $c$, and that as $c \to 0$, $a(c) \to a_1^P$.

The continuity of all functions in $c$ guarantees that there exists numbers $\hat{c} > 0$ and $\hat{a}_1^E < a_1^P$ such that for all $c \in (0, \hat{c})$, $a(c) \in (\hat{a}_1^E, a_1^P)$.

For a given $c > 0$, there exists $\delta \in (0, 1)$ such that an accommodating equilibrium exists if, and only if, (2.3) holds. Note that in the limit, as $c \to 0$, (2.3) holds with equality. Taking the derivative of the LHS of (2.3) yields,

$$\frac{d}{dc} \pi[a(c), R_c(a(c))] = a'R_c \frac{\partial \pi(a_i, a_j)}{\partial a_i} + a'R_c \frac{\partial \pi(a_i, a_j)}{\partial a_j} \bigg|_{(a_i, a_j) = [a(c), R_c(a(c))]}.$$

(2.5)

The first term on the RHS of (2.5) is unambiguously negative as $a(c) < R_c(a(c))$, and approaches zero as $c \to 0$. The second term is negative (positive) where the actions of firms are strategic substitutes (complements).

It follows that where actions are strategic substitutes the LHS of (2.3) is decreasing in $c$. As (2.3) holds with equality in the limit as $c \to 0$, (2.3) is never satisfied for $c > 0$, proving (b).

For (a) note that for arbitrarily small $c > 0$, the first term on the RHS of (2.5) is likewise arbitrarily close to zero and therefore (2.5) is positive. It follows that there exists $\tilde{c} > 0$ such that (2.3) is satisfied for all $c \in (0, \tilde{c})$. Defining $\bar{c} = \min\{\tilde{c}, \bar{c}\}$ completes the proof.

Proposition 1 proves that strategic complementarity is a necessary condition for the existence of an accommodating equilibrium. Moreover, it is clear from the proof
that the distortion away from static equilibrium behaviour, necessary to implement
the accommodating equilibrium, is increasing in $c$. The following propositions provide
a comparison of firm competition and payoffs under the competing equilibria.

**Proposition 2.** *For sufficiently high $\delta \in (0, 1)$, an accommodating equilibrium delivers firms 1 and 2 outcomes that are strictly preferred to a competitive equilibrium.*

*Proof.* It is straightforward to see that firm 2 prefers the accommodating equilibrium. To see that firm 1 is likewise better off note that (2.3) must hold in an accommodating equilibrium.

**Proposition 3.** *In an accommodating equilibrium competition is at its lowest in the entry phase.*

*Proof.* Given the necessity of strategic complementarity $a_1^E < a_1^D < a_1^P$ and $a_2^E < a_2^D$.

An accommodating equilibrium is not a collusive outcome. The fall in competition
that coincides with the arrival of a potential entrant can be entirely attributed to
unilateral optimisation by firm 1. This is clearly illustrated by two key features of
the model: First, firms are playing Markov strategies and are therefore unable to
play strategies that are contingent on the past behaviour of rival firms. Specifically,
the threat of exit is never used by firm 2 to discipline firm 1. Second, firm 2 does
nothing to facilitate the accommodation. In a Markov perfect equilibrium firm 2
always plays its static best response. This failure to assist firm 1 is the reason why
an accommodating equilibrium cannot exist where actions are strategic substitutes.

3 Examples

This section presents two simple examples that illustrate the results of the previous
section. The first is price competition on the Hotelling line, an illustration of
strategic complements. The Hotelling example demonstrates that in an accommodating equilibrium, not only is competition reduced, but firm profits increase with the emergence of a potential entrant. The second example is Cournot competition. Illustrating the non-existence of an accommodating equilibrium where actions are strategic substitutes.

3.1 Hotelling Competition

Two firms are located at either end of a unit line. A unit mass of consumers is uniformly distributed along the line with each consumer having unit demand and linear transport cost $t > 0$ per unit travelled. The market is assumed to be covered in each phase. Firm 1 is located at the left end of the line and faces a marginal cost of zero, while firm 2 is located at the right end of the line and faces a marginal cost of $c \in (0, 3t)$.\(^9\) If firm 3 replaces firm 2 in the entry phase it takes firm 2’s position at the right end of the line.

Each period firms compete by simultaneously setting prices. Given that low prices represent aggressive behaviour, a firm’s action is interpreted as being the negative of its price. The instantaneous profit functions for this game are,

$$
\pi = p_i p_j - p_i + \frac{t}{2t}, \quad \pi_c = (p_i - c)p_j - p_i + \frac{t}{2t},
$$

where $p_i$ is the firm’s own price and $p_j$ is the price set by the rival firm. The corresponding best responses are given by the functions,

$$
R(p_j) = \frac{p_j + t}{2}, \quad R_c(p_j) = \frac{p_j + t + c}{2}.
$$

Employing the results of the previous section, this structure is sufficient to establish the Markov perfect equilibrium behaviour of firms in both the duopoly and post-entry

\(^9\)The upper-bound on $c$ ensures that firm 2 will always produce a strictly positive quantity in a static equilibrium.
phases. Lemma 1 implies,

\[ p_1^D = t + \frac{1}{3}c, \quad \pi^D = \frac{1}{2t} \left( t + \frac{1}{3}c \right)^2, \]
\[ p_2^D = t + \frac{2}{3}c, \quad \pi_c^D = \frac{1}{2t} \left( t - \frac{1}{3}c \right)^2, \]
\[ p_i^P = t, \quad \pi^P = \frac{t}{2}. \]

It is straightforward to see that \( \pi^P > \pi_c^D \) and \( \pi^D > \pi^P \).

The value of the variables in the entry phase depend on the nature of the equilibrium. If an accommodation equilibrium exists \( p_1^E \) is defined implicitly by,

\[ \pi_c(R_c(p_1^E), p_1^E) = (R_c(p_1^E) - c) \frac{p_1^E - R_c(p_1^E)}{2t} + t = \frac{t}{2} = \pi^P. \]

The unique solution to this equation is \( p_1^E = t + c \). Moreover, \( p_2^E = R_c(p_1^E) = t + c \) yielding instantaneous profits of,

\[ \pi^E = \frac{t + c}{2}, \quad \pi_c^E = \frac{t}{2}. \]

To prove the existence of an accommodating equilibrium it only remains to show that there exists \( \delta \in (0, 1) \) such that (2.2) holds. \( R(t + c) = t + c/2 \) thus (2.2) becomes,

\[ \frac{1}{1 - \delta} \left( \frac{t + c}{2} \right) \geq \frac{1}{2t} \left( t + \frac{1}{2}c \right)^2 + \frac{\delta}{1 - \delta} \left( \frac{t}{2} \right), \]
\[ \frac{\delta}{1 - \delta} \geq \frac{c^2}{4t^4}, \]
\[ \delta \geq \frac{c}{4t + c} \in \left( 0, \frac{1}{7} \right). \]

In words, an accommodating equilibrium exists for all \( c \in (0, 3t) \) and \( \delta \in (1/7, 1) \); a wide range of discount factors.

Surprisingly, both incumbent firms benefit from the emergence of a potential entrant in an accommodating equilibrium. Given that \( c < 3t \),

\[ \frac{t + c}{2} > \frac{1}{2t} \left( t + \frac{1}{3}c \right)^2, \quad (3.1) \]
indicating that firm 1 enjoys higher profits in the entry phase than it does in either the post-entry or duopoly phases. Intuitively, with prices strategic complements in Hotelling competition, the inefficient incumbent responds to a relaxation of competition by itself reducing competition. In turn this reduces the cost to firm 1 of accommodating firm 2. Absent a potential entrant firm 1 cannot credibly commit to this reduction in competition as \( R_1(R_2(p_1)) < p_1 \) for \( p_1 > t + c/3 \) implies that firm 1 has the incentive to unilaterally lower its price. This incentive is eliminated by the fact that firm 2 will sell out to an efficient entrant should the flow of profits to the firm fail to match the value of the access rights to the entrant.

### 3.2 Welfare in an Accommodating Equilibrium

A welfare analysis of the accommodating equilibrium in the Hotelling case is straightforward. The twin assumptions of market coverage and unit demand imply that social welfare is a function of productive efficiency alone.

Productive efficiency, and therefore welfare, is highest in the post-entry phase where the efficiency of both firms gives rise to a zero cost of production. In the duopoly phase the inefficient firm serves a fraction \( q_2^D = (3t - c)/6t \) of the market at a marginal cost of \( c \). In other words, going forward the welfare gain that results where the inefficient firm is replaced by an efficient entrant is thus,

\[
\Delta W_{\text{Gain}} = \frac{\delta}{1 - \delta} \left( \frac{c}{2} - \frac{c^2}{6t} \right).
\]

By choosing to accommodate its rival in the entry phase the efficient incumbent not only prevents the efficiency gain \( \Delta W_{\text{Gain}} \) but also concedes market share to its inefficient rival. In turn this results in the cost of production rising to \( c/2 \) each period; higher than the cost of production in either the duopoly or post-entry phases. In fact, in an accommodating equilibrium, the emergence of a potential entrant results in the welfare loss,

\[
\Delta W_{\text{Loss}} = \frac{\delta}{1 - \delta} \left( \frac{c^2}{6t} \right).
\]
The $c^2$ term represents the fact that as the inefficient firm’s cost disadvantage increases, so too does the share of the market that the efficient firm must sacrifice to the inefficient firm in an accommodating equilibrium. At the same time the higher value of $c$ increases the welfare cost of any given market share that efficient firm sacrifices.

The order of prices — implied by proposition 3 and illustrated in this example — have implications for cases in which demand is downward sloping. In an accommodating equilibrium prices are highest in the entry phase and lowest in the post-entry phase. This suggest that in addition to the loss of productive efficiency, accommodation may also increase the magnitude of the dead-weight loss in the market.

### 3.3 Cournot Competition

The second example considered in this section is that of Cournot competition in a market for a homogeneous good. The model retains the same basic structure as the preceding Hotelling example with each firm’s action now the quantity that it selects. Firms 1 and 3 have a constant marginal cost of zero, while firm 2 has a constant marginal cost of $c \in (0, 1/3)$. Each period demand in the market is $D(p) = 1 - p$.

In each period, firms compete by simultaneously selecting quantities. Instantaneous profit functions of each firm are given by,

$$\pi = q_i(1 - q_i - q_j), \quad \pi_c = q_i(1 - q_i - q_j - c),$$

where $q_i$ is the firm’s own quantity and $q_j$ is the quantity set by the rival firm. The corresponding best response functions are,

$$R(q_j) = \frac{1 - q_j}{2}, \quad R_c(q_j) = \frac{1 - q_j - c}{2}.$$

The Markov perfect equilibrium behaviour of firms in both the duopoly and post-entry
phases is summarised by,

\[ q_D^1 = \frac{1 + c}{3}, \quad \pi^D = \left(\frac{1 + c}{3}\right)^2, \]
\[ q_D^2 = \frac{1 - 2c}{3}, \quad \pi_c^D = \left(\frac{1 - 2c}{3}\right)^2, \]
\[ q_i^P = \frac{1}{3}, \quad \pi^P = \frac{1}{9}. \]

In order to accommodate firm 2, firm 1 must select its quantity such that,

\[ \pi_c^c(R_c(q_1^E), q_1^E) = R_c(q_1^E)(1 - q_1^E - c - R_c(q_1^E)) = \frac{1}{9} = \pi^P, \]

which in turn implies \( q_1^E = \frac{1}{3} - c \) and \( q_2^E = R_c\left(\frac{1}{3} - c\right) = \frac{1}{3} \). To see that these quantities cannot be supported in equilibrium note that for the Cournot case,

\[ \pi(a_1^E, R_c(a_1^E)) = \frac{1}{9} - c^2 < \frac{1}{9} = \pi^P, \]

for all \( c \in (0, 1/3) \), contradicting condition (2.3).

The non-existence of an accommodation equilibrium can be put down to the nature of strategic interaction in Cournot competition. Quantities are strategic substitutes and as such when firm 1 relaxes competition to accommodate firm 2, firm 2 responds by adopting a more aggressive position. This increases the cost to firm 1 of accommodating firm 2 into the region where accommodation ceases to be a profitable approach. Thus, in this model, potential entry performs its conventional role of having no impact on behavior until such time as that entry becomes actual.

4 An Investment Game

Replacement is only one mechanism by which the technological efficiency of a restricted access market can be improved. A second possibility is for an inefficient incumbent to invest in a cost reducing technology. This section presents a modified version of the model developed in section 2 in which the inefficient incumbent firm is
given the opportunity to invest in a cost reducing technology. This might arise, for example, if the incumbent’s patent on a process innovation expires.

Once again there are two possible Markov perfect equilibria: A competitive equilibrium in which all players play their static best responses throughout, resulting in firm 2 investing in the technology and the efficiency of the market increasing. The second is an accommodating equilibrium in which firm 1 directs a stream of profits to firm 2 that is sufficient to remove the incentive to invest. It is shown that for a market in which an accommodating equilibrium exists to the entry game, an accommodating equilibrium will also exist to the investment game. Moreover, for sufficiently high $\delta$, the accommodating equilibrium is preferred by both firms to the competitive equilibrium.

4.1 The Modified Model

The model follows the structure laid out in section 2 with the following amendments:

1. The game only involves two players, firms 1 and 2. As before firm 1 is efficient with profit function $\pi$, while firm 2 is inefficient with profit function $\pi_c$.

2. The duopoly phase occurs unless a new technology emerges in which case an investment phase takes place.

3. At the end of each period of the investment phase firm 2 decides whether or not to invest in the new technology. If firm 2 invests, it pays the one time cost $C \geq 0$ of implementing the technology, and the game proceeds to the post-investment (formerly post-entry) phase.

4. As a consequence of implementing the cost reducing technology, the more efficient technology $\pi$ replaces $\pi_c$ as firm 2’s profit function in the post-investment phase. Consequently, competition in the post-investment phase occurs between two equally efficient firms.
It follows that firm 1’s Markov strategy remains a triplet \((a_1^D, a_1^I, a_1^P)\), where \(a_1^I\) is firm 1’s action in each period of the investment phase. While firm 2’s Markov strategy is a tuple \((a_2^D, a_2^I, a_2^P, \pi_c)\) where \(\pi_c\) is the set of investment phase instantaneous profits that would lead firm 2 to implement the cost reducing technology.

4.2 Markov Perfect Equilibria

Amendments notwithstanding, the results developed in section 2 carry over to investment problem almost unchanged. Lemma 1 continues to hold and as such firms play their static best responses in the duopoly and post-investment phases. Once again lemma 1 defines duopoly and post-investment actions and profits in both the duopoly and post-investment phases.

Lemma 3, which defines the range of instantaneous profits which will cause firm 2 to exit, must be modified for the investment game. Due to the cost of implementing the cost reducing technology, the return to investment is,

\[
\Pi_2 = \frac{\delta}{1 - \delta} \pi^P - C.
\]

It follows that firm 2 will make the investment if, and only if,

\[
\pi_c^E < \pi^P - \frac{1 - \delta}{\delta} C.
\]

Once again it is assumed that firm 2 weakly prefers not to make the investment.

From lemmas 4 and 5 it follows that in order to accommodate firm 2, firm 1 must select \(a_1^E\) such that,

\[
\pi_c(R_c(a_1^E), a_1^E) = \pi^P - \frac{1 - \delta}{\delta} C \leq \pi^P. \tag{4.1}
\]

The inequality in (4.1) indicates that it is (weakly) easier for firm 1 to accommodate firm 2 in the investment game than the entry game. Given this structure the condition 2.3 is once again necessary for the existence of an accommodating equilibrium.
Proposition 4. Suppose that the action of firms are strategic complements and that $c \in (0, \bar{c})$, where $\bar{c}$ is defined as in proposition 1. For all $C \geq 0$ there exists an accommodating equilibrium to the investment game.

Proof. The case in which $C = 0$ holds by proposition 1. Similarly, the case in which, 

$$C \geq \frac{\delta}{1 - \delta} \pi_{c}^{D},$$

can be dismissed as $a_{1}^{E} = a_{1}^{D}$ is sufficient to accommodate firm 2. For the remaining cases note that by the monotonicity of $\pi_{c}(R_c(a_{1}^{E}), a_{1}^{E})$ in $a_{1}^{E}$, and the proof to proposition 1, there exists $a_{1}^{E} \in (a(\bar{c}), a_{1}^{D})$ that satisfies (4.1) and therefore, by the proof to proposition 1, also satisfies (2.3).

Proposition 4 provides an explanation for a firm gaining a persistent technical advantage in a restricted access market. One firm achieves a technological lead over its rival, defending it by offering its rival an inducement not to match the efficiency gain. So long as further entry into the market is not viable this accommodation is not only stable, but an outcome that is preferred by both firms.

Interestingly, this provides a rationale for incumbent firms with a technological advantage not pursuing patent protection but instead committing to an open technology. In this respect, it is related to Gallini (1984) who showed that an incumbent might license its technology to an entrant to prevent that entrant from leap-frogging the incumbent’s technological leadership. A key difference between Gallini and the result presented here is that the presence of the R&D opportunity from the entrant harms the incumbent. In contrast, here that potential improves the profits of both firms by allowing the efficient firm to commit to a softer competitive environment.

The proofs presented in this section also have implications for the entry model outlined in section 2. A direct corollary of proposition 4 is that accommodation remains an equilibrium outcome where an entrant faces a non-zero entry cost in addition to the cost of acquiring the access right. Likewise, entry can be prevented
via accommodation where the efficiency of the entrant lies strictly between that of the two incumbents. In each of these cases the willingness to pay off the potential entrant is reduced; either by the need to cover the cost of entry or reduced post-entry profits resulting from reduced efficiency. Consequently, the stream of profits that the efficient firm must direct to its rival — in order to prevent the inefficient firm from selling out to the entrant — is reduced, and the magnitude of the distortion that must be created in order facilitate accommodation is likewise smaller. In other words, the more efficient is a potential entrant, and the is lower the cost of entry that the entrant faces, the greater will be the distortion away from a competitive outcome in an accommodating equilibrium.

5 Efficient Mergers

This section presents a simple example in which firms can employ an accommodation strategy to prevent a socially efficient merger. Unlike the model of the previous sections, three ex-ante identical firms are active in the market. A new class of accommodation is possible; one in which all three firms set prices that are greater than their instantaneous best responses in order to prevent any two firms from merging.

5.1 The Model

The model presented in this section is based on Salop’s (1979) circular city. Three identical firms — indexed 1, 2 and 3 — are evenly spaced around the unit circle. In each period firms compete by setting prices. Firms are assumed to have unlimited capacity and a common constant marginal cost that is normalized to zero. The circle is populated by a unit mass of consumers, likewise uniformly distributed around the circle. Consumers have unit demand and face a transport cost of $t$ per unit travelled. It is assumed that the market is covered.

Otherwise the model has the same structure as the models of the previous sections.
There is an *oligopoly phase* \((O)\) in which the three firms are engaged in a repeated price setting game. A merger between two (or more) firms is not a possibility during the oligopoly phase. This prohibition against mergers can be interpreted as the product of regulatory policy. If the prohibition against mergers is dropped the game transitions to the *merger phase* \((M)\).

Each period of the merger phase begins with a competition stage; setting prices and realising of instantaneous profits. The two firms with the lowest instantaneous profits — the firms that create the greatest surplus by merging — are then given the opportunity to merge. In this way the identity of the two firms which will have the opportunity to merge is endogenously determined by firm actions in the competition stage. If the highest instantaneous profit is realized by more than one firm then two firms are randomly matched.\(^{10}\) If the two firms who achieved the lowest instantaneous profits fail to reach an agreement, the game repeats. On the other hand, a merger causes the game to transition to the *post-merger phase* \((P)\).

As was the case in the preceding models, the post-merger phase is an infinitely repeated price setting game with no further opportunity for entry or merger. It can be assumed that anti-trust regulations prevent a merger from creating a monopoly, and that the returns to entry are insufficient to cover the cost.

Post-merger, a merged entity is assumed to occupy both locations of its constituent firms but sets a common price across locations. It follows from the welfare analysis of the Hotelling line example that for a merger to be efficient in this market it must produce cost savings for the merged entity. Suppose that, if two firms merge, a synergy of magnitude \(s \in [0, t)\) is created within the firm. The synergy is interpreted as the reduction in the marginal cost of production resulting from efficiencies achieved through the merger. In this way the merger creates the classic dilemma for an anti-trust authority. The merger reduces the cost of production — an increase

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\(^{10}\) i.e. If all firms set the same price each firm has a \(2/3\) probability of being one of the two firms to enter merger negotiations in any given period.
in productive efficiency — however consumers on the short arc between the merged firm’s locations become captives of that firm, giving the merged firm an incentive to raise its price. Once again the assumption that the market is covered means that prices have no impact on efficiency. More generally, where demand is downward sloping the incentive for the merged firm to raise prices creates an efficiency loss that offsets the reduced cost of production to some degree.

5.2 Demand and Firm Profits

The demand for firm $i$’s product, in any given period, is given by the expression,

$$q_i = 2 \frac{p_i + p_k}{2} - p_i + \frac{t}{3},$$

where $p_i$ is the firm’s own price and $p_j$ and $p_k$ represent the prices posted by the rival firms. Firm $i$’s instantaneous profit and best response functions are therefore,

$$\pi_i(p_i, p_j, p_k) = p_i \frac{p_j + p_k}{2} - p_i + \frac{t}{3}, \quad R(p_j, p_k) = \frac{p_j + p_k}{4} + \frac{t}{6}.$$

It follows from lemma 1 that in a Markov Perfect equilibrium all firms set the price $p^O = t/3$ and receive instantaneous profit $\pi^O = t/9$ in each period of the oligopoly phase.

Following a merger, the merged firm’s market share in any given period is given by the expression,

$$q_m = \frac{1}{3} + 2 \frac{p_i - p_m + \frac{t}{3}}{2t},$$

where $p_m$ is the merged firm’s price while $p_i$ is the price of the remaining firm. The first term on the RHS of this equation represents the consumers who are captives of the merged firm, while the second term represents the set of consumers on the contested arcs that purchase from the merged firm. The profit and best response functions for the merged firm are thus,

$$\pi_m = (p_m + s) \left( \frac{1}{3} + \frac{p_i - p_m + \frac{t}{3}}{t} \right), \quad R_m(p_i) = \frac{p_i - s}{2} + \frac{t}{3}. $$
It follows that in a Markov perfect equilibrium,

\[ p^p = \frac{4}{9} t - \frac{1}{3} s, \quad \pi^p = \frac{1}{t} \left( \frac{4}{9} t - \frac{1}{3} s \right)^2, \]

\[ p^m = \frac{5}{9} t - \frac{2}{3} s, \quad \pi^m = \frac{1}{t} \left( \frac{5}{9} t + \frac{1}{3} s \right)^2. \]

Note that \( \pi^p \), as well as all prices, are strictly decreasing in \( s \) for all \( s \in [0, t) \).

Interestingly, where the value of the synergy is sufficiently low, the firm excluded from the merger benefits from being excluded. Formally, \( \pi^p \geq \pi^m / 2 \) for all \( s \in [0, s^*] \) where,

\[ s^* = t \frac{8 - 5 \sqrt{2}}{6 + 3 \sqrt{2}}. \]

For larger values of \( s \) the excluded firm receives strictly less than half the merged firm’s profit.

Another significant value of the synergy is \( s^{**} = t/3 \). The synergy \( s^{**} \) is significant for two reasons: First, for all \( s \in [0, s^{**}) \) the firm excluded from the merger is strictly better off post-merger. This is true even in the region \((s^*, s^{**})\). Intuitively, for small synergies the incentive for the merged firm to raise prices, exploiting captive consumers, is greater than its incentive to lower prices to take advantage of the reduction in marginal cost. Conversely, for all \( s \in (s^{**}, t) \) the excluded firm is unambiguously worse off as a result of the merger. Second, for synergies less (greater) than \( s^{**} \), post-merger equilibrium prices are greater (less) than pre-merger prices. It follows that where \( s \in (s^{**}, t) \) the effects of a merger are unambiguously socially desirable.

### 5.3 The Merger Phase

Before proceeding with the analysis of pricing behaviour in the merger phase it is necessary to make some assumptions concerning the process of negotiation between the two partners to the potential merger. An agreement to merge is essentially a
division of the post merger value of the firm,

\[ \Pi_m = \frac{\delta}{1 - \delta} \pi_m^P. \]

Suppose that firm’s \( i \) and \( j \) have the lowest instantaneous profits and therefore enter into merger negotiations at the end of a period. The surplus created by a merger is therefore,

\[ S = \frac{\delta}{1 - \delta} \left( \pi_m^P - \pi_i(p_i^P, p_j^P, p_k^P) - \pi_j(p_j^P, p_i^P, p_k^P) \right). \]

In line with the Nash bargaining solution it is assumed that the this surplus is divided evenly between the two partners to the merger. As previously, it is assumed that firms weakly prefer not to merge and, therefore, the sum of the merger phase instantaneous profits of any two firms must be at least equal to \( \pi_m^P \) in order to facilitate an accommodating equilibrium.

**Proposition 5.** For all \( s \in (s^*, t) \) there exists \( \delta \in (0, 1) \) such that symmetric accommodation is a Markov perfect equilibrium. In this accommodating equilibrium all three firms set the price,

\[ p^M = \frac{3}{2t} \left( \frac{5}{9}t + \frac{1}{3}s \right)^2, \]

in each period of the merger phase.

**Proof.** First note that if all firms set the price \( p^M \) in the merger phase all firms realise the instantaneous profit,

\[ \pi^M = \frac{1}{2t} \left( \frac{5}{9}t + \frac{1}{3}s \right)^2 = \frac{\pi_m^P}{2}. \]

No firm can unilaterally lower its price from this level without in turn leaving the remaining two firms with instantaneous profits that sum to less than \( \pi_m^P \) which in turn will result in the remaining firms merging, transitioning the game to the post-merger phase. It follows that a unilateral deviation by firm \( i \), to its best response, will not improve firm \( i \)'s payoff where,

\[ \frac{1}{1 - \delta} \frac{\pi_m^P}{2} \geq \pi_i(R(p_j, p_k), p_j, p_k) + \frac{\delta}{1 - \delta} \pi_j^P. \quad (5.1) \]
The necessary and sufficient condition for the existence of a $\delta \in (0, 1)$ that satisfies (5.1) is $\pi^P_m/2 > \pi^P$ which holds for all $s \in (s^*, t)$.

Proposition 6 illustrates that accommodation is not confined to action by a single firm defending an advantageous position in a market. Ex-ante identical firms can engage in symmetric accommodation with much the same result. As was the case in previous examples, in an accommodating equilibrium, prices are highest and competition lowest in the merger phase. Moreover, accommodation prevents the realisation of synergies that would reduce the cost of production.

Intriguingly, an accommodating equilibrium is possible where $s \in (s^*, s^{**})$ even though the firm excluded from the merger earns a profit in the post-merger phase that is higher than its profit in the oligopoly phase. It is only where the synergy is so low that the excluded firm earns a profit in excess of $\pi^P_m/2$ — that is where $s \in [0, s^*]$ — that accommodation ceases to be viable.

Unlike the entry and investment games described above, the merger case requires all players to take actions that deviate from their static best responses in the merger phase. The behavior of firms in this example has a lot in common with multi firm collusion enforced by a grim strategy. As is the case in collusion, a deviation in any period is punished by the remaining firms merging, reducing the flow of profits to the deviating firm in all subsequent periods. However, unlike collusion, the folk theorem does not operate here and there is only one profit level that can be support in an accommodating equilibrium: The threat of a merger is only credible where a profitable deviation by one firm necessarily causes the combined profits of the remaining firms to fall below the post merger level.

6 Free Riding with Three Firms

The possibility that more than two firms can operate in a market also raises questions for the entry model outlined in section 2. Specifically, can multiple firms collectively
engage in accommodating behaviour? And, is there the possibility that an efficient firm will free ride on the accommodating behaviour of its rival? In this section we again employ Salop’s (1979) unit circle in order to develop an elementary extension of the Hotelling example in which 3 firms may operate in the market simultaneously. It is shown that both symmetric and asymmetric accommodation is possible, however free riding has the effect of reducing the range of parameter values over which accommodation is possible.

Once again, the model sees three firm’s evenly distributed around the circular city. Suppose that two of the firms are efficient with marginal costs of zero, while the remaining inefficient firm has a marginal cost of \(c = (0, 5t/6)\).\(^{11}\) In the entry phase a third efficient firm emerges which can only enter the market by purchasing the inefficient incumbent’s access rights.

The profit and best response function of firms in this market are,

\[
\pi(p_i, p_j, p_k) = p_i \frac{p_j + p_k}{2} - p_i + \frac{t}{3}, \quad R(p_j, p_k) = \frac{p_j + p_k}{4} + \frac{t}{6},
\]

for the efficient firms and,

\[
\pi_c(p_i, p_j, p_k) = (p_i - c) \frac{p_j + p_k}{2} - p_i + \frac{t}{3}, \quad R(p_j, p_k) = \frac{p_j + p_k}{4} + \frac{t}{6} + \frac{c}{2},
\]

for the inefficient incumbent. It follows that in the oligopoly and post-merger phases,

\[
p^O = \frac{t}{3} + \frac{c}{5}, \quad \pi^O = \frac{1}{t} \left( \frac{t}{3} + \frac{c}{5} \right)^2,
\]

\[
p^C = \frac{t}{3} + \frac{3c}{5}, \quad \pi^C = \frac{1}{t} \left( \frac{t}{3} - \frac{2c}{5} \right)^2,
\]

\[
p^P = \frac{t}{3}, \quad \pi^P = \frac{t}{9}.
\]

The following proposition characterises the symmetric and free rider accommodating equilibria.

---

\(^{11}\)At \(c \geq 5t/6\) the inefficient firm will not trade with any consumers in a static equilibrium.
Proposition 6. There exists $\delta \in (0,1)$ such that (a) for all $c \in (0,5t/6)$ there exists a Markov perfect equilibrium in which both efficient firms set the price $p^E = \frac{t}{3} + c$ in the entry phase; (b) for all $c \in (0,20t/63)$ there exists a Markov perfect equilibrium in which one efficient firm, firm $i$, sets the price $p^E_i = \frac{t}{3} + \frac{7c}{5}$ in the entry phase, while the remaining efficient firm, firm $j$, sets its price according to their its best response function. In both cases the inefficient firms remains in the market indefinitely.

Proof. From lemma 4 we know that the inefficient firm plays its static best response in the entry phase. For (a) first note that $p^E_c = R_c\left(\frac{t}{3} + c, \frac{t}{3} + c\right) = \frac{t}{3}$ yielding the inefficient incumbent $\pi^E_c = \pi^P = \frac{t}{9}$. These prices constitute a Markov perfect equilibrium as $\pi^E = \frac{t}{9} + \frac{c}{3} > \pi^P$.

For (b) note that in a Markov perfect equilibrium,

$$p^E_j = R\left(\frac{t}{3} + \frac{7c}{5}, \frac{t}{3} + c\right) = \frac{t}{3} + \frac{3c}{5}, \quad p^E_c = R_c\left(\frac{t}{3} + \frac{7c}{5}, \frac{t}{3} + \frac{3c}{5}\right) = \frac{t}{3} + c.$$

These prices yield a profit to the inefficient incumbent of $\pi^E_c = \pi^P = \frac{t}{9}$, while firm $i$ receives,

$$\pi^E_i = \frac{1}{t}\left(\frac{t}{3} + \frac{7c}{5}\right)\left(\frac{t}{3} - \frac{3c}{5}\right),$$

which is strictly greater than $\pi^P$ if, and only if, $c \in (0,20t/63)$.

Proposition 6 demonstrates that in an oligopoly accommodation can take two forms: Symmetric accommodation in which the burden of accommodating the inefficient firms is shared, and accommodation with a free rider in which one efficient incumbent opportunistically free rides on the efforts of its efficient rival. In this latter equilibrium the free rider earns an instantaneous profit of,

$$\pi^E_j = \frac{1}{t}\left(\frac{t}{3} + \frac{3c}{5}\right)^2,$$

in each period of the entry phase. This profit is greater than both firm $i$’s instantaneous profit in the free rider equilibrium, and the profit of either efficient firm in a symmetric accommodating equilibrium. It follows that an efficient firm will always
prefer to free ride in equilibrium. However, free riding has consequences for the viability of accommodation. As proposition 6 demonstrates, symmetric accommodation is possible for all $c \in (0, 5t/6)$ while accommodation with free riding can only be supported where $c \in (0, 20t/63)$.\textsuperscript{12}

It follows that there is a sense in which accommodation becomes more difficult where one or more firms free ride. However, for values of $c$ where accommodation with free riding is not an equilibrium, there remains the possibility that efficient firms will symmetrically accommodate their inefficient rival.

7 Conclusion

This paper provides a general context where potential entry can reduce the intensity of competition. The mechanism for this result is the combination of an environment where entry requires the acquisition of critical incumbent assets and key competitive variables are strategic complements. In this situation, an inefficient incumbent may be deterred from selling assets to an efficient entrant by the accommodating actions of a more efficient incumbent. That incumbent sacrifices short-term profits to improve the profitability its rival to such an extent that there are no gains to trade in selling out to a more efficient entrant. We demonstrated that the same mechanism might drive deterred investment and merger activity.

As a novel result in economics, this paper might explain incumbent behavior in the face of potential entry. For instance, in 2008, Microsoft proposed to take-over Yahoo! and create a stronger second force to Google in the search engine market.\textsuperscript{12} Partial free riding is also a possibility here. It is straightforward to verify that any price pair which satisfies,

$$\frac{p_i^E + p_j^E}{2} = \frac{t}{3} + c, \quad \min\{p_i^E, p_j^E\} \geq \frac{t}{3} + \frac{3c}{5},$$

will support an accommodating equilibrium. The firm with the low price can be regarded as partially free riding on the efforts of the higher priced firm, even where its price is above its static best response. Across the admissible range of prices the profit of each firm is decreasing in its own price, and the maximum value of $c$ for which an accommodation can be sustained is increasing in the minimum of the two prices.
That market is limited by customer attention that itself was constrained by switching costs or behavioral inertia. In response, Google proposed a more limited joint venture deal with Yahoo! causing it to reject Microsoft’s offers. The Google response could be viewed as an accommodating one designed to remove the gains of trade between its two rivals (as per the merger model presented in the paper). In the end, anti-trust concerns prevented the Google-Yahoo! deal and arguably the Microsoft-Yahoo! merger as well.

Similarly, the result in this paper should make economists somewhat more cautious in advocating reforms that will improve potential entry in markets. In some circumstances, we have shown that those reforms may lead to reduced welfare. Of course, the extent to which the theoretical conclusions reached here are of relevance in policy-making is ultimately an empirical question that we leave for future researchers.

8 Bibliography


Gilbert, R. & D. Newbery (1992), Alternative Entry Paths: The Build or Buy


