TiVoed: The Effects of Ad-Avoidance Technologies on Broadcaster Behaviour

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The business model of commercial (free-to-air) television relies on advertisers to pay for programming. Viewers ‘inadvertently’ watch advertisements that are bundled with programming. Advertisers have no reason to pay to have their ads embedded if the viewers succeed in unbundling the advertisements from the entertainment content (advertising bypass). TiVo (Digital Video Recorder) machines, remote controls, and pop-up ad blockers are all examples of ad-avoidance technologies whose deployment detracts from the willingness to pay of advertisers for audience since a smaller audience is actually exposed to the ads. However, viewer purchases of devices to avoid ads may cause a disproportionate share of the ad nuisance to fall on the remaining audience. As these are views less adverse to ads, this causes broadcasters to increase advertising levels. This result is in line with observed facts. The bypass option may cause total welfare to fall. We demonstrate that higher penetration of such technologies may cause program content to be of lower quality as well as to appeal to a broader range of viewers (rather than niches). In addition, we cast doubt on the profitability of using subscriptions to counter the impact of ad-avoidance.
1. **Introduction**

Commercial (or free-to-air) television is a two-sided market. The broadcaster provides programming which bundles entertainment content with advertising messages. The programming simultaneously serves two groups of consumers; the viewers who want to enjoy the content, and the advertisers who want to reach prospective customers with their messages. This business model may be undermined by ad-avoidance technologies (AATs), such as Digital Video Recorders (the most famous of which is TiVo), which enable the viewers to unbundle the content from the messages.\(^1\) Since the advertisers no longer reach viewers who strip the messages from the programming, they are willing to pay less for placing their advertisements. This may impact the programming choices. In extremis, the business model will no longer work if no messages are viewed because the advertisers will no longer pay anything for the programming. Even when only some of the viewers watch in the old style, programming will be affected through changes in the amount of advertising because only those viewers less averse to advertising will remain. The incentives to provide quality programming will be diminished because the returns from quality will fall, and the type of programming may be altered too.

The aim of this paper is to analyze the effects on equilibrium advertising levels, programming, and welfare in the face of such unbundling (or “siphoning”). The ability to bypass the advertising will make better off those who are most annoyed by it. But it will also harm the broadcaster’s profit. The broadcaster will be faced with a lower audience, but one that is less sensitive to advertising clutter. This may cause the broadcaster to put on more advertisements – not to recoup lost revenues per se, but because the marginal advertisement is less likely to cause viewers to switch off (or switch over to a rival channel). The welfare economics of the two-sided market with a bypass factor weighs the benefits to viewers who screen out the ads with the costs to those who are subjected to more ads. The advertisers lose from a reduction in the effective viewer base, but gain from the lower price per ad per viewer as the broadcaster raises ad levels.

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\(^1\) A similar threat arises with computer software (e.g., pop-up blockers) that allows people to browse web pages without the more invasive advertising or RSS readers who can bypass some advertising content.
The broadcaster response to the advent of bypass technology may help explain the rise in ads broadcast in the US over recent years (as documented in Wilbur, 2005). The observed pattern of rising ad levels might be explained by increasing market penetration of bypass technology causing broadcasters to focus on those viewers with least ad aversion.\(^2\)

The literature on two-sided markets loosely falls into two categories (see the overview paper of Rochet and Tirole, 2006, and the papers they introduce in the Special Issue of the RAND Journal of Economics, most notably Armstrong, 2006). One branch, following Caillaud and Julien (2001) addresses platforms that bring on board two groups of agents where each group’s utility is positively affected by the number in the other group. This case is less relevant for platform bypass (although some examples might be appropriate, like store credit cards which siphon some clients away from regular credit cards).

The second branch involves one party that benefits positively from the other, and one party that benefits negatively. The leading example is in Media Economics, and commercial broadcasting in particular. The recent contributions to this literature (reviewed in Anderson and Gabszewicz, 2006) treat the advertisers as benefiting from more viewers since they are prospective customers, but the ads themselves are a net nuisance to viewers. Various alternative models of the two sides of the market have been proposed. Kind, Nilsson, and Sorgard (2004) study a representative agent approach. We use a micro-founded approach that builds up from aggregating individuals’ preferences, along the lines in Anderson and Coate (2005), Choi (2003), Crampes, Haritchablet and Julien (2008), and Gabzewicz, Laussel, and Sonnac (2004), among others.

Little work has addressed the effects of AAT on the two-sided market business model. In particular, there exists no theoretical model that establishes and examines the equilibrium in a two-sided model of broadcasting in the presence of an AAT. Wilbur (2008a) conducts an empirical analysis based on a structural model of broadcaster

\(^2\) Ad levels (per hour) rose quite substantially after the entry of Fox television. Ceteris paribus, entry might be expected to reduce ad levels: ads are a nuisance to viewers (who would rather see an extra 30 seconds of content than an ad) and broadcasters compete in nuisance levels. More competition would usually be expected to reduce nuisance, just like equilibrium oligopoly prices (price is also a nuisance) typically fall with the number of competitors. This effect could be offset by an increase in siphoning causing the higher ad levels.
behavior (building on Wilbur, 2005). Our results on the direction of ad level responses to AAT are also consistent with his findings that suggest that ad levels will increase, and broadcasters are worse off as AAT penetration increases. Of course, in our case, the level of AAT penetration is endogenous to the model (something we show has important impacts on the nature of the result equilibrium) while we consider a more general specification of the impact of AAT on consumers.

Theoretically, the closest paper to this one is provided by Shah (2008). Following from Wilbur (2008a), he assumes that viewers with AAT still get exposed to a (fixed) fraction of the advertising that occurs; we set this fraction equal to zero. Shah also considers alternative specifications of marginal nuisance costs from advertising. He finds that, when MNCs are increasing, the broadcaster need not be made worse off by the availability of AAT, as viewers that switch from not watching TV to watching with AAT benefit the network. In this case, FTA viewers see more commercials than would be aired if there were no AAT (and hence bear a larger ad burden), whereas AAT users see fewer commercials. However, the network is still necessarily worse off when MNCs are constant or decreasing.

An alternative perspective of AAT as a second degree price discrimination device is provided by Tag (2006). He considers a website where an internet surfer can pay for an ad-free version or surf an ad-filled version for free. Paying for an ad-free version is like paying the TV company for AAT to watch a TV show ad-free. Johnson (2008) also considers the role of AAT. However, his research agenda is to examine incentives for targeted advertising per se and demonstrate, as part of that, what the impact of AATs are on these types of advertising strategies.

The outline of the paper is as follows. Section 2 presents our baseline model with a monopoly broadcaster/content provider who sells advertising to firms and access to viewers. The model builds on that of Anderson and Coate (2005) by considering viewers who are heterogeneous both in the preference for the media itself as well as their distaste for advertising. The latter heterogeneity is critical in generating demand for ad-

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3 See also, Wilbur (2008b) for an informal discussion of the likely effects of AAT from a Marketing perspective.

4 Wilbur (2008a) also gives useful numbers on the size of viewer turn-off effects in the absence of AAT: he estimates that a 10% increase in ads will cause a 25% decrease in viewership.
avoidance. Section 3 then examines the introduction of an ad-avoidance technology (or AAT) that viewers purchase and are able to use to eliminate the nuisance of advertising completely while still consuming the media. We show that as AAT penetration rises, the broadcaster chooses a higher level of advertising, and we examine the welfare implications of this. Surprisingly, some AAT penetration can benefit advertisers.

Section 4 then considers extensions including endogenizing AAT pricing and programming quality, the introduction of competition, pay television, subscription based AATs and a variant on the way advertising is avoided. The main results for our baseline case continue to hold under these extensions along with further insights generated. A final section concludes.

2. Baseline Model

Our baseline case concerns a monopolist which transmits media to consumers and sells advertising space to firms. We will refer to this firm as a broadcaster (as in television or radio) throughout the paper although it could also be a publisher (of newspapers, magazines, journals, books or websites) or a studio (for movies and DVDs). We will refer to consumers as viewers (as in television, DVDs or movies) although they could be readers (as in print or digital media). Finally, the purchasers of advertising space will be referred to as advertisers.

**Broadcaster**

We assume that there are no marginal costs to the broadcaster for expanding viewership or advertising. There may be costs associated with acquiring media content. However, we leave the specification of these until they become material in our analysis.

**Viewers**

A viewer of type \((x, \gamma) \in [0, \overline{x}] \times [0, \overline{y}]\) will receive utility

\[
U_{x,\gamma} = \theta + \lambda(1-x) - s - \gamma a
\]  

(1)
if choosing to view, and zero otherwise.\(^5\) Common to all viewers is a horizontal quality component \((\lambda > 0)\), a vertical quality component \(\theta \in [\lambda, \lambda(\bar{x} - 1)]\),\(^6\) a subscription fee \((s \geq 0\) if applicable) and a level of advertising \((a \geq 0)\). The latter two variables are (short-run) choices of the broadcaster while in parts of the model we allow the quality components to be (long-run) choices of the broadcaster. Viewers are differentiated by their preference for the horizontal quality component (which is a function of their position, \(x\)) and their marginal disutility from advertising \((\gamma)\). Initially, we assume that \(x\) is distributed uniformly on \([0, \bar{x}]\) (where \(\bar{x} > 1\)) while \(\gamma\) is distributed uniformly on \([0, \bar{\gamma}]\). We will also normalize the population space to unity by dividing through by \(\bar{x}\bar{\gamma}\). This set-up is similar to that of Anderson and Coate (2005) except that we here allow the disutility of advertising to differ amongst consumers.

**Advertisers**

Following Anderson and Coate (2005), we assume that the advertiser demand price per viewer reached is \(r(a)\) with \(r'(a) < 0\) when \(r(a) > 0\). We also assume that \(r(a)\) is log-concave (which means simply that \(\ln r(a)\) is concave). At some junctures, we will use the stronger property that \(r(a)\) is a concave function; at some points we present proofs under the weaker property that \(1/r(a)\) is convex (which is known as \(-1\)-concavity).

The corresponding revenue per viewer, \(R(a) = r(a)a\) has the property that \(\ln R(a)\) is concave under the assumption of log-concavity (and hence also concavity) of \(r(a)\). This is important for profit quasi-concavity and, hence, to equilibrium existence. To see the implication that log-concave \(r(a)\) implies log-concave \(R(a)\) (and hence, that \((R'(a)/R(a)\) is decreasing), it suffices to note that the product of two log-concave functions is log-concave (because the sum of two concave functions is concave). The two concave functions in question are \(r(.)\) and \(a\).

When we come to the welfare analysis, we assume that the private demand for advertising is also the social demand for advertising. This is a useful benchmark against which we can judge differing views on externalities in advertising.

\(^5\) Setting \(\theta + \lambda\) as a vertical “quality” component, and \(\lambda\) as a linear transport cost rate yields a familiar utility form. We retain the current version because we endogenise these parameters in Section 4.

\(^6\) The bounds imply that some, but not all, viewers with a zero advertising nuisance cost would watch were advertising and subscription fees zero.
Equilibrium without Ad-Avoidance

In this paper, our main focus is on the case of ‘free-to-air’ (or FTA) where $s = 0$. Below we consider how the analysis changes when $s$ can be positive. In the FTA model, for a given advertising level, $a$, the number of viewers is:

$$N_{FTA} = \begin{cases} \frac{(\theta + \lambda)^2}{2(\theta + \lambda + \gamma a)} & \text{if } \theta + \lambda < \gamma a \\ \frac{2(\theta + \lambda + \gamma a)}{2\lambda} & \text{if } \theta + \lambda \geq \gamma a \end{cases} \quad (2)$$

The two cases are distinguished upon whether at $x = 0$, some or all viewers across the range of advertising dis-utilities consume the media or not. When $\theta + \lambda \geq \gamma a$, some viewers with the maximum possible advertising dis-utility still watch. It will turn out that, in equilibrium, this is always the case.

Define $\varepsilon_a = \frac{\partial R}{\partial a}$ as the (per viewer) elasticity of advertising revenue, and $\varepsilon_N = -\frac{\partial N}{\partial a}$ as the aggregate viewer demand elasticity (with respect to advertising). The simple relation between these two variables encapsulates the structure of the two-sided market structure. The broadcaster chooses $a$ to maximize $R(a)N_{FTA}$. Since the solution to this problem is equivalent to the solution to the problem $\max_a \ln R(a) + \ln N_{FTA}$, this immediately yields the first order condition:

$$\varepsilon_a(a) = \varepsilon_N(a) \quad (3)$$

This condition equates the relevant elasticities on the two sides of the market, which are the revenue elasticity and the viewership elasticity. It determines the equilibrium level of advertising, $a_{\text{NoAdT}}$.

Using (2), the viewership elasticity is given as:

$$\varepsilon_N(a) = \begin{cases} \frac{1}{\gamma a} & \text{if } \theta + \lambda \leq \gamma a \\ \frac{\theta + \lambda + \gamma a}{2(\theta + \lambda + \gamma a)} & \text{if } \theta + \lambda > \gamma a \end{cases} \quad (4)$$

Recall now that the advertising revenue elasticity is $\varepsilon_a(a) = \frac{r'(a) + r(a)}{r(a)}$. This is always below 1 for $a$ positive since marginal revenue to the demand $r(a)$ is below average revenue. Hence, the relevant case of (4) that satisfies the first order condition (3) is the second one, i.e., $\theta + \lambda \geq \gamma a$. There is necessarily at least one solution to the first-order condition in the relevant range since $\varepsilon_a(a)$ and $\varepsilon_N(a)$ are continuous functions with $\varepsilon_a(0) = 1 > \varepsilon_N(0)$ and $\varepsilon_a\left(\frac{2\lambda}{\gamma}\right) < \varepsilon_N\left(\frac{2\lambda}{\gamma}\right) = 1$. It remains to show the solution is a
maximum and is unique. Both tasks are accomplished by showing that $\varepsilon_a(a)$ must cross $\varepsilon_N(a)$ from above at any crossing point; i.e., $\frac{\partial \varepsilon_a(\hat{a})}{\partial a} < \frac{\partial \varepsilon_N(\hat{a})}{\partial a}$ for any $a$ satisfying (3). This is done in the Appendix, which also contains the proofs of the subsequent Propositions.

**Proposition 1.** Let $r(a)$ be strictly $(-1)$-concave. With FTA broadcasting and in the absence of AAT, there is a unique equilibrium level of advertising, $\hat{a}_{\text{NoAAT}}$. It equates the revenue elasticity of the advertiser side of the market to the viewership elasticity on the consumer side, with:

$$
\varepsilon_a(\hat{a}_{\text{NoAAT}}) = \varepsilon_N(\hat{a}_{\text{NoAAT}}) = \frac{\bar{y} \hat{a}_{\text{NoAAT}}}{2(\theta + \lambda) - \bar{y} \hat{a}_{\text{NoAAT}}} < 1.
$$

Notice that viewers with lower advertising nuisance dis-utilities are more likely to watch. Advertising nuisance acts like a “price” to watching, although an individual-specific price that is lower to low-$\gamma$ viewers. Put differently, a subscription price on top of the advertising level depicted in Figure 1 would shift the dividing line inwards in parallel, but an ad-level increase would pivot the line down around the horizontal intercept. The subscription price analysis is explored further later in the paper.

**Figure 1: Viewer partition, no AAT**

![Figure 1: Viewer partition, no AAT](image)
3. Ad-Avoidance

We model ad-avoidance as a viewer decision to purchase of a durable appliance. The clearest example of this would be a VCR or a DVR such as a TiVo. However, software to avoid pop-up ads would also fall into this class.\(^7\)

Since broadcasters can choose the advertising level flexibly, the durability assumption gives us a particular timing structure of the order of moves. First, and given a price, \(p\), viewers choose whether or not to buy an AAT.\(^8\) Given the number of machines purchased (and the type of viewers who purchased), the broadcaster chooses how many ads to screen. The structure of the game is interesting because it means that viewers must rationally anticipate the subsequent choice of equilibrium ad levels. This means that the viewers must figure out the number and types of other viewers who buy the machines. As we note below, individual choices impose externalities on others.

The model is also consistent with a game in which viewers choose simultaneously whether to buy AAT and the broadcaster chooses an advertising level. In equilibrium, each agent rationally and correctly anticipates the actions of the others. This means that the viewers anticipate the advertising choice of the broadcaster (which, indeed, depends on the choices of all the other viewers), and the broadcaster anticipates which viewers choose AAT. It is this particular game structure which makes the current set-up quite different from the rest of the literature in broadcasting economics. Indeed, in much of Industrial Organization, consumers are passive followers (price takers, say): here they are not strategic players, since each is “small”, but the expectation of their collective action determines the broadcaster’s action.

**Equilibrium outcome**

In equilibrium, viewers anticipate a level of advertising: call it \(\hat{a}(p)\). We first need to find how many (and which) viewers buy the AAT machine, at price \(p\), and then we must determine the broadcaster’s advertising choice. Finally, we must ensure that the advertising level chosen is indeed \(\hat{a}(p)\), the one anticipated by the consumers/viewers.

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\(^7\) In some situations, ad-avoidance technology also requires an on-going subscription fee. We will examine that situation in a later section.

\(^8\) Initially, we hold \(p\) constant, assuming it is driven by, say, cost considerations only.
Given this structure, we first present a preliminary result which now follows from Proposition 1.

**Corollary 1.** Suppose that the price, \( p \), of the AAT machine satisfies \( \bar{y} \hat{a}_{\text{NoAAT}} < p \) where \( \hat{a}_{\text{NoAAT}} \) is defined in Proposition 1. Then there is an equilibrium at which AAT is not purchased by anyone.

Clearly, no one will buy AAT if the price is too high, and the Corollary gives the exact condition. As will become apparent from the analysis below, this is the unique equilibrium: there cannot be an equilibrium for the same parameter values at which some viewers do buy AAT.

Conversely, if \( \bar{y} \hat{a}_{\text{NoAAT}} > p \), the only equilibrium will involve AAT usage. Notice that the condition given implies (from Proposition 1) that \( \theta + \lambda < \bar{y} \hat{a}_{\text{NoAAT}} \leq p \). This means that we will need

\[
\theta + \lambda > p
\]

in order to have AAT used in equilibrium, which condition we henceforth assume.

To begin, suppose that an advertising level of \( \hat{a}(p) \) is anticipated by consumers, and assume the price of the AAT is sufficiently low, namely that \( \bar{y} \hat{a}_{\text{NoAAT}} > p \) and \( \theta + \lambda > p \) (or else no viewer would buy the AAT). Then, all consumers for whom \( \gamma \hat{a}(p) \geq p \) and \( \theta + \lambda (1 - x) > p \) will find it is worth paying \( p \) to avoid the nuisance of ads. This leaves the consumers with \( \gamma \hat{a}(p) < p \) who will choose either to watch with ads or not watch at all. Figure 2 (a) and (b) depicts the division between the three groups.
Figure 2: Non-Equilibrium Outcomes

(a) $a < \hat{a}(p)$

(b) $a > \hat{a}(p)$
Neither panel of Figure 2 depicts an equilibrium situation. In 2(a), the choice of \( a \) by the broadcaster (given \( \hat{y} = p / \hat{a}(p) \)) is less than \( \hat{a}(p) \). In this case, the number of AAT viewers would fall. In 2(b), the choice of \( a \) by the broadcaster (given \( \hat{y} = p / \hat{a}(p) \)) is greater than \( \hat{a}(p) \). In this case, the number of AAT viewers would rise. An equilibrium requires that the choice of \( a \) by the broadcaster (given \( \hat{y} = p / \hat{a}(p) \)) is indeed equal to \( \hat{a}(p) \). This outcome is depicted in Figure 3.

**Figure 3: Equilibrium Outcome with AAT**

\[
a = \hat{a}(p)
\]

Notice that, given the potential out-of-equilibrium advertising choices, this equilibrium is qualitatively different from the outcome without AATs. Importantly, there is a potential existence issue for an interior equilibrium. For example, when the broadcaster chooses \( a \) greater than \( \hat{a}(p) \), viewers will respond by increasing their AAT purchases. However, if this causes the advertising level to rise further, this will drive more AAT purchases. It is, therefore, possible that an interior equilibrium may not exist and that a sufficiently low \( p \) may lead to very high advertising levels and no FTA viewership. Similarly, a high \( p \) may lead to no take up of AATs at all.
Nonetheless, the proposition below demonstrates that these types of vicious cycles do not arise, an interior equilibrium exists and it is unique. To characterise the equilibrium outcome requires working backwards. We can restrict attention to the analysis of the choice of advertising whereby some rectangular space of viewers have purchased an AAT (namely consumers with advertising dis-utilities above some threshold, $\gamma = \frac{p}{\lambda}$, and who have TV preferences below $\frac{\lambda + p}{\lambda}$). Given the durable nature of the AAT technology, the broadcaster will take this as given when choosing its advertising level.

Given this, the number of FTA viewers will be qualitatively different depending upon whether the broadcaster chooses $a$ greater than or less than $\hat{a}$ (where for ease of exposition we drop the qualifier $(p)$). For a choice $a \geq \hat{a}$, the number of FTA viewers is:

$$N_{a,\hat{a}} = \frac{\gamma}{\lambda} \left( \theta + \lambda - \frac{1}{2} \hat{a} \right).$$

(6)

In contrast, if $a \leq \hat{a}$, we have:

$$N_{a,\hat{a}} = \frac{1}{\lambda} \left( \left( \theta + \lambda - p \right) \hat{a} + \frac{1}{2} \lambda p^2 \right).$$

(7)

Notice that for $a = \hat{a}$, (6) and (7) are the same. Thus, given $\hat{a}$, the broadcaster will choose $a$ to maximize:

$$R(a)N_{a,\hat{a}} \quad \text{for} \quad a \leq \hat{a}$$

(8)

$$R(a)N_{a,\hat{a}} \quad \text{for} \quad a \geq \hat{a}.$$

The solution is described below.

**Proposition 2.** Suppose that $r(a)$ is concave. For a given AAT price, $p$, there exists a unique advertising level $\hat{a}(p) > 0$ such that the broadcaster is maximizing profits and consumers for whom $\{x, \gamma \} | \gamma \geq p / \hat{a}(p)$ and $x \leq \frac{\gamma + p}{\lambda}$ purchase an AAT. Moreover, $\hat{a}(p)$ satisfies:

$$\varepsilon_{\gamma}(\hat{a}(p)) = \frac{p}{2(\theta + \lambda) - p}.$$  

(9)

The properties of the solution are described below.

**Impact on advertising levels**

We are now in a position to examine the comparative statics of the advertising level with respect to the penetration of AATs. Their penetration level is indexed by $-p$ (the lower the AAT price, the more that will be bought). Note, first, that when
\( p \geq \bar{\gamma} \hat{a}(p) \), where \( \hat{a}(p) \) is defined as in Proposition 2, there are no AAT viewers. When this condition holds with equality, the ad level in Proposition 2 (see (9)) becomes:

\[
\epsilon_a(\hat{a}) = \frac{\bar{\gamma} \hat{a}(p)}{2(\theta + \lambda) - \bar{\gamma} \hat{a}(p)}
\]  

which is the same outcome as the equilibrium in Proposition 1. Thus, as \( p \) falls from a high level to a level where there is some positive demand for AATs, the equilibrium impact on advertising levels is smooth.

We next characterize the impact on advertising from increased AAT penetration. The equilibrium relation between the AAT price and advertising is given in (9). The RHS is clearly increasing in \( p \). The LHS, \( \epsilon_a \) is decreasing in \( a \) (when \( r(a) \) is log-concave).

Hence, the equilibrium level of advertising will rise as the price of AAT’s falls:

**Proposition 3.** Suppose that \( r(a) \) is log concave. A lower price of AAT increases the equilibrium amount of advertising.

In many respects, this result seems counter-intuitive. AATs represent a substitution possibility for viewers and one might consider them, therefore, as competing with broadcasters: that is, in response to cheaper AATs broadcasters would have to work harder to attract viewers by lowering advertising levels and hence, viewer disutility. However, this simple intuition does not take into account who would be purchasing AATs. When there is heterogeneity amongst viewers in terms of their preferences against advertising, those who enjoy television the most and who dislike advertising the most will purchase AATs. From the broadcaster’s perspective, it was these viewers who – in the absence of AATs – where causing them to constrain advertising levels; they were the marginal viewers. With AATs, their disutility is no longer an issue and the average disutility of a viewer without AATs is lower. Hence, the broadcaster faces fewer costs in expending advertising levels and does so.

As noted in the introduction, the proliferation of ad avoidance technology over the past two and a half decades (from VCRs to DVRs) has occurred at the same time as increasing levels of advertising on television (especially, the share of non-program to program content). Our result here that lower prices for AATs leads to high advertising levels suggests that these trends may be linked; that is, the penetration of AATs may be driving the greater levels of advertising on television. This is because AAT proliferation
changes the nature of the pool of viewers; reducing the elasticity of viewership with respect to advertising levels.

Impact on welfare

Turning now to welfare, it is instructive to consider who are the broad winners and losers as AAT penetration increases. Figure 4 overlays Figures 1 and 3. Notice that the impact of AATs is to shrink the total volume of viewers watching FTA television but it also means that some viewers previously not watching television do so (the shaded blue triangle). The impact on this group is a strict welfare gain from the introduction of AATs.

**Figure 4: Comparison**

For other viewers the effect may be negative. For viewers still watching FTA and not purchasing an AAT, the effect of AATs is strictly negative. This is because every additional AAT viewer causes the broadcaster to increase advertising levels; increasing FTA viewers’ dis-utilities from television viewing. Some of these choose to opt out of watching television altogether. Others choose to purchase an AAT. However, for this group, the presence of AATs may be negative in that they may not have chosen to
purchase these but for the level of penetration of AATs and the consequent advertising level. In Figure 4, AAT viewers in the orange shaded area are worse off as a result of the presence of AATs while others are better off.

The following proposition demonstrates that the impact of AAT penetration on the broadcaster is strictly negative. While this might appear to be obvious, technically, demonstrating this is difficult as we cannot simply apply the envelope theorem as the value of $\hat{a}(p)$ is not directly determined by the broadcaster. Moreover, as $p$ rises, we know the advertising levels fall. This per se reduces profit since we move further away from the level that maximizes profit per viewer (i.e., further away from the captive audience). Viewer demand appears to unambiguously rise: there are fewer ads (the indifferent consumer rotates clockwise) and the critical ad disutility ($\hat{\gamma}$) rises further increasing ad viewers.

**Proposition 4.** The broadcaster’s profits are decreasing in AAT penetration.

Notice that this implies that, if there were fixed costs associated with being a broadcaster, then the broadcaster may shut down if $p$ falls below some critical level. However, if viewers anticipate the close-down possibility, in equilibrium, some of them will not purchase AATs and the broadcaster would just earn a break-even profit. Thus, in the context of our model, dire predictions that the broadcasting industry would be destroyed by AATs do not occur for the simple reason that without broadcasting there is no demand for AATs.

Finally and most interestingly, the effect of AATs on advertisers is ambiguous. AAT penetration reduces total FTA viewership and hence, the impact of advertising. However, it also leads the broadcaster to increase advertising levels and reduce advertising prices. It is, therefore, possible that this latter effect could outweigh the former for advertisers.

To examine this possibility, we consider a specific functional form of $r(a)$ that is consistent with our assumptions; namely, $r(a) = 1 - a$. With this assumption we can demonstrate the following:

**Proposition 5.** For $r(a) = 1 - a$, then if $\theta + \lambda < 0.649018$, an increase in AAT penetration can increase advertiser surplus.
Hence, for sufficiently low quality \((\theta + \lambda)\), a reduction in \(p\) from its highest level that could attract some AAT purchasers can raise advertiser surplus. This is because at that level, with low quality, the loss in viewership from AATs is small relative to the fact that those viewers have high ad disutility. Hence, advertisers benefit more from the per viewer increase in surplus across most viewers than the loss in viewers from AATs. Of course, as \(p\) falls further, this balance shifts and advertiser surplus will decrease.

Even though advertisers can be better off when AAT is introduced (Proposition 5), the broadcaster is necessarily worse off (Proposition 4). This leads us to consider the effects on total surplus from AAT introduction. For simplicity, we again use the advertiser demand specification \(r(a) = 1 - a\).

Under this specification, the gross advertiser surplus per FTA viewer\(^9\) is \(\hat{a}(1 - \frac{\hat{a}}{2})\), with the equilibrium ad level given as \(a = \frac{2(\theta + \lambda - p)}{\hat{a}(\theta + \lambda - 3p)}\).\(^{10}\) From (6) and (7) we have the mass of viewers watching FTA as \(N(p) = \frac{p}{\lambda(\theta + \lambda - \frac{p}{2})}\).

As regards viewers, the utility of those with AAT is \(\theta + \lambda(1 - x)\). There are \(\frac{1}{\gamma x}(\gamma - \hat{\gamma})\left(\frac{\theta + \lambda - p}{2}\right)^2\) watchers using AAT, and their utility varies uniformly from \(\theta + \lambda - p\) (at \(x = 0\)) down to zero. This means the average utility is \(\frac{\theta + \lambda - p}{2}\) for this group, implying a total group utility of \(\frac{1}{2\gamma x}(\gamma - \hat{\gamma})\left(\frac{\theta + \lambda - p}{2}\right)^2\). For the FTA viewers, the utility of a type \(\gamma\) varies uniformly from \((\theta + \lambda - \gamma \hat{a})\) down to zero, for an average (conditional on nuisance annoyance, \(\gamma\)) of \(\frac{\theta + \lambda - \gamma \hat{a}}{2}\). The mass of those of type \(\gamma\) watching is up to type \(\bar{\gamma}\) such that \(\theta + \lambda(1 - \bar{x}) = \gamma \hat{a}\), so there are \(\frac{\bar{x}}{\bar{\gamma}} = \frac{\theta + \lambda - \gamma \hat{a}}{\bar{x}\lambda}\) of them. Integrating over \(\gamma \in [0, \hat{\gamma}]\) yields the aggregate surplus to FTA viewers as

\[
\frac{1}{\gamma x} \int_{0}^{\hat{\gamma}} \left(\frac{\theta + \lambda - \gamma \hat{a}}{2\lambda}\right)^2 d\gamma
\]

Adding together these various surpluses gives the welfare function as:

\[
W(p) = \frac{1}{\gamma x} \int_{0}^{\hat{\gamma}} \left(\frac{\theta + \lambda - \gamma \hat{a}}{2\lambda}\right)^2 d\gamma + \frac{1}{2\gamma x} (\gamma - \hat{\gamma}) \left(\frac{\theta + \lambda - p}{\lambda}\right)^2 + \hat{a}(1 - \frac{\hat{a}}{2}) \frac{p}{\lambda(\theta + \lambda - \frac{p}{2})}
\]

where we note that \(\hat{\gamma} \hat{a} = p\).

---

\(^9\) This includes advertiser surplus per viewer, \(\frac{1}{2} \hat{a}^2\) and broadcaster profit \(\hat{a}(1 - \hat{a})\).

\(^{10}\) See (51) in the Appendix.
The proof of the next result relies on examining total welfare around the point whereby AATs just become affordable for viewers with a disutility close to $\bar{v}$.

**Proposition 6.** Let $r(a) = 1 - a$. A marginal reduction in the AAT price $p$ that just renders it attractive to some users reduces aggregate surplus. The conclusion is that the total surplus is reduced by AAT penetration in the neighborhood of the preliminary incursion. In that neighborhood too, even aggregate viewer surplus is decreased. To put this result in perspective though, the viewers who (initially) sign up are broadly indifferent between adopting AAT and not, and so it is scarcely surprising that total viewer surplus falls as all the other viewers suffer from the increased ad levels on FTA. As the price of AAT falls below the initial incursion level, there is more viewer surplus from lower prices, and more "high-nuisance" viewers now tune in, registering a greater surplus gain.

However, not only is the broadcaster harmed by AAT (at all levels), but so is total advertiser surplus (at all levels). As we noted in Proposition 5 though, there may be advantages to advertisers from lower ad prices, even though the broadcaster suffers.

Some important caveats are in order regarding this last Proposition. First, it is for a specific ad demand function, and for a particular model of preferences. Second, it is not a global result, but in the neighborhood of no AAT. Nevertheless, it does indicate that the business model of FTA might be quite vulnerable to welfare-reducing siphoning. Note that this is quite different from individuals who pirate cable television, or shoplifting, or other forms of stealing, legalized or not. First, one might argue that the monopoly of the airwaves given to certain select broadcasters is abused by bundling content with ads and not letting viewers 'opt-out,' so that AAT offers them a break on the force-feeding of commercials. Second though, the interesting economic effect, for the economics of platforms in two-sided markets, is the selection effect. This is that AAT allows opt-out for those most put off by commercials. There remains a viewer base which is less sensitive, and the optimal response for the broadcaster is to ramp up commercials to them.

This leads obviously to the question of how easily the result above can be overturned, and some reflection indicates it is quite brittle. Indeed, it is quite easy to configure model specifications where AAT improves welfare. For example, if there were
viewer types with advertising nuisance costs way above the level $\gamma_1$, then these would be ignored by the broadcaster, they would not watch and the equilibrium would look just the same as in the current model. However, the introduction of AAT would enable these types to enjoy the TV entertainment without the shackles of ads. This would be a pure welfare gain, and existing viewers would be completely unaffected because there would be no siphoning of an existing viewer base.

4. Extensions

Here we consider several extensions of our baseline model to explore in more detail some of the implications of the spread of AATs on broadcaster behavior.

*Endogenous programming content*

So far, we have taken as given the programming characteristics offered by the broadcaster. But these too might be affected by the incursion and penetration of AAT. As we shall show, endogenising the program type can be viewed as the simple choice by the broadcaster of one of the parameters of the model ($\lambda$). In the analysis that follows, we will show how the program choice may “tip” towards broad-based programming or “lowest common denominator” programming, to borrow the phrase from the analysis of Beebe (1977).

In the model thus far, we have taken the broadcaster’s program to be at ‘type,’ $x = 0$, with viewer’s preferences given as $U_{x,r} = \theta + \lambda(1 - x) - \gamma a$. As we have noted already, we might usually interpret $\theta + \lambda$ as a “vertical quality” of programming, and then $\lambda$ is a (linear) transport rate traditional in models of product differentiation. However, this set-up also admits another interpretation in which we can naturally take the $\lambda$ parameter as endogenous. The back-drop follows along the lines of a model developed in Anderson and Neven (1989) in which consumers can combine different products (this was colorfully called “roll-your-own” preferences by Richardson, 2006) in order to consume an ideal type reflecting watching some of two different channels. This model was recently used by Hoernig and Valletti (2007) in order to consider alternative financing methods in (duopolistically) competitive media markets. The current
specification is similar in spirit to the Anderson-Neven one, but differs because the broadcaster determines the mix rather than the consumer/viewer, and here utilities are defined over the convex combination of the tastes for end-point specifications rather than the taste for the convex combination of the end-points.

We first introduce the idea with a separate set of parameters, and then show the transformation under which they are equivalent to the current set. In this approach, individuals are assumed to have ideal points, which are viewed as mixes of extreme programming types. That is, the broadcaster is assumed to provide a fraction of the broadcast time for each of two extreme (pure) programming types. For example, these could be comedy or drama: and the broadcaster has to choose the mix. Suppose it chooses a fraction $\alpha$ of the type 0 (comedy) and, residually, fraction $1-\alpha$ is drama. Consumers derive utility from the mix in the following way. A consumer of type $x$ has utility $v - t\alpha x - t(1-\alpha)(\bar{x} - x)$ where $v$ is quality; and the other parts are distance components with parameter $t$. The idea here is that the consumer has utility from the two separate parts of the programming, and therefore has a utility from the mixture.$^{11}$

This model has the broadcaster choosing $\alpha$, the fraction of the time to devote to each type: note that the choice of $\alpha$ is naturally bounded between 0 and 1 because it is not possible to “sell short” either programming type (and have it aired more that 100% or less that 0% of the time). The choice of $\alpha$ can be further restricted to $\alpha \in [\frac{1}{2}, 1]$, by symmetry of the audience distribution: at one limit the programming is fully specialized ($\alpha = 1$), at the other, it is fully mixed ($\alpha = \frac{1}{2}$). Note that the case $\alpha = \frac{1}{2}$ (and in its neighborhood) necessitates us considering another case of the demand model, where the market is “covered” for $\gamma = 0$.

The parameter matching now works as follows. We wish to match the utility (non-ad component) $v - t\alpha x - t(1-\alpha)(\bar{x} - x)$ with the utility $\theta + \lambda(1-x)$. Rearranging the first of these to $v - \bar{x}t(1-\alpha) - xt(2\alpha - 1)$, this is clearly effectuated by setting $\theta + \lambda = v - \bar{x}t(1-\alpha)$ and $\lambda = t(2\alpha - 1)$. Hence the relevant range for $\lambda$, given that $\alpha \in [\frac{1}{2}, 1]$, is $\lambda \in [0, t]$.

$^{11}$ In the example above, the specification in the text so far corresponds to $\alpha = 1$, so only comedy is shown.
We are now in a position to think about directly choosing $\lambda$, now we have established that this choice is equivalent to the choice of a program format, as expressed in terms of the percentage of programs of each type.

**Choosing $\lambda$, no AAT case**

We first find the equilibrium choice of $\lambda$ when AAT is unavailable (equivalently, prohibitively expensive). Following the discussion above, we will assume the choice of $\lambda$ lies between 0 and $t$.

We shall assume that $\theta + \lambda \leq \hat{\gamma}a$ for all feasible $\lambda \in [0, t]$, which means that the highest nuisance viewer is so put off that they do not watch even if their preferred program is aired. Now there are two cases for where the indifferent viewer type $\gamma = 0$ is, either at $x < \bar{\alpha}$ or $x = \bar{\alpha}$. The former case corresponds to the analysis up till now in the main text, and holds for $\frac{\theta}{\lambda} + 1 < \bar{\alpha}$. Over this region, the market space is a triangle, and the value of demand is:

$$D_T(\lambda) = \frac{1}{2\alpha\beta} \frac{\theta + \lambda}{\lambda} \frac{\theta + \lambda}{\lambda}$$

which is a convex function of $\lambda$, indicating that demand (and hence, profit) is convex in this region. This convexity property will be important in the later analysis with AAT which is not a trivial extension of the previous exogenous quality case because the rational expectations requirement means the broadcaster needs to take account of the installed base of AAT users.

The other demand region (which arises for lower $\lambda$) has all types $x$ watching, and constitutes a trapezoid in $(x, y)$ space, with vertical intercepts $\frac{\theta + \lambda}{\lambda}$ at $x = 0$ (as just shown) and $\frac{\theta + \lambda(2 - \tau)}{\lambda}$ at $x = \bar{\alpha}$. Taking the average of these two and dividing by the conditional density at $x$ yields the demand expression as:

$$D_{T_{\alpha\beta}}(\lambda) = \frac{1}{\bar{\alpha}} \frac{\theta + \lambda(2 - \tau)}{2\alpha}$$

which has derivative:

$$D'_{T_{\alpha\beta}}(\lambda) = \frac{\theta + \lambda(2 - \tau)}{2\alpha\bar{\alpha}}$$
which is positive (negative) depending on whether \(2 - \bar{x} > (\leq) 0\).\(^{12}\)

We can now bring this all together. Since \(D'_r(\lambda)\) is increasing, then if \(2 - \bar{x} \geq 0\) the demand derivative is non-negative and non-decreasing throughout as \(\lambda\) rises, so the optimal choice is as large as possible, \(\lambda = t\). On the other hand, if \(2 - \bar{x} < 0\), the demand derivative starts out negative: if it eventually goes positive,\(^{13}\) then the solution is in one end or the other of the feasible range, i.e., either at \(\lambda = 0\) or at \(\lambda = t\). The solution is whichever gives higher demand: comparing (11) with (12) shows the Niche market is preferred as long as

\[
\frac{(\theta + t)^2}{2\bar{x}t} \geq \theta
\]  

(14)

In summary, the demand, and hence profit, is a convex function of \(\lambda^{14}\). The optimal choice is the (extreme) Niche market if and only if \(\bar{x} \leq \frac{(\theta + t)^2}{2\bar{x}t}\) (a sufficient condition for a Niche market is that \(\bar{x} < 2\)). We next determine how the equilibrium depends upon an active AAT price.

**Choosing \(\lambda\), active AAT case**

We are most interested in whether the equilibrium can involve LCD programming \((\lambda = 0)\) with AAT, and so we establish conditions under which that is an equilibrium. There are two cases. First, if \(\theta \leq p\), nobody buys AAT because no-one finds the programming worthwhile. Then \(\lambda = 0\) is an equilibrium only if it is an equilibrium when no AAT exists, the case we just analyzed. Since there is no “lock-in” to AAT, nothing has changed from the analysis above, and \(\lambda = 0\) is an equilibrium if \(\theta \geq \frac{(\theta + t)^2}{2\bar{x}t}\); otherwise, the broadcaster deviates to \(\lambda = t\).

\(^{12}\) The demand derivative is continuous through the point where the consumer type \((x,y) = (\bar{x},0)\) is just indifferent between buying or not. To see this, note the "corner" indifferent individual satisfies \(\theta + \lambda(1 - \bar{x}) = 0\), corresponding to \(\lambda = \frac{\theta}{2\bar{x}}\). Inserting this value gives

\(D'_r(\frac{\theta}{2\bar{x}}) = \frac{-\bar{x}}{2\bar{x}^2}(1 - (\bar{x} - 1)^2) = \frac{-\bar{x} + \bar{x}^2 - 1}{2\bar{x}^2} = D'_{\text{Niche}}(\frac{\theta}{2\bar{x}})\).

\(^{13}\) A necessary condition is that \(D'_r(t) = \frac{1 - (\theta + t)^2}{2\bar{x}t} > 0\), or \(t > \theta\), and hence \(t < \theta\) is a sufficient condition for this not to happen.

\(^{14}\) Johnson and Myatt (2006) stress a monopolist’s preference for extremes.
So now suppose that $\theta > p$ (otherwise, watching LCD TV with AAT is not worthwhile to anyone). As long as $p < \bar{y}a$, which we assume or else AAT is not worthwhile (for anyone), then the AAT buyers are all those with $\gamma > p/a$. Put another way, the high nuisance types all buy AAT. The consequence is that the broadcaster has all those without AATs anyway, and changing $\lambda$ cannot bring in new viewer types because there are none left (without AATs) that the broadcaster does not already have.

We next show that the other extreme, $\lambda = t$, or full Niche programming, is not an equilibrium in the presence of AAT. Assume that $p < \theta + t$; otherwise no-one ever buys AAT (it costs more than the value the happiest person places on television). Continue to assume too that $\bar{y}$ is large enough that $p < \bar{y}a$. Then no type with $\gamma > p/a$ will watch free-to-air, but types below that value will, with their $x$ values low enough. (The situation is akin to that in Figure 3.) Now, the type $(x, \gamma) = \left(\frac{\theta + p}{t}, \frac{p}{a}\right)$ is the crucial “three-way” type indifferent between AAT, FTA and not viewing; the type $(x, \gamma) = \left(\frac{\theta + t}{t}, 0\right)$ is the type with most extreme preference still watching. Demand $D(\lambda; p)$ is then given by (a trapezoid) $\frac{1}{t^2} \cdot \frac{p}{a} \cdot \frac{2\theta + 2t - p}{2\theta t}$. Now, a necessary condition for an equilibrium with $\lambda = t$ is that demand cannot be increased by reducing $\lambda$ below $t$, given the lock-in of the viewers with AAT. Whenever $p < 2\theta$, the condition cannot hold.\(^1\)

The above analysis can be summarized in the following proposition:

**Proposition 7.** Fix $a > \frac{\theta + \lambda}{\bar{y}}$, and consider the equilibrium choice of $\lambda$. In the absence of AAT, the equilibrium is Full Niche programming ($\lambda = t$) if $\bar{x} \leq \frac{\theta + \gamma}{2\theta t}$. If AAT is introduced with $p < \theta$, there is an equilibrium with $\lambda = 0$ (LCD programming), and no equilibrium with Full Niche programming.

The intuition here is that changing horizontal quality ($\lambda$) rotates the ‘demand curve.’\(^2\)

Increasing $\lambda$ means that those viewers closest to $x = 0$ are willing to bear more advertising while those further away are not. A higher $\lambda$ is as if the broadcaster chosen

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\(^1\) Indeed, given the number of viewers buying AAT, the demand for FTA is $D(rp)$ as just given. For $\lambda$ (unexpectedly) lowered below $t$, and given the set of AAT buyers, the indifferent FTA viewer of type $\gamma = 0$ has location $x = (\theta + \lambda)/\lambda$ (as always) while the indifferent FTA viewer of type $\gamma = p/a$ (who is the boundary type for the AAT region) has location $x = (\theta + \lambda - p)/\lambda$. For $\theta > p$, both expressions increase as $\lambda$ falls, so demand rises and $\lambda = t$ cannot be an equilibrium. For $p > \theta$, the FTA demand is proportional to $(2\theta - p + 2\lambda)/2\lambda$ and therefore rises as $\lambda$ falls when $p < 2\theta$.

\(^2\) Johnson and Myatt (2006) provide extensive analysis of the incentives to rotate and shift demand.
content with greater ‘niche’ appeal while a lower $\lambda$ would orient it towards a mass market. Recalling that AAT penetration is concentrated amongst those with high utility of viewing (the primary targets of Niche programming), as AAT penetration increases, the incentives of the broadcaster tips towards LCD programming.

Finally, it is worth observing that greater penetration of AAT’s always reduces the incentives to provide vertical quality. Put simply, AAT’s reduce the scale of viewership, particularly amongst those with a higher value on television. Consequently, shrinking the advertising viewership reduces incentives to raise the vertical quality of programming. Interestingly, if this is anticipated, it will provide a check on the growth of AAT adoption.

*Endogenous AAT pricing*

Up until now, we have treated $p$ as exogenous. In many respects this is reasonable as for the most part AATs are electronic appliances the supply of which is arguably competitive. Hence, $p$ will be driven by cost considerations independent of the behaviour of broadcasters.

Of course, if the AAT were provided by a monopolist, it might internalize the equilibrium advertising effects in its demand. We can illustrate the effects by taking as a benchmark case when the advertising level is exogenous. Then, if the monopoly AAT provider takes into account the induced changes in advertising level, its demand curve will be more elastic than in the benchmark case. This is because a lower price for the AAT induces more ads, which in turn further raises the demand for AAT. Internalizing this effect suggests the monopolist should set a relatively low price to trigger the (rational) expectation of high ad rates, and hence a large market share of viewers, along with substantial damage to the FTA viewer base of the broadcaster. The issue of dynamic pricing of introducing AAT is an interesting one for the tension between the effects just mentioned and the desire to extract rents from the most ad-averse consumers, but this dynamic issue is beyond our current scope.
Competition

Following Anderson and Coate (2005), now suppose that there are two broadcasters; one located at $x = 0$ and the other located at $\bar{x}$. A viewer $(x, \gamma)$ who watches broadcaster $i \in \{0, 1\}$, gets utility:

$$U_i = \begin{cases} \theta + \lambda(1-x) - \gamma a_i & \text{for } i = 0 \\ \theta + \lambda(1-(\bar{x}-x)) - \gamma a_i & \text{for } i = 1 \end{cases}$$ (15)

In the absence of AATs, viewers will watch the broadcaster that gives them the higher utility. Suppose that $\theta$ is sufficiently high so that each viewer chooses one broadcaster rather than not viewing at all (as in Anderson and Coate, 2005). Then the marginal consumer for any given $\gamma$-type will be defined by:

$$\theta + \lambda(1-x) - \gamma a_0 = \theta + \lambda(1-(\bar{x}-x)) - \gamma a_1 \Rightarrow \hat{x} = \frac{\lambda \gamma (a_0 - a_1)}{2 \lambda}$$ (16)

Thus, broadcaster 0’s demand will be $N_0 = \frac{1}{\gamma} \int_0^x \hat{\gamma} d\gamma = \frac{1}{2} - \frac{\gamma}{4\lambda}(a_0 - a_1)$. It will choose $a_0$ to maximize $R(a_0)N_0$. This yields the first order condition:

$$\frac{R'(a_0)}{R(a_0)} = \frac{\gamma}{2\lambda - \gamma (a_0 - a_1)}$$ (17)

The equilibrium must necessarily be symmetric,\(^{17}\) so the equilibrium condition is:

$$\frac{R'(a)}{R(a)} = \frac{\gamma}{2\lambda}$$ (18)

Now suppose that viewers consider purchasing an AAT for a price of $p$. Suppose also that they anticipate a symmetric level of advertising, $\hat{a}$. In this case, for broadcaster 0, their demand will become: $N_0 = \frac{1}{\gamma} \int_0^{\hat{a}} \hat{x} d\gamma = \frac{p(\lambda \gamma (a_0 - a_1))}{2\lambda \gamma \hat{a}}$. This implies that, in a symmetric equilibrium,

$$\frac{R'(\hat{a})}{R(\hat{a})} = \frac{p}{2\lambda}$$ (19)

Thus, as $\epsilon_a$ is decreasing in $a$, an increase in $p$ will lead to an increase in the equilibrium level of advertising; just as in the monopoly broadcaster case.

\(^{17}\) Indeed, if $a_0 > a_1$ then broadcaster 0 would serve less than half the market, from the demand equation, $N_0$. However, the RHS of the first order condition (17) would be larger for broadcaster 0 than for broadcaster 1, implying $R'/R$ on the LHS would be larger for 0 than 1. However, since $R'/R$ is decreasing under the assumption of -1-concavity, this means that $a_0 < a_1$, a contradiction.
Pay Television

We now turn to the case of pay television (or PAD) where consumers pay a subscription fee to access television content including advertising. In this case, in the absence of AATs, the broadcaster chooses \( a \) and \( s \) to maximize \((R(a) + s)N_{PAD}\) where (cf. (2)):

\[
N_{PAD} = \begin{cases} 
\frac{(\frac{\theta + \lambda - s}{s})^2}{2\lambda s} & \text{if } \theta + \lambda < s + \overline{\eta}a \\
\frac{2(\theta + \lambda - s) - \overline{\eta}a}{2\lambda} & \text{if } \theta + \lambda \geq s + \overline{\eta}a
\end{cases}
\]

(20)

This yields the first order conditions:

\[
R'(a)N_{PAD} + (R(a) + s)\frac{\partial N_{PAD}}{\partial a} = 0
\]

(21)

\[
N_{PAD} + (R(a) + s)\frac{\partial N_{PAD}}{\partial s} = 0
\]

(22)

These can be combined to give:

\[
R'(a) = \frac{\partial N_{PAD}}{\partial a} \frac{1}{\frac{\partial N_{PAD}}{\partial s} s}
\]

(23)

and we can rewrite this condition as (where \( \varepsilon_s = -\frac{s}{N_{PAD}} \frac{\partial N_{PAD}}{\partial s} \)):

\[
\frac{R'(a)a}{s} = \frac{\varepsilon_N}{\varepsilon_s}
\]

(24)

the ratio of the elasticities of viewership with respect to advertising and price. Writing the elasticity of revenue with respect to advertising as \( \varepsilon_a \) yields the equivalent condition as

\[
\frac{R(a)}{s} = \frac{\varepsilon_N}{\varepsilon_s \varepsilon_a}
\]

(25)

which neatly relates the ratio of revenues per viewer from the two different sources to the various elasticities at play in the two-sided market.\(^\text{18}\)

Using the uniform distribution, the RHS of (23) becomes:

\[
\frac{\theta + \lambda - s}{s} \frac{1}{\frac{\theta + \lambda - s - \overline{\eta}a}{2\lambda}} \text{ if } \theta + \lambda < s + \overline{\eta}a \\
\frac{\theta + \lambda - s - \overline{\eta}a}{2\lambda} \frac{1}{s} \text{ if } \theta + \lambda \geq s + \overline{\eta}a
\]

(26)

We first show that the first case is inconsistent with profit maximization. Since we impose \( s \geq 0 \), the advertising first-order condition (21) implies that \( R'(a) = \frac{R(a) + s}{a} \); the LHS is marginal revenue, which lies below average revenue, which is on the RHS. But

\(^\text{18}\) This condition is reminiscent of the Dorfman-Steiner condition for advertising a good in a market. If demand is \( D(p, A) \), with \( p \) product price and \( A \) advertising expenditure, the DS condition describing monopoly advertising levels is \( \frac{pD}{A} = \frac{-\varepsilon_p}{\varepsilon_a} \), where the LHS is the sales to advertising ratio.
this is impossible with \( s \) non-negative. Thus, the second case is the germane one: just as without subscriptions (see Proposition 1), the equilibrium advertising level is set to involve all nuisance types at \( x = 0 \) watching.

The advertising rate is then determined only by \( \bar{\gamma} \) according to:

\[
R'(a_s) = \frac{1}{2} \bar{\gamma}
\]  

(27)
as long as \( s > 0 \). This is just the average \( \gamma \) in the population, and the current formulation reflects the Anderson-Coate (2005) result for subscription pricing when all viewers have the same nuisance cost, namely that marginal revenue equal that common cost. That result comes about because for any given total nuisance, \( s + \gamma a \), faced by consumers, revenue is maximized where the marginal nuisance cost is monetized, i.e., \( R'(a) = \gamma \). The logic is similar here because each high nuisance marginal consumer (marginal between watching and not) has a low nuisance marginal counterpart, indicating the average nuisance as the relevant statistic since the equilibrium condition involves all \( \gamma \)-types watching.

We can then solve the monopoly broadcaster’s problem sequentially given that (27) holds. Using the second case of (20), then \( \frac{\delta N_{\text{ad}}}{\delta a} = -\frac{1}{\lambda} \). Hence, substituting into (21) gives:

\[
s = \frac{1}{2} \left( \theta + \lambda - R(a_s) - \frac{1}{2} \bar{\gamma} a_s \right)
\]  

(28)
Hence, \( s \) is positive when the LHS of this expression is positive. Otherwise, the subscription price is zero and the advertising rate is given by Proposition 1.\(^{19}\)

The other extreme case where only one type of finance is used is when advertising demand is relatively weak. If \( R'(0) < \frac{1}{2} \bar{\gamma} \), then the sole financing method will be subscription pricing.\(^{20}\) In this case, the subscription price is simply that of the monopoly spatial model with a low reservation price.

---

\(^{19}\) Hence \( s > 0 \) for \( \theta + \lambda > R(a') + \frac{1}{2} \bar{\gamma} a' \). Consider the boundary case, where this holds with equality. When there is no subscription fee, Proposition 1 applies and \( \frac{K_a}{\lambda} = \frac{r_0}{2(\theta + \lambda) - \bar{\gamma} a} \). Substituting in the boundary case, \( R'(a) = \frac{1}{2} \bar{\gamma} \), as expected, and so the cases paste smoothly.

\(^{20}\) In terms of the demand curve for advertising, the condition is \( R'(0) - r(0) \leq \frac{1}{2} \bar{\gamma} \), so if the advertising demand curve intercept is below the average nuisance value there will be no advertising finance.
With AATs at a price, $p$, in contrast to the FTA case, with PAD, a viewer who avoids ads still has to pay the subscription fee, $s$. This means that the choice of a viewer of nuisance type $\gamma$ between purchasing an AAT or not depends upon whether $p < \gamma \hat{a}$ or not; regardless of the subscription fee. The number of viewers watching ads will be (cf. (6) and (7)):

$$N_{a \neq \hat{a}} = \frac{1}{\lambda s^2} \left( \theta + \lambda s - \frac{1}{2} \hat{\gamma} a \right)$$

$$N_{a = \hat{a}} = \frac{1}{\lambda s^2} \left( (\theta + \lambda s - p) \hat{\gamma} + \frac{1}{2a} p^2 \right)$$

For $a = \hat{a}$, these are the same. With subscription pricing, the broadcaster now also earns money from viewers who purchase AAT.\(^{21}\)

Given a subscription price $s$ and advertising level $\hat{a}$, this number, $n$, is (for $p$ sufficiently low):

$$n = \frac{\bar{\gamma} - \hat{\gamma}}{s} \left( \frac{\theta + \lambda \hat{a} - p}{\lambda} \right)$$

Thus, given $\hat{\gamma}$, the broadcaster will choose $a$ and $s$ to maximize:

$$sn + (s + R(a)) N_{a \neq \hat{a}}$$

for $a \leq \hat{a}$

$$sn + (s + R(a)) N_{a = \hat{a}}$$

for $a \geq \hat{a}$

By our earlier logic in the baseline model, the broadcaster would never choose $a < \hat{a}$. Thus, if $a \geq \hat{a}$, then the first order conditions for the broadcaster are (cf. (21) and (22)):

$$R'(a) N_{a = \hat{a}} + (s + R(a)) \frac{\partial N_{a = \hat{a}}}{\partial a} = 0$$

$$n + N_{a = \hat{a}} + (s + R(a)) \frac{\partial N_{a = \hat{a}}}{\partial s} = 0$$

Together these imply that:

$$R'(a) = \frac{\partial N_{a = \hat{a}}}{\partial a} \left( \frac{n + N_{a = \hat{a}}}{N_{a = \hat{a}}} \right) = \frac{\bar{\gamma}}{2} \left( \frac{n + N_{a = \hat{a}}}{N_{a = \hat{a}}} \right)$$

In contrast to that situation where there are no AAT’s (equation (23)), here the advertising rate is not simply determined by the average nuisance (independent of $s$ or program quality). In equilibrium, the subscription rate is determined by:

\(^{21}\) In this section we assume that the broadcaster sets $s$, which the consumers then observe (along with $p$) and they then choose whether to buy a subscription and AAT. The latter choices rationally anticipate (or are simultaneous with) the broadcaster’s choice of $a$. In the next section we analyze an alternative timing game (without the subscription choice) in which the broadcaster’s choice of $a$ is made before consumers have chosen AAT, and we show that this formulation unravels the AAT market (under the current distribution assumptions).
Given this, we can now compare subscription and advertising rates to those when there were no AATs.

**Proposition 8.** Comparing an equilibrium with positive AAT penetration \((n > 0)\) with one where AATs are unavailable, if AAT penetration is low \((p \text{ is high})\) advertising rates are higher and subscription rates are lower when AATs are available. As AAT penetration becomes high \((p \text{ is very small})\), advertising rates are lower and subscription rates are higher when AATs are available.

The intuition for this result is straightforward. When viewers purchase AATs, the broadcaster can still make money from them (and not drive them away) by putting up subscription charges. When AAT penetration is low, however, this benefit does not outweigh our earlier identified effect that such penetration causes broadcasters to increase advertising rates. In this situation, they do that and, to maintain viewership, lower subscription rates. In contrast, when AAT penetration is very high, most of the broadcaster’s revenue is earned from subscription fees rather than advertising. For this reason, they rely on that instrument and relax advertising levels to encourage those with AATs to bear those higher fees.

**Subscription-Based AAT**

Thus far, we have modeled AATs as a durable good. This was a realistic assumption given that many AATs are electronic appliances that last many years compared with advertising levels that can be more readily changed.

However, recent moves by cable television operators in the United States and elsewhere have seen AATs begun to be marketed as subscription based. The AAT is provided alongside cable service at a more expensive on-going rate. In Australia, the dominant cable television provider – Foxtel – only rents out its Foxtel IQ DVR. Thus, a viewer that does not continue rental payments or their cable television service cannot utilise the DVR.\(^{22}\) Similarly, TiVo has recently introduced plans to move to a fully subscription-based plan based on renting their appliance.

\[
s = \frac{p^2 + 2\bar{a}(\theta + \lambda - p) - 2pR(\bar{a})}{2(\bar{a} + p)}
\]

\(^{22}\) Related are moves to sell television in download format. Apple’s iTunes, for example, sells television programs without advertising for $1.99 per episode. In principle, this is like a subscription-based AAT as these downloads substitute viewership potentially away from broadcast television. However, their on-going
For these reasons, it is instructive to consider what happens when the payment for an AAT, \( p \), is on-going rather than once-off. This means that the broadcaster will no long hold the AAT penetration level as given when choosing its advertising level. Instead, it knows that for any advertising level it might choose, viewers for whom \( \gamma > p/a \) will subscribe to the AAT service while those for whom \( \gamma \leq p/a \) will watch FTA television and ads. Thus, the number of AAT viewers will not be given even if the AAT subscription rate, \( p \), is.

This change has a significant impact on the equilibrium outcome. In particular, as the following proposition demonstrates, there is no interior equilibrium involving non-trivial AAT viewership.

**Proposition 9.** For a given subscription rate, \( p \), either the unique equilibrium advertising level is as described in Proposition 1 and there is no AAT penetration or \( \bar{a} = p/\bar{\gamma} \) and there is no non-trivial AAT penetration.

Proposition 9 says that for any given \( p \), either the advertising level is set such that no viewer would want to subscribe the AAT service or the advertising level will be set such that the demand for this service is zero.

Significantly, this means that our earlier result that an increase in AAT penetration will lead to higher advertising rates will not hold for a subscription service. The lower the price of the subscription service, \( p \), the lower the advertising level. Thus, the subscription service constrains the broadcaster into choosing reduced advertising levels but higher advertising rates.

Importantly, this suggests that moves by AAT providers such as TiVo to switch from a durable appliance model to a rental or subscription model will actually harm them as the competitive response from broadcasters will be stronger rather than accommodating.

This variant of the model has, however, another interpretation. It could be considered as a model of ‘traditional’ ad-avoidance (e.g., going to the bathroom). Actively not paying attention when ads come on often means getting up and one could think of \( p \) as the cost of this. It is an on-going cost of ad-avoidance. Hence, viewers’

---

nature means that they have non-durable elements. Of course, you need a device to play the downloaded programs such as an iPod or a computer. In that respect, it has a significant durable quality to it.
incentives to incur \( p \) will be related to the length of ad breaks. Proposition 9 shows that broadcasters will want to set ad break length to deter this behavior. This is very different from their reaction to durable AATs where they move to accommodate them.

**Time management extension**

In the main model of the paper we have assumed that utility depends on the gross viewing evaluation from which we subtract the advertising nuisance. An alternative assumption is that the quality evaluation only accrues on the actual program content and an hour’s worth of television means only a fraction \( (1 - a) \) of actual programming. To capture this, suppose that viewer utility was:

\[
U_{x,y} = (\theta + \lambda(1-x))(1-a) - \gamma a
\]

(37)

With AAT, if the same amount of actual programming content were watched (i.e., if the broadcaster adapts program length to fit in the commercials), utility might become \( U_{x,y} = (\theta + \lambda(1-x))(1-a) - p \). In this case the main result still holds that ads increase with AAT penetration. However, there is an additional welfare cost involved with the introduction of AATs. Utility will fall even for those adopting them because program contents decrease.

Alternatively, if we were to assume the viewer will watch a fixed amount (one hour) of programming. utility with AATs might become \( U_{x,y} = \theta + \lambda(1-x) - p \). In this case, those purchasing AATs may be all of the viewers located around \( x = 0 \) because they are the ones who value actual program time most, and they get more concentrated programming per hour of watching if they screen out the commercials. In this case, advertising increases will shift the demand curve as well as pivot it.

5. **Conclusion**

This paper has demonstrated that the penetration of AATs like TiVo is unlikely to be innocuous for broadcaster behavior. Not only do these reduce the rents to broadcasters and, therefore, the incentives to invest in certain forms of broadcast content, they change

\[\text{\[23\text{That is, for a given value of ad-disutility, } \gamma, \text{ the FTA viewers are ones who like TV relatively less.}\]}


the mix of viewers. With TiVo though, the individuals who are siphoned off from ad exposure are those who are most annoyed by ads. The selective siphoning enhances the welfare of those who siphon, and enables those most annoyed by ads to opt out, but it weakens the two-sided business model of commercial television. Moreover, the TV platform’s response is to raise the ad level. This, we stress, is not per se an attempt to recapture the lost revenues, but rather it comes from the revealed preference of those who do not invest in TiVo: they are revealed to be less sensitive to ad nuisance and so the marginal incentive to raise the ad level is increased. Arguably, this effect has contributed to the larger number of ads per hour observed recently in US television (the US does not impose caps on the number of commercial minutes, in contrast to the EU).

We have shown that the advent of TiVo can nonetheless raise overall viewer surplus, despite the loss on the account of the viewers who remain watching more ads. Moreover, advertiser surplus can also go up: despite a lower viewer base, the larger volume of ads means a lower price per viewer reached, and the latter effect can dominate. However, it is likely (but by no means always so) that gains to viewers and advertisers are more than outweighed by the reduction in broadcaster profit as the business model is eroded.24,25

Other performance dimensions chosen by the broadcaster will also be affected. There is a lower incentive to provide vertical quality because the viewer footprint is reduced. This has negative feedback effects on the demand for AAT. There may also be a shift in program type offered towards Lowest Common Denominator programming as opposed to more specialty tastes.

The introduction of AAT might also tip the platform’s reliance from ad finance towards subscription pricing. Instead of delivering eyeballs to advertisers, if the consumers are screening out the ads, the platform can instead have them pay directly for access to the entertainment content. This means a tilt towards a more traditional (one-sided) market structure as the ability to strip out the financing side improves.

24 One might nonetheless be less concerned about the broadcaster insofar as it likely enjoys an excessive surplus due to barriers to entry anyway.
25 In the central case of the uniform preference distribution, we showed that a marginal AAT incursion reduces total welfare.
References


Steinberg, Brian, and Andrew Hampp (2007), “DVR Ad Skipping Happens, but Not Always” *Advertising Age* (May 31)


Appendix

Proof of Proposition 1:

Note first that:

\[
\frac{\partial \varepsilon_a}{\partial a} = \frac{r(r' + ar^2) - ar^2}{r^2} = \frac{ar^2}{r} - \frac{(1-\varepsilon_a)(2-\varepsilon_a)}{a}
\] (38)

From (4) with \(a \in (0, \frac{1}{7}(\theta + \lambda)]\), we have:

\[
\frac{\partial \varepsilon_a}{\partial a} = \frac{\bar{y}^2}{2(\theta + \lambda) - \bar{y}a} + \frac{\bar{y}^2 a}{(2(\theta + \lambda) - \bar{y}a)^2} = \frac{\varepsilon}{a} (1 + \varepsilon_N)
\] (39)

(so that \(\varepsilon_N(a)\) is strictly increasing in \(a\) in this range). Setting \(\varepsilon_a = \varepsilon_N = \varepsilon\) and comparing (38) and (39), \(\frac{\partial \varepsilon_a}{\partial a} < \frac{\partial \varepsilon_N}{\partial a}\) becomes:

\[
\frac{a^2 r''}{r} < \varepsilon(1 + \varepsilon) + (1 - \varepsilon)(2 - \varepsilon) = 2(1 - \varepsilon + \varepsilon^2)
\] (40)

The RHS of this expression exceeds \(2\varepsilon^2\) (since \(\varepsilon < 1\)). Hence, it suffices to prove that

\[
\frac{a^2 r''}{r} < 2\varepsilon^2 = \frac{2(r')^2 a^2}{r^2},
\] (41)

which is the same as \(2(r')^2 - r'' > 0\). But this is simply the condition that \(r\) is strictly \((-1\)-concave (as is implied by log-concavity and concavity\(^{26}\)).

Proof of Proposition 2:

First note that (from (6) and (7)):

\[
\frac{\partial N_a k}{\partial a} / N a \dot{a} = -\frac{\dot{y}}{2(\theta + \lambda) - \bar{y}a}
\] (42)

\[
\frac{\partial N_a k}{\partial a} / N a \dot{a} = \frac{p^2}{d(2\bar{y}(\theta + \lambda) - p^2)}
\] (43)

These imply that \(\varepsilon_N > 0\) is decreasing for \(a < \dot{a}\) and increasing thereafter.

In contrast, \(\varepsilon_a\) is decreasing over all \(a\) (by the log-concavity of \(r(a)\)) while at \(a = 0\), \(\varepsilon_a = \varepsilon_N = 1\) and as \(a \to \infty\), \(\varepsilon_a < 0 < \varepsilon_N\). Finally, we note that the broadcaster’s profit derivative (with respect to \(a\)) has the sign of

\[
(\varepsilon_a - \varepsilon_N)
\]

Thus, an equilibrium will involve \(\varepsilon_a = \varepsilon_N\) for some \(a > 0\) at a point where \(a = \dot{a}\).

Substituting \(\dot{y} = p / \dot{a}\) into (42) and (43) implies that, in equilibrium:

\(^{26}\) Indeed, if \(r(a)\) is concave, then the LHS of (41) is non-positive (since \(r'(a) \leq 0\)), while the RHS is positive.
\[
\frac{R'(\hat{a})\hat{a}}{R(\hat{a})} = \frac{p}{2(\theta + \lambda) - p}, \tag{44}
\]
as per the proposition. There is a unique solution with \(\hat{a} > 0\) to this equation (and hence a unique candidate equilibrium) since the LHS is decreasing from 1 through 0, while the RHS is strictly between 0 and 1 (by (5)).

To demonstrate that this is an equilibrium, we need to show that profit is quasiconcave and maximized at \(\hat{a}\). This property is ensured if, for a given \(\hat{a}\), if \(\varepsilon_a(a)\) crosses \(\varepsilon_N\), it does so only from above in the domain \(a \in (0, \hat{a}]\). Using (38), note that for \(a < \hat{a}\), \(\varepsilon_N = \frac{p^2}{2\gamma a(\theta + \lambda - \rho) + p^2}\) and so:
\[
\frac{\partial \varepsilon_N}{\partial a} = -2\hat{\gamma}(\theta + \lambda - p)\left(\frac{\varepsilon_N}{p}\right)^2. \tag{45}
\]
Now note that \(2\hat{\gamma}(\theta + \lambda - p) = \frac{p}{\varepsilon_a} - 1\) so that (45) becomes:
\[
\frac{\partial \varepsilon_N}{\partial a} = -\frac{1}{a}(1 - \varepsilon_N)\varepsilon_N \tag{46}
\]
(which is negative as we know that \(\varepsilon_N < 1\)).

Now suppose there is an \(a\) such that \(\varepsilon = \varepsilon_a = \varepsilon_N\). Then we can show that, at that point:
\[
\frac{\partial \varepsilon_a}{\partial a} < \frac{\partial \varepsilon_N}{\partial a}
\]
\[\Leftrightarrow r''(a)a < \frac{(\varepsilon - 1)(2 - \varepsilon)}{\varepsilon} < -\frac{1}{a}(1 - \varepsilon)\varepsilon \tag{47}
\]
\[\Leftrightarrow \frac{r''(a)a^2}{r(a)} < 2(\varepsilon - 1)^2
\]
The right hand side of this expression is positive, so that the desired inequality holds as \(r''(a) \leq 0\).\(^{27}\)

\(^{27}\) The assumption of \(r\) concave is stronger than the log-concavity assumption used elsewhere. However, log-concavity of \(R\) (which, admittedly, is a weaker assumption than log-concavity of \(r\)) is insufficient to do the trick. To see this point, note that the condition \(\frac{\partial R}{\partial a} < 0\) can be written as \(\frac{\partial R}{\partial a} = \frac{\partial R}{\partial a} + \frac{\partial R}{\partial a} - \frac{\partial R}{\partial a} < -\frac{1}{a}(1 - \varepsilon)\varepsilon\) at a point where \(\varepsilon_a = \varepsilon_N\) and where we have used (46) on the RHS and substituted in \(\varepsilon_a\). Rearranging, we would like to show that \((R' - R)\alpha < 0\). However, while the LHS is non-positive under log-concave \(R\), it can be zero if \(R\) is log-linear; the RHS is negative (since \(R' < \frac{\partial R}{\partial a}\)). Hence we use the stronger condition \(r\) concave in the Proposition. (Note that even the condition of \(R\) concave does not suffice, since we want to show \(R' Ra < 2R'(R'a - R)\). Even though the LHS is then negative, so is the RHS.)
Proof of Proposition 4:

Recall that the broadcaster’s profit is: \( \hat{x} = R(\hat{a}) \frac{\hat{\gamma}}{\hat{\gamma} x} \left( \theta + \lambda - \frac{1}{2} \hat{\gamma} \hat{a} \right) \). A shift in \( p \) shifts \( \hat{a} \) and \( \hat{\gamma} \). Since we know \( \hat{\gamma} \) shifts monotonically with \( p \), we treat this as the exogenous variable in the expression for \( \hat{x} \). Thus, we have:

\[
\frac{d\hat{x}}{d\hat{\gamma}} = R(\hat{a}) \frac{\partial N(\hat{a})}{\partial \hat{\gamma}} + R(\hat{a}) \frac{\partial N(\hat{a})}{\partial \hat{a}} + R(\hat{a})N(\hat{a}) \frac{d\hat{a}}{d\hat{\gamma}}
\]

(48)

The term in the brackets is identically zero by the first order conditions for the choice of \( \hat{a} \) given \( \hat{\gamma} \). The expression thus has the sign of:

\[
\frac{\partial N(\hat{a})}{\partial \hat{\gamma}} = \frac{\theta + \lambda - \hat{\gamma} \hat{a}}{\hat{\gamma} x} > 0
\]

(49)

The desired result then follows as \( \hat{\gamma} \) is increasing in \( p \).

Proof of Proposition 5:

The total surplus accruing to advertisers is:

\[
A = N(p) \int_a^\hat{a} (r(a) - r(\hat{a})) da = \frac{p}{\hat{\gamma} x} \left( \theta + \frac{1}{2} \right) p \frac{\hat{\gamma}}{2}
\]

(50)

where in the second step we have used the specific advertising demand function and we have used (6) (equivalently, (7)) to substitute for the viewer expression \( N(p) \). For this advertising demand function, revenue is \( R(a) = a(1-a) \), and, hence, it can readily be shown (using (9)) that:

\[
\hat{a} = \frac{2(\theta + \lambda - p)}{4(\theta + \lambda) - 3p}
\]

(51)

where both numerator and denominator are positive. Substituting into (50) yields:

\[
A = \frac{p}{\lambda \hat{\gamma} x} \frac{(\theta + \lambda - \frac{1}{2} p)(\theta + \lambda - p)}{4(\theta + \lambda) - 3p}
\]

(52)

Differentiating:

\[
\frac{\partial A}{\partial p} = \frac{8(\theta + \lambda)^3 - 24 p(\theta + \lambda)^2 + 21 p^2(\theta + \lambda) - 6 p^3}{2\lambda \hat{\gamma} x (4(\theta + \lambda) - 3p)^2}
\]

(53)

The sign of (53) depends on the sign of the numerator, which can be written as a cubic function of \( \frac{p}{\theta + \lambda} \). The sign is negative if:

\[
\theta + \lambda < p^\frac{1}{3} \left( 4 + \left( 2 + \sqrt{2} \right)^{\frac{1}{3}} + \frac{2^{\frac{2}{3}}}{(2 + \sqrt{2})^{\frac{1}{3}}} \right) = p(1.73784)
\]

(54)

We want to establish that (54) may hold for a relevant range of prices. To do this, we consider the highest possible AAT price \( p \) consistent with DVR use, such that \( p = \hat{\gamma} \hat{a} \). Using (51) and solving, this gives:

\[
p = \frac{1}{3} \left( \hat{\gamma} + 2(\theta + \lambda) - \sqrt{\hat{\gamma}^2 - 2\hat{\gamma}(\theta + \lambda) + (\theta + \lambda)^2} \right)
\]

Substituting this expression into (54) and re-arranging gives the condition of the proposition.
Proof of Proposition 6:

We first extract a factor $\frac{1}{2p^{\lambda}}$ from the welfare expression in the text, so that welfare is proportional to

$$\hat{W} = \int_{0}^{\gamma} (\theta + \lambda - \gamma \hat{a})^2 d\gamma + (\gamma - \hat{\gamma})(\theta + \lambda - p)^2 + 2(1 - \frac{\hat{a}}{2}) p(\theta + \lambda - \frac{p}{2})$$

The partial derivative with respect to $\hat{\gamma}$ is

$$\frac{\partial \hat{W}}{\partial \hat{\gamma}} = (\theta + \lambda - \hat{\gamma} \hat{a})^2 - (\theta + \lambda - p)^2,$$

which is identically zero: this term just represents transfers of indifferent viewers. The partial derivative with respect to $\hat{a}$ is

$$\frac{\partial \hat{W}}{\partial \hat{a}} = -2\int_{0}^{\gamma} \gamma(\theta + \lambda - \gamma \hat{a})d\gamma - p(\theta + \lambda - \frac{p}{2})$$

(55)

of which both terms are negative. This indicates first the deleterious effect to FTA viewers from a higher ad level. Second, gross advertiser surplus is reduced (recalling that $\theta + \lambda > p$ for AAT to be used).

Finally, evaluating around $\bar{\gamma} = \hat{\gamma}$, which is where AAT just becomes palatable to some viewers (and so the middle term's contribution vanishes) the partial derivative with respect to $p$ yields.28

$$\frac{\partial \hat{W}}{\partial p} = 2(1 - \frac{\hat{a}}{2})(\theta + \lambda - p)$$

(56)

Hence the welfare derivative boils down to

$$\frac{d\hat{W}}{dp} = \frac{\partial \hat{W}}{\partial \hat{a}} \frac{d\hat{a}}{dp} + \frac{\partial \hat{W}}{\partial p},$$

with $\frac{\partial \hat{W}}{\partial \hat{a}}$ given by (55), $\frac{\partial \hat{W}}{\partial p}$ given by (56), and hence $\frac{d\hat{W}}{dp}$ is proportional to:29

$$\left(-2\gamma^2 \left(\frac{\theta + \lambda}{2} - \frac{p}{2}\right) - p(\theta + \lambda - \frac{p}{2})\right) \frac{d\hat{a}}{dp} + 2(1 - \frac{\hat{a}}{2})(\theta + \lambda - p)$$

Substituting now $\hat{a} = \frac{2(\theta + \lambda - p)}{4(\theta + \lambda - 3p)}$, then $\frac{d\hat{a}}{dp} = -\frac{2(\theta + \lambda)}{4(\theta + \lambda - 3p)}$, and so the desired welfare derivative is proportional to

$$\frac{\theta + \lambda}{(\theta + \lambda - p)^2} \left(\frac{\theta + \lambda}{2} - \frac{p}{2}\right) + p(\theta + \lambda - \frac{p}{2}) \frac{2(\theta + \lambda)}{4(\theta + \lambda - 3p)} + 2(1 - \frac{\hat{a}}{2})(\theta + \lambda - p).$$

Since $\theta + \lambda > p$ (which is needed for a positive AAT segment), each of these three terms is positive. This means that in the neighborhood of no AAT usage, a price rise that forces out AAT improves aggregate surplus.

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28 The positive sign of this derivative, which stems solely from the total advertising surplus side, in conjunction with the negative effect on total advertising surplus through the $\hat{a}$ channel, means that gross advertising surplus is reduced by all incremental levels of AAT penetration—i.e., this result is not a local one.

29 Here we have used $\int_{0}^{\gamma} \gamma(\theta + \lambda - \gamma \hat{a})d\gamma = \hat{\gamma}^2 \left(\frac{1}{2}(\theta + \lambda) - \frac{1}{2} \hat{a} \hat{a}\right)$ and since $\hat{\gamma} \hat{a} = p$, then $\hat{\gamma}^2 = \frac{p}{2} \frac{\frac{\theta + \lambda}{2} - \frac{p}{2}}{4(\theta + \lambda - p)^3}$. 
Proof of Proposition 8:

Comparing (27) and (35), note that (at \( a = \hat{a} \)):

\[
\frac{\tau}{\tau} > \frac{\hat{\gamma} \left( \frac{n + N_{a_s}}{N_{a_s}} \right)}{\theta + \lambda - \frac{1}{2} p} = \frac{\hat{\gamma} \left( \frac{\theta + \lambda - \frac{1}{2} p}{\theta + \lambda - \frac{1}{2} p} + 1 \right)}{\theta + \lambda - \frac{1}{2} p}
\]

\[
\Rightarrow \frac{\tau}{\tau} > \frac{\theta + \lambda - \frac{1}{2} p}{\theta + \lambda - \frac{1}{2} p}
\]

\[
\Rightarrow \hat{\gamma} (\theta + \lambda - s - \frac{1}{2} p) > (\tau - \hat{\gamma} ) (\theta + \lambda - s - p)
\]

\[
\Rightarrow \hat{\gamma} (2 (\theta + \lambda - s) - \frac{1}{2} p) > \tau (\theta + \lambda - s - p)
\]

\[
\Rightarrow (2 (\theta + \lambda - s) - \frac{1}{2} p) > \frac{\tau a (\theta + \lambda - s - p)}{p}
\]  

(57)

Taking limits as \( p \) approaches \( \tau a \), it is easy to see that this inequality holds and \( R'(a_s) > R' (\hat{a}) \) implying that \( \hat{a} > a_s \). As \( p \) approaches 0, the reverse is true.

Looking at \( s \),

\[
s > s_{a_s} \Rightarrow \frac{1}{2} \left( \theta + \lambda - R(a^*) - \frac{1}{2} \tau a^* \right) > \frac{p^2 + 2 \tau \hat{a} (\theta + \lambda - p)}{2 (\tau \hat{a} + p)}
\]

\[
\Rightarrow \left( \theta + \lambda - R(a^*) - \frac{1}{2} \tau a^* \right) (\tau \hat{a} + p) > p^2 + 2 \tau \hat{a} (\theta + \lambda - p) - 2 p R(\hat{a})
\]

As \( p \) approaches \( \tau a \), \( R(\hat{a}) - R(a_s) > \frac{1}{2} \tau (a_s - \hat{a}) \) which given that \( \hat{a} > a_s \), implies that the inequality holds. As \( p \) approaches 0, this inequality becomes \( - R(a^*) - \frac{1}{2} \tau a^* > \theta + \lambda \); which cannot hold.

Proof of Proposition 9:

For a given \( a \), only viewers with \( \gamma \leq p / a \) will watch FTA television. Thus,

\[
N_{FTA} = \begin{cases} 
\frac{p (\theta + \lambda - \frac{1}{2} p)}{a \lambda \tau} & \text{if } p \leq \tau a \\
\frac{2 (\theta + \lambda - \frac{1}{2} \tau a)}{2 \lambda \tau} & \text{if } p > \tau a 
\end{cases}
\]

(58)

With this demand, \( e_N = 1 \) for \( p \leq \tau a \). In this case, \( e_a < e_N \) and so \( a \) will be set as low as possible in this range i.e., \( \hat{a} = p / \tau \) or alternatively, \( a < p / \tau \). If, however, \( \hat{a} \) as defined by \( e_a (\hat{a}) = \frac{\tau a}{2 (\theta + \lambda - \tau a)} \) exceeds \( p / \tau \) then the equilibrium will involve \( \hat{a} = p / \tau \). In either case, total AAT viewership equals 0.