A Note on Uncertain Lifetimes: A Comment

Joseph Pelzman, George Washington University
Don Rousslang
In a recent article in this Journal, Katz (1979) claimed that with actuarially fair life insurance and annuity policies available, “an increase in the probability of survival into the future may reduce an individual’s welfare.” This surprising result is derived using a simple two-period model based on earlier work by Yaari (1965) and Barro and Friedman (1977). The purpose of this comment is to demonstrate that Katz’s result is not possible given the assumptions of his model unless the income endowment in the second period is negative, in which case his result is not surprising. Therefore, his warning that “the expected utility hypothesis is not geared to dealing with problems involving uncertain lifetime” (Katz 1979, p. 193) is unwarranted.

Katz posits a two-period model in which the individual’s level of satisfaction from consumption in each period \( i \) is given by \( u(c_i) \) \( (i = 1, 2) \) with \( u' > 0 \) and \( u'' < 0 \). The individual has an endowed income of \( y_1 \) in period 1 and \( y_2 \) in period 2 and a probability \( p \) of surviving to period 2. Hence the individual’s expected utility is given by \( V \) where

\[
V = u(c_1) + pu(c_2) + (1 - p) u(0). \tag{1}
\]

Given the availability of actuarially fair life insurance policies, the individual may transfer income from period 1 to period 2 and vice versa according to the budget constraint,

\[
c_1 + pc_2 = y_1 + py_2. \tag{2}
\]

We wish to express our gratitude to the editor for helpful comments on an earlier draft.
Maximizing equation (1) subject to equation (2) yields \( u'(c_1) = u'(c_2) \), implying \( c_1 = c_2 \). Hence solving equation (2) for \( c = c_1 = c_2 \) yields

\[
c = \frac{(y_1 + py_2)}{(1 + p)}.
\] (3)

Substituting equation (3) into equation (1) gives the individual’s expected utility function,

\[
\bar{V} = u\left[ \frac{(y_1 + py_2)}{(1 + p)} \right] + pu\left[ \frac{(y_1 + py_2)}{(1 + p)} \right] + (1 - p)u(0). \] (4)

In order to determine the effect on the individual’s welfare of an increase in the probability that he will survive into period 2, differentiate (4) with respect to \( p \). This yields

\[
\frac{\partial \bar{V}}{\partial p} = u\left[ \frac{(y_1 + py_2)}{(1 + p)} \right] - \frac{(y_1 - y_2)}{(1 + p)} u'\left[ \frac{(y_1 + py_2)}{(1 + p)} \right] - u(0). \] (5)

Katz believes the sign of (5) is ambiguous. In particular, he posits that if

\[
\frac{(y_1 - y_2)}{(1 + p)} > \frac{u[(y_1 + y_2)/(1 + p)] - u(0)}{u'[(y_1 + py_2)/(1 + p)]},
\]
then “the individual is better off with a lower probability of survival and with the implicit shorter life expectancy.” However, on closer observation we see that the sign of (5) is not ambiguous but is in fact positive for nonnegative \( y_1 \) and \( y_2 \). In this case we have:

\[
c = \frac{(y_1 + py_2)}{(1 + p)} \geq \frac{(y_1 - y_2)}{(1 + p)}.
\]

Substituting \( c \) for \( (y_1 + py_2)/(1 + p) \) in equation (5) yields

\[
u(c) - \frac{(y_1 - y_2)}{(1 + p)} u'(c) - u(0) \geq u(c) - cu'(c) - u(0) > 0. \] (6)

In order to demonstrate the last inequality in (6), we consider a Taylor series expansion of the utility function around a level of consumption \( c \): \( u(t) = u(c) + (t - c)u'(c) + \frac{(t - c)^2}{2!}u''(c) + \text{negligible terms} \). Let \( t = 0 \); then

\[
u(c) - cu'(c) - u(0) = -(c^2/2!u''(c) > 0.
\]

Katz has suggested in correspondence that the individual may derive no utility from consumption below some level \( s \) necessary for survival, so that \( u(c) = 0 \) for \( c \leq s \). For such an individual, \( [u(c)/c] - u'(c) < 0 \) could easily hold for some level of consumption only slightly greater than \( s \). In this case, the sign of the last inequality in (6) may be reversed. However, the utility function of such an individual obviously violates the assumptions of Katz’s original model that marginal utility is always positive and declining.
It is also possible for (6) to be negative if the endowment in period 2, \( y_2 \), is negative and sufficiently large.\(^1\) In this case an increased probability of survival into period 2 may decrease the expected value of the individual’s utility. In addition to being a trivial case it should be noted that this result is due to the fact that the increased probability of survival decreases the expected value of the individual’s income and consumption and not because “the loss in welfare due to the worsened terms offered to an individual transferring money to the future outweighs the gain in welfare due to the increased probability of consumption in the future” as explained by Katz (1979, p. 195).

References


\(^1\) Note that \( [(y_1 - y_2)/(1 + p)] = c - y_2 \). Therefore (6) may be written as \( u(c) - cu'(c) + y_2 u'(c) - u(0) \). If \( y_2 \) is negative and sufficiently large, (6) can clearly be negative.