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Nice Guys Finish Fast and Bad Guys Finish Last: Facilitatory vs. Inhibitory Interaction in Parallel Systems

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Running Head: Interactive Parallel Models

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Abstract

Systems Factorial Technology is a powerful framework for investigating the fundamental properties of human information processing such as architecture (i.e., serial or parallel processing) and capacity (how processing efficiency is affected by increased workload). The Survivor Interaction Contrast (SIC) and the Capacity Coefficient are effective measures in determining these underlying properties, based on response-time data. Each of the different architectures, under the assumption of independent processing, predicts a specific form of the SIC along with some range of capacity. In this study, we explored SIC predictions of discrete (Poisson) and continuous (Linear Dynamic) models that allow for certain types of cross-channel interaction. The interaction can be facilitatory or inhibitory: one channel can either facilitate, or slow down processing in its counterpart. Despite the relative generality of these models, they predict a restricted range of SIC function and capacity coefficient values.

[Word count: 140]

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The issue of how we process multiple signals or multiple attributes of a given object is of considerable interest to psychologists. Different signals can be processed simultaneously (i.e., in a parallel manner) or sequentially (i.e. in a serial manner). Additionally, the signals can be processed in independent channels, or alternatively, the channels can somehow communicate with each other in such a way that one channel facilitates or inhibits processing in the other channel. In this paper we explore response-time (RT) predictions of parallel models that allow some degree of cross-channel interactions.

The following example will serve us throughout this report: suppose that two sources of information, say, an auditory and a visual signal, are processed in parallel channels 1 and 2 respectively. The channels can operate independently from one another, as shown in Figure 1A. That is, the activation in channel 1 does not affect the activation level in channel 2, and vice versa. Conversely, the channels may interact, as in Figure 1B. The interaction can be positive where each channel facilitates the processing of its counterpart causing an overall reduction in the time it takes to finish the processing of the incoming information. Hence, <u>nice guys finish fast</u>. Alternatively, the channels may inhibit each other's activity causing a slowdown in performance and hence, <u>bad guys finish last</u>.

In the absence of direct access to the underlying mental processes, researchers have traditionally adopted behavioral measures such as mean RTs to assess how different, most often simultaneously presented signals are processed (e.g., Donders, 1869; Sternberg, 1969). Investigators have generally been concerned with broad information processing issues such as whether multiple sources of information are processed in serial or in parallel. However, these techniques typically assume independent processing in the respective channels and little research has been carried out to investigate the effects of dependencies between processing channels.

One shortcoming of methodologies traditionally used to asses parallel versus serial processing is that mean RTs alone often cannot differentiate between competing models. Serial and parallel systems may mimic each other by exhibiting the same pattern of observed response times (e.g., Snodgrass & Townsend, 1980; Townsend, 1972, 1990a). For example, Snodgrass and Townsend proved that parallel models with limited capacity can easily mimic broad classes of serial models. A related issue is a possible trade-off between processing capacity and architecture, in which RT measures are consistent with parallel processing while capacity is in some sense 'limited' and consistent with serial processing (c.f., Townsend & Ashby, 1983). Workload capacity, or simply capacity, refers to the system's performance when the load is varied. If the processing rate on one channel remains invariant when another signal is added, then the capacity of the system is unlimited. Alternatively, if increasing the work load by presenting an additional signal slows down processing in a given channel then capacity is limited.

To overcome the problem of model mimicking, Townsend and colleagues (e.g., Schweickert & Townsend, 1989; Townsend, 1984; Townsend & Nozawa, 1995; Townsend & Schweickert, 1989) developed robust statistical measures that utilize entire RT distributions rather than mean RTs. Townsend & Nozawa (1995) developed a mathematical theory (and a related methodology), dubbed Systems Factorial Technology. Specifically, they provided a test that employs an interaction contrast between RT distributions from different experimental conditions -- known as the survivor interaction contrast (SIC) -- to distinguish between serial and parallel processing architectures (and within the latter category, independent-parallel from coactive-parallel models).[1] This index will be defined formally in the next section.

Parallel and serial models predict unique functional forms for the SIC. For example, suppose that a human observer is asked to respond affirmatively if an auditory signal <u>and</u> visual signal both appear. To respond correctly, the observer must exhaustively process both modalities. Under this regime, if the two signals are processed in parallel, then the predicted survivor contrast is negative, as depicted in Figure 2A. If the signals are processed serially, then the predicted SIC has a distinctive S-shaped curve that begins at zero and then becomes negative, crosses the abscissa, and then becomes positive before returning to zero. The SIC signatures for serial models are presented elsewhere (Townsend & Nozawa, 1995). In this paper we examined the effects that different levels of cross-channel interaction have on the SIC signature of parallel models.

[Figure 1 here]

Although SIC predictions do not depend on distributional or parametric assumptions, they do depend on the assumption of <u>selective influence</u> of experimental factors (see e.g., Sternberg, 1969). For an experimental manipulation to 'selectively influence' a particular process, the manipulation must affect the target process and no other process. For example, a sound intensity manipulation is said to selectively influence the auditory channel if it affects processing of the auditory signal but has no effect on processing of the visual signal. Townsend, Dzhafarov, and colleagues (Dzhafarov, 2003; Kujala & Dzhafarov, 2008; Townsend &

[1] In a coactive model, activation from multiple channels is summed and compared to a single threshold prior to decision. In the case of the Poisson coactive model, for example, counts from two or more channels can accumulate in a common buffer, in which the overall amount of counts is subsequently compared to the decision criterion.

Schweickert, 1989; Schweickert & Townsend, 1989; see also Townsend, 1990b) have provided robust theoretical assays in addition to statistical tests for assessing whether the conditions for selective influence are present.

If the channels in a parallel system interact with each other then the experimental manipulation targeted on one channel will have an effect on the other, violating the assumption of selective influence. Unlike ‘pure’ parallel or serial models, the channels are no longer independent; activation from one channel, such as the auditory channel, may be sent to the other channel and vice versa. The outcome of this cross-channel communication may be facilitatory or inhibitory depending on the nature of the interaction. In the current study we examined several classes of formal and computational parallel-interactive models, and explored their predictions with respect to the SIC and workload capacity, beyond the cases where selective influence holds.

The SIC test is traditionally employed within the context of a factorial design. We begin by outlining the paradigm often referred to as “the double factorial design.” We then explain the basic methodology for calculating the SIC and discuss the predictions for parallel independent models. Next, we describe two types of models, discrete state and continuous state, that are used to explore early cross-channel interactions (pre-accumulator) and late interactions (post-accumulator).[2] We then report simulation results of these models in terms of the SIC and workload capacity patterns they predict. Finally, we discuss the similarities and differences in the predicted SICs due to changes in the locus in which interactions occur.

The Double Factorial Design

[2] Cross-channel interaction may be early on in the process, representing perhaps a dependence of the activation in one channel on the input from the other. Or else, the interaction may occur at a later stage, for example if the activation in a channel depends on the activation in the other.

The double factorial design combines two levels of manipulation. The first manipulation is concerned with the presence versus absence of target items. For instance, in a target detection task with auditory and visual targets, four types of trials exist: double target trials, in which an auditory signal and a visual signal are presented at the same time, visual target alone, auditory target alone, and finally target absent trials. This manipulation of presence versus absence is used to create double versus single target conditions, which are necessary for the calculations of our capacity measure, as we shall see in later sections. A second manipulation of salience performed on the subset of double target trials yields four sub-types of trials: HH trials, where both the visual and the auditory target appear in their highly salient form (for example, a loud beep sound and a bright dot of light), HL and LH trials, where one target is highly salient whereas the salience level of the other target is low (e.g., loud sound and a dim dot, or a bright dot and weak sound) , and LL trials where both targets have low salience.

The survivor function for each of the factorial conditions (HH, LH, HL, and LL) can then be estimated from response times to yield the SIC. The survivor function is the complement of the cumulative distribution function, such that $S(t)=1-F(t)$. While the cumulative distribution function, $F(t)$, tells us the probability that processing of a given stimulus is finished before or at time *t*, the survivor function marks the probability that processing has not yet terminated. The SIC is computed by taking a double difference of survivor functions from the different factorial conditions, $\mathrm{SIC}(t) = [\mathrm{S}_{\mathrm{LL}}(t) - \mathrm{S}_{\mathrm{LH}}(t)] - [\mathrm{S}_{\mathrm{HL}}(t) - \mathrm{S}_{\mathrm{HH}}(t)]$.

The SIC predictions for two independent parallel models are presented in panels A and B of Figure 2 (for formal proofs and predictions for serial models, we refer the reader to Townsend & Nozawa, 1995). Townsend and Nozawa also derived predictions for a special case of parallel processing, referred to as coactive processing, in which information from two channels converges to satisfy a single criterion. A schematic of such a model is presented in Figure 1B, and the SIC prediction is plotted in Figure 2C. Under some assumptions, which we discuss later, the coactive model is in fact a special case of an interactive-facilitatory model.

[Figure 2 here]

We systematically varied the degree of cross-channel interaction within several classes of simulated models, and tested how it affects the form of the SIC. Varying the level of interaction makes parallel models flexible in terms of their predictions. In particular, it allows the model to mimic a range of architectures from independent-parallel (when the level of interaction is negligible or effectively null) to coactive. Consequently, parallel interactive models can predict a range of SIC signatures. Nonetheless, we found that despite the inherent flexibility of interactive-parallel models, their SIC functions do in fact span a finite range, thus allowing the falsification of certain classes of models based on observed data. For example, a facilitatory AND model (a system with two parallel channels which facilitate each other and stops as soon as the slower of the two finishes processing) can produce a range of SIC functions from completely negative to mostly positive. An entirely positive SIC, often observed in some of our studies (e.g., Eidels & Townsend, 2009; Eidels, Townsend, & Algom, in press; Townsend & Nozawa, 1995), would allow one to reject this broad class of parallel models.

We explored, in this paper, both continuous and discrete models of parallel processing with two varieties of interaction, one at the input stage (pre-accumulator) and one during the accumulation stage. For each model, we assumed that processing of two or more sources of information is carried out simultaneously in parallel channels. We allowed either first-termination (i.e., terminate processing when either channel 1 or 2 finishes; OR rule) or exhaustive processing (i.e., terminate processing when both 1 and 2 channels finish; AND rule). Furthermore, in all models, we manipulated the level of excitatory and inhibitory cross-channel interactions. However, the exact manner by which one channel affects the other differed across the two varieties.

Next, we present the models in greater detail and explain how the cross-channel interaction is realized in the discrete and continuous classes of models. The interaction can be facilitatory, with one channel 'helping' the other, or inhibitory, where one channel slows down the processing of its counterpart. Therefore, for each class of models there exist four cases of interest: facilitatory interaction associated with an OR rule, facilitatory with an AND rule, inhibitory OR, and inhibitory AND. After describing the models we present the simulation results showing the SIC functions for different levels of interactions for each of these four cases.

'Early' and 'Late' Cross-Channel Interactions

Each channel can interact with its counterpart in different loci. In Figure 1C and 1D we illustrate two possible loci of interaction, which we have explored in detail. In Figure1C, 'early' interaction, the interaction occurs before the accumulator in both channels. We refer to these models as "<u>pre-accumulator interaction</u>" models. This type of interaction is a model for dependent inputs. In facilitatory models, higher input in one channel leads to more activation feeding into the accumulator of the other channel. In inhibitory models the higher input in one channel leads to lower input to the accumulator of the other channel.

In Figure 1D, 'late' interaction, accumulated activation on one channel is added to- (in case of facilitation) or subtracted from- (in case of inhibition) the input of the other channel. In this type of model, it is the total activation, not just the input level of one channel that affects the other. We refer to these models as “post-accumulator interaction” models. Naturally, in facilitatory models higher total activation on one channel leads to higher input level in the other channel’s accumulator, whereas in inhibitory models higher total activation leads to lower input.

Discrete and Continuous Activation Models

The pre- and post-accumulator types of interaction were realized in this study within two types of models: A discrete state model, based on a Poisson process, and a continuous state model, which is based on a stochastic linear dynamic system.

Discrete Activation Models

We modeled discrete-state parallel-interactive processes with two parallel counting processes or channels. The input to each channel was treated as a Poisson process, with the rate determined by the salience level of an assumed stimulus processed by that channel (salient stimulus = high rate, faint stimulus = low rate). Each channel in the model accumulates counts until a prescribed criterion is reached. Channels could facilitate or inhibit each other by sharing positive or negative counts, respectively. For models of pre-accumulator interaction, only the most recent count could be shared. For models of post-accumulator interaction, any amount of the previously accumulated counts could be shared. In the AND case (“detect signal 1 and 2”), overall processing in the system ceased only when both channels reach their respective criterion. In the OR case, overall processing stopped once either channel 1 or 2 reaches its criterion. The following examples illustrate the process of counting with facilitatory versus inhibitory channel interaction.

Consider first a facilitatory model, where the probability of cross-channel interaction is 1 in both directions -- from channel 1 to 2, and from channel 2 to 1. This means that activation is fully shared between channels, but the exact manner differs across pre- and post-accumulation models. Both model varieties start with [0, 0]. On the first step, a count occurs on both of the channels. In the pre-accumulator models, each incoming count on a given channel is also added to the other channel, setting the state of the system to [2, 2]. On the second step, a count occurs on the first channel but not on the second. Nonetheless, due to the interaction, the same count is also sent from the first to the second channel, updating the state of the model to [3, 3]. Notice that in this extreme case the channels are perfectly correlated and will terminate processing at the same time (as long as their criterion values are identical). In the post-accumulator models, all accumulated counts are shared. If the state of the model is [2, 2], then all counts are shared from both channels to the other, increasing the state to [4, 4].

Alternatively, consider an inhibitory model where the probability of channels' interaction is again symmetric and equal to 1. Suppose that the model state is [2, 2] and a count is added to channel 1. With cross-channel inhibition, activation added to one channel is subtracted from the other in one of two ways, depending on the locus on interaction: In the pre-accumulator models the added count to channel 1 is simultaneously subtracted from channel 2, so the new state would be [3, 1]. In the post-accumulator model, in contrast, a count would be subtracted from channel 2 due to sharing from channel 1 a rate proportional to $2p$ (since there are two counts in channel 1) and likewise for decreases in channel 1 due to sharing from channel 2. By assumption, a channel cannot have fewer than zero counts. For instance, if the model starts at [0, 0] and a count is added to channel 1, a count would not be subtracted from channel 2 even if the probability of interaction is $p = 1$. In that case, the updated state of the model becomes [1, 0].

A formal description of the discrete activation models is provided in Appendix A. We investigated the RT predictions, and in particular the SIC predictions of these models by carrying out computer simulations and, in some cases, examining numerical computations based on analytic solutions. We tested both facilitatory and inhibitory models with varying levels of cross-channel interaction starting with completely independent channels, where the probability of interaction was null, p=0, all the way through p=1. In Appendix A we present the general model, but for brevity report results in which the sharing between channels is symmetric and the criteria are equal.

Continuous State Models

We modeled continuous-state parallel-interactive processes with linear dynamic systems. Similar to the discrete state models, we specified a state space describing the accumulation of perceptual or cognitive activation in a channel at each point in time. The process of accumulation began when input entered the system from the environment or from another internal system. Again the salience level determined the magnitude of the input. To make the process stochastic we added independent white noise processes to the input. Pre-accumulator interactions were modeled by adding a multiple of the input of each channel to the other. Post-accumulator interaction was modeled by adding a multiple of the total activation of each channel to the other. The level of interaction was determined by the magnitude of the multiplier in either case. In facilitatory models the multiplier was positive, while in inhibitory models the multiplier was negative.

We simulated the models with varying levels of cross-channel interaction starting with completely independent channels and gradually increasing the extent of the interaction. To obtain the necessary estimate of the CDF in each condition, we simulated a series of trials with the model to get a sample of predicted RTs. From those estimated CDFs we computed and plotted the SIC. For simplicity, the interaction parameters were set to be equal across channels. For a formal explication of the continuous state models see Appendix B.

Results and Discussion

Simulation results for the models presented above are summarized in Figure 3. The qualitative SIC predictions of the discrete space and continuous space models were the same. To avoid redundancy, we only included figures of the former. The SIC pattern predicted by pre- and post-accumulator models were often the same but differed on some aspects. Therefore we included figures of both, and compare their results shortly.

The SIC functions for four types of pre-accumulator model (facilitatory AND, facilitatory OR, inhibitory AND, inhibitory OR) are presented in the first column of Figure 3. The corresponding SIC functions for the post-accumulator models are shown in the second column of Figure 3. The solid black line in each panel corresponds to the SIC function of the parallel independent model. A lighter shade represents more interaction, with the lightest line representing the SIC function with the highest level of interaction. While the Poisson models have a clear maximum level of interaction ($p = 1$), the linear dynamic models are only bounded by the constraint on facilitation that the system remain stable and the constraint on inhibition that the system should complete processing in a finite time. For the parameters used in the simulation of the post-accumulator linear-dynamic models, this corresponded to cross-channel interaction values of $\mathrm{a}_{12} = \mathrm{a}_{21} = \pm 4.8$.

A cursory comparison between the first and second columns of Figure 3 reveals that the patterns of results predicted by pre- and post-accumulator model are qualitatively quite similar.

Next, we survey the results of each class in more detail and point out discrepancies, when exist. The order of discussion coarsely follows the difficulty for interpretation, from easy to more difficult, and not necessarily the order of presentation in Figure 3.

Pre-Accumulator Models

For both facilitatory models (AND, OR; top two rows of Figure 3), increasing the probability of interaction resulted in faster completion times. The corresponding curves shifted farther to the left as the level of facilitation increases (as the shade lightens). For the inhibitory model (bottom panels), increased interaction resulted in slower processing, and the corresponding SIC functions shifted to the right.

Figure 3A shows the SIC functions for a facilitatory exhaustive (AND) model where two parallel channels facilitated each other and stopped as soon as both channels finished processing. For the independent parallel-exhaustive models (i.e., $p = 0$), the SIC function was entirely negative, like Figure 2A, and commensurate with Townsend and Nozawa's (1995) Proposition 2. As the probability of cross-channel interaction increased, the early part of the survivor contrast function (i.e., for small t) remained negative, but the later part became more and more positive until, for p close to or equal to 1, the size of the positive area exceeded that of the early negative area. It is important to note that the facilitatory exhaustive model failed to produce a completely positive SIC function regardless of the amount of interaction. In fact, for the highest level of interaction the curve took the form of the SIC function predicted by a coactive model presented in Figure 2C (see Townsend & Nozawa's Proposition 5). This result is predictable because perfectly correlated channels (cross-channel interactions of $p = 1$ in the Poisson model) mean that all activation from one channel is sent to the other channel and vice versa. Hence, termination of processing on each channel occurred when the sum of counts from the two channels exceeds the criterion value, exactly as in a coactive (channel-summation) model.

Figure 3B shows the SIC function for a facilitatory first terminating (OR) model. For $p = 0$ (i.e., no cross channel interaction) the SIC remained entirely positive for all t, as predicted by an independent parallel first-terminating model (Figure 2A; see also Townsend and Nozawa, 1995, Proposition 1). As interaction increased, the early part of the function turned negative, but the total negative area was smaller than the positive area for all levels of interaction. At the maximum value, the SIC was mostly positive with an early negative blip, again the signature of a coactive model (cf. Figure 2C).

Regardless of the termination rule then, perfect sharing of counts between channels is structurally identical to coactive processing. The SIC signatures of the two facilitatory models are therefore bounded (from opposite directions) by the SIC signature of the coactive model. This observation is of extreme importance as it allows the researcher to reject certain classes of models. The facilitatory-first-terminating (OR) model, for example, predicted a range of SIC functions that span a finite range from total positivity to mostly positive with an early negative region (Figure 3B). If an entirely negative SIC function is observed in experimental data, facilitatory first-terminating models can be safely rejected.

Next, consider the forms of the SIC functions produced by parallel-inhibitory models. For the OR case (Figure 3D), the SIC functions were always positive regardless of the probability of cross-channel interaction. Increasing the level of interaction resulted in an overall slowdown of processing, as demonstrated by the horizontal stretching of the SIC function for high levels of interaction (to the extent that the SIC for the highest level had to be truncated in the figure).

However, the qualitative form of the SIC remained unaffected. Any negativity in the observed SIC rules out inhibitory first-terminating models.

The SIC results of the inhibitory exhaustive (AND) model, in Figure 3C, pose a more serious challenge for interpretation. The SIC was entirely negative for independent processes, while the right tail gradually became positive as the level of interaction increased. For large amounts of interaction, the positive area exceeded the negative area, and with the maximum amount of interaction the function was almost entirely positive.

<u>Post-Accumulator Models and Comparisons with Predictions of Pre-Accumulator Model</u>

Beginning with the inhibitory OR case (Figure 3D), the SIC predictions for the pre- and post-accumulator models were qualitatively similar. With increased interaction, the SIC function shifted to the right but always remained positive. Thus, any observed negativity in an empirical SIC function immediately rules out inhibitory first-terminating models, regardless of the level, and locus of interaction.

Next, consider the facilitatory OR case in Figure 3B. Once again, the qualitative predictions of pre- and post-accumulator models were similar. In the absence of cross-channel interaction, the SIC function was entirely positive. With increased interaction it gradually shifted to the left and was increasingly negative for early processing times. Even for the highest levels of interaction, though, it was mostly positive. Therefore, observing a completely negative SIC function, or even mostly negative function, excludes the facilitatory first-terminating model, again regardless of the locus of interaction.

For the facilitatory AND case (Figure 3A), the SIC functions predicted by the pre- and post-accumulator models were slightly different. The pre-accumulator model generated a range of SIC functions, from completely negative when processing in the two channels occurs independently, to mostly positive with an early negative blip when interaction was maximal. The post-accumulator model produced SIC functions which were negative across all tested parameter values, and thus comprised only a subset of the pre-accumulator predictions. Observing a completely positive SIC function rules out the facilitatory exhaustive model regardless of its class.

Finally, the predictions of the inhibitory AND model (Figure 3C) were somewhat similar across both classes. The SIC function was completely negative for independent processing, and its right tail gradually became positive as we increased the level of interaction. For the pre-accumulator model, the function was almost totally positive for the highest possible level of interaction. This model poses a challenge for interpretation as it predicted a wide range of function forms from totally negative to nearly totally positive. To overcome this problem and in general to increase one's ability to discriminate between models based on observed data, one needs to execute the second branch of systems factorial technology -- estimating the capacity coefficient, which we shall discuss shortly.

Summarizing the results, most models predicted a finite range of SIC forms. Observing an empirical SIC function that does not fall within the range predicted by a particular model allows the investigators to reject that model. Nonetheless, certain models had overlapping predictions of the SIC function. For concreteness, suppose that you observe an empirical SIC which is completely positive for all time *t*. One can immediately rule out the facilitatory exhaustive model (Figure 3A), as none of the SIC curves constantly stay above the abscissa, regardless of the level and locus of the interaction. However, the facilitatory first-terminating model (Figure 3B; for $p = 0$, which is an independent model), the inhibitory exhaustive model (figure 3C; for $p = 1$) and the inhibitory first-terminating model (figure 3D; for all p values including $p = 0$ which is an independent model) could predict a completely positive SIC function. What methodology can be utilized to distinguish between them? At this point, we shall discuss how workload capacity can help distinguishing between inhibitory, facilitatory, and independent parallel models.

[Figure 3 here]

Distinguishing between Facilitatory and Inhibitory Models that have Similar SIC Forms

The Capacity Coefficient. Inhibitory, facilitatory, and independent-channels models make different predictions with regard to a measure of processing efficiency that gauges workload capacity. By workload capacity, we refer to the processing efficiency of the system as we increase the load of information by, say, increasing the number of the to-be-processed targets. Townsend and Nozawa (1995) proposed a measure of workload capacity -- the capacity coefficient. For OR processes, the appropriate version is computed as the ratio between the integrated hazard function of the double target condition (i.e., two targets presented simultaneously) and the sum of the integrated hazard functions of the single target conditions:

$$C_{\mathrm{OR}} = \frac{\mathrm{H}_{12}(t)}{\mathrm{H}_1(t) + \mathrm{H}_2(t)}.$$

If the survivor function is the complement of the cumulative distribution function $S(t) = 1 - F(t)$, and the hazard function is the probability density function over the survivor function, $h(t) = \frac{f(t)}{S(t)}$, then the integrated hazard function, $H(t)$ is the integral of the hazard function from zero to *t*. The subscripts OR indicate that this index is calculated for the OR task.

Recently, Townsend & Wenger (2004) developed a comparable capacity index for the AND task, $C_{\mathrm{AND}}(t) = \dfrac{\mathrm{K}_1(t) + \mathrm{K}_2(t)}{\mathrm{K}_{12}(t)}$, where $\mathrm{K}(t)$ is analogous to the integrated hazard function, H(t). If we let k(t) be equal to the density over the distribution function, $k(t) = \dfrac{f(t)}{F(t)}$, then $\mathrm{K}(t)$ is defined as the integral of $k(t)$ from zero to t.

The interpretation of the two indices for both OR and AND conditions is the same (so we can momentarily ignore the subscripts): $C(t)$ values of 1 imply that the system has an unlimited capacity, such that processing in a given channel is not affected by the increase in workload due to the increase in the number of targets; i.e., a given channel has the same processing rate whether a target is presented to the other channel or not. $C(t)$ values that are below 1 suggest that capacity is limited, such that increasing the processing load (e.g., by increasing the number of targets on the display) takes a toll on the performance of each channel. Finally, if $C(t) > 1$ then the system is said to have super-capacity; processing efficiency of individual channels actually increases as we increase the workload.

The capacity coefficient gauges the processing efficiency of the system relative to the performance expected from an unlimited capacity independent parallel model. At the same time it indirectly provides information about architecture and channel (in)dependence. For example, the prediction of a parallel-independent model is, by definition, $C(t) = 1$, whereas a standard serial model roughly predicts $C(t) = .5$. The prediction of a parallel model with positive cross-channel interactions is $C(t) > 1$, as is the prediction of a coactive model.[3] Very strong inhibitory

[3] Townsend & Wenger (2004) simulated linear dynamic parallel-interactive models and showed that positive channel interactions have a facilitatory effect on workload capacity (C(t)>1) and that negative interactions have an inhibitory effect of capacity (C(t)<1). An unlimited capacity parallel model without cross-channel interactions ($a_{12} = a_{21} = 0$) produces capacity coefficient values of 1. Notably, coactive models in which activation from each

cross-channel interactions, in either parallel or serial mode of processing, may lead to severely limited capacity, such that $C(t) < .5$. The more inhibition there is between channels, the slower each channel is relative to its performance in isolation, and this slowdown is reflected in smaller values of the capacity coefficient. Conversely, with more cross-channel facilitation, each channel is faster than it would be in isolation, and the coefficient values increase. Thus, the capacity coefficient provides an indication of the degree of facilitation or inhibition.

Independent models, with different combinations of architecture (serial, parallel) and stopping rule (exhaustive, first terminating) predict unique forms of SIC functions (cf. Figure 2). When different models predict similar survivor contrasts at least one of the models must have high levels of cross-channel interactions. Examining the SIC and the capacity coefficient in tandem provides (in some cases) a decisive test for the architecture and possible dependencies between the processing channels.

In the third column of Figure 3 we present, for each of the four models, the predictions of the capacity coefficient for various degrees of cross-channel interaction (based on simulations of the pre-accumulator Poisson model). Like the SIC plots on the same figure, the black line in each panel represents the function for an independent model and as the probability of interaction increases, the shade gets lighter. Under the assumption of parsimony, we can assume that the same underlying processing system generates the data used for estimating $SIC(t)$ and $C(t)$.[4]

channels is summed together produces extremely super capacity values, higher than those observed in parallel models with positive channel interaction.

[4] That is, architecture does not change when we estimate these two statistics in a single experiment. For example, if two parallel channels operate independently, then they should be independent whether we use the data to estimate the capacity coefficient or whether we use just a subset of this data to estimate the SIC. Within a given experiment, a processing system of some kind cannot exhibit the signatures of independence on one measure (say, SIC(t) function which is all positive or all negative) and an interaction signature on the other (say, C(t) values much greater than, or much smaller than 1). In fact, the actual level of interaction (*p* value, in case of the Poisson model) is said to be invariant across the two measures.

With this assumption in mind, we provide the reader with a decision tree (Figure 4) in which, given the observed $SIC(t)$ and $C(t)$ patterns, one can decisively rule out certain models that fail to accommodate the observed pattern. The decision tree in restricted to the models tested in this study so choosing a particular model, as opposed to rejecting an unsuitable model, presents more difficulties; other models may exist that can exhibit similar SIC(t) and $C(t)$ patterns.

Choosing the Appropriate C(t) Formula. Given two different formulas, one for $C_{\text{OR}}(t)$ and another for $C_{\text{AND}}(t)$, how do we know which one to use for our data? In some cases we know what the stopping rule should be and the appropriate measure is clear. For instance, when exhaustive processing is called for by the instructions of a detection task, a failure to comply with the instructions will lead to noticeable proportion of errors. The participants must use the appropriate rule (AND) in order to perform accurately, and the appropriate capacity measure should be $C_{\text{AND}}(t)$.

If we do not know the stopping rule in advance, the form of the SIC can be helpful in determining the appropriate capacity coefficient. Observing a completely negative SIC for all time t rules out the two candidate OR models (left branch of Figure 4; compare with SIC predictions of OR models in Figure 3); in this case, the appropriate capacity coefficient would be $C_{\text{AND}}(t)$. $C_{\text{AND}}(t)$ values greater than 1 rule out inhibitory and independent models, $C_{\text{AND}}(t)$ values less than 1 rule out facilitatory and independent models, and $C_{\text{AND}}(t) = 1$ is only predicted by independent models.

When the SIC does not give enough evidence to determine the stopping rule then it is best to choose the most informative version of the capacity coefficient. For example, when the

SIC is positive for all t (middle branch of Figure 4), then we cannot determine the stopping rule. Both inhibitory AND and OR models predict $C_{\mathrm{OR}}(t) < 1$, while facilitatory OR models predict $C_{\mathrm{OR}}(t) > 1$ but not necessarily $C_{\mathrm{AND}}(t) > 1$. Thus $C_{\mathrm{OR}}(t)$ is more informative in this case.[5]

In conclusion, models that predict the same form of survivor contrast may be distinguished by observing their $C(t)$ predictions (and vice versa). Within the restricted universe of parallel-interactive models tested here, and given experimentally observed SIC and the capacity coefficient functions in tandem, one can identify a unique candidate processing model (end boxes of each of the paths in Figure 4). There are only two non-unique cases, but even in these paths the decision tree ends in two candidate models instead of many.

[Figure 4 here]

Conclusions

In this study, we explored SIC predictions of several classes of interactive parallel models: models with either discrete or continuous activation states, where the locus of interaction can be either pre-accumulation or post-accumulation. For each class, we simulated facilitatory and inhibitory models with OR (inclusive disjunctive) and AND (conjunctive) stopping rules, and generated SIC functions for various levels of cross-channel interactions.

The SIC as a tool for identifying the architecture of underlying processing systems was first introduced by Townsend and Nozawa (1995). These researchers showed that different

[5] In some cases both $C_{\mathrm{AND}}(t)$ and $C_{\mathrm{OR}}(t)$ are informative such as when the SIC function is negative for early times and positive for late times (Figure 4, right branch). In this case, if $C_{\mathrm{AND}}(t) > 1$ then inhibitory AND models can be rejected, leaving both facilitatory models. If, additionally, $C_{\mathrm{OR}}(t) < 1$ then facilitatory OR models are rejected in favor of facilitatory AND models.

processing models predict distinctive shapes of the SIC function. Thus, by estimating the SIC directly from data, one can rule out models that fail to predict the observed shape of the contrast function. Townsend and Nozawa limited their exploration to processing models with independent channels. Townsend and Wenger (2004) studied parallel models with interactions, but focused solely on workload capacity using linear dynamic systems. In this paper we provided a theoretically important generalization of the results of Townsend and colleagues by investigating the SIC predictions of parallel models with cross-channel interaction.

Two important types of parallel models were scrutinized in this paper: discrete space and continuous space models. The discrete state model was constructed as a two channel counting model, in which the probability of a single count within each channel was given by a Poisson distribution. The probability of sharing or sending a count from one channel to another was treated as a Bernoulli trial. The continuous state model, on the other hand, was formulated as a set of linear differential equations with additive noise. There were no qualitative differences between the results of the discrete and continuous space models.

Using both continuous state models and discrete state models, we modeled the effects of pre- and post-accumulation interaction between channels on the form of the SIC. Despite differences in the formulation of the models, their results were very similar as we demonstrated in Figure 3.

Although we explored a wide range of parallel interactive models, they predicted a limited range of SIC forms, thereby allowing for the falsification of certain model architectures. Even in the case where different models predict identical SICs, Systems Factorial Technology still provides powerful non-parametric methods for distinguishing among the models. Every pair of facilitatory and inhibitory models that share the same SIC, for instance, can be distinguished by analyzing their capacity predictions. Therefore, combined analysis of empirical SICs and capacity coefficients has proven to be a useful tool in model diagnosis as we demonstrated in the decision tree shown in Figure 4.

Systems factorial technology is a powerful modeling technique that relies on analytically proven theorems without making parametric assumptions about the underlying distributions responsible for generating the data. As such, its predictions are general and hold for any type of processing model with a particular architecture and stopping rule, regardless of the exact way in which individual channels of the model accumulate evidence over time. For example, a two-channel parallel-independent model always predicts a completely positive SIC, whether the accumulation of evidence towards decision within a channel is based on a diffusion process (e.g., Ratcliff, 1978) or a Poisson process (e.g., Smith & Van Zandt, 2002). The results reported in this paper therefore show that certain types of interactive-parallel models produce typical signatures (or a limited range of signatures). And, when the empirical survivor contrast and capacity coefficient functions are different from the predicted signatures, certain classes of models that fail to produce the observed outcome can be safely rejected.

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Appendix A: Formal Description of Discrete State Models

In this appendix we present the formal description of the discrete activation models discussed in the text. Processing channels in these models simultaneously (i.e., in parallel) accumulate evidence, in form of counts, toward some threshold. Via cross-channel interaction channels can also send counts to each other. Therefore, counts in each channel could be from two sources: (i) Within channel counts, based on the channel's response to some external stimulus or stimulus attribute, that represent the channel independent process of accumulating evidence from the environment. (ii) Shared counts, which were sent by the other channel, and represent the interaction across channels.

Modeling Within- and Between-Channel Counts

Within channel counts. In both pre- and post- accumulator models, the accumulation of information within a channel is modeled as a Poisson process. Hence, the amount of counts accumulated within a channel up to time t, denoted by $u_1(t)$ and $u_2(t)$, has a Poisson distribution. To model the difference between high and low salience conditions, the rate for the high condition (H), and thus the probability of accumulating a count in an interval, was set to be higher than the rate for the low condition (L), implying a shorter processing time for the H condition.

Between channel counts. The level of interaction between channels is set by the probability p_{12} of sending a count from channel 1 to channel 2 and probability p_{21} of sending a count from channel 2 to channel 1. In the pre-accumulator models, each new within-channel count is shared with probability. Because the count sharing in each time interval is a Bernoulli trial, we can model the process of sharing over time with a Binomial distribution. We use $k_{ij}(t)$ to denote the number of counts sent from channel i to channel j by time t.

In the post-accumulator models, the sharing follows a pure birth process, in which shared counts from channel i to channel j arrive according to an exponential distribution with rate $\mu_{ij}(t) = x_i(t) p_{ij} \mu$. $x_i(t)$ is the total activation (shared and within channel) in channel i at time t. p_{ij} is a probability that is varied to model degrees of interaction. The variable μ, with no subscripts, is a constant rate that is independent of the degree of interaction or direction of sharing. In general the sharing rate can be set to any positive number and it does not affect the qualitative aspects of the SIC. For the purposes of this paper, we set it to be in a similar range as the input rate.

Whether count sharing (cross-channel interaction) happens before or after the accumulation of counts, in the facilitatory models the shared count is added to the total activation of the receiving channel so the total activation at time t is the sum of the accumulated within-channel counts and the accumulated shared counts, $x_i(t) = u_i(t) + k_{ji}(t)$. In the inhibitory models, the shared counts are subtracted rather than added. The total activation $x_i(t)$ is then the total accumulated within-channel counts $u_i(t)$ minus the shared counts, $x_i(t) = u_i(t) - \sum_{t' \in \Omega} \Delta k_{ji}(t')$; $\Omega = \{t' \mid 0 < x_i(t') < \gamma_i ; 0 \le t' \le t\}$, where Ω ranges over positive times for which x_i is above zero and below criterion (if activation is zero, or if that channel had reached its criterion, then the shared counts bear no effect).

A channel completes processing when the total activation reaches threshold γ_i. If the system is an OR system, then the system also finishes processing at this point. If it is an AND system, then the other channel will continue <u>unaffected</u> by the completed channel. A channel is assumed to have between 0 and γ_i counts, so the model is defined over the $\gamma_1 \times \gamma_2$ state space.

The cumulative distribution function for the AND rule is given by $P_{\text{AND}}(RT \le t) = P\{T_1 \le t \text{ AND } T_2 \le t\}$, and the distribution for the OR rule is given by $P_{\text{OR}}(RT \le t) = P\{T_1 \le t \text{ OR } T_2 \le t\}$, where T_1 and T_2 are the random variables for processing times on the two channels. The probability that a channel finished processing at or before time t is equivalent to the probability that the total number of counts in the channel is at or above its criterion. Consequently, the cumulative distribution function for the AND rule can also be written as $P_{\text{AND}}(RT \le t) = P\{X_1(t) \ge \gamma_1 \text{ AND } X_2(t) \ge \gamma_2\}$ and the distribution for the OR rule is given by $P_{\text{OR}}(RT \le t) = P\{X_1(t) \ge \gamma_1 \text{ OR } X_2(t) \ge \gamma_2\}$.

The above discrete state models are all Markov processes and thus can be analyzed using the general tools associated with that class of models. In particular, we can use a matrix, $\mathbf{R}$, of the transition rates to specify the model and to calculate the distribution of completion times. Formally, the transition rate matrix if defined as follows. Suppose v_i is the rate at which the state changes from state i, and q_{ij} is the transition rate from state i to state j. Then the entries of the transition rate matrix are given by $r_{ij} = \begin{cases} q_{ij} & \text{if } i \ne j \\ v_i & \text{if } i = j \end{cases}$. If $P_{ij}(t) = P\{X(t) = j \mid X(0) = i\}$, then the matrix of probabilities with entries P_{ij} can be approximated by the equation,

$$\mathbf{P}(t) \approx (\mathbf{I} + \mathbf{R}t / n)^n \qquad \text{(A1)}$$

for large n (Busemeyer & Diederich, 2009, pp. 104-117; Ross, 1995). The only difference between the models is in the specification of the transition rate matrix.

In the special case of the facilitatory, pre-accumulator models the equations for the completion time distributions are relatively straightforward to derive directly. Depending on the level of precision desired, these equations can be used for more efficient computation. We begin by deriving these equations for the facilitatory, pre-accumulator models, followed by descriptions of the transition rate matrix $\mathbf{R}$ for each of the other models.

<u>Facilitatory Models</u>

<u>Facilitatory exhaustive (AND) model.</u> Figure A1 illustrates the state space for such model. The state of the model in the figure is represented by the number of counts on channel 1 (y axis) and the number of counts on channel 2 (x axis). The model starts without any counts, at [0, 0], and gradually accumulates evidence towards the thresholds γ_1 and γ_2, thus moving in the state space up and right towards the bounds. At each point of time, the state of the model must fall within one of the five areas in the figure. A pre-accumulator model cannot complete processing if its state is within area 5 of Figure A1, as there are not enough counts to reach either criterion. However, there are four ways in which a facilitatory AND model can in fact complete processing, corresponding to areas 1 through 4 in Figure A1.

In the first case, both channels may have enough counts on their own to satisfy criterion ($u_1 = \gamma_1, u_2 = \gamma_2$) which corresponds to the upper right corner of Figure A1 in the space marked by "1". Stated in terms of the completion time distribution,

$$\mathrm{P}_1(\mathrm{RT} \le t) = \mathrm{P}\{U_1(t) = \gamma_1\}\mathrm{P}\{U_2(t) = \gamma_2\}. \qquad \text{(A2)}$$

Alternatively, one of the channels may have enough within channel counts to reach criterion, while the other may not ($u_1 = \gamma_1, u_2 < \gamma_2$ or $u_1 < \gamma_1, u_2 = \gamma_2$). These possibilities correspond to the upper and right borders of Figure A1 (marked by 2 and 3). Finally, it is possible that neither channel has enough within channel counts to reach criterion. It that case, the channel(s) can reach criterion with the aid of the counts shared by the other channel (area 4), or not reach criterion at all (such that the model does not complete processing; area 5). Next we express the probability distribution for each case.

$$\mathrm{P}_2(\mathrm{RT} \le t) = \mathrm{P}\{U_1(t) = \gamma_1\}\mathrm{P}\{U_2(t) < \gamma_2 \text{ AND } K_{21}(t) = \gamma_2 - U_2(t) \mid U_1(t) = \gamma_1\} \quad \text{(A3)}$$

$$\mathrm{P}_3(\mathrm{RT} \le t) = \mathrm{P}\{U_2(t) = \gamma_2\}\mathrm{P}\{U_1(t) < \gamma_1 \text{ AND } K_{12}(t) = \gamma_1 - U_1(t) \mid U_2(t) = \gamma_2\} \quad \text{(A4)}$$

When neither channel has enough within-channel activation to complete ($U_1 < \gamma_1, U_2 < \gamma_2$; "4" in Figure 3), then there must be enough shared activation to make up the difference for the model to complete ($U_1 + K_{12} = \gamma_1, U_2 + K_{21} = \gamma_2$).

$$\mathrm{P}_4(\mathrm{RT} \le t) = \mathrm{P}\{U_1(t) < \gamma_1, U_2(t) < \gamma_2 \text{ AND } K_{21}(t) = \gamma_2 - U_2(t), K_{12}(t) = \gamma_1 - U_1(t)\} \quad \text{(A5)}$$

Equation A5 also holds for area 5, although for the post-accumulator models this probability is necessarily 0 as stated above.

As the first four events cover all possible values of the within channel counts, without overlap, their sum is the probability of the model completing:

$$\begin{aligned}
&\mathrm{P}_{\mathrm{AND}}(\mathrm{RT} \le t) \\
&= \mathrm{P}_1(\mathrm{RT} \le t) + \mathrm{P}_2(\mathrm{RT} \le t) + \mathrm{P}_3(\mathrm{RT} \le t) + \mathrm{P}_4(\mathrm{RT} \le t) \\
&= \mathrm{P}(U_1 = \gamma_1; t)\,\mathrm{P}(U_2 = \gamma_2; t) \\
&+ \mathrm{P}(U_1 = \gamma_1; t)\,\mathrm{P}(U_2 < \gamma_2 \text{ AND } K_{21} = \gamma_2 - U_2 \mid U_1 = \gamma_1; t) \\
&+ \mathrm{P}(U_2 = \gamma_2; t)\,\mathrm{P}(U_1 < \gamma_1 \text{ AND } K_{12} = \gamma_1 - U_1 \mid U_2 = \gamma_2; t) \\
&+ \mathrm{P}(U_1 < \gamma_1, U_2 < \gamma_2 \text{ AND } K_{21} = \gamma_2 - U_2, K_{12} = \gamma_1 - U_1; t)
\end{aligned} \quad \text{(A6)}$$

The pre-accumulator model can thus be written in a closed form by inserting the appropriate probabilities (recall: Poisson distribution for the within-channel counts, binomial distributions for between-channel counts):

$$
\begin{aligned}
&\mathrm{P}_{\mathrm{AND}}(RT \le t) = \\
&\left[1-\sum_{u_1=0}^{\gamma_1-1}\frac{e^{-\lambda_1 t}(\lambda_1 t)^{u_1}}{u_1!}\right]\left[1-\sum_{u_2=0}^{\gamma_2-1}\frac{e^{-\lambda_2 t}(\lambda_2 t)^{u_2}}{u_2!}\right] \\
&+\left[1-\sum_{u_1=0}^{\gamma_1-1}\frac{e^{-\lambda_1 t}(\lambda_1 t)^{u_1}}{u_1!}\right]\sum_{u_2=0}^{\lambda_2-1}\frac{e^{-\lambda_2 t}(\lambda_2 t)^{u_2}}{u_2!}\left[1-\sum_{k_{21}=0}^{\min(u_1,\gamma_2-u_2-1)}\binom{u_1}{k_{21}}p_{21}^{k_{21}}(1-p_{21})^{u_1-k_{21}}\right] \\
&+\left[1-\sum_{u_2=0}^{\gamma_2-1}\frac{e^{-\lambda_2 t}(\lambda_2 t)^{u_2}}{u_2!}\right]\sum_{u_1=0}^{\gamma_1-1}\frac{e^{-\lambda_1 t}(\lambda_1 t)^{u_1}}{u_1!}\left[1-\sum_{k_{12}=0}^{\min(u_2,\gamma_1-u_1-1)}\binom{u_2}{k_{12}}p_{12}^{k_{12}}(1-p_{12})^{u_2-k_{12}}\right] \\
&+\sum_{u_1=0}^{\lambda_1-1}\sum_{u_2=0}^{\gamma_2-1}\frac{e^{-\lambda_1 t}(\lambda_1 t)^{u_1}}{u_1!}\frac{e^{-\lambda_2 t}(\lambda_2 t)^{u_2}}{u_2!}\left[1-\sum_{k_{12}=0}^{\min(u_2,\gamma_1-u_1-1)}\binom{u_2}{k_{12}}p_{12}^{k_{12}}(1-p_{12})^{u_2-k_{12}}\right]\left[1-\sum_{k_{21}=0}^{\min(u_1,\gamma_2-u_2-1)}\binom{u_1}{k_{21}}p_{21}^{k_{21}}(1-p_{21})^{u_1-k_{21}}\right]
\end{aligned}
$$

To use equation A1 for this model, the following equations give the entries for transition rate matrix:

$$
v_{(x_1,x_2)(x_1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1; x_2 = \gamma_2 \\ \lambda_i & \text{if } x_j = \gamma_j \\ \lambda_1 + \lambda_2 & \text{otherwise} \end{cases},
$$

$$
q_{(x_1,x_2)(x_1+1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \\ \lambda_1 & \text{if } x_1 \neq \gamma_1 \text{ and } x_2 = \gamma_2 \\ (1-p_{12})\lambda_1 & \text{otherwise} \end{cases},
$$

$$
q_{(x_1,x_2)(x_1,x_2+1)} = \begin{cases} 0 & \text{if } x_2 = \gamma_2 \\ \lambda_2 & \text{if } x_2 \neq \gamma_2 \text{ and } x_1 = \gamma_1 \\ (1-p_{21})\lambda_2 & \text{otherwise} \end{cases},
$$

$$
q_{(x_1,x_2)(x_1+1,x_2+1)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ p_{21}\lambda_1 + p_{21}\lambda_2 & \text{otherwise} \end{cases}.
$$

In the post-accumulator model, any of the counts acquired so far may be shared. Hence, the rate of transition increases as the number of counts increase. The corresponding entries in transition rate matrix are:

$$v_{(x_1,x_2)(x_1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ and } x_2 = \gamma_2 \\ \lambda_i & \text{if } x_i \neq \gamma_i \text{ and } x_j = \gamma_j \text{ ,} \\ \lambda_1 + \lambda_2 + \mu(p_{12}x_1 + p_{21}x_2) & \text{otherwise} \end{cases}$$

$$q_{(x_1,x_2)(x_1+1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \\ \lambda_1 & \text{if } x_1 \neq \gamma_1 \text{ and } x_2 = \gamma_2 \text{ ,} \\ \lambda_1 + \mu p_{21}x_2 & \text{otherwise} \end{cases}$$

$$q_{(x_1,x_2)(x_1,x_2+1)} = \begin{cases} 0 & \text{if } x_2 = \gamma_2 \\ \lambda_2 & \text{if } x_2 \neq \gamma_2 \text{ and } x_1 = \gamma_1 \text{ .} \\ \lambda_2 + \mu p_{21}x_1 & \text{otherwise} \end{cases}$$

<u>Facilitatory first-terminating (OR) model.</u> In a first-terminating model, only one channel must reach criterion ($u_1 + k_{12} = \gamma_1$ or $u_2 + k_{21} = \gamma_2$). Mathematically, this can be stated more simply as the complement of ‘both channels are less than criterion’.

$$\begin{aligned} &\mathrm{P}(T_1 < t \text{ OR } T_2 < t) \\ &= \mathrm{P}(X_1(t) = \gamma_1 \text{ OR } X_2(t) = \gamma_2) \\ &= 1 - \mathrm{P}(U_1(t) + K_{12}(t) < \gamma_1 \text{ AND } U_2(t) + K_{21}(t) < \gamma_2) \end{aligned} \qquad \text{(A7)}$$

The pre-accumulator model can again be written in a closed form by inserting the appropriate probabilities.

$$\mathrm{P}(T_1 < t \text{ OR } T_2 < t) =$$
$$1 - \sum_{u_1-0}^{\gamma_1-1}\left[\sum_{u_2-0}^{\gamma_2-1}\left[\frac{(\lambda_1 t)^{u_1} e^{-\lambda_1 t}}{u_1!}\frac{(\lambda_2 t)^{u_2} e^{-\lambda_{21} t}}{u_2!}\sum_{k_{21}-0}^{\min(u_2,\gamma_1-1-u_1)}\binom{u_2}{k_{21}}(p_{21})^{k_{21}}(1-p_{21})^{u_2\ k_{21}}\sum_{k_{12}-0}^{\min(u_1,\gamma_2-1-u_2)}\binom{u_1}{k_{12}}(p_{12})^{k_{12}}(1-p_{12})^{u_1\ k_{12}}\right]\right]$$

The transition rate matrices representing the pre-and post-accumulator, facilitatory OR models are quite close to the corresponding matrices for the facilitatory AND models. The only difference is that once either one of the channels has reached its criterion, the transition rate is zero. For the pre-accumulator model:

$$v_{(x_1,x_2)(x_1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \lambda_1 + \lambda_2 & \text{otherwise} \end{cases},$$

$$q_{(x_1,x_2)(x_1+1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ (1-p_{12})\lambda_1 & \text{otherwise} \end{cases},$$

$$q_{(x_1,x_2)(x_1,x_2+1)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ (1-p_{21})\lambda_2 & \text{otherwise} \end{cases}.$$

For the post-accumulator model:

$$v_{(x_1,x_2)(x_1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \lambda_1 + \lambda_2 + \mu(p_{12}x_1 + p_{21}x_2) & \text{otherwise} \end{cases},$$

$$q_{(x_1,x_2)(x_1+1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \lambda_1 + \mu p_{21} x_2 & \text{otherwise} \end{cases},$$

$$q_{(x_1,x_2)(x_1,x_2+1)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \lambda_2 + \mu p_{21} x_1 & \text{otherwise} \end{cases}.$$

Inhibitory Models

In an inhibitory model the shared counts are subtracted from the total activation of the receiving channel. An additional assumption of the inhibitory models is that the total activation of a channel cannot go below zero (cf. Usher & McClelland 2001). Such an assumption is not necessary in facilitatory models, because channels' activation cannot be negative. Since the shared counts do not always contribute to the total activation the inhibitory model cannot be stated with the relatively simple equations of the pre-accumulator facilitatory model. Instead we use a random walk process to describe the inhibitory model. We begin by illustrating the state space and possible processing steps in such models (Figure A2). In keeping with the intended

Poisson nature of the model, we treat the probability of two counts occurring in the same miniscule time increment as zero.

Panel A in Figure A2 depicts the initial state of the model and the possible transitions from that state. Initially, there is no activation in either channel so the model starts at [0, 0]. When a channel gains a count, it may or may not share that count. We assume that a channel cannot have negative activation, so if a channel with zero counts receives a shared count, the shared count will have no effect. Therefore, at [0, 0] a count that is added to one channel is not subtracted from its counterpart, and the new state of the model is [1, 0], or [0, 1].

If both channels have at least one count, but neither channel has completed processing, there exist other possible transitions, as depicted in Figure A2 -- Panel B. The model can stay in the same state, both channels could increase, or one channel could increase while the other remains constant.

It is impossible for both channels to lose a count simultaneously. It is also impossible for one channel to lose a count while the other stays the same. This is because for a channel to lose a count, it must receive a shared count from the other channel and not gain a within-channel count. Since the first channel does not gain a within-channel count, it cannot share. However, the other channel must gain a within-channel count to share. This other channel cannot have received a shared count from the first channel, meaning that its total activation must also increase. Thus, for one channel to decrease, the other must increase.

Once a channel reaches criterion, the behavior of the first-terminating and the exhaustive models diverges. The first-terminating model finishes processing at this point (so the model will not transition to any other state). The exhaustive model must continue processing until both channels reach criterion. After one channel completes processing it can no longer affect processing in the other channel. The unfinished channel will continue accumulating counts as an independent Poisson process until it reaches its criterion. When both channels reach criteria the model reaches its final state (and processing is completed).

We are now in a position to specify the transition rate matrix for these models. In most cases, the transition rate depends on the current state. Cases where at least one channel is zero or at criterion, pictured in Figure A2 – Panels A, B and D, are dealt with first, then we specify transition rates from states in which both channels have at least one count and neither channel has reached criterion. These are the states exemplified in Figure A2 -- Panel C. As we described above, the model can transition to a state where only a single channel increased while the other channel either stayed the same or decreased, or to the same state in which both channels have the same amount of activation.

Inhibitory exhaustive (AND) model. In the AND model, the unfinished channel will continue independently until it finishes.

For one channel to increase while the other decreases, the first channel must gain a count, then share it.

$$q_{(x_1,x_2)(x_1+1,x_2-1)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1, x_2 = \gamma_2 \text{ or } x_2 = 0 \\ p_{12}\lambda_1 & \text{otherwise} \end{cases}$$

$$q_{(x_1,x_2)(x_1-1,x_2+1)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1, x_2 = \gamma_2 \text{ or } x_1 = 0 \\ p_{21}\lambda_2 & \text{otherwise} \end{cases}$$

For the total activation in one channel to increase while the other remains the same, the first channel must obtain a within-channel count but not share it.

$$q_{(x_1,x_2)(x_1+1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \\ \lambda_1 & x_2 = \gamma_2 \text{ or } x_2 = 0 \text{ and } x_1 \neq \gamma_1 \\ (1-p_{12})\lambda_1 & \text{otherwise} \end{cases}$$

$$q_{(x_1,x_2)(x_1,x_2+1)} = \begin{cases} 0 & \text{if } x_2 = \gamma_2 \\ \lambda_2 & x_1 = \gamma_1 \text{ or } x_1 = 0 \text{ and } x_2 \neq \gamma_2 \\ (1-p_{21})\lambda_2 & \text{otherwise} \end{cases}$$

If neither channel gains a count then no counts had been shared and the activation simply stays the same.

$$v_{(x_1,x_2)(x_1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ and } x_2 = \gamma_2 \\ \lambda_i & \text{if } x_i \neq \gamma_i \text{ and } x_j = \gamma_j \\ \lambda_1 + \lambda_2 & \text{otherwise} \end{cases}$$

The post accumulator models are similar to facilitatory post-accumulator models. The diagonal entries to the transition rate matrix are the same.

$$v_{(x_1,x_2)(x_1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ and } x_2 = \gamma_2 \\ \lambda_i & \text{if } x_i \neq \gamma_i \text{ and } x_j = \gamma_j \\ \lambda_1 + \lambda_2 + \mu(p_{12}x_1 + p_{21}x_2) & \text{otherwise} \end{cases}.$$

The difference is that when a count is shared, the receiving channel decreases. Hence, the transition rates for gaining a count in a channel are,

$$q_{(x_1,x_2)(x_1+1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \\ \lambda_1 & \text{otherwise} \end{cases} \text{ and}$$

$$q_{(x_1,x_2)(x_1,x_2+1)} = \begin{cases} 0 & \text{if } x_2 = \gamma_2 \\ \lambda_2 & \text{otherwise} \end{cases}.$$

The transition rates for losing a count are,

$$q_{(x_1,x_2)(x_1,x_2-1)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \mu p_{21} x_2 & \text{otherwise} \end{cases} \text{ and}$$

$$q_{(x_1,x_2)(x_1-1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \mu p_{12} x_1 & \text{otherwise} \end{cases}.$$

Inhibitory first-terminating (OR) model. The transition probabilities listed up to this point apply to both the OR and AND models. As discussed earlier, the two models differ once one of the channels finishes. In this case, the OR model does not change states. In all other cases, the behavior of the two models is identical.

For one channel to increase while the other decreases, the first channel must gain a count, then share it.

$$q_{(x_1,x_2)(x_1+1,x_2-1)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1, x_2 = \gamma_2 \text{ or } x_2 = 0 \\ p_{12}\lambda_1 & \text{otherwise} \end{cases}$$

$$q_{(x_1,x_2)(x_1-1,x_2+1)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1, x_2 = \gamma_2 \text{ or } x_1 = 0 \\ p_{21}\lambda_2 & \text{otherwise} \end{cases}$$

For the total activation in one channel to increase while the other remains the same, the first channel must obtain a within-channel count but not share it.

$$q_{(x_1,x_2)(x_1+1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \lambda_1 & \text{if } x_2 = 0 \text{ and } x_1 \neq \gamma_1 \\ (1-p_{12})\lambda_1 & \text{otherwise} \end{cases}$$

$$q_{(x_1,x_2)(x_1,x_2+1)} = \begin{cases} 0 & \text{if } x_2 = \gamma_2 \text{ or } x_1 = \gamma_1 \\ \lambda_2 & \text{if } x_1 = 0 \text{ and } x_2 \neq \gamma_2 \\ (1-p_{21})\lambda_2 & \text{otherwise} \end{cases}$$

If neither channel gains a count then no counts had been shared and the activation simply stays the same.

$$v_{(x_1,x_2)(x_1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \lambda_1 + \lambda_2 & \text{otherwise} \end{cases}.$$

Like the post-accumulator AND models, the post-accumulator inhibitory OR models are quite similar to the post- accumulator facilitatory OR models,

$$v_{(x_1,x_2)(x_1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \lambda_1 + \lambda_2 + \mu(p_{12}x_1 + p_{21}x_2) & \text{otherwise} \end{cases},$$

$$q_{(x_1,x_2)(x_1+1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \lambda_1 & \text{otherwise} \end{cases},$$

$$q_{(x_1,x_2)(x_1,x_2+1)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \lambda_2 & \text{otherwise} \end{cases}.$$

$$q_{(x_1,x_2)(x_1-1,x_2)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \mu p_{21}x_2 & \text{otherwise} \end{cases}$$

$$q_{(x_1,x_2)(x_1,x_2-1)} = \begin{cases} 0 & \text{if } x_1 = \gamma_1 \text{ or } x_2 = \gamma_2 \\ \mu p_{12}x_1 & \text{otherwise} \end{cases}.$$

Appendix B: Formal Description of Continuous State Models

In this appendix we present the formal description of the continuous state models discussed in the text. Like the discrete models in Appendix A, we assume two parallel processing channels, but now we allow the state to be any positive real number, as opposed to just integer value. The total activation in each channel is represented by $x_i(t)$, although to conform to the standard presentation of linear dynamic systems (e.g., Townsend & Wenger, 2004), we use vector and matrix notation, i.e. $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$. Each channel has some input, $\mathbf{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$, corresponding to the within-channel counts in the discrete state model.

To represent cross-channel interactions, we use a matrix of coefficients indicating the values of the activation weights. Following Ashby's model for stochastic general recognition theory (Ashby, 1989), we use $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ for pre-accumulator interactions, and $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ for post-accumulator interactions. The off diagonal coefficients represent the amount of between-channel cross talk, or information sharing, so a_{12} determines the amount of cross-talk from channel 2 to channel 1 and a_{21} determines the amount of cross-talk from channel 1 to channel 2. For those unfamiliar with linear dynamic system notation, it may seem odd to use a_{12} for the sharing from channel 2 to channel 1 rather then vice versa. In keeping with the standard notation of this class of models, we use the subscripts to denote the row and column of the matrix $\mathbf{A}$.

By setting the off diagonal coefficients of matrix $\mathbf{A}$ to zero, cross-channel sharing is completely eliminated, thereby making the model equivalent to an independent-parallel model. Activation in the model is then solely dependent on the diagonal elements, representing within-channel contribution. The diagonal elements b_{11} and b_{22} are parameters denoting gain or loss applied to the within channel input. Since changing the diagonal elements of $\mathbf{B}$ is equivalent to rescaling the inputs, we fixed them to 1, $b_{11} = b_{22} = 1$. The diagonal elements a_{11} and a_{22} are parameters denoting the feedback rate for a particular channel. As we shall see shortly, these values can be used to ensure that the system is stable. Townsend and Wenger (2004) used parameter values that maintained stability in the system, a property that is often assumed for natural systems (cf. Usher & McClelland 2001).

Deterministic Pre-Accumulator Model

The two-channel pre-accumulator interactive parallel model, with no post-accumulator interaction, is given by:

$$\frac{d}{dt}\mathbf{x}(t) = \mathbf{Ax}(t) + \mathbf{Bu}(t) = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}. \quad \text{(B1)}$$

We refer to the above version of the model as deterministic, because it has no source of noise or variability. We shall shortly present the stochastic version of the model, which includes a noise term.

The magnitude of the interaction parameters (off diagonal elements of $\mathbf{B}$) was varied between 0 and 1 to represent the range between complete independence and total information sharing. Similar to our explorations with the discrete state models, we set the interaction to be symmetric so that $b_{12} = b_{21}$. Assuming a constant input, the solution to this differential equation is

$$x_1(t) = \frac{u_1 + b_{12}u_2}{a_{11}}\left[\exp(a_{11}t) - 1\right]$$
$$x_2(t) = \frac{b_{21}u_1 + u_2}{a_{22}}\left[\exp(a_{22}t) - 1\right]. \qquad \text{(B2)}$$

Deterministic Post-Accumulator Model

The (deterministic) two-channel post-accumulator interactive parallel model is given by:

$$\frac{d}{dt}\mathbf{x}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{u}(t) = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}. \qquad \text{(B3)}$$

In accordance with Townsend and Wenger (2004), we further simplified the model with the assumption that the activation rates within each channel are equal, $a_{11} = a_{22}$, and as above, cross-channel interaction coefficients are equal, $a_{12} = a_{21}$. Furthermore, we assumed that the input to each channels is constant (for $t > 0$, $u_1(t) = u_1; u_2(t) = u_2$), making the system time invariant.

In this case there exists a closed form solution that describes the activation level in each of the channels at time $t \geq 0$:

$$x_1(t) = \frac{u_1 + u_2}{2(a_{11} + a_{12})}\left[\exp[(a_{11} + a_{12})t] - 1\right] + \frac{u_1 - u_2}{2(a_{11} - a_{12})}\left[\exp[(a_{11} - a_{12})t] - 1\right]$$
$$x_2(t) = \frac{u_1 + u_2}{2(a_{11} + a_{12})}\left[\exp[(a_{11} + a_{12})t] - 1\right] + \frac{u_2 - u_1}{2(a_{11} - a_{12})}\left[\exp[(a_{11} - a_{12})t] - 1\right] \qquad \text{(B4)}$$

The channel's activation is an exponential expression, meaning that if the sum $a_{11} + a_{12}$ or $a_{11} - a_{12}$ is positive, the activation increases without bound. To stabilize the system, we set $a_{11} = a_{22} < 0$; and $|a_{12}| = |a_{21}| < |a_{11}| = |a_{22}|$ to prevent the sum from being positive.

Stochastic Pre- and Post-Accumulator Models

To make the model stochastic, we added Gaussian white noise, $\eta(t)$, to the inputs. The added noise is independently and identically distributed such that it is uncorrelated over time and across channels. The differential equation that describes channel activation in a stochastic model with two parallel channels that interact pre-accumulation is:

$$\frac{d}{dt}\mathbf{x}(t) = \mathbf{Ax}(t) + \mathbf{Bu}(t) = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\begin{bmatrix} u_1(t) + \eta_1(t) \\ u_2(t) + \eta_2(t) \end{bmatrix}. \tag{B5}$$

When the interaction occurs post-accumulation, the equation is:

$$\frac{d}{dt}\mathbf{x}(t) = \mathbf{Ax}(t) + \mathbf{u}(t) = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} u_1(t) + \eta_1(t) \\ u_2(t) + \eta_2(t) \end{bmatrix}. \tag{B6}$$

As in the Poisson counting models, we allowed the interaction parameters to be either positive (facilitation) or negative (inhibition), and manipulated the magnitude of the cross-channel interaction. The actual interaction parameters for the post-accumulator, facilitatory models were set to be 0 (independent channels), 1.2, 2.4 and 3.6 and 4.8, with the stabilizing parameter set to -10. For simplicity, both the interaction parameters and stabilizing parameters were set to be equal across channels ($a_{12} = a_{21}$; $a_{11} = a_{22}$). The particular range of parameter values was chosen to ensure the stability of the model. As stated above, the cross channel interaction in the pre-accumulator models varied in magnitude from 0 to 1, with positive values for facilitatory values and negative values for inhibitory models.

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Figure Captions

Figure 1. Schematics of four types of parallel processing models: independent parallel channels (panel A), parallel coactive model (panel B), parallel channels with pre-accumulator interaction (panel C), and parallel channels with post-accumulator interaction (panel D).

Figure 2. Survivor functions (left column) and SIC predictions (right column) for different processing models: parallel first-terminating (panel A), parallel exhaustive (panel B), and coactive (panel C). To calculate the SIC, one first estimates the survivor functions for each of the four factorial conditions (HH, HL, LH, and LL), and then calculates the double difference: $SIC(t) = [S_{LL}(t) - S_{LH}(t)] - [S_{HL}(t) - S_{HH}(t)]$.

Figure 3. Simulated SIC results from four types of pre-accumulator parallel interactive models (first column) and post-accumulator models (second column): Facilitatory AND (panel A), Facilitatory OR (panel B), Inhibitory AND (panel C), and Inhibitory OR (panel D). The third column shows the predicted $C(t)$ values, which are similar for pre- and post-accumulator interaction models. In each panel, the thick dark line represents the independent model and as the probability of interaction increases, the lines become lighter.

Figure 4. A decision tree for parallel-interactive model diagnosis. Given both empirical survivor interaction contrast [SIC(t)] and capacity coefficient [C(t)] estimates, one can analyze the diagram from top to bottom to rule out models that fail to predict the observed functions. The decision tree accommodates the models tested in this paper. * A coactive model is a candidate.

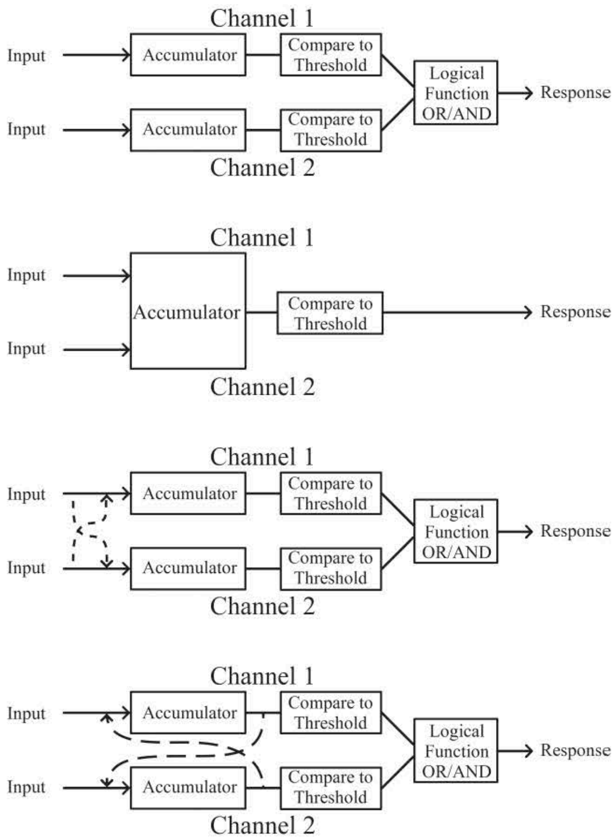

Channel 1
Input
Accumulator
Compare to Threshold
Logical Function OR/AND
Response
Input
Accumulator
Compare to Threshold
Channel 2
Channel 1
Input
Accumulator
Compare to Threshold
Response
Input
Channel 2
Channel 1
Input
Accumulator
Compare to Threshold
Logical Function OR/AND
Response
Input
Accumulator
Compare to Threshold
Channel 2
Channel 1
Input
Accumulator
Compare to Threshold
Logical Function OR/AND
Response
Input
Accumulator
Compare to Threshold
Channel 2

Figure 1

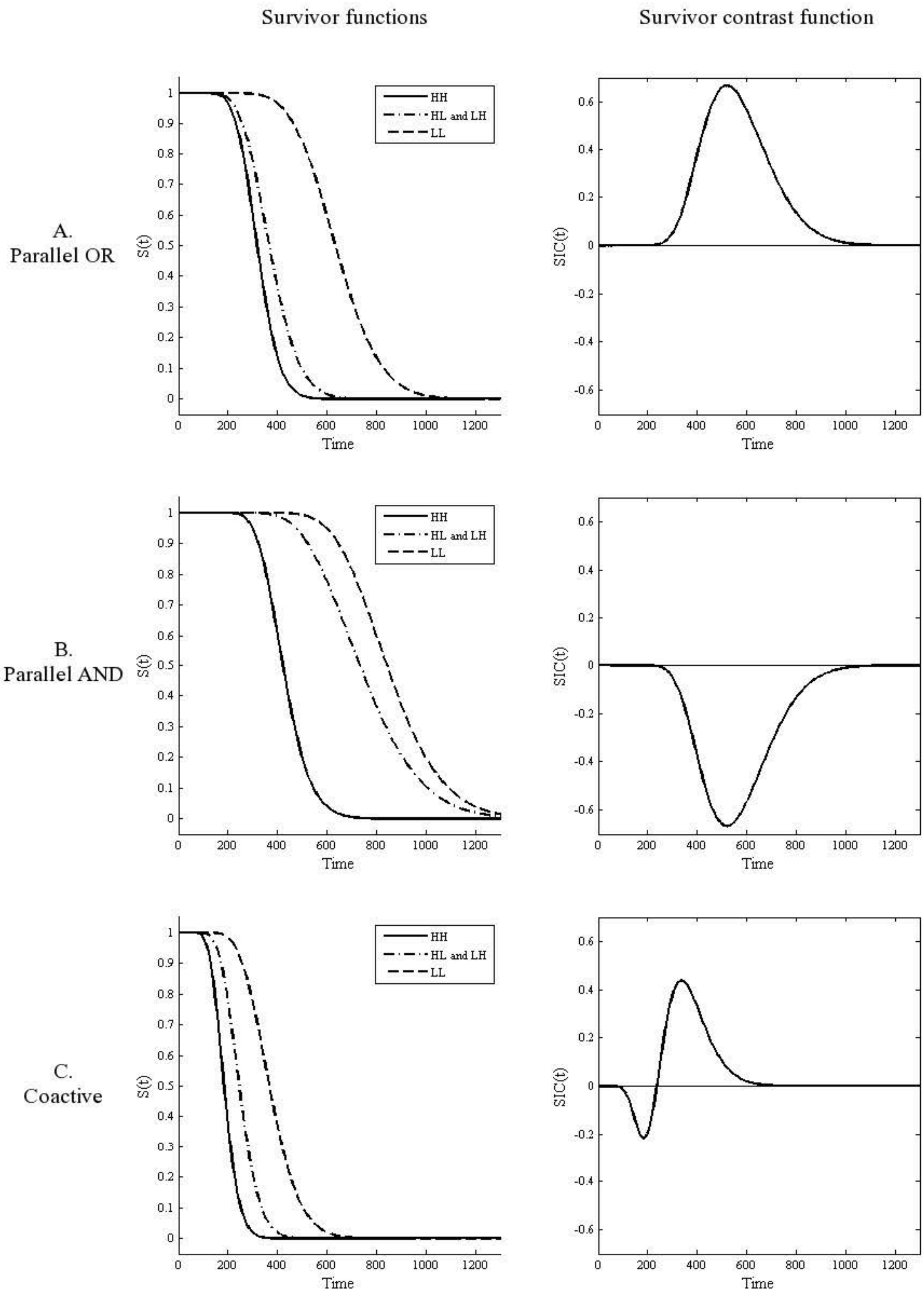
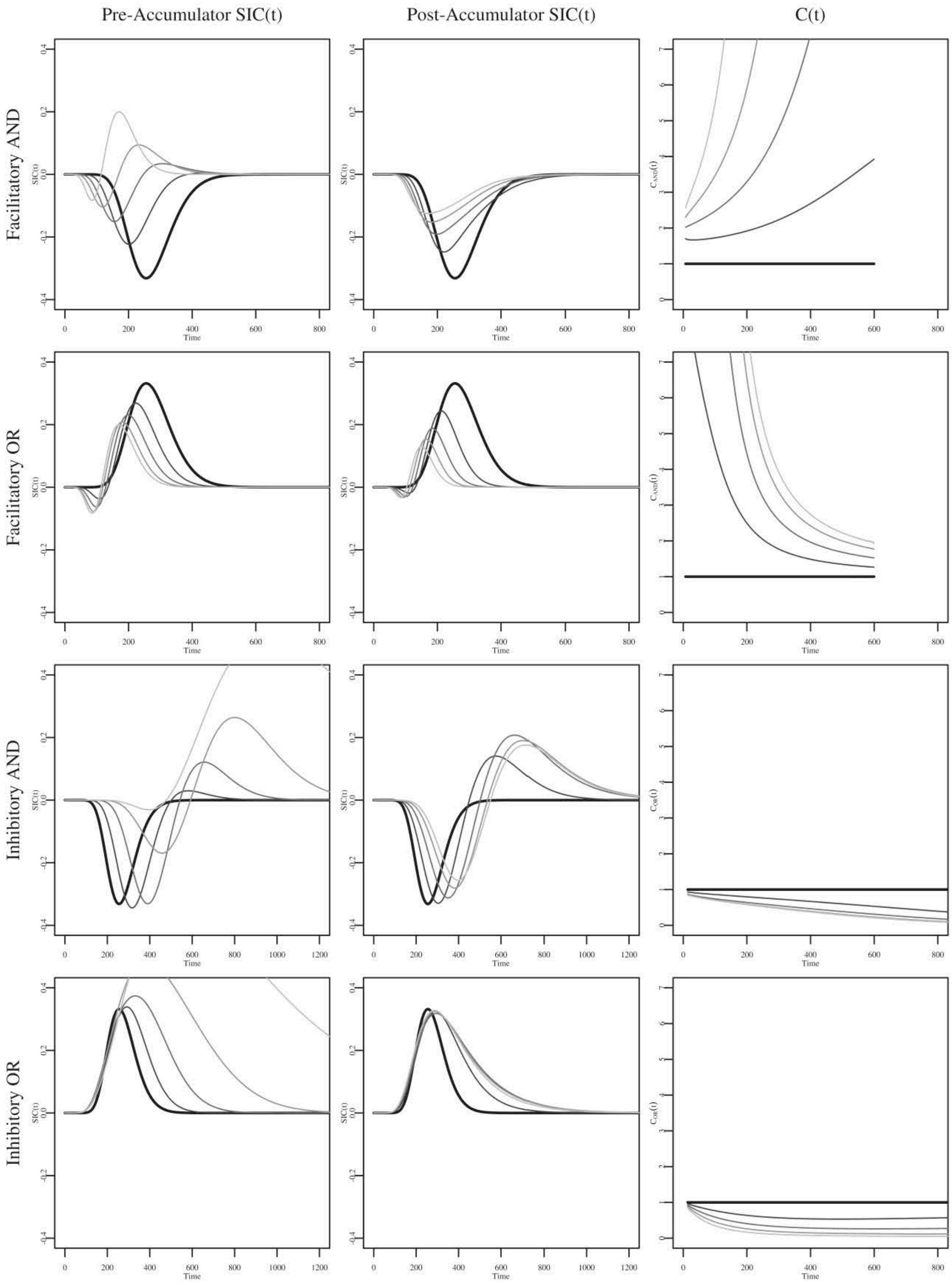

Survivor functions
Survivor contrast function
A.
Parallel OR
B.
Parallel AND
C.
Coactive
S(t)
SIC(t)
Time
HH
HL and LH
LL

Figure 2



Pre-Accumulator SIC(t)
Post-Accumulator SIC(t)
C(t)
Facilitatory AND
Facilitatory OR
Inhibitory AND
Inhibitory OR
SIC(t)
Time

Figure 3


Examine the empirical SIC(t) and C(t)
Is SIC(t) all negative? all positive? negative for small t and positive for large t?
SIC(t)<0 for all t
SIC(t)>0 for all t
SIC(t)<0 for small t, and SIC(t)>0 for large t
Rule out: 1. Facilitatory OR model 2. Inhibitory OR model
Rule out: Facilitatory AND model
Rule out: Inhibitory OR model
Is C(t) equal to, smaller, or larger than 1?
C(t)=1
C(t)<1
C(t)>1
Independent AND (no interaction)
Facilitatory AND model
Inhibitory AND model
Independent OR (no interaction)
Facilitatory OR model
Possible candidates: 1. Inhibitory AND model 2. Inhibitory OR model
??? None of the Interactive models makes such prediction
Possible candidates:* 1. Facilitatory AND model 2. Facilitatory OR model

Figure 4

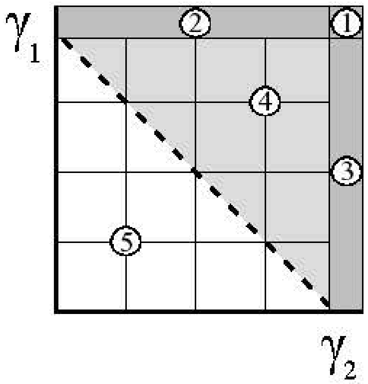


Figure A1. The state space of within-channel activation of the discrete state, pre-accumulator models. The y axis corresponds to the level of within-channel activation in channel 1 while the x axis corresponds to channel 2. Area 1 represents the case in which Facilitatory AND and OR models have completed processing. In areas 2 and 3, Faciliatory OR models have terminated and facilitatory AND models may be finished if there is enough between-channel sharing. In area 4, facilitatory AND and OR models may finish, but only with enough sharing. In area 5, the pre-accumulator models cannot finish processing, regardless of the amount of sharing.

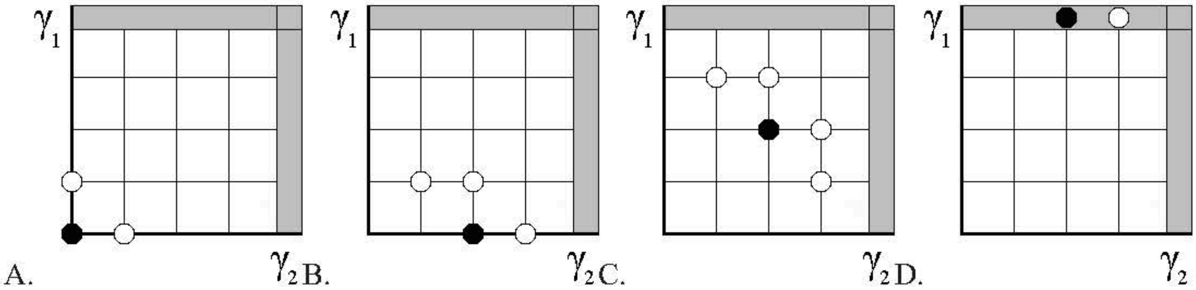


Figure A2. The state space of total channel activation of inhibitory Poisson models. The y axis corresponds to the level of activation in channel 1 and the x axis corresponds to channel 2. If the model is in the state marked by the black dot, then the possible states in the next time step are depicted by all of the dots, including the possibility of staying in the current state. Panel A shows the initial state of the model. Panel B shows an example of a state in which one channel has acquired some activation while the other has none. Panel C shows an example of a state in which both channels have some activation, but neither has reached its criterion. Panel D shows an example of a state in which one channel has reached criterion but the other has not. If the model is an OR model, processing has terminated. If it is an AND model, then only the activation in the channel that is still below criterion can change.