General Recognition Theory Extended to Include Response Times: Predictions for a Class of Parallel Systems

Joseph W Houpt, Wright State University - Main Campus
James T Townsend, Indiana University - Bloomington
Noah H Silbert

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General Recognition Theory Extended to Include Response Times: Predictions for a Class of Parallel Systems

James T. Townsend\textsuperscript{1}
Joseph W. Houpt\textsuperscript{2}  Noah H. Silbert\textsuperscript{3}

\textsuperscript{1}Indiana University  \textsuperscript{2}Wright State University  \textsuperscript{3}Center for Advanced Study of Language

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We are interested in the nature of dependencies in perception.

green /ba/
Traditional (Static) GRT
Failure of Static Perceptual Separability
Failure of Static Perceptual Independence
Overview

- We seek, in the present work, to extend GRT by combining patterns of accuracy with response times (RTs).
  - There are many models that use both response time and accuracy to understand cognitive processes, but they are rarely used to explore perceptual dependencies.
- We extend perceptual independence, perceptual separability, and decisional separability to a general stochastic-dynamic model.
- We establish extensions of report independence and marginal response invariance that account for both RT and accuracy.
Two channels operating in parallel \((X(t), Y(t))\)

- Each is composed of two subchannels (e.g.,
  \(X = (X_{brown}(t), X_{blue}(t))\))
- A decision is made on a channel when either of the subchannels reaches its bound
- The decision of the system is the combination of the first subchannel to reach its bound in each channel

For example

- \(X(t)\) is the process representing color while \(Y(t)\) represents shape
- \(X_{brown}(t)\) is evidence accumulated for brown; \(X_{blue}(t)\) is evidence accumulated for blue
- \(Y\square(t)\) is evidence accumulated for square; \(Y\triangle(t)\) is evidence accumulated for triangle
Decisional Separability (DS)

Definition
Decisional separability holds on dimension $X$ if and only if
\[ P\{C_{\text{color}} = \gamma | Y(t), C\blacksquare, C\blacktriangle \} = P\{C_{\text{color}} = \gamma \} \]
for color $\in \{\text{brown, blue}\}$ and all $t$. Similarly, decisional separability holds on dimension $Y$ if and only if
\[ P\{C_{\text{shape}} = \gamma | X(t), C_{\text{brown}}, C_{\text{blue}} \} = P\{C_{\text{shape}} = \gamma \} \]
for shape $\in \{\blacksquare, \blacktriangle\}$ and all $t$. Although there are possibly some very interesting effects of failures of decisional separability on these models, we do not explore these effects in this paper.
Decisional Separability (DS)

**Definition**

Decisional separability holds on dimension $X$ if and only if
\[ P\{C_{\textcolor{red}{\text{color}}} = \gamma| Y(t), C_{\text{■}}, C_{\text{▲}}\} = P\{C_{\textcolor{red}{\text{color}}} = \gamma\} \]
for color $\in \{\text{brown, blue}\}$ and all $t$. Similarly, decisional separability holds on dimension $Y$ if and only if
\[ P\{C_{\textcolor{red}{\text{shape}}} = \gamma| X(t), C_{\text{brown}}, C_{\text{blue}}\} = P\{C_{\textcolor{red}{\text{shape}}} = \gamma\} \]
for shape $\in \{\text{■, ▲}\}$ and all $t$.

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Perceptual Separability (PS)

**Definition**
Perceptual separability of one channel at a particular stimulus level is defined as invariance of the marginal processes of that channel over changes in the stimulus level of the other channel. Thus, for perceptual separability to hold on $X$ at $s_x = brown$, then for all $x, t$,

\[
P\{X_{brown}(t) \leq x; s = \square\} = P\{X_{brown}(t) \leq x; s = \triangle\}
\]

\[
P\{X_{blue}(t) \leq x; s = \square\} = P\{X_{blue}(t) \leq x; s = \triangle\}.
\]
Example model with perceptual separability

Within channel cross-talk does not affect perceptual separability or independence.
Example model without perceptual separability

Independent Noise
Marginal Response Invariance (MRI)

Definition
Marginal response invariance on a channel holds if and only if the marginal probability of a particular response on that dimension is invariant across the level of the other stimulus dimension. For marginal response invariance to hold on channel $X$,

$$P\{R_{\text{color}} = \text{brown}; s = \text{■}\}$$

$$= P\{R = \text{■}; s = \text{■}\} + P\{R = \text{▲}; s = \text{■}\}$$

$$= P\{R = \text{■}; s = \text{▲}\} + P\{R = \text{▲}; s = \text{▲}\}$$

$$= P\{R_{\text{color}} = \text{brown}; s = \text{▲}\}.$$
Timed Marginal Response Invariance (tMRI)

Definition
Timed marginal response invariance is defined by satisfaction of the condition for all $t > 0$,

$$P\{R_{\text{color}} = \text{brown}, T \leq t; s = \blacksquare\}$$

$$= P\{R = \blacksquare, T \leq t; s = \blacksquare\} + P\{R = \triangle, T \leq t; s = \blacksquare\}$$

$$= P\{R = \blacksquare, T \leq t; s = \triangle\} + P\{R = \triangle, T \leq t; s = \triangle\}$$

$$= P\{R_{\text{color}} = \text{brown}, T \leq t; s = \triangle\}.$$
Proposition

Perceptual separability and decisional separability imply marginal response invariance in accrual halting parallel models.

Proposition

Timed marginal response invariance implies static marginal response invariance but not conversely.

Proposition

Perceptual separability, decisional separability and speed invariance imply timed marginal response invariance in accrual halting parallel models.
Perceptual Independence (PI)

Definition
Two channels are said to be perceptually independent if \( \{X(t); t \geq 0\} \) and \( \{Y(t); t \geq 0\} \) are independent.
Example model with perceptual independence
Example model without perceptual independence

(Perceptual separability holds if noise is only dependent at B).
Example model without perceptual independence

(If noise is at A).
**Report Independence (RI)**

**Definition**
We say that report independence holds for a particular stimulus–response combination if the probability of that response is equal to the product of the marginal probability of each of the response dimensions. Formally, report independence holds for $R = \blacklozenge$ with stimulus $R = \blacklozenge$ if,

$$P\{R = \blacklozenge; s = \blacklozenge\} =$$

$$\left[ P\{R = \blacklozenge; s = \blacklozenge\} + P\{R = \blacktriangle; s = \blacklozenge\} \right]$$

$$\times \left[ P\{R = \blacklozenge; s = \blacklozenge\} + P\{R = \blacklozenge; s = \blacklozenge\} \right].$$
Timed Report Independence (tRI)

Definition
We say that timed report independence holds for a particular stimulus–response combination if the probability of a particular response, given that the response was made by $t$, is equal to the product of the marginal probabilities on each response dimension given the response was made by $t$, for all $t > 0$. Formally, report independence holds for $R = \square$ with stimulus $R = \square$ if for all $t > 0$,

$$P\{ R = \square | RT \leq t; s = \square \}$$

$$= P\{R_{\text{color}} = \text{brown} | RT \leq t; s = \square \} P\{R_{\text{shape}} = \square | RT \leq t; s = \square \}.$$

Equivalently,

$$P\{ R = \square, RT \leq t; s = \square \} P\{RT \leq t; s = \square \}$$

$$= P\{R_{\text{color}} = \text{brown}, RT \leq t; s = \square \} P\{R_{\text{shape}} = \square, RT \leq t; s = \square \}.$$
tRI Theorems

Proposition

*Decisional separability and perceptual independence imply report independence.*

Proposition

*Timed report independence implies ordinary report independence but not conversely.*

Proposition

*Perceptual independence and decisional separability imply timed report independence in accrual halting parallel models.*
Summary

- Described a general extension of static GRT to the time domain.
  - A generalization of earlier efforts (e.g., Ashby, 1989; Ashby, 2000).

Thank you.

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Summary

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- Timed MRI is stronger than static MRI.
- Timed RI is stronger than static RI.

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- Perceptual separability, decisional separability and speed invariance imply timed MRI.
- Perceptual independence and decisional separability imply timed RI.

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