A Problem Of Risk Aversion: An Insurance Model With Excel Solver

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Abstract: In this paper we present a tutorial exercise where students can solve a particular problem of insurance demand using the Excel Solver, and explore the main features of this type of problem. The document pretends to be self-contained since it includes an introductory theoretical background on choice under uncertainty with the most important tools and concepts required. Next, we introduce an exercise that solves the model in an Excel spreadsheet and do several comparative statics analysis. Finally, we present some practice exercises and provide a worksheet that allows the student to visualize the analysis with an interactive graph.

Keywords: Uncertainty, von Neumann-Morgenstern expected utility theory, Certainty equivalent, Risk premium, Insurance, Excel Solver.

JEL codes: A22, C65, D89

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1. Introduction

The objective of this document is to describe and develop a self-contained exercise to understand the decision problem of agents facing uncertainty, using the theory of von Neumann-Morgenstern expected utility as a theoretical background and Excel Solver as the analytical and graphical tool. We concentrate in an insurance problem. The basic model and main concepts presented below are standard in the chapters devoted to choice under uncertainty in microeconomics textbooks. For example, the essential aspects of this topic can be found in Jehle and Reny (2011). Gravelle and Rees (2007) and Mas-Colell, Whinston, and Green (1995) make a more extensive analysis.

Theoretical textbooks in economics do not usually cover information technology tools in order to solve choice under uncertainty exercises. Excel tools are increasingly being used for teaching economics, helping to improve learning outcomes (Barreto, 2015). Along this line, many authors have used Excel Solver (see, for example, Houston, 1997; Cahill and Kosicki, 2000; Barreto, 2009; Benninga, 2010; Strulik, 2012; and Silva and Xabadia, 2013), which is a user-friendly and flexible tool for economic optimization problems (MacDonald, 1996). Thus, Excel Solver can help to solve choice under uncertainty exercises and, in our case, to teach the insurance demand model. As far as we know, documents using Excel Solver do not address such type of exercises apart from Barreto (2009), who presents a somewhat similar problem using an endowment model instead of the expected utility model used by us.

Similar to Silva and Xabadia (2013), we introduce a tutorial exercise that solves the insurance model using an empty Excel spreadsheet and then present some comparative statics analysis
in order to help the students to understand the role of each one of the parameters included in the problem. Finally, we provide an Excel worksheet that allows the student to visualize the analysis with an interactive graph.

The document is structured as follows. For the rest of Section 1, we introduce some of the important concepts of the topic of choice under uncertainty, concentrating on simple lotteries. In Section 2, we write the insurance demand model formally and its graphical interpretation. Then, Section 3 solves the exercise and proposes the comparative statics analysis. Finally, Section 4 proposes some additional exercises and further questions for students practicing.

1.1. Choice under uncertainty: The expected utility theory

Many important agents’ decisions involve choices with uncertain consequences. For example, to buy insurance to cover the risk for home flooding, to decide the investment in a portfolio containing risky and non-risky assets, etc. In this exercise we will concentrate on the insurance model as an application. To handle this problem we need to know the fundamentals of the expected utility theory leading to the market for risk. Expected utility theory does not replace consumer theory with certainty, but extends the model to choices over risky outcomes. Choices under uncertainty mean that the agent (decision maker) faces a choice among a number of risky alternatives. Each of them may result in one of a number of possible outcomes or payoffs. Which outcome occurs is uncertain at the time of choice. That is to say that people choose among “bundles” that have uncertain payoffs. Therefore preferences are made over “lotteries” instead of consumption bundles, as in the deterministic utility theory.

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2 Silva and Xabadia (2013) present a different tutorial where students can solve and explore the basic two-period consumer choice model using Excel Solver.
Formally, here is the model. Let \( \mathcal{W} \) be the set of possible outcomes in a consumer decision problem. The set \( \mathcal{W} \) could be commodity bundles, as in the classical consumer choice problem with certainty, or amount of wealth as in an insurance decision problem, or monetary payoffs as in the portfolio selection problem. From now on, we focus on wealth, with positive real values, to cope with our insurance problem.

We consider a lottery as an object which specifies a finite set of outcomes \( W = (w_1, \ldots, w_n) \) and a probability distribution on \( W \), say \( P = (p_1, \ldots, p_n) \). Formally, we define a lottery as \( L = (P, W) = (p_1, \ldots, p_n, w_1, \ldots, w_n) \) with \( p_j > 0 \) for all \( j \in N \), and \( \sum_{j=1}^{n} p_j = 1 \), where \( p_j \) is the probability of outcome \( j \) occurring, and where the outcome \( j \) provides a monetary payoff \( w_j \). The expected value of the lottery \( L \) is \( E(L) = \sum_{j=1}^{n} p_j w_j \).

We stick to the analysis of simple lotteries and do not consider compound lotteries (that is, convex combinations of simple lotteries) since following the standard literature we assume the reduction property: The consumer is indifferent between any two compound lotteries that can be reduced to the same simple lottery.

Concerning preferences on outcomes, we assume that the consumer has a rational preference relation \( R \) which is complete and transitive with the same specific meaning as in the deterministic theory. Other assumptions required can be seen in any textbook covering this subject as the ones mentioned in the list of references.
By these assumptions on preferences, we know that we can extend the representation theorem in classical consumer choice with certainty to our model with uncertainty by means of a von Neumann-Morgenstern (VNM) expected utility representation. Any outcome \( w \in W \) has a utility level \( u(w) \), and for the analysis we want to carry out, it is useful to assume that \( u(w) \) is at least twice differentiable at all wealth levels. The expected utility assigned to a lottery is measured by the expected level of utility that it provides. Furthermore, these utility functions are unique up to a positive affine transformation. Formally, an **expected utility function** \( (U) \) is a function that assigns to any lottery \( L = (P, W) = (p_1, ..., p_n; w_1, ..., w_n) \) a real number, its utility, as follows: \( U(L) = \sum_{j=1}^{n} p_j u(w_j) \). In words, the utility assigned to a lottery is the expected value of the utilities \( u(w_j) \) of the \( n \) outcomes, and it is the weighted average of utilities, weighted by the outcomes’ probability.

Based on the assumptions of the rational preference relation, a lottery \( L_1 \) is strictly preferred to a lottery \( L_2 \) if and only if the utility assigned to \( L_1 \) is strictly higher than the utility assigned to \( L_2 \). Therefore, VNM utility functions represent an ordinal utility ranking.

### 1.2. Risk Aversion

Agents’ decisions under uncertainty depend on their attitude towards risk. To define risk aversion we compare the utility assigned to a lottery \( L \) (that is, the expected value of the utilities of outcomes \( U(L) \)) with the utility assigned to the lottery’s expected value, say \( U(E(L)) \), where the lottery \( E(L) \) can be thought of receiving the value \( E(L) \) with certainty, with probability 1, thus, \( U(E(L)) = u\left(\sum_{j=1}^{n} p_j w_j\right) \).
We say that an agent is **risk averse** if \( U(L) < U(E(L)) \) holds for all lottery \( L \); the agent is **risk loving** if \( U(L) > U(E(L)) \) for all lottery \( L \), and the agent is **risk neutral** in case of equality. In words, for a risk averse agent, the expected utility assigned to the lottery is less than the utility assigned to its expected wealth. The contrary applies to the risk loving agent.

Each one of the three attitudes towards risk is related with the shape of the agent’s utility function. The utility function of a risk averse agent is strictly concave. In this case wealth has diminishing marginal utility, and losses are more costly than gains in terms of utility (see Figure 1 below). A risk loving agent has a strictly convex utility function, while a risk neutral agent has a linear utility function. In the following Example 1 we show the crucial concepts defined till now and in Figure 1 we depict the case of a risk averse agent.

**EXAMPLE 1**: Consider a lottery defined by the following probability distribution and outcomes: \( L = (P, W) = (0.5, 0.5; 2, 16) \). The expected value of this lottery is 9. Suppose that the agent’s utility function on wealth is \( u(w) = \frac{1}{w^2} \). Then, \( U(L) < U(E(L)) \) as we check below:

\[
E(L) = \sum_{j=1}^{n} p_j w_j = 0.5 \cdot 2 + 0.5 \cdot 16 = 9,
\]

\[
U(L) = \sum_{j=1}^{n} p_j u(w_j) = 0.5 \cdot u(2) + 0.5 \cdot u(16) = 2.71,
\]

\[
U(E(L)) = u \left( \sum_{j=1}^{n} p_j w_j \right) = 9^{\frac{1}{2}} = 3.
\]

Because of strict concavity of \( u \), the same inequality holds for each lottery, thus, the agent is risk averse.
Note also that it is easy to prove that if the utility is $u(w) = w$, the agent is risk neutral, and for this particular lottery $L$, the following holds: $U(L) = U(E(L)) = 9$. Moreover, if we consider $u(w) = w^2$, then the agent is risk loving and for the particular lottery $L$ its utility level is $U(L) = 130 > U(E(L)) = 81$.

Figure 1: Risk aversion in Example 1

An interactive graph in Excel can be downloaded at https://works.bepress.com/jose_silva/31/. The graph allows the student to modify the parameters of the lottery as well as the degree of attitude towards risk and, therefore, shows what happens to the utility outcomes under different scenarios, including the ones where individuals are risk loving or risk neutral.
1.3. Certainty Equivalent and Risk Premium

For the analysis of agents’ attitude towards risk, the following concepts are useful. The certainty equivalent of a lottery \( L \), denoted as \( CE(L) \), is the amount of wealth for which the agent is indifferent between choosing the lottery \( L \) and receiving that certainty amount. Formally:

\[
U(L) = U(CE(L)).
\]

In words, is the amount of money which, if received for certain would be regarded by the agent as just as good as the lottery \( L \).

Each one of the three attitudes towards risk is related with some relationship between \( CE(L) \) and \( E(L) \). If \( CE(L) = E(L) \) the agent values the lottery as its expected value and it represents risk neutral agents. If \( CE(L) < E(L) \) the agent values the lottery less than its expected values and the agent is risk averse. Otherwise, if \( CE(L) > E(L) \) the agent values the lottery more than its expected values and the agent is risk loving.

A risk averse agent will sacrifice some positive amount of wealth to avoid the lottery’s inherent risk. The risk premium \( (r(L)) \) is the difference between the lottery expected value and the certainty equivalent. It makes the agent indifferent between participating in the lottery or receiving an amount equal to the lottery’s expected value minus the risk premium. Therefore, the risk premium determines the amount of wealth the risk averse agent will sacrifice to avoid the risk. Formally,

\[
r(L) = E(L) - CE(L),
\]

\[
U(L) = U(E(L) - r(L)).
\]
Again, each one of the three attitudes towards risk is related with the sign of \( r(L) \). If \( r(L) = 0 \), the agent is not willing to give up/sacrifice any amount of wealth to avoid the (risky) lottery, and thus, it represents a risk neutral agent. If \( r(L) > 0 \), the agent is willing to give up/sacrifice a positive amount of wealth to avoid the (risky) lottery, and thus, it represents a risk averse agent. Otherwise, if \( r(L) < 0 \), the agent has to be paid to avoid the lottery, this represents a risk loving agent.

Going back to Example 1, notice that the amount of wealth \( c = 7.33 \) in Figure 1 is the certainty equivalent of the lottery \( L \). Moreover, the horizontal distance from \( c \) to \( E(L) \), that is, \( E(L) - c = 9 - 7.33 = 1.67 \) is the risk premium.

### 1.4. Arrow-Pratt Measures of Risk Aversion

The decision made by an agent varies with the degree of aversion and we need a measure for such a degree. Arrow-Pratt coefficients of absolute and relative risk aversion are defined as follows.

The **Arrow-Pratt coefficient of absolute risk aversion** is defined as \( A(w) = -\frac{u''(w)}{u'(w)} \). This measure is independent on the utility representation of each agent’s preferences.

The **Arrow-Pratt coefficient of relative risk aversion** is defined as \( R(w) = -\frac{w\cdot u''(w)}{u'(w)} \). This measure is independent on the utility representation of agents’ preferences and also on the units in which income is measured.
For both measures, it is a local measure of risk aversion at \( w \) and if current wealth \( w \) changes, the measure changes and comparisons between agents could switch. The coefficient is positive for risk averse agents, it is zero for risk neutral agent, and it is negative for risk loving agents.

Concerning Example 1, observe that if \( u(w) = w^{\frac{1}{2}} \), \( A(w) = -\frac{u''(w)}{u'(w)} = -\frac{\frac{1}{2}w^{-\frac{3}{2}}}{\frac{1}{2}w^{-1}} = \frac{1}{2}w^{-1} > 0 \).

If \( u(w) = w^2 \), \( A(w) = -\frac{u''(w)}{u'(w)} = -\frac{2}{2w} = -w^{-1} < 0 \). And for the case \( u(w) = w \), \( A(w) = -\frac{u''(w)}{u'(w)} = -\frac{0}{3} = 0 \).

2. The Market for Risk – The Insurance Model

In this section we present and formally analyze, mathematically and graphically, the problem of the demand for insurance in the setting defined below. In Section 3, we will propose several activities for Excel Solver to help the readers with the understanding of the role of each one of the parameters defined in the model.

Let us start with the explanation of the problem. Consider a risk averse agent that has to decide whether to contract an insurance, totally or partially, against a possible robbery. The agent has an initial wealth of \( w_0 \), and \( \pi \) is the probability of being stolen an amount \( l \) of wealth. The uncertainty is captured by the existence of two possible situations or states of the world: there is a robbery with probability \( \pi \) or there is no robbery with probability \((1 - \pi)\).\(^3\)

We assume that there is an insurance company that is willing to sell insurance cover against

\(^3\) We assume that the agent can affect neither \( l \) nor \( \pi \).
the robbery at a premium rate of $p$, $0 < p < 1$. This means that the agent pays $p \cdot q$ to the insurer in exchange for the insurer’s payment of $q$ if the robbery occurs.

As we said, we concentrate on the problem of the demand for insurance. Note that we consider the insurer decision fixed and represented by the risk premium parameter $p$. Thus, the agent decision problem is to decide which is the amount $q \in [0,l]$ to contract as insurance cover (the decision variable) given $w_0, l, \pi, p$ as parameters of the problem, and with the objective to maximize his expected utility.

The insurance decision problem can be stated mathematically as an optimization problem $[P1]$ with a single variable, $q$, and several constraints:

$$[P1]$$

$$\max_{q} \{ u(w_1(q)) + (1 - \pi)u(w_2(q)) \}$$

$$w_1(q) = w_0 - l + q - p \cdot q = w_0 - l + q \cdot (1 - p), \quad (1)$$

$$w_2(q) = w_0 - p \cdot q, \quad (2)$$

where $w_1(q)$ and $w_2(q)$ are the agent’s contingent wealth level in each state of the world, satisfying the following conditions:

$$w_0 - l \leq w_1(q) \leq w_0 - p \cdot l, \text{ and} \quad (3)$$

$$w_0 - p \cdot l \leq w_2(q) \leq w_0. \quad (4)$$

The feasibility constraint (3) says that $w_1(q)$, the agent’s wealth if robbery takes place, has a lower bound $w_0 - l$ when no insurance is contracted ($q = 0$), and an upper bound $w_0 - p \cdot l$, when full insurance is contracted ($q = l$). Similarly, the feasibility constraint (4) says that $w_2(q)$, agent’s wealth with no robbery, has a lower bound $w_0 - p \cdot l$, when full insurance is contracted ($q = 0$). We do not analyze the insurance company problem. An analysis of insurer’s behavior can be seen in Gravelle and Rees (2007), Chapter 19, page 120.
contracted and an upper bound $w_0$, when no insurance is contracted. See Figure 2 for a graphical example of these constraints.

In this section we will tackle the problem using the two intermediate variables: the wealth level in each state of the world. The solution and interpretation of the problem using such two variables have similarities with the standard consumer choice problem with two goods (under certainty), a basic topic in microeconomics. However, Excel Solver is able to handle both approaches.

We can now formally state the agent’s insurance problem as [P2]. The agent chooses the levels of wealth $w_1$ and $w_2$ such that his expected utility is maximized subject to the constraints faced. That is:

\[
\text{[P2]}
\max_{(w_1, w_2)} \pi u(w_1) + (1 - \pi) u(w_2)
\]

subject to (3), (4) and

\[
p \cdot w_1 + (1 - p) \cdot w_2 = w_0 - p \cdot l \tag{5}
\]

Note that [P2] is obtained from [P1] as follows: Isolate $q$ from Equations (1) and (2) and equate them, obtaining what we call the agent budget constraint (5).\(^5\)

To solve the optimization problem [P2], we get $w_1$ and $w_2$ solving a system of the following four equations:

\[
\frac{\pi u(w_1)}{(1 - \pi) u(w_2)} = \frac{p}{1 - p}, \tag{6}
\]

\(^5\) We call it a budget constraint because it can be thought of as the one in a standard consumer decision problem over bundles of goods. See, for example, page 508 in Gravelle and Rees (2007).
the agent budget constraint in Equation (5), and the feasibility constraints in Equations (3) and (4). The economic interpretation of this problem can be easily handled with a graphical analysis. Figure 2 shows the main elements of the decision problem [P2].\textsuperscript{6} The indifference curve and the budget constraint (5) are depicted in red and black, respectively. The optimal levels of wealth $w_1^*$ and $w_2^*$ correspond to the unique point where the two curves cut tangently, meaning that Equation (6) holds.

\textbf{Figure 2: Graphical representation of the insurance optimization problem [P2]}

\textsuperscript{6} See, for example, Appendix E (page 675) in Gravelle and Rees (2007) for topics about uniqueness of solutions in optimization problems.
3. Solving the Insurance Decision Problem with Excel Solver

The insurance decision problem can also be solved numerically in a classroom exercise using the Excel Solver. To do this we will ask Excel to solve the optimization problem [P2] with the two agent’s contingent wealth levels $w_1$ and $w_2$ as variables of decisions subject to several constraints obtained from Equations (3)-(5). Additionally, the amount of insurance $q$ can be obtained from Equation (2) in [P1]. More precisely, we solve [P3] since notice that the constraints (3) and (4) in [P1] have been modified to introduce one inequality for each constraint.

\[ \text{[P3]} \]
\[
\max_{(w_1, w_2)} \pi u(w_1) + (1 - \pi) u(w_2)
\]

subject to:
\[
p \cdot w_1 + (1 - p) \cdot w_2 = w_0 - p \cdot l,
\]
\[
w_0 - l - w_1 \leq 0,
\]
\[
w_0 - p \cdot l - w_1 \geq 0,
\]
\[
w_0 - p \cdot l - w_2 \leq 0,
\]
\[
w_0 - w_2 \geq 0,
\]

with
\[
q = \frac{w_0 - w_2}{p}.
\]

Since our optimization problem is not linear, we implement the Generalized Reduced Gradient Nonlinear Optimization Method (GRG Nonlinear). Next, we present a tutorial that first builds a table of functions, variables, parameters, and constraints for the initial stage of the insurance model. And then, we introduce an additional table to do the comparative static
analysis. We will also provide instructors and students the Excel worksheet that can be modified. This worksheet contains the basic set-up for doing comparative statics and interactive graphical analysis.

(a) Initial stage solution

Table 1 that appears in Figure 3 sets up the initial optimization insurance problem of the representative agent. Rows 5 to 9 in column C include standard parameter values that we introduce. There is a probability \( \pi = 0.20 \) of being robbed the amount \( l = 700 \) with initial wealth \( w_0 = 1,000 \). The insurance company offers to cover the individual against the robbery at a premium rate of \( p = 0.30 \). Finally, the parameter of the expected utility function is \( \alpha = \frac{1}{3} \), implying that the individual is risk adverse.

Figure 3: Setting up the problem for the initial solution
We introduce the initial values for the decision variables $w_1$ and $w_2$ in rows 12 and 13 of column C (we set them at 100 monetary units but this can be somewhat arbitrary). Just note that the expected utility function is included in cell C25 while the constraints are introduced in cells C16-C20 without including the equalities or inequalities that appear at the end of them. Finally, the function $q$ is included in row 22 of column C.

Now we can use the Solver. Choose **Solver** from the **Data** menu in Excel. The **Solver Parameters** window will open. Set the **Objective Cell C25** to the location of the objective function value, which is the expected utility function in our case, select **Max**, and set the **Changing Variable Cells** C12 and C13 to add the decisions variables $w_1$ and $w_2$ in Table 1. To introduce the constraints in [P3], go to the **Subject to Constraints** box and select Add. The **Add Constraint** window will appear. In this window, we tell the solver that cell C16 is " = 0", cells C17 and C19 are " ≤ 0" while cells C18 and C20 are " ≥ 0". Then select OK since there are no more constraints to add. You will return to the **Solver Parameters** window as shown in Figure 3. Select the GRG Nonlinear as the **Solving Method**.

Once we have defined all the necessary components to solve the model click **Solve** in the **Solver Parameters** window. A window will appear telling us that Solver has found a solution. Select Keep Solver Solution and click OK. We just solved the insurance decision problem as shown in Table 1 of Figure 4. As you can see, the individual has maximized his expected utility when the agent’s wealth spent in both states of the world are $w_1 = 422.2$ and $w_2 = 947.6$. This implies an amount of insurance $q$ of 175 monetary units. Also notice that the

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7 For example, the restriction $w_0-w_2 \geq 0$ is introduced in C20 just as “= C7-C13”. The inequality $\geq 0$ will be included later with Solver as shown in the **Solver Parameters** window in Figure 3.

8 If the command Solver does not appears in the Data, you can follow the instructions that appear in Excel help writing Load the Solver Add-in.
values of the cells C16-C20 show that the problem satisfies the constraints in [P3]. In line with Figure 2, Figure 4 also shows an Excel-graph with the optimal solution. The initial budget constraint in cell C16 and the initial indifference curve in cell C25 appear in black and red curves, respectively.

**Figure 4: Finding the initial solution**

(b) **Comparative static analysis**

Next, we can do some comparative static analysis by modifying the parameters of the model. In order to do that we duplicate all the components from Table I to a new Table II in the same worksheet as shown in Figure 5. To change the optimization problem open Solver and change the **Objective Cell** from C26 to F26 as well as the **Changing Variable Cells** from C12 and C13 to F12 and F13, respectively. Then, go to the **Subject to Constraints** box and select **Change**. The **Change Constraint** window will appear. In this window, we tell the solver that, in each constraint, the latter C must be changed to F. Then select OK. You will return to the Excel sheet as shown in Figure 5. An Excel worksheet with an additional interactive graph can be
The initial and final indifference curves as well as the initial and final budget constraints will appear in the graph. Now, we can include the new values of the parameters in the corresponding cells of Table II and compare the new solution with the initial one in Table I.

**Figure 5: Setting up the comparative static analysis**

![Figure 5: Setting up the comparative static analysis](image)

**Comparative analysis 1:** What happens if the probability of a robbery increases from \( \pi = 0.2 \) to \( \pi = 0.4 \)?
In this case, it is necessary to change only the value of cell F5 from 0.2 to 0.4. Then call Solver again and, click Solve in the Solver Parameters window.

**Figure 6: The effect of an increase in the probability of robbery $\pi$**

In Figure 6 we can see how the increase in the probability of robbery from $\pi = 0.2$ to $\pi = 0.4$ increases optimal level of wealth $w_1$ from 422.2 to 790.0 but reduces $w_2$ from 947.6 to 790.0. This new solution corresponds to an amount of insurance $q$, which increases from 175 to its maximum level 700 corresponding to the total loss in case of robbery $l = 700$. In other words, the individual has decided to drastically increase the
amount of insurance since the probability of being stolen has increased considerably. Notice that the individual is worse off since the level of the utility falls from 9.4 to 9.2. The Excel-Graph shows both the final budget constraint and the final indifference curve in dashed lines that can be compared with the initial solution shown in continues lines. Note that $\pi$ only changes the expected utility but not the budget constraint.

**Comparative analysis 2:** What happens if initial wealth increases from $w_0 = 1,000$ to $w_0 = 1,100$?

**Figure 7:** The effect of an increase in the initial wealth $w_0$
Proceeding as in the previous case, it is easy to see in Figure 7, that an increase in the initial wealth from $w_0 = 1,000$ in cell C7 to $w_0 = 1,100$ in F7 reduces the insurance amount from $q = 174.6$ to $q = 108.1$.

**Comparative analysis 3:** What happens if the probability of robbery increases from $\pi = 0.2$ to $\pi = 0.4$ and the company offers an actuarially fair premium $p = \pi$, in both cases?

Figure 8: The effect of an increase in the probability of robbery $\pi$ with an actuarially fair premium
This case requires changing some parameter values in both scenarios, and then finding the new and final solutions in both tables (see Figure 8). First, we can set \( \pi = p = 0.4 \) in cells F5 and F6 and solve the solution in Table II going to Solver and clicking \texttt{Solve} in the \textit{Solver Parameters} window. Next, to solve the new initial solution in Table I, we first set \( p = 0.2 \) in cell C6 and then open the Solver again, change the letter F for C in all cells of the \textit{Solver Parameters} window. Additionally, change the letter F for C in all constraints by selecting Change and, finally, click Solve. Figure 8 shows what happens when the probability of robbery increases under the presence of an actuarially fair premium. In both cases, and always when \( \pi = p \), the individual chooses full insurance \( q = 700 \).

\textbf{Comparative analysis 4: What happens if the probability of robbery increases from } \pi = 0.2 \text{ to } \pi = 0.4 \text{ when the individual is risk neutral } u(w) = w? \]

In this case, we will use the same parameters value as in Comparative analysis 1 (see Figure 6) except for the parameter of the expected utility function \( \alpha \), which will be set equal to one in both tables to change the individual from being risk adverse to be risk neutral. First, we set \( \alpha = 1 \) in cells C8 and F8. Next, to solve the solution in Table II go to Solver and click \texttt{Solve} in the \textit{Solver Parameters} window. Finally, to solve the new initial solution we need to open the Solver again, change the letter F for C in all cells of the \textit{Solver Parameters} window and, finally, click Solve. Figure 9 shows the effects of an increase in the probability of robbery when the individual is neutral to the risk.

The first observation is that the indifference curves become linear implying that natural candidates for the agent’s decision are the corner solutions which correspond to \( q=0 \) or \( q=l \). The graph in Figure 9 shows, precisely, these two possibilities. That is, when \( \pi = 0.2 \) the agent doesn’t spend on insurance while when \( \pi = 0.4 \) full insurance is chosen.
4. Practice Exercises

We propose now several exercises consisting on comparative statics analysis additional to the ones in Section 3. To solve these proposed exercises we can depart from the Excel worksheet in Section 3 by clicking Figure 5’s title.
Practice exercise 1:

In comparative analysis 1 (Section 3), we saw that when $\pi$ increases from $\pi = 0.2$ to $\pi = 0.4$, then the amount of insurance $q$ also increases. Check this fact holds for different values of $\pi$. And find the value of $\pi$ above which the agent demands full insurance.

Practice exercise 2:

In comparative analysis 2 (Section 3), we saw that when $w_0$ increases from $w_0 = 1,000$ to $w_0 = 1,100$, then the amount of insurance $q$ decreases. Check this fact holds for different values of initial wealth until you get null insurance. Find the value for $w_0$ above which null insurance holds.

Practice exercise 3:

In comparative analysis 3 (Section 3), we saw that full insurance takes place if the probability of robbery increases from $\pi = 0.2$ to $\pi = 0.4$ and the company offers in both cases an actuarially fair premium $p = \pi$. In fact, there is always full insurance if $\pi = p$, whatever their values are. Check this fact for another value for $\pi = p$.

Practice exercise 4:

In comparative analysis 4 (Section 3), we saw the presence of full insurance when the probability of robbery increases from $\pi = 0.2$ to $\pi = 0.4$ and the individual is risk neutral $u(w) = w$. In fact, it is easy to see that full insurance will hold for any $\pi > p$ and no insurance for any $\pi < p$. Now, for $p = 0.3$, check that this statement holds for one particular value of $\pi$ strictly bigger than $p$ and for one value of $\pi$ strictly lower than $p$. Include the graphical analysis for both cases.

Practice exercise 5:

In section 3, we asked Excel Solver to solve the optimization problem [P2] with the two agent’s contingent wealth levels $w_1$ and $w_2$ as variables of decisions. Using the same parameters values from the initial solution in section 3, this exercise consists on solving the optimization problem [P1] with insurance amount $q$ as the only decision variable. (Hint: To obtain the initial solution, you need to express $w_1(q)$ and $w_2(q)$ as a function of
by using equations (1) and (2). Additionally, you need to modify constraints (3) and (4) in [P1] to introduce one inequality for each restriction.

**Practice exercise 6:**

Once you have found the initial solution in Practice exercise 5, then, repeat the static comparative analysis done in Section 3 and check that the results are identical.

*Note: Do not include the graphical analysis.*

**REFERENCES**


