A Comparison of Two Regression Procedures Used to Test Single-Case Designs: A Monte Carlo Study

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Abstract

This paper presents the results of a Monte Carlo study designed to estimate the relative likelihoods of type I errors and type II errors that a researcher may experience when either of two regression procedures is applied to data obtained from a single-case research design. The first regression procedure, which will be referred to as the Equal Slopes Method, requires that a full regression model and a restricted regression model be used to test whether the slopes of Treatments A and B in a single-case study were equal. The second regression procedure, which will be referred to as the Over and Above Method, requires that a full regression model and a restricted regression model be utilized to test whether the slope of Treatment B accounts for a significant amount of variation in the criterion variable over and above the variation that can be accounted for by the slope of Treatment A. The results obtained from this study indicated that the type I error rates were lower for the Equal Slopes Method. The type II error rates were lower, however, for the Over and Above Method. Since this study was limited in scope, additional research on these two methods is needed.
A Comparison of Two Regression Procedures Used to Test Single-Case Designs:

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There has been a resurgence of interest among researchers in the use of single-case designs as an alternative to the more traditional group experiments. A portion of this renewed interest can be attributed to behavior modification researchers who believe that while the aggregation of scores may provide information about the group, such an analysis may not indicate how an individual changes.

Another portion of this renewed interest in single-case designs can be attributed to the increased use of qualitative research techniques coupled with the acceptance by an increasing number of researchers that qualitative and quantitative research methodologies can be used in a multi-method approach to increase the accuracy of the interpretation of one's data (Newman & Benz, In Press). Such a coupling of quantitative and qualitative methods can be found in some of the visual analysis techniques used to identify trends in the qualitative data that lend themselves to quantitative analysis (Miles & Huberman, 1994).

Many research studies in the behavioral sciences employ between-group designs. In such studies, multiple subjects are commonly observed at one or several points in time. In single-case subject designs, one or a few subjects are tested, observed, or measured at numerous points in time.

The methods used to statistically analyze the data obtained from between-group designs, such as analysis of variance, are well documented. Kazdin (1987) noted,
however, that "statistical test applicable to group studies may not be appropriate for single cases where data are collected over time" (p. 287).

A number of statistical regression techniques have been recommended when a researcher is faced with single-case data. For example, Jones, Vaught, and Weinrott (1977) suggest the use of Box Jenkins (1976) autoregressive integrated moving average (ARIMA) models be used to analyze single-case data. As noted by Glass, Willson and Gottman (1975), however, time-series analyses often require at least 50 observations. Few behavioral researches will have the luxury of obtaining that number of observations.

Another regression approach used to analyze single-case data, which is the focus of this research, is referred to as trend analysis. Green (1978) and Kelly, McNeil and Newman (1973) suggested that data obtained from a single-case design could be analyzed through a regression procedure that would test whether the slope of Treatment B differed from the slope of Treatment A in an A-B design. This procedure will be referred to as the Equal Slopes Method.

Kelly, McNeil and Newman (1973) described another trend analysis approach. In their approach a researcher utilizes full and restricted regression models to determine whether the slope of the regression line for Treatment B accounts for a significant amount of variation in the criterion variable over and above the amount of variation accounted for by the slope and of the regression line for Treatment A. This procedure will be referred to as the Over and Above Method.

Although these two approaches may at first seem identical, they are conceptually
and mathematically different. Thus, the likelihood that the researcher may commit either a type I error or a type II error may vary depending on the regression procedure that is employed.

Purpose and Regression Procedures

The question that we are attempting to answer in this study is: What are the relative type I and type II error rates that a researcher may encounter when using either the Equal Slopes Method or the Over and Above Method to analyze data collected in A-B design studies? A Monte Carlo study was conducted to answer this question.

Regression Procedures

Both of the regression procedures used in this study utilize a full regression model and a restricted regression model. The variables contained in the various models used by both regression procedures are listed in Table 1.

Insert Table 1 about here

The Equal Slopes Method attempts to answer the question: Does the slope of the regression line estimated for Treatment A differ from the slope of the regression line estimated for Treatment B? The full regression model used to test this question measures the amount of variation in the criterion variable that can be accounted for by the slopes
and Y-intercept points of the regression lines for Treatments A and B. The full model is constructed as follows:

\[ Y = a + b_1x_1 + b_4x_4 + b_5x_5 + e \]  \hspace{1cm} \text{(Model 1)}

The restricted model is designed as follows:

\[ Y = a + b_1x_1 + b_3x_3 + e \]  \hspace{1cm} \text{(Model 2)}

An F test is conducted on the difference between the R-squared value of the full model and the R-squared value of the restricted model using the following formula:

\[ F = \frac{(R_F^2 - R_R^2) / df_n}{(1 - R_F^2) / df_d} \]  \hspace{1cm} \text{(Equation 1)}

where:

1. \( R_F^2 \) and \( R_R^2 \) are the \( R \)-squared values for the full and restricted models, respectively.

2. \( df_n \) is equal to the number of linearly independent parameter estimates in the full model minus the number of linearly independent parameter estimates in the restricted model.

3. \( df_d \) is equal to the number of observations minus the number of linearly independent estimates in the full model.

If the probability of the \( F \) value generated from Equation 1 is less than the alpha value of .05, the difference between the slopes of the regression lines for the two treatments is declared to be statistically significant.

The other regression approach, i.e., the Over and Above Method, addressed the
question: Does the slope of the regression line for Treatment B account for a significant amount of variation in the criterion variable over and above the amount of variation accounted for by the slope of the regression line for Treatment A. As was the case for the Equal Slopes Method, the Over and Above Method uses a full regression model and a restricted regression model. The full model is identical to Model 1, which was previously listed. The restricted model is constructed as follows:

\[ Y = a + b_1x_1 + b_4x_4 + e \]

(Model 3)

Once again, Equation 1 is used to statistically test the difference between the R-squared value of the full model and the R-squared value of the restricted model. If the probability of the F value is less than the alpha value of .05, the amount of variation accounted for by the slope of Treatment B over and above the amount of variation accounted for by the slope of Treatment A is declared to be statistically significant.

Methodology

Twelve populations were generated in this Monte Carlo study to determine the type I and type II error rates for the two regression procedures. The total number of observations for each of the Populations 1 through 6 was 30. Of these 30 observations, 10 were recorded under Treatment A and 20 were recorded for Treatment B. The total number of observations for each of the Populations 7 through 12 was 15. In these populations, the first 5 observations were recorded under Treatment A and the last 10 values represented values for Treatment B. The characteristics for the 12 populations were as follows:
Populations 1 and 7: The correlation values between the time period values and the criterion variable values were .00 for both Treatments A and B.

Populations 2 and 8: The correlation values between the time period values and the criterion variable values were .49 for both Treatments A and B.

Populations 3 and 9: The correlation values between the time period values and the criterion variable values were .38 for both Treatments A and B.

Populations 4 and 10: The correlations values between the time period values and the criterion variable values for Treatments A and B were .00 and .49, respectively.

Populations 5 and 11: The correlation value between the time period values and the criterion variable values for Treatment A was .49. The criterion variable values for Treatment B consisted of values that would have produced a line with the same slope as the line for the values in Treatment A except that they were multiplied by 2.

Populations 6 and 12: The correlation between the time period values and the criterion variable values for Treatment A was .38. The criterion variable values for Treatment B consisted of values that would have produced a line with the same slope as the line for the values under Treatment A except that they were multiplied by 1.1.

Representative graphs of Populations 1 through 3 and Populations 7 through 9, which are the populations that contained no treatment effects, are depicted in Figure 1. Likewise,
representative graphs of Populations 4 through 6 and Populations 10 through 12, which do contain treatment effects, are depicted in Figure 2.

Insert Figures 1 and 2 about here

One thousand samples were randomly selected from each of the 12 populations. The Equal Slopes and the Over and Above Methods were used to analyze the data contained in each sample. The type I error rates that were committed with each method were recorded for each of the populations that contained no treatment effects. Populations 1 through 3 and Populations 7 through 9 were involved in these analyses. Likewise, the type II error rates recorded for each method were recorded for each of the populations that did contain treatment effects. Populations 4 through 6 and Populations 10 through 12 were involved in these analyses.

Results

The type I error rates recorded for each regression method when applied to Populations 1 through 3 and Populations 7 through 9 are listed in Table 2. As indicated by the results obtained for the samples selected from Populations 1 and 7, which had no treatment effect and no slope, the type I error rates for both methods are very close to .05. For the populations that had no treatment effect but possessed positive slopes, which were Populations 2, 3, 8 and 9, the type I error rates obtained with the Equal Slopes Method were close to the .05 level. The type I error rates obtained when the Over and
Above Method was used, however, were higher than .05. As the slope increased the type I error rate increased when this method was used.

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Insert Table 2 about here

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The type II error rates recorded for each of the two regression procedures when applied to the populations that contained treatment effects, which were Populations 4 through 6 and Populations 10 through 12, are listed in Table 3. The results indicate that the Over and Above Method produced lower type II error rates than did the Equal Slopes Method when a treatment effect was present regardless of the degree of the treatment effect.

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Insert Table 3 about here

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Conclusions

Various methods may be used to analyze single-case designs. Researchers must be aware that the type I and type II error rates of the various methods may differ. This study attempted to compare the type I and type II error rates of two regression procedures that can be used to analyze the data obtained from single-case designs. The results of this study indicated that the Equal Slopes Method had lower type I error rates than did the Over and Above Method. The Over and Above Method, however, had lower type II error
rates than did the Equal Slopes Method.

Since this was a very limited study, one should be careful not to generalize beyond the types of populations used. In addition to various degrees of treatment effects and sample sizes, the variability of the data and the degree of autocorrelation may influence the relative rates of type I and type II error rates that a researcher may experience when using these methods. It is important that additional studies be conducted with different parameters, i.e., various sample sizes, treatment effects, autocorrelation, and data variability.
References


<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>Criterion variable</td>
</tr>
<tr>
<td>$X_1$</td>
<td>Treatment A (If the time period is in Treatment A = 1; 0 otherwise)</td>
</tr>
<tr>
<td>$X_2$</td>
<td>Treatment B (If the time period is in Treatment B = 1; 0 otherwise)</td>
</tr>
<tr>
<td>$X_3$</td>
<td>time periods contained in Treatments A and B (1,2, ..., $n_3$; where $n_3$ is the number of total time periods)</td>
</tr>
<tr>
<td>$X_4$</td>
<td>$X_1 \times X_3$</td>
</tr>
<tr>
<td>$X_5$</td>
<td>$X_2 \times X_3$</td>
</tr>
<tr>
<td>a</td>
<td>Y-intercept value</td>
</tr>
<tr>
<td>$b_1, b_3, b_4, b_5$</td>
<td>regression coefficients</td>
</tr>
<tr>
<td>e</td>
<td>error term</td>
</tr>
</tbody>
</table>
Table 2

**Type I Error Rates**

<table>
<thead>
<tr>
<th>Population</th>
<th>Relationship between Y and time</th>
<th>Method</th>
<th>Equal slopes</th>
<th>Over and above</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ( (n_1=10) ) ( (n_2=20) )</td>
<td>( r = 0 )</td>
<td>.055(^2)</td>
<td>.052</td>
<td></td>
</tr>
<tr>
<td>7 ( (n_1=5) ) ( (n_2=10) )</td>
<td>( r = 0 )</td>
<td>.046</td>
<td>.055</td>
<td></td>
</tr>
<tr>
<td>2 ( (n_1=10) ) ( (n_2=20) )</td>
<td>( r = .49 )</td>
<td>.056</td>
<td>.247</td>
<td></td>
</tr>
<tr>
<td>8 ( (n_1=5) ) ( (n_2=10) )</td>
<td>( r = .49 )</td>
<td>.046</td>
<td>.081</td>
<td></td>
</tr>
<tr>
<td>3 ( (n_1=10) ) ( (n_2=20) )</td>
<td>( r = .38 )</td>
<td>.055</td>
<td>.104</td>
<td></td>
</tr>
<tr>
<td>9 ( (n_1=5) ) ( (n_2=10) )</td>
<td>( r = .38 )</td>
<td>.046</td>
<td>.060</td>
<td></td>
</tr>
</tbody>
</table>

\(^1\)The \( n_1 \) and \( n_2 \) values are equal to the number of observations in Treatments A and B, respectively.

\(^2\)Proportion of type I errors for the 1000 samples analyzed.
Table 3

**Type II Error Rates**

<table>
<thead>
<tr>
<th>Population</th>
<th>Treatment A</th>
<th>Treatment B</th>
<th>Equal slopes</th>
<th>Over and</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 ( (n_1=10) ) ( (n_2=20) )</td>
<td>( r = 0 )</td>
<td>( r = .49 )</td>
<td>.915 (^2)</td>
<td>.853</td>
</tr>
<tr>
<td>10 ( (n_1=5) ) ( (n_2=10) )</td>
<td>( r = 0 )</td>
<td>( r = .49 )</td>
<td>.947</td>
<td>.919</td>
</tr>
</tbody>
</table>

**Multiplication Factor\(^3\)**

<table>
<thead>
<tr>
<th>Population</th>
<th>Treatment A</th>
<th>Treatment B</th>
<th>Equal slopes</th>
<th>Over and</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 ( (n_1=10) ) ( (n_2=20) )</td>
<td>( r = .49 )</td>
<td>2</td>
<td>.996</td>
<td>.681</td>
</tr>
<tr>
<td>11 ( (n_1=5) ) ( (n_2=10) )</td>
<td>( r = .49 )</td>
<td>2</td>
<td>.995</td>
<td>.891</td>
</tr>
<tr>
<td>6 ( (n_1=10) ) ( (n_2=20) )</td>
<td>( r = .38 )</td>
<td>1.1</td>
<td>.954</td>
<td>.883</td>
</tr>
<tr>
<td>12 ( (n_1=5) ) ( (n_2=10) )</td>
<td>( r = .38 )</td>
<td>1.1</td>
<td>.965</td>
<td>.933</td>
</tr>
</tbody>
</table>

\(^1\) The \( n_1 \) and \( n_2 \) values are equal to the number of observations in Treatments A and B respectively.

\(^2\) Proportion of type II errors for the 1000 samples analyzed.

\(^3\) The value by which the progression of values in Treatment A is multiplied.
Figure 1

Populations With No Treatment Effects Present
Figure 2

Populations With Treatment Effects Present