The Quark Sea and the $\Delta I = 1/2$ rule

John Donoghue, University of Massachusetts - Amherst
Eugene Golowich, University of Massachusetts - Amherst
THE QUARK SEA AND THE $\Delta I = \frac{1}{2}$ RULE

John F. DONOGHUE
Department of Physics, Carnegie Mellon University, Pittsburgh, PA 15213, USA

and

Eugene GOLOWICH
Department of Physics, University of Massachusetts, Amherst, MA 01003, USA

Received 10 June 1977

The effect on nonleptonic processes of quark-antiquark pairs due to quantum chromodynamics is studied. Their presence improves agreement between theory and experiment for hyperon decays. In kaon decays a new $\Delta I = \frac{1}{2}$ contribution is found, but $\Delta I = \frac{3}{2}$ effects are still too large to be in agreement.

In a recent calculation [1], it was shown how to unambiguously construct a hadron state containing quark-antiquark pairs in addition to the usual "valence" configuration. The effect of these pairs on static properties of hadrons was estimated in the MIT bag model and found to be reasonable. Here, we describe the effect of this hadron "sea" upon nonleptonic decay processes. The new contribution substantially improves the agreement between experiment and previous theoretical models for hyperon decay amplitudes, and allows for the first time the effect of heavy quarks upon nonleptonic processes to be explicitly estimated.

Although we direct the reader to ref. [1] for details, we can display the sea configuration due to the interaction of quantum chromodynamics as

$$\langle P \rangle = |3q\rangle + \sum_{\alpha} \frac{\langle 5q, \alpha |5q, \alpha |T3q\rangle}{E_V - E_{\nu}, -E_S - E_{\bar{S}}}$$

(1)

where $|3q\rangle$ is the usual valence configuration and

$$T = g^2 \int d^3x d^3x' \bar{\psi}(x') \gamma_{\mu} \frac{\lambda^A}{2} \psi(x):$$

$$\times G(x', x) \bar{\psi}(x) \gamma_{\mu} \frac{\lambda^A}{2} \psi(x):$$

(2)

with $G(x', x)$ being a gluon propagator and $g$ the quark-gluon coupling constant. The sea wave function is depicted in fig. 1(a), which also serves to define the subscripts $V, V', S, \bar{S}$ of eq. (1). Using the MIT bag model as an illustration, each of the quarks $V^', S, \bar{S}$ occupies an arbitrary bag mode, constrained only by parity conservation and by the Pauli principle. The sum on $\alpha$ in eq. (1) includes these modes and in addition, the various quark flavors carried by $S$ and $\bar{S}$.

All of the nonleptonic decays exhibiting the $\Delta I = \frac{1}{2}$ rule involve pion emission. Given the success of current algebra methods and our present inability to explicitly construct realistic multihadron states, we employ the soft pion theorem

$$\lim_{q^\mu \to 0} \langle \beta, \pi_i(q) | H^{\Delta S = 1} | \alpha \rangle = \frac{-i}{F_\pi} \langle \beta | F_i^{\Delta S = 1} | \alpha \rangle \quad (3)$$

to approximate the physical decay amplitudes [2]. A calculation of these matrix elements depends not only on the wave functions of $\alpha$ and $\beta$, but also on the structure of $H^{\Delta S = 1}$.

The nonleptonic Lagrangian due to W-boson exchange is given by

$$H^{\Delta S = 1} = \frac{G M^2}{\sqrt{2}} \int d^4x D(x, M_w) T \{ J^4(x/2) J^4(x/2) \}$$

(4)

At present, only the left-handed Cabibbo currents (including charm) are known to definitely exist in nature.
Fig. 1. (a) The creation of a quark pair; (b) – (f) Contributions to matrix elements of the weak Hamiltonian.

We consider only such empirically determined currents. The resulting form for \( H^{\Delta S=1} \) can thus be shown in asymptotically free theories to be [3,4]

\[
H^{\Delta S=1} = \frac{G_F \sin \theta \cos \theta}{2\sqrt{2}} \{c_+ H_+ + c_- H_- \},
\]

where, summing on color indices \( i, j \) and letting \( \Gamma^\mu \equiv \gamma^\mu(1 + \gamma_5) \),

\[
H_\pm = \mp \frac{1}{2} \Gamma^\mu u_i \bar{u}_j \Gamma_\mu s_i \bar{s}_j : + \frac{1}{2} \Gamma^\mu s_i \bar{s}_j \Gamma_\mu u_i \bar{u}_j - (u \leftrightarrow c) + \text{h.c.}
\]

The enhancement and suppression factors \( c_\pm \) depend upon the quark-gluon fine structure constant \( \alpha_S \equiv g^2/4\pi \) determined at mass scale \( \mu \) and the number of quark flavors \( n \) and colors \( N \),

\[
c_\pm = \left( 1 + \frac{\alpha_S}{\pi} \frac{11N - 2n}{3} \ln \frac{M_w^2}{\mu^2} \right) \pm \frac{9(N \pm 1)}{N(11N - 2n)}
\]

The \( \Delta I = 1/2 \) operator \( H_- \) is enhanced, but not enough to explain the \( \Delta I = 1/2 \) rule by itself.

Our program is then to calculate appropriate single-particle matrix elements of \( H^{\Delta S=1} \) and to compare the results with corresponding amplitudes gleaned from experiment. In each case, we distinguish between the valence-quark contribution and the new sea contribution to the hadron wave function. In order to cast our results in the most general possible light, we express calculated quantities in table 1 as explicit functions of the factors \( c_\pm \). Reasonable values of \( c_- = 2 \rightarrow 3 \), \( c_+ = 0.7 \rightarrow 0.5 \). For convenience, the \( c_+ \) dependence is normalized to unity in the limit \( g \rightarrow 0 \) (i.e., \( c_+ = c_- = 1 \)). Throughout we employ the dynamical wave functions of the MIT bag model [5].

At this point, let us summarize the pertinent experimental findings [6]. For hyperon decay, the quantities of interest are the dimensionless matrix elements corresponding to \( \Lambda \rightarrow n, \Sigma^+ \rightarrow p, \) and \( \Xi^0 \rightarrow \Lambda \) transitions. These may be parameterized in terms of the dominant octet amplitudes \( f, d \) as \( f \approx 10^{-7} \) and \( d/f \approx -0.4 \). The absence of a sizable 27-plet amplitude is a manifestation of the \( \Delta I = 1/2 \) rule for hyperons. An analogous reduction of \( K \rightarrow \pi \pi \) decay amplitudes to \( K \rightarrow \pi \) matrix elements is unfortunately less straightforward due to the presence of only light particles in the final state [7].

We estimate that values for the isospin 1/2 and 3/2 \( K \rightarrow \pi \) amplitudes [8] \( \alpha_1 \) and \( \alpha_3 \) lie in the range \( \alpha_1 \approx 20 - 70 \times 10^{-9} \text{ GeV}^2 \) and \( \alpha_3 \approx 0.5 - 1.0 \times 10^{-9} \text{ GeV}^2 \). Evidently the isospin 1/2 dominance of the \( K \rightarrow \pi \pi \) amplitudes carries over to the \( K \rightarrow \pi \) matrix elements.
Table 1

<table>
<thead>
<tr>
<th>Contribution</th>
<th>$f$</th>
<th>$d$</th>
<th>$\sigma_{27}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>valence</td>
<td>$25 c_-$</td>
<td>$-25 c_-$</td>
<td>0</td>
</tr>
<tr>
<td>$uu$ sea (totally sea)</td>
<td>$34 \frac{c_+ + c_-}{2}$</td>
<td>$0.4 \frac{c_+ + c_-}{2}$</td>
<td>0</td>
</tr>
<tr>
<td>$cc$ sea (totally sea)</td>
<td>$3.4 \frac{c_+ + c_-}{2}$</td>
<td>$-0.3 \frac{c_+ + c_-}{2}$</td>
<td>0</td>
</tr>
<tr>
<td>$uu$ sea (sea-valence)</td>
<td>$9.6 \frac{c_+ + 2c_-}{3}$</td>
<td>$-4.5 \frac{3c_+ - 3c_-}{7}$</td>
<td>$3.8 c_+$</td>
</tr>
</tbody>
</table>

(a) Baryons

(b) Mesons

References

[4] The interplay between the renormalization group and mass scales associated with heavy quarks has been addressed by M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, ITEP preprint 63, Moscow (1976).
We define \( \alpha_1 + \alpha_3 \) and \( \sqrt{2} (\alpha_0 |\psi_1 \rangle) = -\alpha_1 + 2\alpha_3 \).


F. Wilczek and A. Zee, \( \Delta I = \frac{1}{2} \) rule and right handed currents, Princeton preprint (1976).