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We explore the possibility that pairs of quarks will form diquark clusters in the density regime above the deconfinement transition for hadronic matter at finite density. We use a simple model and find that the spin interactions of quarks provide a significant lowering of the energy for diquark combinations. A modified P -matrix method is used in order to calculate the density dependence of this effect. It is found that, within the model, a diquark configuration has a lower energy than a quark gas for a significant range of densities above deconfinement, although at very high densities a free quark phase is preferred.

I. INTRODUCTION

The possibility exists that at high temperature and/or high density, hadronic matter will undergo a phase transition to an unconfined phase of quarks.¹ This idea has been supported by analytic calculation and by lattice Monte Carlo simulations.¹⁻³ The quark phase at very high temperatures and densities is expected to resemble an almost free-quark gas interacting weakly via gluon exchange. The physical conditions of quark matter just above the deconfinement transition are, however, not as clear.⁴ In this paper we report on a preliminary investigation on whether there is a range of densities when the quark matter could exist primarily in a configuration of diquark clusters. Our results are that such a diquark phase may indeed be possible and that further investigation of this picture is warranted.

Our motivation comes from the fact that the energy that can be gained from such clustering is in fact sizable on the scale of energies relevant for quark matter just above the deconfinement transitions. In what follows, we will be concerned primarily with isoscalar matter at finite density and zero temperature—i.e., cold quark matter. In this case, deconfinement occurs when the chemical potential is of the order of a few hundred MeV (Ref. 4). However, even in the unconfined phase, the interaction of gluons will provide attractive forces for some configurations of quarks. For example, three quarks in the configuration of a nucleon are about 300 MeV lower in energy than three quarks in the configuration of a Δ , due to the spin-spin interaction from gluon exchange.⁵ It is thus energetically disfavored to flip the spin of one of the quarks in the nucleonlike cluster, and despite the lack of total confinement, quarks may be at lower energy if they remain clustered and correlated in spin. In fact, diquark clusters could lower the energy even further. The quarks in the nucleon are combined with two quarks in a spin-zero color $\bar{3}$ combination, with the third quark carrying the overall spin of the nucleon. All of the attractive spin-spin energy comes from the diquark combinations, as the net spin interaction of the third quark with the diquark combination vanishes because the net spin of the diquark is zero. From the spin interaction at least there

is no energetic advantage to having this third quark grouped with the others, and it would be better if this quark were to find another unpaired quark and form a spin-zero diquark with it. Clearly in the confined hadron regime this rearrangement is impossible because the diquark clusters carry a net color. However, in the deconfined regime at higher densities, this presents no problem. The picture that emerges from the above heuristic description is one where quarks are paired off in diquark clusters in order to maximize the attractive potential energy.

We will attempt to assess the viability of this picture by utilizing a calculation based on the P -matrix methods of Jaffe and Low.⁶ Basically we will transform the quark interactions into an effective ϕ^4 theory for the diquark. This effective theory is then used to calculate the energy dependence of an interacting diquark gas as a function of density. We will see that at low densities the diquark gas is energetically preferred over an uncorrelated gas of quarks, while at very high densities the free-quark gas becomes favorable. The densities at which the diquark phase is preferred include those above the deconfinement transition and this would suggest that hadronic matter passes from nucleons to diquarks to single quarks as one increases the density. This is not to say that the passage from diquarks to quarks is necessarily a phase transition in the technical sense, but rather that the description of the unconfined phase is predominantly diquarks at lower densities and largely uncorrelated quarks at high densities.⁷

During the course of this work, a report appeared on the role of diquark at finite temperature and density.⁸ However, the author considered the unrealistic case of noninteracting diquarks, so that a Bose condensation is inevitable, and, hence, obtained results which necessarily differed from ours.

II. DIQUARK ENERGY

Quarks are color-triplet, spin- $\frac{1}{2}$ objects, and, hence, the possible states of a pair of quarks include color 6 and $\bar{3}$ and spin zero and one, i.e., SU(3) and spin quantum numbers $(M, J) = (\bar{3}, 0)$, $(\bar{3}, 1)$, $(6, 0)$, and $(6, 1)$. If the quarks

are in a spatially symmetric configuration then the $(\bar{3},0)$ and $(6,1)$ options are required by Fermi statistics to contain two distinct quarks, i.e., u and d , while either like or unlike quarks are possible for $(\bar{3},1)$ and $(6,0)$.

The interaction energy which we will consider is the spin-spin energy of the form⁹⁻¹¹

$$H_s = -A \sum_{i,j} b_i^\dagger \sigma^a \lambda^B b_i b_j^\dagger \sigma^a \lambda^B b_j, \quad (1)$$

where i,j labels the interacting quark, $b_i^\dagger$ is the creation operator for that quark, σ^a , $a=1,2,3$, are the Pauli spin matrices, and λ^B , $B=1,2,\dots,8$, are the Gell-Mann matrices for color SU(3). It is this interaction which generates the N - Δ mass difference as

$$\langle \Delta | H_s | \Delta \rangle = 16A, \quad (2a)$$

$$\langle N | H_s | N \rangle = -16A, \quad (2b)$$

which implies that

$$M_\Delta - M_N = 32A = 300 \text{ MeV}. \quad (3)$$

Of the diquark states, the most attractive is the color-triplet spin-zero combination $(\bar{3},0)$, with results¹²

$$\langle \bar{3},0 | H_s | \bar{3},0 \rangle = -16A, \quad (4a)$$

$$\langle 6,1 | H_s | 6,1 \rangle = -\frac{8}{3}A, \quad (4b)$$

$$\langle \bar{3},1 | H_s | \bar{3},1 \rangle = \frac{16}{3}A, \quad (4c)$$

$$\langle 6,0 | H_s | 6,0 \rangle = 8A. \quad (4d)$$

We assume that the wave function of the diquark cluster is approximately the same as that of the diquarks in a nucleon so that the value of A does not change drastically. From now on we will focus exclusively on the $(\bar{3},0)$ state and will refer to it as the diquark. Its explicit spin and color wave function is given by

$$|\alpha\rangle = \frac{1}{2} \epsilon^{\alpha\beta\gamma} u_\beta^\dagger (i\sigma^2)_{mm'} d_{\gamma m'}^\dagger |0\rangle \quad (5)$$

with α, β, γ being color indices and m, m' referring to spin. Note that, as mentioned in the Introduction, the spin energy of the diquark is equal to that of the nucleon:

$$\langle \beta | H_s | \alpha \rangle = \delta_{\alpha\beta} \langle N | H_s | N \rangle = -\delta_{\alpha\beta} \left[\frac{m_\Delta - m_N}{2} \right]. \quad (6)$$

We use this as an estimate of the binding energy of two quarks into a diquark versus the energy of unpaired quarks. The diquark energy is then roughly

$$M_{dq} \approx \frac{2}{3} \left[\frac{M_\Delta + M_N}{2} \right] - \left[\frac{M_\Delta - M_N}{2} \right] \approx 575 \text{ MeV}, \quad (7)$$

where the first term is the quark energy and the second is the spin energy.

III. THE EFFECTIVE FIELD THEORY

We are interested in studying the properties of a gas of diquark clusters. Present techniques using only quark fields do not allow a treatment at the quark level any more than it would be possible for a gas of nucleons. In

the latter case, one treats the problem by introducing an effective field theory for nucleon fields, with interactions introduced in order to reproduce what we know about the two nucleon systems. This effective theory is then used to calculate the properties of a nucleon gas at nonzero density. We will follow the same course with the case of diquarks. The diquarks are spin zero and, hence, would be represented by a boson field ϕ^α with α being a color index ($\alpha=1,2,3$). Conservation of quark number forbids any trilinear coupling such as $\phi^* \phi \phi$, so that the effective theory should reduce to a ϕ^4 field theory:

$$L_{\text{eff}} = \frac{1}{2} (\partial_\mu \phi^\dagger \partial^\mu \phi - M^2 \phi^\dagger \phi) - \lambda (\phi^\dagger \phi)^2. \quad (8)$$

There could, in principle, also be ϕ^6 , ϕ^8 , $\dots$ interactions, because this is only an effective theory, but, as usual in the nucleon case,¹³ we will ignore such potential three-body, four-body, etc., forces. This model could also be extended by including couplings to various mesonic excitations and utilizing one-meson-exchange contribution. At the present exploratory level such an elaboration is not yet called for.

At this stage the effective interaction parameter λ is not known. We will estimate it by using a method inspired by the P -matrix formalism of Jaffe and Low.⁶ These authors consider a world where all fields are artificially confined to be within a cavity of radius R . The hadronic fields, even if noninteracting, would have discrete eigenfrequencies corresponding to the allowed energy levels in a cavity. If there is an interaction, these energy levels will shift to higher or lower values depending on whether the interaction is repulsive or attractive, respectively. In our diquark case, the energy value will be a direct function of the variable λ . If the cavity is small enough, however, the contents could equally well be treated as quarks. The energy shift of the quark configurations can be calculated in the quark model, and the overall energy found. Jaffe and Low then identify the full quark energy levels with those of the interacting hadrons and from the locations of the eigenenergies they deduce quantitative results on the nature of the hadronic interaction. They deal directly with the S matrix and phase shifts. We will use a variant of these methods which directly yields an estimate for λ .

First consider the quark-level interactions. Here we define a normalized two-diquark state by multiplying together the creation operators for two individual diquark states

$$|\alpha, \eta\rangle \equiv \frac{1}{\sqrt{12}} \epsilon^{\alpha\beta\gamma} u_\beta^\dagger (i\sigma^2) d_\gamma^\dagger \epsilon^{\eta\rho\sigma} u_\sigma^\dagger (i\sigma^2) d_\rho^\dagger |0\rangle \quad (9)$$

such that

$$\langle \epsilon, \lambda | \alpha, \eta \rangle = (\delta_{\alpha\lambda} \delta_{\eta\epsilon} + \delta_{\alpha\epsilon} \delta_{\eta\lambda}).$$

The effect of the spin interactions here will simply not be twice that of a single diquark because the Pauli principle will correlate the spin states of the quarks, in that whenever two identical quarks appear to be in the same state the fermion character of the creation operators will ensure that the state vanishes. This still leads to a net spin attraction, but only weakly so. We find

$$\Delta E_2 = \frac{\langle \epsilon, \lambda | H_s | \alpha, \eta \rangle}{\langle \epsilon, \lambda | \alpha, \eta \rangle} = -8A = \frac{(M_\Delta - M_N)}{4}. \quad (10)$$

This is to be contrasted with the sum of the separate diquark energy shifts, which would have totaled $-(M_\Delta - M_N)$. Thus, the effect of the interaction between the diquarks is a net repulsion, of magnitude

$$\Delta E_{\text{rep}} = \Delta E_2 - 2(\Delta E_1) = -\frac{M_\Delta - M_N}{4} - [-(M_\Delta - M_N)] = \frac{3}{4}(M_\Delta - M_N). \quad (11)$$

This effect is also similar to the explanation of the nucleon-nucleon repulsive core.

To match this with the effective field theory, we consider diquarks confined to a cavity of radius R . The ground-state wave function is of the form

$$\phi(x) = N j_0(pr) e^{-iEt}, \quad E = (p^2 + m^2)^{1/2}, \quad (12)$$

with $j_0(k)$ being a spherical Bessel function. The vanishing of ϕ at the boundary tells us that

$$p = \frac{\pi}{R}$$

and the normalization condition

$$1 = \int d^3x \phi^\dagger(x) i \vec{\partial}_0 \phi(x)$$

implies that

$$N = \frac{1}{(4\pi R^3)^{1/2}} \frac{1}{\sqrt{E}} \frac{1}{j_1(p)}. \quad (13)$$

The interaction energy is given by the form $\lambda \phi^4$ term and we find

$$\begin{aligned} \frac{\langle \epsilon, \lambda | H_{\text{int}} | \alpha, \eta \rangle}{\langle \epsilon, \lambda | \alpha, \eta \rangle} &= 2\lambda \int d^3x [\phi^\dagger(x) \phi(x)]^2 \\ &= \frac{0.34\lambda}{E^2 R^2}. \end{aligned} \quad (14)$$

At this stage, we identify the energy shift due to λ and the quark-model energy shift in order to obtain

$$\lambda = \frac{3}{4} \frac{(m_\Delta - m_N) E^2 R^3}{0.34}. \quad (15)$$

The choice of R should be such that the quark-model states fit well into the cavity, which we take to be $R \approx 1$ fm. This yields

$$\lambda = 27.8, \quad (16)$$

which we will use in our numerical work.

Note that this method already gives a rough idea of the fate of the diquark gas. Well-separated diquarks have considerable attractive spin energy (neglecting confinement problems). At a density of 4 quarks $(4\pi/3)$ $(\text{fm})^3 \approx 1$ quark/fm³ the energy is in the region of the deconfinement transition. One would expect the attractive character to persist to somewhat higher densities.

IV. THE DIQUARK GAS

We consider isoscalar nucleonic matter at high density and zero temperature. At the highest densities, one would expect this to be basically a nearly free Fermi gas of equal numbers of up and down quarks. For a total number of quarks N in a volume V , the Fermi momentum will be

$$k_f = \left[\frac{\pi^2}{2} \rho \right]^{1/3}$$

and the energy per quark will be

$$\begin{aligned} \frac{E}{N} &= \frac{3m_q}{4\pi^2} \left[\frac{m_q^3}{\rho} \right] \left\{ \left[\left[\frac{\pi^2}{2} \frac{\rho}{m_q^3} \right]^{2/3} + 1 \right]^{1/2} \right. \\ &\quad \times \left[\left[\frac{\pi^2}{2} \frac{\rho}{m_q^3} \right]^{1/3} + \frac{\pi^2 \rho}{m_q^3} \right] \\ &\quad \left. - \text{arcsinh} \left[\frac{\pi^2}{2} \frac{\rho}{m_q^3} \right]^{1/3} \right\}, \end{aligned} \quad (17)$$

where $\rho \equiv N/V$ is the number density.

In the limit of small ρ ,

$$\frac{E}{N} \rightarrow m_q + \frac{3}{10} m_q \left[\frac{\pi^2}{2} \frac{\rho}{m_q^3} \right]^{2/3}$$

and, for very large values of ρ ,

$$\frac{E}{N} \rightarrow \frac{3m_q}{4} \left[\frac{\pi^2}{2} \frac{\rho}{m_q^3} \right]^{1/3}.$$

We are interested in contrasting the quark gas picture with a system consisting of diquark clusters, treated at lower densities appropriate for the region directly above the deconfinement transition. It is most convenient to start this by initially quantizing in a box of volume V , such that the diquarks field operator has the form

$$\phi^\alpha(x) = \frac{1}{\sqrt{V}} \sum_i \frac{1}{\sqrt{2\omega_i}} (a_i^\alpha e^{-ik_i \cdot x} + b_i^{\dagger\alpha} e^{ik_i \cdot x}), \quad (18)$$

where a_i^α is the destruction operator for a diquark of color α and energy quantum number i , while $b_i^{\dagger\alpha}$ is the creation operator for the corresponding antidiquark. A state of N particles will be specified by the occupation number of each of the modes n_i , yielding a state of the form

$$\frac{1}{(n_0!)^{3/2} (n_1!)^{3/2} \dots} \times (a_0^{\dagger 1})^{n_0} (a_0^{\dagger 2})^{n_0} (a_0^{\dagger 3})^{n_0} (a_1^{\dagger 1})^{n_1} (a_1^{\dagger 2})^{n_1} \dots |0\rangle,$$

where the three colors enter symmetrically. We will later adjust the occupation number in order to minimize the energy. The total number of quarks is given by $N = 2 \sum_i (3n_i)$ where the factor of 3 is for color and 2 is the number of quarks per diquark.

If one mistakenly did not include the repulsive interactions between diquarks, the ground state of the diquark gas would be one with all particles in the zero-momentum state. However, at realistic densities, the repulsive interaction will populate the other energy levels in addition. With the Hamiltonian

$$H_I = \lambda \int d^3x : \phi^{\dagger\alpha}(x) \phi^\alpha(x) \phi^{\dagger\beta}(x) \phi^\beta(x) : \quad (19)$$

we find an interaction energy

$$\langle H_I \rangle = \frac{\lambda}{4V} \left[\sum_{i,j} \frac{2(3n_i)(3n_j)}{\omega_i \omega_j} - \frac{4(3n_i)\delta_{ij}}{\omega_i^2} \right]. \quad (20)$$

In order to transform to the continuum limit, we will use a diquark distribution $n(k)$ in place of $(3n_i)$, normalized such that

$$2 \int d^3k n(k) = N. \quad (21)$$

Using this, the second term in Eq. (20) becomes negligible in the large- N , large- V limit and we obtain

$$\langle H_I \rangle = \frac{\lambda}{2V} \int \frac{d^3k d^3k'}{\omega(k)\omega(k')} n(k)n(k'). \quad (22)$$

The total energy can then be written as

$$E_{dq} = \int d^3k (k^2 + m_{dq}^2)^{1/2} n(k) + \frac{\lambda}{2V} \left[\int d^3k \frac{n(k)}{(k^2 + m_{dq}^2)^{1/2}} \right]^2. \quad (23)$$

In order to evaluate this, we assume that $n(k)$ may be approximated by a Gaussian distribution with width am_{dq} (chosen for convenience),

$$n(k) = \frac{N}{2} \frac{1}{(2\pi a^2 m_{dq}^2)^{3/2}} \exp \left[-\frac{1}{2} \frac{k^2}{a^2 m_{dq}^2} \right], \quad (24)$$

subject to the normalization conditions Eq. (21). This change produces a resultant energy (rescaling the momenta)

$$E_{dq} = 0.79 N m_q \left\{ \left[\frac{2}{\pi} \right]^{1/2} a^{-3} \int_0^\infty dk k^2 (k^2 + 1)^{1/2} e^{-k^2/2a^2} + 0.062 \lambda \frac{\rho}{m_q^3} \left[\left[\frac{2}{\pi} \right]^{1/2} a^{-3} \int_0^\infty \frac{dk k^2}{(k^2 + 1)^{1/2}} e^{-k^2/2a^2} \right]^2 \right\}. \quad (25)$$

In this expression we have used quark masses, and the quark density ρ , utilizing Eq. (10).

In order to calculate the energy of the diquark gas, we numerically find the value of the parameter a which minimizes the energy at fixed ρ . For extreme values of ρ , the solution is given by

$$a = \begin{cases} 0, & \rho \rightarrow 0, \\ \left[\frac{0.062 \lambda \rho}{m^3} \right]^{1/3}, & \rho \rightarrow \infty \end{cases}$$

and

$$\frac{E_{dq}}{N} \sim \begin{cases} 0.79 m \left[1 + 0.062 \frac{\lambda \rho}{m^3} \right], & \rho \rightarrow 0, \\ 0.79 \left[\frac{2}{\pi} \right]^{1/2} m \left[2 + \left[\frac{2}{\pi} \right]^{1/6} \right] \left[\frac{0.062 \lambda \rho}{m^3} \right]^{1/3}, & \rho \rightarrow \infty, \end{cases}$$

while for intermediate values, the solution is displayed in Fig. 1. With the appropriate value of a , the energy per quark is found and is given in Fig. 2. Also plotted is the quark energy under the assumption of free Fermi-gas behavior. The hatched region to the left indicates values of

density below that of close-packed nucleons. In the hatched region it is clear that both of the calculations do not apply, as confinement into color singlets is crucial.

The interesting feature of the results is that the diquark phase is favored for a significant range of densities above

FIG. 1. Parameter a for which E_{dq} [Eq. (25)] is a minimum, as a function of density.

FIG. 2. Energy per quark for the diquark gas as a function of density, and the free-quark gas energy.

the confinement regime. That the diquark phase should be lower in energy at very low densities (neglecting confinement) is essentially required because of the attractive spin interaction. Our calculation provides encouraging evidence which suggests that the diquark phase may be important for moderate and, physically interesting, densities.

We can display the dependence on the choice of λ by plotting the resulting energies for several values of λ , as in Fig. 3. This indicates that our qualitative conclusions are unchanged under reasonable modifications in λ .

V. SUMMARY

We have utilized a simplified model in order to assess the reasonableness of the notion that diquark clusters may be an important aspect of hadronic matter at finite density above the deconfinement transition. Within the model, diquark clusters do lead to a significant lowering of the energy when compared to a free-quark gas. We feel that this is sufficient reason to begin to take the idea seriously and to explore more refined calculations. It is clear that analytic calculations can make some improvements over the simple model used here. However, in the long run all analytic calculations will suffer from the problem that these regions of density do not allow rigorous QCD methods and hence, models must be utilized. These can be useful in identifying important physics, but may not provide an unambiguous answer. Lattice Monte Carlo techniques may ultimately provide the most complete test of this idea. Here the first obstacle which must be overcome is the identification of signals of

FIG. 3. Sensitivity of the diquark energy to the choice of λ .

the proposed diquark nature. Perhaps one method which could be used is to compare the real QCD lattice results with an effective diquark theory which is also similar in spirit to that of this paper. For example, one could define an effective diquark theory, perhaps using ϕ^4 version of this paper. This theory could then be calculated in a lattice Monte Carlo simulation, and the parameters of the model could be adjusted to match some property of the QCD results. The test of the idea would then be obtained by seeing if the model, thus defined, did a good job of reproducing other features of the QCD Monte Carlo simulation. If this were to work well and the existence of a diquark phase were confirmed, there could be the extra practical advantage of using the diquark model in more detailed studies, as the bosonic nature of the diquarks would make them easier to handle on the lattice than fermions.

There remains also the goal of identifying experimental consequences of this idea. The analogy with the pairing phenomena of superconductivity raises obvious interesting possibilities.^{14,15} The early heavy-ion experiments may probe the finite density regime even more than finite temperature. The cores of neutron stars may possibly be relevant for such a diquark phase. If confirmed, the diquark idea will likely prove quite important in these contexts.

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