University of Massachusetts Amherst

From the Selected Works of John Donoghue

October 1, 1987

Electric dipole moments of nuclei

John Donoghue, *University of Massachusetts - Amherst*

Barry Holstein, *University of Massachusetts - Amherst*

M.J. Musolf, *Princeton University*

Available at: https://works.bepress.com/john_donoghue/83/

ELECTRIC DIPOLE MOMENTS OF NUCLEI ☆

John F. DONOGHUE, Barry R. HOLSTEIN

Department of Physics and Astronomy, University of Massachusetts, Amherst, MA 01003, USA

and

M.J. MUSOLF

Department of Physics, Princeton University, Princeton, NJ 98540, USA

Received 1 July 1987

Recent calculations by Sushkov, Flambaum and Khriplovich have suggested that the existence of the possibility of kaon exchange within nuclei allows *nuclear* electric dipole moments within the standard model to be enhanced substantially over those of *nucleons*. This suggestion is analysed carefully and it is found that the constraints of chiral symmetry, which were not correctly included in the earlier calculation, reduce this prediction substantially.

1. *Introduction.* The origin of *CP* violation is one of the fundamental problems of particle physics [1]. As of this writing, despite twenty years of endeavor, no experiment has provided a convincing *CP* violating signal outside the $K \rightarrow 2\pi$ system in which the nonconservation was originally detected (and the related semileptonic K_{L3} process). Because of this scarcity of information, although many models have been proposed in order to explain the phenomenon, it has proved very difficult to isolate the source (or sources) responsible.

One of the most promising avenues in this regard are those experiments which search for electric dipole moments of elementary particles, exploiting the features that an interaction of the form $\sigma \cdot E$ is odd under time reversal (and therefore by the *CPT* theorem odd under *CP*). Thus ongoing measurements of the neutron EDM by both Soviet and western investigators have yielded the incredibly precise result [2]

$$d_n \lesssim 10^{-25} \text{ e cm} \quad \text{at 90\% CL,}$$

which may prove fatal to at least one proposed *CP* nonconserving model.

A second promising area is that of atomic EDM experiments, where present results on ^{199}Hg are already at the 10^{-26} e cm level and improvement by several orders of magnitude appears feasible [3]. A problem here is interpretation, however. Thus, the measured atomic moment is a (model dependent) linear combination of electron and nuclear moments which must be disentangled. However, even in the case of the standard or KM model, wherein the electron moment vanishes, there is considerable model dependence. The problem here is due to Schiff's theorem, which guarantees the vanishing of the atomic moment for the case of a *point* nucleus [4]. Finite nuclear size abrogates this to some extent, but at the cost of a factor of 10^3 – 10^4 suppression of atomic versus nuclear EDM. Since in the standard model the expected size of the nuclear moment is only $\sim 10^{-32} \text{ e cm}$, this is a discouraging prospect. Nevertheless there are two encouraging developments in this regard.

(i) Haxton and Henley [5], following a suggestion by Feinberg [6], have shown that in selected nuclei mixing by the *CP* violating interaction between the ground state and a nearby level of opposite parity can yield enhancements by factors of 10^2 – 10^3 ;

☆ Research supported in part by the National Science Foundation.

(ii) a calculation by Sushkov, Flambaum and Khriplovich (SFK) suggests that the existence of kaon exchange between nucleons within the nucleus yields values of nuclear moments which are a factor of sixty or so larger than the corresponding neutron moment [7].

It is the goal of the present work to study this claim by Sushkov et al. Their calculation is reviewed and analysed in section 2 and it is found to be inconsistent with constraints of chiral invariance. We correct the analysis by the inclusion of new diagrams and find a considerably smaller result. We summarize our findings in section 3.

2. Chiral symmetry and the NNK matrix element. In the standard or KM model CP violation arises via the phase δ which is present in the heavy quark section of the unitary matrix connecting weak and mass eigenstates of the quarks. Thus CP violation does not arise in tree level for light quark (u, d, s) systems. Nevertheless, at one-loop levels it was pointed out by Gilman and Wise that the so-called "penguin" diagram gives rise to an effective $\Delta S=1$ interaction [8]

$$L_{\text{eff}} = (G_1/\sqrt{2}) \bar{s}_i \gamma_\mu (1 + \gamma_5) d_j \sum_q \bar{q}_k \gamma^\mu q_l \lambda_{ij}^a \lambda_{kl}^a + \text{h.c.}, \quad (1)$$

with

$$G_1 = G s_1 s_2 s_3 s_\delta (\alpha_s A / 12\pi) \ln(m_t^2/m_c^2). \quad (2)$$

Here $G \simeq 10^{-5} m_p^{-2}$ is the usual weak coupling constant, α_s is the strong QCD coupling, and $s_i \equiv \sin \theta_i$ ($i=1, 2, 3$) being the usual KM angles. The parameter A is generated by the short-distance gluonic corrections to this form and takes the value [9]

$$1.3 \lesssim A \lesssim 2.5 \quad (3)$$

for reasonable assumptions concerning mass scale effects. In fact the penguin operator would be better written in the Gilman-Wise parameterization ($c_5 O_5 + c_6 O_6$ in the notation of ref. [8]). However, the form used here is sufficient for our purposes, and allows a direct comparison with SFK. Since this interaction carries $\Delta S=1$ it cannot contribute directly to a nonzero dipole moment of the neutron in first-order perturbation theory but *can* in second order as, for example, when a kaon is exchanged between a pair of nucleons. Sushkov et al. argue that the dominant contribution in this regard comes from exchange of neutral kaons and estimate the CP violation vertices, e.g.

$$\langle \bar{K}^0 p | H_w | p \rangle$$

by means of the so-called "factorization" approximation, wherein one inserts the vacuum in all possible ways between the pair of currents. Specifically, using the Fierz identity, these authors find the leading terms to be

$$\begin{aligned} \langle \bar{K}^0 p | H_w | p \rangle_{1a} &= (G_1/\sqrt{2}) \langle K^0 | \bar{s}_i \gamma_5 d_l | 0 \rangle \langle p | \bar{d}_k \gamma_5 d_j + \bar{s}_k \gamma_5 s_j | p \rangle \lambda_{ij}^a \lambda_{kl}^a \\ &\simeq (G_1/\sqrt{2}) \frac{16}{9} \langle \bar{K}^0 | \bar{s} \gamma_5 d | 0 \rangle \langle p | \bar{d} \gamma_5 d + \bar{s} \gamma_5 s | p \rangle, \end{aligned} \quad (4)$$

where here we have exploited the feature that physical states are color singlets and the result $\text{Tr} \lambda^a \lambda^b = 2\delta^{ab}$. Pseudoscalar density matrix elements are evaluated using the divergence condition

$$\langle \bar{K}^0 | \bar{s} \gamma_5 d | 0 \rangle = \frac{\sqrt{2} F_K m_K^2}{m_s + m_d} \quad (5)$$

and pion pole dominance

$$\langle p | \bar{d} \gamma_5 d + \bar{s} \gamma_5 s | p \rangle \simeq -\frac{1}{2} \langle p | \bar{u} \gamma_5 u - \bar{d} \gamma_5 d | 0 \rangle = \frac{m_\pi^2}{m_u + m_d} \frac{m_N g_A}{m_\pi^2 - q^2} \bar{u}(p_2) \gamma_5 u(p_1), \quad (6)$$

Fig. 1. Pion pole contributions to the ppK weak vertex.

yielding the result

$$\langle \bar{K}^0 p | H_w | p \rangle_{1a} \simeq -G_1 \frac{16}{9} \frac{F_K m_\pi^2 m_K^2 m_N g_A}{(m_u + m_d)(m_s + m_d)(m_\pi^2 - q^2)} \bar{u}(p_2) \gamma_5 u(p_1), \quad (7)$$

which is a factor of four *larger* than found by SFK. (From comparison with SFK, we feel that the factor of four is a mathematical error in ref. [7] associated with the color counting factors.)

Use of this form then would seem to lead to an enhancement even larger than found by Sushkov et al. However, additional thought about the form of this result gives one pause about questions of consistency. The point is that what is being calculated here is essentially the pion pole diagram shown in fig. 1, with the $K-\pi$ weak matrix element evaluated via factorization. However, this result is then inconsistent with the approximate chiral symmetry of the QCD lagrangian. Indeed in terms of the nonlinear pseudoscalar matrix

$$U = \exp(i\lambda^a \phi^a / F_\pi), \quad (8)$$

the lowest-order effective lagrangian which transforms as $(8_L, 1_R)$ under chiral $SU(3)_L \times SU(3)_R$ is given by [10]

$$L_w \sim g_w \text{Tr} \lambda_6 \partial^\mu U \partial_\mu U^\dagger. \quad (9)$$

Thus the $K-\pi$ matrix element should be proportional to q^2 , a feature which is lacking in eq. (7). The resolution of this problem is the realization that there exists an additional "factorization" contribution which must be included in order to produce the correct chiral structure [11]. This is the term

$$\langle \bar{K}^0 p | H_w | p \rangle_{1b} = (G_1 / \sqrt{2}) \langle 0 | \bar{s}_i s_j + \bar{d}_i d_j | 0 \rangle \langle \bar{K}^0 p | \bar{s}_k d_l | p \rangle \lambda_{il}^a \lambda_{jk}^a \quad (10)$$

involving the vacuum expectation value of the scalar density terms. These can be written in terms of the pseudoscalar masses by use of PCAC and current algebra

$$\langle 0 | \bar{s}s | 0 \rangle = \langle 0 | \bar{d}d | 0 \rangle = -\frac{F_\pi^2 m_\pi^2}{m_u + m_d} = -\frac{F_K^2 m_K^2}{m_s + m_d}. \quad (11)$$

Finally, the soft pion limit determines the $q=0$ value of the remaining scalar density

$$\langle \bar{K}^0 p | \bar{s}d | p \rangle \xrightarrow{q \rightarrow 0} -\frac{1}{\sqrt{2} F_K} \langle p | \bar{d} \gamma_5 d + \bar{s} \gamma_5 s | p \rangle. \quad (12)$$

Thus, again assuming pion pole dominance we find

$$\langle \bar{K}^0 p | H_w | p \rangle_{1b} = G_1 \frac{16}{9} \frac{F_K m_\pi^2 m_K^2 m_N g_A}{(m_u + m_d)(m_s + m_d)(m_\pi^2 - q^2)} \bar{u}(p_2) \gamma_5 u(p_1). \quad (13)$$

We see that this contribution exactly cancels the result of fig. 1a in the chiral limit. This occurs in the same way as has been discussed for the $K \rightarrow \pi$ matrix element [11]. This exact cancellation would be expected to be modified by corrections to the soft pion limit, which would enter proportional to the momentum transfer. These

Fig. 2. Baryon pole contributions to the ppK weak vertex.

are not readily calculable, but their order of magnitude can be estimated from experience with other corrections to chiral predictions. Typically all of the current matrix elements involved have form factors with q^2 variation scaled by a dimensionful quantity Λ^2 where $\Lambda \sim 4\pi F_\pi \sim 1$ GeV is the chiral scale parameter [12]. These modify diagrams 1a and 1b differently and their sum will no longer cancel exactly, but will be of order

$$\langle \bar{K}^0 p | H_w | p \rangle_{1a+1b} \sim G_1 \frac{16}{9} \frac{F_K m_K^2 m_\pi^2 m_N g_A}{(m_u + m_d)(m_s + m_d)(m_\pi^2 - q^2)} \frac{q^2}{\Lambda^2} \bar{u}(p_2) \gamma_5 u(p_1). \quad (14)$$

In this expression q_μ is the four-momentum carried by the virtual kaon. Chiral symmetry has told us that the π pole contributions will vanish as $q^2 \rightarrow 0$, instead of being constant as found in SFK. Since in nuclei the q^2 associated with meson exchange is small, this is a major suppression. Even if we use the maximal value

$$q^2 \sim -p_F^2, \quad (15)$$

where $p_F \sim 200$ MeV is the Fermi momentum of a typical nucleus, we find that the kaon exchange contribution due to this term is vastly ($\leq \frac{1}{25}$) suppressed compared to its earlier estimated value.

If the pion pole contributions to the NNK vertex are no longer enhanced, there may be other competing diagrams which must be included. In particular, the baryon pole diagrams, indicated in fig. 2, must also be accounted for. These involve a weak $N\Sigma$ or NA transition together with a strong ΣNK or NAK vertex. These poles play a major role in the analysis of P wave hyperon decay. These can be estimated using vacuum saturation plus a quark model calculation.

Thus considering first the scalar density vacuum expectation value term (fig. 2a), we find

$$\langle \bar{K}^0 p | H_w | p \rangle_{2a} = \frac{16}{9} \frac{G_1}{\sqrt{2}} \langle 0 | \bar{d}d + \bar{s}s | 0 \rangle \bar{u}(p_2) \gamma_5 u(p_1) \frac{1}{\sqrt{2}} \frac{g_A}{F_K} (d-f) \frac{(m_\Sigma + m_p)}{(m_s - m_d)}, \quad (16)$$

where we have utilized the generalized Goldberger-Treiman relation [13] and have defined the strong BBM vertices in terms of the conventional f and d type couplings, with the normalization

$$f=0.344, \quad d=0.656. \quad (17)$$

On the other hand for the four-quark baryon pole diagram (fig. 2b) we have

$$\langle \bar{K}^0 p | H_w | p \rangle_{2b} = \bar{u}(p_2) \gamma_5 u(p_1) a_{\Sigma^+ p} \frac{1}{m_p - m_\Sigma} (m_\Sigma + m_p) g_A \frac{1}{2F_K} (d-f), \quad (18)$$

where here

$$a_{\Sigma^+ p} = \langle p | H_w | \Sigma^+ \rangle, \quad (19)$$

and can be evaluated in a particular model. Thus in the MIT bag model [14] one finds [15]

$$a_{\Sigma^+ p} = (G_1/\sqrt{2}) \cdot 6 \times 10^{-3} \text{ GeV}^3. \quad (20)$$

Comparing with its baryon pole-scalar density counterpart we find that

$$\frac{\langle p\bar{K}^0 | H_w | p \rangle_{2b}}{\langle p\bar{K}^0 | H_w | p \rangle_{2a}} \ll 1, \quad (21)$$

so that the contribution from fig. 2b can be safely neglected.

As an aside for the careful reader we note that the overall scalar density contribution – figs. 1b, 2a –, which can be written as

$$\langle p\bar{K}^0 | H_w | p \rangle_{1b+2a} = \frac{16}{9} (G_1/\sqrt{2}) \langle 0 | \bar{d}d + \bar{s}s | 0 \rangle \langle p\bar{K}^0 | \bar{s}d | p \rangle, \quad (22)$$

must vanish when the kaon is on its mass shell according to the Feinberg–Weinberg–Kabir theorem [16]. In fact this feature is easily verified when we, for consistency, include also the η pole contribution. Thus, neglecting the $O(q^2/\Lambda^2)$ correction we have

$$\begin{aligned} & \langle \bar{K}^0 p | H_w | p \rangle_{1b-\pi} + \langle \bar{K}^0 p | H_w | p \rangle_{1b-\eta} + \langle \bar{K}^0 p | H_w | p \rangle_{2a} \\ &= \bar{u}(p_2) \gamma_5 u(p_1) G_1 \frac{2F_\pi^2 m_\pi^2}{(m_u + m_d)} \frac{16}{9\sqrt{2}} \left(\frac{g_{pp\pi^0} \langle K^0 | \bar{s}d | \pi^0 \rangle}{m_\pi^2 - q^2} + \frac{g_{pp\eta^0} \langle K^0 | \bar{s}d | \eta^0 \rangle}{m_\eta^2 - q^2} - \frac{g_{\Sigma^+ p K^0} \langle p | \bar{s}d | \Sigma^+ \rangle}{m_\Sigma - m_p} \right) \\ &\sim \bar{u}(p_2) \gamma_5 u(p_1) G_1 \frac{F_\pi^2 m_\pi^2}{m_u + m_d} \frac{16}{9} \frac{g_{pp\pi^0}}{m_s} \left((d+f) \frac{m_\pi^2 - m_K^2}{m_\pi^2 - q^2} + (d-3f) \frac{m_\eta^2 - m_K^2}{m_\eta^2 - q^2} - 2(d-f) \right) \\ &= 0 \quad \text{when } q^2 = m_K^2, \end{aligned} \quad (23)$$

so that an exact cancellation occurs in the on-shell limit, as expected. In our application, however, we look in the region where $|q^2| \lesssim p_F^2$ in which case the η pole is suppressed by a factor

$$\frac{m_\eta^2 - m_K^2}{m_\pi^2 - m_K^2} \frac{m_\pi^2}{m_\eta^2} \ll 1, \quad (24)$$

and may be dropped. The pion pole term cancels the leading contribution from fig. 1a as before, leaving the baryon pole piece – fig. 2a – and the higher-order chiral piece – $\propto q^2/\Lambda^2$ – as the dominant terms. However, compared to the original SFK estimate, these contributions are smaller by the factors

$$\frac{\text{baryon pole}}{\text{SFK}} \simeq \frac{m_\pi^2 - q^2}{m_K^2} \frac{d-f}{d+f} \leq \frac{m_\pi^2 + p_F^2}{m_K^2} \frac{d-f}{d+f} = \frac{1}{15}, \quad \frac{\text{higher chiral}}{\text{SFK}} \simeq \frac{|q^2|}{\Lambda^2} \leq \frac{p_F^2}{\Lambda^2} = \frac{1}{25}. \quad (25)$$

While one could attempt to be more quantitative here, it is not really warranted at this time.

Overall, we have shown that the vacuum saturation estimate of SFK was in error. The corrected matrix element for the NNK vertex is considerably smaller than originally thought. In addition to meson pole diagrams, baryon poles enter with comparable strength. The q^2 variation of the amplitude is important, with the π -K pole term vanishing as $q^2 \rightarrow 0$. Because of this q^2 variation, the amplitudes given above also need to be used in a realistic nuclear calculation in order to determine the actual amount of suppression in the resulting dipole moment.

The above discussion has been concerned with questions of principle, and the chiral constraints are not optional. Before we leave this subject, we would also like to comment on the evaluation of the mass factors in the vacuum saturation formula. This concerns some common practice in the literature where care is not taken in the proper definition of mass parameters. SFK and many others utilize the “Weinberg” values [17] for quark masses

$$m_u^* = 4 \text{ MeV}, \quad m_d^* = 7 \text{ MeV}, \quad m_s^* = 150 \text{ MeV}. \quad (26)$$

However, while the derivation of quark mass ratios is reasonably secure, the same cannot be said for the absolute normalization. The quark masses enter the theory only through the combination $m\bar{\psi}\psi$, so that the magnitude of quark masses can only be extracted if the matrix element of $\bar{\psi}\psi$ is known. The Weinberg method hides this fact by absorbing the matrix element into a new mass parameter m^* . In particular,

$$m_s^* \equiv m_s (\langle \Lambda | \bar{s}s | \Lambda \rangle - \langle p | \bar{s}s | p \rangle) \approx m_\Lambda - m_p \approx 150 \text{ MeV}, \quad (27)$$

which leads to the values given above. However, in the application to vacuum saturation it must be recognized that m^* is not the appropriate mass factor, because it does not enter the equation of motion. To isolate the appropriate m_s , one must divide by the scalar density. Defining

$$S \equiv \langle \Lambda | \bar{s}s | \Lambda \rangle - \langle p | \bar{s}s | p \rangle, \quad (28)$$

we have

$$m_s = \frac{m_s^*}{S} \approx \frac{150 \text{ MeV}}{S}. \quad (29)$$

All quark models predict $S < 1$, with the MIT bag model giving $S \approx \frac{1}{2}$. Since in all vacuum saturation formulas the quark masses enter in the denominator, the change from the use of m^* to the use of m in the numerical evaluation leads to an extra factor of S^2 suppression of the matrix element, with $S^2 \approx \frac{1}{4}$. Use of S^2 cancels the factor of four enhancement noted in eq. (7). (We note also that use of S^2 in hadronic matrix elements has support from recent experiments concerning parity violation in nuclei, where the pion exchange term cannot be understood without this factor [18,19].)

3. Conclusions. We have examined the claim by Sushkov, Flambaum and Khriplovich that the kaon exchange process in nuclei allows standard model *nuclear* electric dipole moments to be enhanced considerably with respect to corresponding *nucleon* values. We unfortunately do not agree with this conclusion. The NNK vertices on which the SFK claim is based were evaluated incorrectly using a simple "factorization" scheme which did not respect the underlying chiral symmetry. We have shown that inclusion of additional diagrams involving vacuum expectation values of scalar quark densities and baryon pole contributions reduce the expected size of the NNK coupling to a value roughly 20 times smaller than that originally claimed. This result, when combined with recent work by McKellar suggesting that the neutron dipole moment used by SFK may have been underestimated [20], argues that it is unlikely to expect a nuclear/nucleon enhancement in the standard model *via this mechanism*. While this is somewhat discouraging, it is to be hoped that improvements in atomic EDM measurements will continue, as only by such careful and precise experimental probes will the *CP* mystery be solved.

References

- [1] See e.g. J.F. Donoghue, B.R. Holstein and G. Valencia, Intern. J. Mod. Phys. A 2 (1987) 319, and references therein.
- [2] I.S. Alterev et al., Phys. Lett. B 102 (1981) 13;
N.F. Ramsey, Rep. Progr. Phys. 45 (1982) 95.
- [3] T.G. Vold et al., Phys. Rev. Lett. 52 (1984) 2229;
E.N. Fortson et al., to be published;
F.P. Calaprice, Nucl. Phys. A 434 (1985) 515.
- [4] L.I. Schiff, Phys. Rev. 132 (1963) 2194.
- [5] W.C. Haxton and E.M. Henley, Phys. Rev. Lett. 51 (1983) 1937.
- [6] G. Feinberg, Trans. N.Y. Acad. Sci. 28 (1977) 6.
- [7] V.V. Flambaum, I.B. Khriplovich and O.P. Sushkov, Zh. Eksp. Teor. Fiz. 87 (1984) 1521 [Sov. Phys. JETP 60 (1984) 873].
- [8] F. Gilman and M. Wise, Phys. Lett. B 93 (1980) 129; Phys. Rev. D 27 (1985) 1128.
- [9] A.I. Vainshtein, V.I. Zakharov and M.A. Shifman, Zh. Eksp. Teor. Fiz. 72 (1977) 1275 [Sov. Phys. JETP 45 (1977) 670].
- [10] J. Cronin, Phys. Rev. 161 (1967) 1483.

- [11] See e.g. J.F. Donoghue, Phys. Rev. D30 (1984) 1499;
M.B. Gavela, A. Le Yaouanc, L. Oliver, O. Pene and J.C. Raynal, Phys. Lett. B 148 (1984) 225.
- [12] J.F. Donoghue, E. Golowich and B.R. Holstein, Phys. Rev. D 30 (1984) 587;
A. Manohar and H. Georgi, Nucl. Phys. B 234 (1984) 189.
- [13] M.L. Goldberger and S.B. Treiman, Phys. Rev. 111 (1958) 354.
- [14] T. De Grand, R. Jaffe, K. Johnson and J. Kiskis, Phys. Rev. D 12 (1975) 2060.
- [15] J.F. Donoghue, E. Golowich, B.R. Holstein and W. Ponce, Phys. Rev. D 23 (1981) 1213.
- [16] G. Feinberg, P. Kabir and S. Weinberg, Phys. Rev. Lett. 3 (1959) 527.
- [17] S. Weinberg, in: Festschrift for I.I. Rabi (NY Academy of Sciences, New York, 1977).
- [18] J.F. Donoghue and B.R. Holstein, Phys. Rev. Lett. 46 (1981) 1603;
B.R. Holstein, Can. J. Phys., to be published.
- [19] B. Desplanques, J.F. Donoghue and B.R. Holstein, Ann. Phys. (NY) 124 (1980) 449.
- [20] B.H.J. McKellar, Proc. Workshop on Neutron physics (Chapel Hill, 1987), to be published.