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SURVEY OF PRESENT AND FUTURE TESTS OF CP VIOLATION*

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Abstract

We review the present status of CP violation and critically assess the various tests of this symmetry which have been proposed. Future progress in the subject requires new experimental information distinct from the present measurement of the parameter ϵ . We find that there exist experiments which are potentially feasible, although all such experiments are reasonably difficult. The theoretical analysis of the various CP-odd observables is reviewed and in many cases updated.

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I. Introduction

CP violation is one of the most fascinating phenomena in particle physics. Its existence is well documented in kaon decays,¹ but its origin is far from clear. Although it is possible to be explained within the standard model it also could be a sensitive low energy probe for physics beyond the standard model.

Thus far CP nonconservation has been seen only in the kaon sector, but it would also be potentially visible in a wide variety of other processes. It is the purpose of this article to collect together all of the suggestions for new observations of CP violation and to assess the theory behind them and their prospects for success.

There exist several good reviews of the present status of CP violation.^{2,3} Although we shall update these analyses, our main focus will be on the future of the field. With the present knowledge, and even with an improved measurement of ϵ'/ϵ , we are unable to isolate the origin of this phenomenon. For this goal to succeed, we require evidence of CP nonconservation in additional processes. Thus we feel that the prime role of theory at present is to explore and develop experimental possibilities and hopefully to provide guidance and direction for experimenters. This motivates our attempt to survey the entire field. Indeed, the theory of CP violation is remarkably subtle and in many topics we have found that the standard analysis requires some modifications. These have been incorporated in this survey and have sometimes been explained more fully in separate papers. At the least, our survey provides an overview of all topics from a unified theoretical perspective, and represents the status of theory at the present time.

We begin by describing a few of the models of CP violation in Sec. II. These by no means exhaust the possible list of theories, but they do provide a set of examples which demonstrates a range of the possible patterns of CP-violating phenomena. In Sec. III we review $K \rightarrow 2\pi$, which updates the status of

theory and experiment. Following this discussion, the remainder of the paper is devoted to new tests, beginning with kaon decays and then progressing into other systems. For the reader interested in a compact summary, Table I provides a guide to the article.

It is, of course, understood that attempts to assess the future of a field are quite possibly in error. Certainly our analysis is based primarily on the present "standard" model. It is possible, and in fact highly desirable, that future discoveries will modify the basis of our expectations. Thus, experimenters are to be encouraged to explore new territory in CP violation via precise experiments independent of any particular model. Nevertheless theoretical guidance can be useful in deciding where limited experimental resources may be most effectively utilized.

Process	Section	Comments
I) <u>LOW ENERGY</u>		
$K_L \rightarrow 2\pi$	III	ϵ' expected to be seen in standard model $\epsilon'/\epsilon = 0.002 \pm 0.01$
$K_S \rightarrow 3\pi$	IV	Leading order signal is ϵ . Non- ϵ signal may be of order $20\epsilon'$
$K^+ \rightarrow \pi^+ \pi^+ \pi^-$	IV	Tiny
$K_L \rightarrow \gamma\gamma$	V	Non- ϵ signal can be of order $20\epsilon'$
$K_L \rightarrow \pi^0 e^+ e^-$	VI	Large non- ϵ signal. B.R. $\sim 10^{-11}$
$K_L \rightarrow \mu^+ \mu^-$	VII	Polarization large in some models
$K_L \rightarrow \pi^+ \pi^- \gamma$	VIII	CP violation is comparable to CP even amplitude
Hyperon decay asymmetries	IX	Tests in $p\bar{p} \rightarrow \Lambda\bar{\Lambda}$. May be possible to see 10^{-4} effects
Electric dipole moments	X	Next generation of limits sensitive to some models
Triple product correlations	XI	Small in most but not all models
II) <u>HEAVY QUARKS</u>		
B decays	XII	Many tests. May be feasible in future
D decays	XIII	Not likely in standard model but perhaps useful in supersymmetric theories
T decays	XIII	Not likely in standard model but perhaps useful in charged Higgs models
III) <u>HIGH ENERGY</u>		
$Z^0 \rightarrow b\bar{s}, \bar{b}s$	XIV	A signal is possible, but not likely, with four generations
$p\bar{p} \rightarrow e^\pm X$	XV	Probe of new physics
Jet physics	XVI	New tool for high energy tests

Table I. Summary of the tests of CP violation discussed in this article

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II. CP Violating Theories

The fundamental principles for constructing gauge field theories allow one to write down the most general Lagrangian consistent with the requirements of renormalizability and gauge symmetry.⁴ Included in such a Lagrangian will be various forms of CP violating interactions, unless the discrete symmetry CP is imposed in addition to the gauge symmetry. In this sense almost any theory can be CP violating. However in some models one has the freedom to redefine the phases of the physical fields such that all interactions are manifestly CP conserving. [This implies, of course, that, even if one does not redefine the fields, all CP-odd observables must vanish.] For example QED can be made totally CP conserving, while QCD will conserve CP except for the θ term (discussed below).⁵ The minimal Weinberg Salam model with only four quarks also automatically conserves CP, although with six quarks it does not.⁶ This freedom to redefine phases often allows the CP violation to be restricted to couplings involving rare particles and hence leads naturally to a small amount of CP violation.

An example is provided by the standard Kobayashi-Maskawa-Glashow-Weinberg-Salam⁶ model of CP violation, which has the only CP-odd coupling being in the interaction of quarks with the charged W boson. Using the standard parameterization of the KM matrix, given by

$$V = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 + s_2 s_3 e^{i\delta} & c_1 c_2 s_3 - s_2 c_3 e^{i\delta} \\ -s_1 s_2 & c_1 s_2 c_3 - s_2 s_3 e^{i\delta} & c_1 s_2 s_3 + c_2 c_3 e^{i\delta} \end{pmatrix} \quad (1)$$

where $c_i, s_i = \cos\theta_i, \sin\theta_i$, the freedom to redefine the phases of quark fields implies that all CP violation in the model is proportional to the rephasing invariant⁷ (setting $\cos\theta_1 \approx 1$)

$$s_1^2 s_2 s_3 \sin \delta \quad (2)$$

The angle s_1 is known from kaon and hyperon decay⁸ to be

$$s_1 = 0.220 \pm 0.003 \quad (3)$$

while s_2 and s_3 are constrained by the b quark decay rate and the $b \rightarrow u/b \rightarrow c$ ratio. Overall this combination of angles is bounded⁹ by (using only $\sin \delta < 1$)

$$s_1^2 s_2 s_3 \sin \delta \lesssim 10^{-4} \quad (4)$$

This small number explains the general pattern of CP violation in the model. In reactions which take place unsuppressed by any small KM angles, such as $c \rightarrow s$ and $t \rightarrow b$ decay, any CP violation must be of $O(10^{-4})$. On the other hand, in KM suppressed processes, where the CP conserving rate is proportional to small angles, the relative CP odd signal may be much more visible. (No single tree level diagram contains the full combination of KM angles, and therefore either loops and/or interference with other diagrams is required in order to generate a CP odd effect.) This theory provides then a natural explanation of why CP violation is small. In fact, in order to accommodate the known CP violation in $K \rightarrow 2\pi$ (see Sec. III), the combination of angles must be close to its present upper bound. Since θ_3 and δ could in fact be much less than the present bound, it is possible that they will ultimately be found to be too small to explain the observed CP violation and the so-called standard model will be ruled out. However, we shall see that the present bound does accommodate the observed magnitude of the CP-odd signal.

Many other models exist which can explain CP violation. It is not our purpose to review all of them, as one does not learn very much from enumerating such theories. Instead we will briefly describe a few theories which illustrate the variety of patterns of CP violation.

The superweak theory of Wolfenstein¹⁰ is one of the standards of the subject and has various gauge theory realizations.¹¹ These involve a $\Delta S=2$ transition of a very heavy Higgs or gauge boson, as in Fig. 1. Such an effect shows up strongly in the mixing of $K_L^0 K_S^0$ because of the small $K_L K_S$ mass difference. How-

Fig. 1. Superweak theories involve direct $\Delta S=2$ transitions

ever in other processes the superweak model predicts virtually no signal. If this class of theories is correct, the only visible CP violation is in the $K^0-\bar{K}^0$ mixing parameter ϵ .

It was pointed out by Weinberg¹² that, if one has more than two Higgs doublets in an $SU(2)_L \times U(1)$ theory, then the Higgs interactions will not in general conserve CP. This situation could easily occur if supersymmetry is present around the weak scale, since supersymmetric theories often have many Higgs bosons. The CP-odd couplings can be placed in the interactions of charged Higgs with quarks.¹³ The important features of these couplings are 1) they are proportional to the masses of the quarks involved, 2) they are proportional to the same KM elements as are the quark couplings to charged W's and 3) they contain extra (unknown) KM-like angles which describe the mixing of the Higgs bosons. It is these latter Higgs angle factors which contain the dominant CP violation in the theory. The types of processes which violate CP are shown in Fig. 2. For light quarks, the loop diagram of Fig. 2b is most important¹⁴ since it can involve the masses of heavy quarks. On the other hand, for heavy quarks, Fig. 2a is dominant. One of the special features of the Higgs model is that the CP violation should become more important for the heavy quarks, due to the large quark mass factors.

Extra gauge bosons can also lead to CP violation. Examples of this are provided by the various $SU(2)_L \times SU(2)_R \times U(1)$ theories.¹⁵ There are many possible patterns of CP nonconservation within such models, as their structure is very rich. Perhaps the simplest one is the "isoconjugate" model¹⁶ which is interesting in that its structure is quite simple and distinct from other models. In this version the left-handed and right-handed interactions are proportional to each other, but there exists an overall relative phase between them. Thus parity conserving interactions have one common overall phase while parity violating ones have the conjugate phase.

Finally it is worth mentioning the strong CP violation which can in principle

9.

(a)

(b)

Fig. 2. Diagrams containing CP violation in theories where charge Higgs exchange is important.

10.

be present in QCD. This occurs if the determinant of the bare quark mass matrix has a nonzero phase, and is manifested in the theory by a P and CP violating component of the Lagrangian

$$\theta E^{\mu\nu\alpha\beta} F_{\mu\nu}^A F_{\alpha\beta}^A \quad (5)$$

with $F_{\mu\nu}^A$ being the gluon field strength tensor. Such a term influences the strong and electromagnetic properties of hadrons, and in particular generates an electric dipole moment for the neutron. The present bound on the neutron's moment requires¹⁸ $\theta < 10^{-8}$. Unfortunately such a small value of θ implies that strong CP violation will be negligibly small in all other processes, including $K \rightarrow 2\pi$. Thus, the θ parameter is not the source of the known CP violation and should not influence any of the tests described in this survey.

11.

III. CP Violation and $K \rightarrow 2\pi$

The phenomenon of CP violation was first discovered¹ in the decay $K \rightarrow 2\pi$ and this mode has received intense scrutiny ever since. The time evolution of the $K^0, \bar{K}^0$ system is described, using second order perturbation theory, in terms of an effective Hamiltonian.¹⁹ That is, writing the wavefunction in two component form

$$\psi(t) = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \equiv a(t)|K^0\rangle + b(t)|\bar{K}^0\rangle \quad (6)$$

we have

$$i \frac{d}{dt} \psi(t) = (M - i \frac{1}{2} \Gamma) \psi(t) \quad (7)$$

where

$$(M - i \frac{1}{2} \Gamma)_{ij} = m_K^{(0)} \delta_{ij} + \langle K_i^0 | H_W | K_j^0 \rangle + \sum_n (2\pi)^3 \delta^3(p_n) \frac{\langle K_i^0 | H_W | n \rangle \langle n | H_W | K_j^0 \rangle}{m_K^{(0)} - E_n + i\eta} \quad (8)$$

is usually called the "mass matrix". Diagonalizing, we find the eigenstates of this system to be

$$\begin{aligned} |K_S^0\rangle &= \frac{|K_+^0\rangle + \sigma |K_-^0\rangle}{\sqrt{1 + |\sigma|^2}} \\ |K_L^0\rangle &= \frac{|K_-^0\rangle + \sigma |K_+^0\rangle}{\sqrt{1 + |\sigma|^2}} \end{aligned} \quad (9)$$

where, if we define $CP|K^0\rangle \equiv |\bar{K}^0\rangle$, then

$$|K_{\pm}^0\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle \pm |\bar{K}^0\rangle) \quad (10)$$

are CP even, odd eigenstates respectively and

$$\sigma \equiv -\frac{1}{2} \frac{M_{12} - M_{21} - \frac{i}{2} (\Gamma_{12} - \Gamma_{21})}{m_L - m_S - i \frac{1}{2} (\Gamma_L - \Gamma_S)} = \frac{\langle K_+^0 | M - i \frac{1}{2} \Gamma | K_-^0 \rangle}{\langle K_-^0 | M - i \frac{1}{2} \Gamma | K_+^0 \rangle - \langle K_+^0 | M - i \frac{1}{2} \Gamma | K_+^0 \rangle} \quad (11)$$

is the mixing parameter. Defining

$$m_L - m_S \equiv \Delta m \quad (12)$$

and noting that, experimentally,²⁰

12.

$$\tan^{-1} \frac{\Gamma_S - \Gamma_L}{2\Delta m} \approx 45^\circ \quad (13)$$

we can write

$$\sigma \approx + \frac{e^{i\frac{\pi}{4}}}{\sqrt{2}} (\frac{m'}{\Delta m} - i\alpha) \quad (14)$$

where

$$\begin{aligned} m' &\equiv -i \langle K_+^0 | M | K_-^0 \rangle \quad \text{and} \quad \alpha = \frac{-i \langle K_+^0 | \Gamma | K_-^0 \rangle}{\Gamma_S - \Gamma_L} \\ &= -\text{Im } M_{12} \end{aligned} \quad (15)$$

We can calculate the parameter α by using the fact that Γ involves only real intermediate states and hence is dominated by 2π . Thus one defines

$$\begin{aligned} \langle \pi^+ \pi^- | H_W | K^0 \rangle &= A_0 e^{i\delta_0} + \frac{1}{\sqrt{2}} A_2 e^{i\delta_2} \\ \langle \pi^0 \pi^0 | H_W | K^0 \rangle &= A_0 e^{i\delta_0} - \frac{1}{\sqrt{2}} A_2 e^{i\delta_2} \end{aligned} \quad (16)$$

where δ_I is the strong interaction phase shift in the $I=0,2$ isotopic spin state and the transition amplitudes A_0, A_2 , which would both be real in the absence of CP violation, are written as

$$A_0 = |A_0| e^{i\xi}, \quad A_2 = |A_2| e^{i\xi'} \quad (17)$$

The experimental validity of the $\Delta I = \frac{1}{2}$ rule, of course, requires that²¹

$$\omega = \frac{\text{Re } A_2}{\text{Re } A_0} = 0.045 \quad (18)$$

is small. Then due to dominance by the $I=0$ state of two pions we find

$$\alpha \approx + \xi = \frac{\text{Im } A_0}{\text{Re } A_0} \quad (19)$$

Overall this result can be rephrased in standard notation as

$$\sigma = - \frac{e^{i\pi/4}}{\sqrt{2}} \left[\frac{\text{Im } M_{12}}{\Delta m} + \frac{\text{Im } A_0}{\text{Re } A_0} \right] \quad (20)$$

with $\Delta m = 2\text{Re } M_{12}$.

The magnitudes of the CP violating transitions follow from the above definitions. Using

$$\eta_{+-} = \frac{\langle \pi^+ \pi^- | H_W | K_L \rangle}{\langle \pi^+ \pi^- | H_W | K_S \rangle} \equiv \epsilon + \epsilon'$$

$$\eta_{00} = \frac{\langle \pi^0 \pi^0 | H_W | K_L \rangle}{\langle \pi^0 \pi^0 | H_W | K_S \rangle} \equiv \epsilon - 2\epsilon'$$
(21)

we find

$$\epsilon = \sigma + i\xi = \frac{e^{i\pi/4}}{\sqrt{2}} \left(\frac{m'}{\Delta m} + \xi \right) = -\frac{e^{i\pi/4}}{\sqrt{2}} \left[\frac{\text{Im } M_{12}}{\Delta m} - \frac{\text{Im } A_0}{\text{Re } A_0} \right]$$
(22)

$$\epsilon' = \frac{i\omega}{\sqrt{2}} e^{i(\delta_2 - \delta_0)} (\xi' - \xi) = \frac{i\omega e^{i(\delta_2 - \delta_0)}}{\sqrt{2}} \left[\frac{\text{Im } A_2}{\text{Re } A_2} - \frac{\text{Im } A_0}{\text{Re } A_0} \right]$$
(23)

Note that experimentally²¹ $(\delta_2 - \delta_0 + \pi/2) \approx 48 \pm 8^\circ$ so that the phase of ϵ and that of ϵ' are very nearly equal. The two sources of CP violation, the $K^0 \bar{K}^0$ mass matrix and the direct $K \rightarrow 2\pi$ transitions are illustrated in Fig. 3.

These formulae exhibit an important theoretical property. One is always free to redefine the phase of states by a global strangeness transformation.

$$|A\rangle \rightarrow e^{i\lambda S} |A\rangle$$

e.g.

$$|\bar{K}^0\rangle \rightarrow e^{i\lambda} |\bar{K}^0\rangle$$
(24)

Such a transformation cannot change measurable quantities such as ϵ and ϵ' , but can modify intermediate results, such as σ in Eq. 14. Under such a transformation

$$\begin{aligned} \xi &\rightarrow \xi + \lambda \\ \xi' &\rightarrow \xi' + \lambda \\ \sigma &\rightarrow \sigma - i\lambda \end{aligned}$$
(25)

and the formulae for ϵ and ϵ' clearly display this invariance. Note that the mixing parameter σ is equal to ϵ in the convention where A_0 is real, i.e. $\lambda = -\xi$, and this "Wu-Yang" convention¹⁹ is often utilized in the literature. We shall, however, quote our results in a conventional independent manner.

On the experimental side the mixing parameter ϵ is well-determined²⁰

(a)

(b)

Fig. 3. The two sources of CP violation in the kaon sector a) the $K^0 \bar{K}^0$ mass matrix ($\Delta S=2$), and b) direct $\Delta S=1$ CP violation in the $K \rightarrow 2\pi$ transition.

$$|\epsilon| = (2.27 \pm 0.02) \times 10^{-3} \quad (26)$$

$$\text{Arg } \epsilon = \text{Arg } \eta_{+-} = 44.6 \pm 1.2^\circ$$

However, increasingly precise experiments have yet to find evidence that ϵ' is nonvanishing, the current limit being²²

$$\left| \frac{\epsilon'}{\epsilon} \right| \leq 0.01 \text{ at } 90^\circ \text{ C.L.} \quad (27)$$

Ongoing efforts at Fermilab and CERN hope to gain yet another order of magnitude of sensitivity in this regard.²³

Now consider what these results have to say about the underlying source of CP violation. The key here is the parameter ϵ' . Note that a non-zero value of ϵ' requires direct CP violation in the $\Delta S=1$ weak decay amplitude — $\xi - \xi' \neq 0$ — as opposed to ϵ , which can arise either from direct CP violation — $\xi \neq 0$ — or from CP nonconservation in the mass matrix — $m' \neq 0$,

Thus the superweak theory predicts a nonvanishing $\Delta S=2$ matrix element

$$\langle \bar{K}^0 | H_W | K^0 \rangle$$

so that $m'_{SW} \neq 0$. However, there exists no direct CP violation in $\Delta S=1$ processes, so $\xi_{SW} = \xi'_{SW} = 0$. In this scenario then

$$\epsilon = \frac{1}{\sqrt{2}} e^{i\pi/4} \frac{m'}{\Delta m} \neq 0 \quad (28)$$

$$\epsilon' = 0$$

in agreement with current experiment. This model, of course, predicts that the improved Fermilab and CERN experiments will also find no evidence for a nonzero value of ϵ' . It should also be noted that the magnitude of ϵ is not predictable. Rather, $|\epsilon|$ is used to fit the size of the CP violating $K^0 - \bar{K}^0$ mixing interaction.

A great deal of effort has gone into the analysis of CP violation within the standard model. In this picture, up to possible small ($\sim 20\%$) dispersive corrections,²⁴ Fig. 4, and the small Siamese penguin diagram,²⁵ Fig. 5, one expects that m' can be reasonably estimated from its short distance contribution arising from the so-called box diagram (Fig. 6), which yields²⁶

Fig. 4. Dispersive contributions to $\text{Im } M_{12}$.

Fig. 5. The "Siamese penguin" or "dipenguin" contribution to $K^0 \bar{K}^0$ mixing

Fig. 6. The box diagram for $K^0 \bar{K}^0$ mixing

$$m' = \frac{G_F^2}{6\pi} F_K^2 m_K^2 s_1^2 s_2 s_3 s_8 [\eta_1 F(x_c) - \eta_2 F(x_t) K - \eta_3 G(x_c, x_t)] M_W^2 \quad (29)$$

where $x_i = m_i^2/M_W^2$, $K = s_2^2 + s_2 s_3 s_8$ and the functions F, G are defined by

$$F(x) = x \left(\frac{1}{4} + \frac{9}{4} \frac{1}{1-x} - \frac{3}{2} \frac{1}{(1-x)^2} \right) + \frac{3}{2} \ln x \left(\frac{x}{x-1} \right)^3 \quad (30)$$

$$G(x, y) = xy \left[\frac{1}{x-y} \left(\frac{1}{4} + \frac{3}{2} \frac{1}{1-x} - \frac{3}{4} \frac{1}{(1-x)^2} \right) \ln x + (y \leftrightarrow x) - \frac{3}{4} \frac{1}{(1-y)(1-x)} \right]$$

Also $G_F \approx 10^{-5} m_p^{-2}$ is the weak coupling constant,

$$\eta_1 \approx 0.7, \quad \eta_2 = 0.6, \quad \eta_3 = 0.5 \quad (31)$$

are QCD enhancement factors as calculated by Gilman and Wise,²⁷ and

$$B = \frac{3}{8 F_K^2 m_K^2} \langle \bar{K}^0 | \bar{s} \gamma_\mu (1 + \gamma_5) d \bar{s} \gamma^\mu (1 + \gamma_5) d | K^0 \rangle \quad (32)$$

is a measure of the matrix element of the $\Delta S=2$ short distance operator compared to its so-called "factorization" value. The parameter B has been extracted from $\Delta I = \frac{3}{2}$ $K\pi\pi$ data by use of PCAC and SU(3) symmetry with the result²⁸

$$B = 0.33 \pm 0.33 \quad (33)$$

where the generous error is assigned given the possibly significant errors associated with SU(3) breaking in the π -K system and is the off-mass-shell extrapolation of the physical $K \rightarrow 2\pi$ amplitude.³ In practice, however, knowing B to within a factor of two is adequate, because the uncertainties due to other effects (m_t and the KM angles) are even larger.

In order to estimate the size of ϵ , we can now insert numerical values. If we use $m_c = 1.5$ GeV and $m_t = 40$ GeV, 60 GeV or 100 GeV, we find

$$\begin{aligned} \frac{\text{Im } M_{12}}{(\Delta m)_{\text{expt}}} &= s_2 s_3 s_8 B [1.9 + 430 K] & (m_t = 40 \text{ GeV}) \\ &= s_2 s_3 s_8 B [2.1 + 880 K] & (m_t = 60 \text{ GeV}) \\ &= s_2 s_3 s_8 B [2.5 + 2200 K] & (m_t = 100 \text{ GeV}) \end{aligned} \quad (34)$$

Note that we compare here to the experimental value of $\Delta m = m_L - m_S$. A number

of authors have tried to estimate Δm from its short distance contribution²⁹

$$\begin{aligned} \Delta m_{\text{Box}} &= \frac{G_F^2}{6\pi} s_1^2 F_K^2 m_K^2 B [\eta_1 F(x_c) + \eta_2 F(x_t) K^2 + 2\eta_3 G(x_c, x_t)] M_W^2 \\ &\approx B \eta_1 \Delta m_{\text{expt}} \end{aligned} \quad (35)$$

However, it is known that long distance (dispersive) contributions are equally important here³⁰ and there is no way in which to reliably calculate these effects.

We will show below that direct CP violation in the $K \rightarrow 2\pi$ amplitude gives only a small contribution to ϵ . Thus the dominant effect is due to the mass matrix, with

$$\begin{aligned} |\epsilon| &\approx 2 \times 10^{-3} \left(\frac{s_1 s_2 s_3 s_8}{5 \times 10^{-4}} \right) \left(\frac{B}{.33} \right) \quad (m_t = 40 \text{ GeV}) \\ 3.3 \times 10^{-3} &\left(\frac{s_1 s_2 s_3 s_8}{5 \times 10^{-4}} \right) \left(\frac{B}{.33} \right) \quad (m_t = 60 \text{ GeV}) \\ 6.7 \times 10^{-3} &\left(\frac{s_1 s_2 s_3 s_8}{5 \times 10^{-4}} \right) \left(\frac{B}{.33} \right) \quad (m_t = 100 \text{ GeV}) \end{aligned} \quad (36)$$

where we have used $K = 0.005$. From these formulae, it is clear that the KM model can accommodate the experimental value of ϵ . If the t quark turns out to be light and the bound on the KM angles is lowered, it is possible that the KM model could be ruled out. However the most conservative interpretation of these results is that they indicate that the product of KM angles should be close to the present bound.

In order to make further contact with experiment we also require the values of "direct" CP violating parameters ξ, ξ' . Since the CP violating penguin interaction, Fig. 7, is purely $\Delta I = \frac{1}{2}$ we find

$$\xi' = 0. \quad (37)$$

This standard result ignores the electromagnetic penguin diagram³¹ and the effects of isospin breaking.³² We will return to these latter. Assuming that the $\Delta I = \frac{1}{2}$ CP nonconservation arises solely from the penguin diagram,²¹ we find

Fig. 7. The $\Delta S=1$ penguin interaction

$$\xi = s_2 s_3 s_\delta \frac{\ln m_t^2 / m_c^2}{\ln m_c^2 / \mu^2} \frac{\text{Re } c_5 \langle \pi\pi | O_5 | K^0 \rangle}{\Lambda_0} \quad (38)$$

where μ is a mass scale characteristic of bound state quark momenta. Taking $\mu \approx 300$ MeV, $m_t = 40$ GeV one finds

$$\xi = -0.1 s_2 s_3 s_\delta P \quad (39)$$

where P depends on the size of the matrix element $\langle \pi\pi | O_5 | K^0 \rangle$ and is found, in a range of different models, to be in the range³³

$$0.7 < P < 2.5 \quad (40)$$

We find then in the standard model (for $m_t = 40$ GeV)

$$\left| \frac{\epsilon'}{\epsilon} \right| = \frac{0.1 P \omega}{B(1.9 + 430 K) - 0.1 P} \quad (41)$$

Using

$$K \approx 0.005 \quad (42)$$

from the B decay lifetime and the $b \rightarrow u/b \rightarrow c$ ratio³⁴ we find

$$\left| \frac{\epsilon'}{\epsilon} \right| = \frac{1}{22 \left(\frac{40 B}{P} - 1 \right)} \quad (43)$$

Thus even in a pessimistic scenario — $B \approx 0.5$, $P \approx 0.7$ — we have

$$\left| \frac{\epsilon'}{\epsilon} \right| = 0.002 \quad (44)$$

which represents a reasonable lower estimate of the size of $|\epsilon'|$. [Due to the uncertainty in the hadronic matrix elements we do not feel that a rigorous lower bound on ϵ'/ϵ exists.]

With other values of B, P , the predicted size of $|\epsilon'/\epsilon|$ can be as large as 0.01. The dependence of this analysis on the top quark mass is such that at large values of m_t , the expected magnitude of $|\epsilon'/\epsilon|$ decreases. Roughly, the penguin operator coefficient increases as $\ln m_t^2 / m_c^2$ while the box diagram goes as m_t^2 for large m_t . Thus the relative size of penguin diagram to box diagram (which is what ϵ'/ϵ represents) decreases.

Recently it was realized that the standard analysis of ϵ'/ϵ given above can be modified by electromagnetic and isospin breaking terms.³²

The point is that CP violation in the $I=2$ final state is 22 times more effective in contributing to ϵ' than that in the $I=0$ final state. This occurs because ϵ' is equally sensitive to $\text{Im } A_0/\text{Re } A_0$ and $\text{Im } A_2/\text{Re } A_2$ (cf. Eq. 23) and $\text{Re } A_2$ is 22 times smaller than $\text{Re } A_0$.

$$\begin{aligned} \epsilon' &= \frac{i\omega e}{\sqrt{2} \text{Re } A_0} \left[\frac{\text{Re } A_0}{\text{Re } A_2} \text{Im } A_2 - \text{Im } A_0 \right] \\ &= \frac{i\omega e}{\sqrt{2} \text{Re } A_0} [22 \text{Im } A_2 - \text{Im } A_0] \end{aligned} \quad (45)$$

Thus any interaction which can give a phase to A_2 is 22 times more effective in producing ϵ' than one which contributes only to A_0 . The usual penguin diagram only generates A_0 . Electromagnetic corrections, which should be suppressed by α/α_s , and isospin breaking effects, suppressed by $(m_d - m_u)/m_s \sim 1/30$, contribute to A_2 . Since $22 \alpha/\alpha_s$ and $22(m_d - m_u)/m_s$ are near unity, this isospin enhancement of small effects could in principle lead to major modifications in the analysis. Because it is important to have these effects under control, we present the revised analysis in some detail.

For the dominant component of the CP violating electromagnetic-weak interaction we utilize the so-called "electromagnetic penguin" diagram, Fig. 8, which has the same topology as the penguin, but with the gluon replaced by a photon. The resulting effective Hamiltonian has the form³¹

$$H_{\text{emp}} = - \frac{G_F}{\sqrt{2}} s_1 c_1 c_3 (c_7 Q_7 + c_8 Q_8) \quad (46)$$

where

$$\begin{aligned} Q_7 &= \bar{s}_a \gamma_\mu (1 + \gamma_5) d_a (\bar{u}_b \gamma^\mu (1 - \gamma_5) u_b - \frac{1}{2} \bar{d}_b \gamma^\mu (1 - \gamma_5) d_b - \frac{1}{2} \bar{s}_b \gamma^\mu (1 - \gamma_5) s_b) \\ Q_8 &= \bar{s}_a \gamma_\mu (1 + \gamma_5) d_b (\bar{u}_b \gamma^\mu (1 - \gamma_5) u_a - \frac{1}{2} \bar{d}_b \gamma^\mu (1 - \gamma_5) d_a - \frac{1}{2} \bar{s}_b \gamma^\mu (1 - \gamma_5) s_a) \end{aligned} \quad (47)$$

and

Fig. 8. The "electromagnetic penguin" diagram in $\Delta S=1$ transitions

$$c_7 \approx (0.037 - 0.067 \tau) \alpha$$

$$c_8 \approx (0.008 - 0.11 \tau) \alpha$$
(48)

Here α is the fine structure constant and

$$\tau = s_2^2 + \frac{s_2 c_2 s_3}{c_1 c_3} e^{-i\delta}$$
(49)

For the iso-spin mixing term we utilize the conventional quark mass term

$$H_{\text{ISB}} = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s$$
(50)

(Electromagnetic contributions to isotopic spin breaking exist also, of course, but are small and can be neglected.)

The important features of these interactions are summarized via their chiral transformation properties. Thus the conventional penguin interaction, the electromagnetic penguin, and the quark mass term transform respectively as $(8, 27_L, 1_R)$, $(8_L, 8_R)$ and $(3_L, \bar{3}_R)$ under chiral rotations. These properties can be described most succinctly in terms of effective Lagrangians involving pseudo-scalar octet fields ϕ^A in the combination³⁵

$$U = \exp(i \sum_a \phi^a \lambda^a / F_\pi)$$
(51)

Thus the three interactions above have the lowest order chiral Lagrangians

$$L_p = g_p \text{Tr} \lambda_6 \partial_\mu U \partial^\mu U^\dagger$$

$$L_{\text{emp}} = g_{\text{emp}} \text{Tr} \lambda_6 U \lambda_8 U^\dagger$$

$$L_m = \text{Tr}(a + b\lambda_3 + c\lambda_8)(U + U^\dagger)$$
(52)

In terms of these forms, we find for A_0, A_2

$$A_0 = A_0^{(0)} + A_p + \frac{2}{3} \theta_\eta A(K^0 + \pi^0 \eta) + \frac{2}{3} \theta_{\eta'} A(K^0 + \pi^0 \eta')$$

$$A_2 = A_2^{(0)} + \frac{1}{3} \sqrt{2} A_{\text{emp}} - \frac{2}{3} \sqrt{2} \theta_\eta A(K^0 + \pi^0 \eta) - \frac{2}{3} \sqrt{2} \theta_{\eta'} A(K^0 + \pi^0 \eta')$$
(53)

where

$$\theta_\eta = \frac{\langle \pi^0 | H_W | \eta \rangle}{\frac{m_\pi^2 - m_\eta^2}{2}}, \quad \theta_{\eta'} = \frac{\langle \pi^0 | H_W | \eta' \rangle}{\frac{m_\pi^2 - m_{\eta'}^2}{2}}$$

$$A_p = \frac{1}{\sqrt{2} F_\pi} \langle \pi^+ | H_p | K^+ \rangle, \quad A_{\text{emp}} = \frac{-1}{\sqrt{2} F_\pi} \langle \pi^+ | H_{\text{emp}} | K^+ \rangle$$
(54)

Here we have also included η' effects, which, strictly speaking, lie outside of the effective Lagrangian framework. Putting in numerical factors one then finds

$$|\epsilon'| = |\epsilon'^{(0)}| (1 - \Omega_{\text{emp}} - \Omega_{\eta, \eta'})$$

where

$$\Omega_{\text{emp}} \approx -0.38(1 \pm 50\%)$$
(55)

and, including η - η' mixing Fig. 9

$$\Omega_{\eta, \eta'} = 0.34 - 0.46$$
(56)

where the range is over different values of the η, η' mixing angle from $\theta=0$ to $\theta = -20^\circ$. We see that although each effect is large, they tend to cancel. [Note that a sign mistake lead to the opposite conclusion in Ref. 32.] The overall estimate is somewhat uncertain because of the strong cancellation; however we find

$$0.7 \lesssim (1 - \Omega_{\text{emp}} - \Omega_{\eta, \eta'}) \lesssim 1.3$$
(57)

This conclusion is reassuring for the attempts to measure ϵ' as it indicates that the signal should be within the reach of the present generation of experiments. The theory described above suggests that

$$0.002 \leq \frac{\epsilon'}{\epsilon} \leq 0.01$$
(58)

although the upper end of this range is already somewhat disfavored by experiment.

A summary of all of the CP violating contributions to $K \rightarrow 2\pi$ within the KM model is given in Table II.

Moving to the Higgs model, several years ago it was suggested that the measured value of ϵ'/ϵ had already ruled out such a picture of CP violation.^{14,36} However, recent analysis indicates that the model is viable.³⁷ The argument here is straightforward. The dominant $\Delta S=0$ and $\Delta S=1$ operators are found to be (here we use the 4-quark version - inclusion of six quarks does not change the conclusions)

Fig. 9. The diagram which generates isospin breaking via η - η' mixing

Summary of Contributions to CP Violation in $K \rightarrow 2\pi$

Process	Figure	Strangeness Change	Comments
Box diagram	6	$\Delta S=2$	Dominant contribution to ϵ $\epsilon \approx 4 s_1 s_2 s_3 s_6$ (B/0.33) m_t is largest unknown at present.
Dispersive effects	4	$\Delta S=2$	Small if ϵ'/ϵ is small
Siamese penguin	5	$\Delta S=2$	Only a few percent of the box diagram
Penguin diagram	7	$\Delta S=1$	Major contribution to ϵ' $\epsilon'_p \approx 0.002 \pm 0.01$
Electromagnetic penguin and isospin breaking	8,9	$\Delta S=1$	$\epsilon' = \epsilon'_p [1 - \Omega_{EMP} - \Omega_{\eta, \eta'}]$ $\approx (0.7 \rightarrow 1.3) \epsilon'_p$

Table II. Summary of contributions to CP violation in $K \rightarrow 2\pi$ within the KM model

$$L^{\Delta S=1} = i \tilde{f} \bar{d} \sigma^{\mu\nu} (1 - \gamma_5) \frac{\lambda^a}{2} s F_{\mu\nu}^a \quad (59)$$

$$L^{\Delta S=2} = i \tilde{g} \bar{d}_i \gamma_\mu (1 + \gamma_5) s_j \partial_\nu d_i \gamma^\mu \gamma^\nu (1 - \gamma_5) s_j \quad (60)$$

with

$$\tilde{f} = \frac{G_F}{\sqrt{2}} \frac{g_s}{32\pi^2} m_c^2 m_c \cos\theta_c \sin\theta_c \sum_{i=1}^2 \frac{\text{Im } \gamma_i}{m_{H_i}} \left(\ln \frac{m_{H_i}^2}{m_c^2} - \frac{3}{2} \right)$$

$$\tilde{g} = \frac{G_F^2}{32\pi^2} m_c^2 m_s \cos^2\theta_c \sum_{i=1}^2 \frac{\text{Im } \gamma_i}{m_{H_i}} \quad (60)$$

and

$$\text{Im } \gamma_1 = -\text{Im } \gamma_2 = \frac{s_2^1 s_3^1 c_1^1}{c_1^1 c_2^1} \quad (61)$$

where the $\Delta S=1$ term arises from the Higgs penguin diagram, Fig. 2b, and the effective $\Delta S=2$ interaction from box diagrams including a single Higgs and single W-boson. The matrix element of $L^{\Delta S=2}$ can be estimated, say in the MIT bag model, yielding the short distance contribution to $\text{Im } M_{12}$,³⁸

$$m' = \tilde{g} \times (6 \times 10^{-3} \text{ GeV}^4) \quad (62)$$

The size of the $\Delta S=1$ interaction can also be calculated using the bag model, yielding³⁶

$$\langle \pi^0 | L^{\Delta S=1} | K^0 \rangle = \tilde{f} (0.4 \text{ GeV}^3) \quad (63)$$

Although it is hard to directly compare quantities with different dimensions, the $\Delta S=1$ effect turns out to be dominant over that with $\Delta S=2$. This can be seen by considering dispersive contributions to the mass matrix as first suggested by Chang, and Dupont and Pham.³⁸ The long distance estimate of $K^0 \rightarrow \pi, \eta, \eta' \rightarrow \bar{K}^0$ using Eq. 63 yields a contribution to $\text{Im } M_{12}$ of

$$\frac{\text{Im } M_{12} \big|_{\text{dispersive}}}{\text{Im } M_{12} \big|_{\text{short distance}}} = 12 \left[\ln \frac{m_H^2}{m_c^2} - \frac{3}{2} \right] \gg 1 \quad (64)$$

which is much more important than the short distance contribution. Thus $\Delta S=1$ CP violation appears dominant in the Higgs model, in strong contrast with the KM model.

CP-odd $\Delta S=1$ effects will also generate ϵ' . This calculation is somewhat subtle, as chiral symmetry requires that the leading order contribution of $L^{\Delta S=1}$ to $\pi^0 \rightarrow \pi^0$ must vanish. This occurs because the operator has a $(3_L, \bar{3}_R)$ transformation property under chiral SU(3) which is the same as the structure of the mass term in the strong interaction. This leads to the ability to remove the lowest order $\Delta S=1$ interactions by using a chiral rotation to diagonalize all $(3, \bar{3})$ terms.³⁹ In more straightforward terms, this implies an exact cancellation between the two diagrams in Fig. 10, when both are evaluated to lowest order in chiral symmetry. Estimates of higher order effects are quite uncertain, but a simple dimensional consideration

$$\text{Amp}(K_L \rightarrow \pi^0 \pi^0) = - \frac{1}{\sqrt{2} F_\pi} \langle \pi^0 | L^{\Delta S=1} | K^0 \rangle \frac{m_K^2}{\Lambda^2} \quad (65)$$

with $\Lambda \approx 1 \text{ GeV}$, was used in Ref. 37 to estimate

$$\epsilon'/\epsilon = -0.006 \quad (66)$$

Earlier estimates³⁶ which obtained much larger values of ϵ'/ϵ were incorrect in their treatment of the chiral properties and/or dispersive effects.

The last picture of CP violation which we consider here is the isoconjugate left-right symmetric model described previously. In this model the imaginary part of $K^0 \bar{K}^0$ mixing proceeds through the box diagram with one W_L and one W_R , as in Fig. 11. This yields¹⁶

$$\epsilon = \frac{G_F^2}{12\sqrt{2} \pi^2} \frac{s_1^2 F_K^2 m_K^2}{\Delta m} B \{ 60 C_{LR} \frac{M_{LR}}{M_{LL}} \eta \beta \} \quad (67)$$

where

$$\eta = \frac{M_{W_L}^2}{M_{W_R}^2} \quad (68)$$

C_{LR} is the QCD correction factor $C_{LR} \sim 3$, M_{LL} and M_{LR} are matrix elements

$$\frac{M_{LR}}{M_{LL}} = \frac{\langle K^0 | \bar{s}_R d_L \bar{s}_L d_R | \bar{K}^0 \rangle}{\langle K^0 | \bar{s}_L \gamma_\mu d_L \bar{s}_L \gamma^\mu d_L | \bar{K}^0 \rangle} \quad (69)$$

(a)

(b)

Fig. 10. Direct and tadpole diagrams which cancel for a $(3, \bar{3})$ operator

and β is the CP violating phase. Despite the small value of η , this can easily fit the value of ϵ .

In $\Delta S=1$ processes the Hamiltonian has the form

$$H_w = \frac{G_F}{\sqrt{2}} s_1 c_1 (O_{LL} + \eta e^{i\beta} O_{RR}) \quad (70)$$

where O_{LL} and O_{RR} are identical operators except that O_{LL} is the product of two left-handed currents, while O_{RR} has two right-handed currents. Because of this structure one can see that all parity violating amplitudes have phase given by $1 - \eta e^{i\beta}$ and parity conserving ones have phase $1 + \eta e^{i\beta}$. [Estimates put $\eta\beta \sim O(10^{-4})$]. Thus both A_0 and A_2 share a common phase

$$\frac{\text{Im } A_0}{\text{Re } A_0} = \xi = \xi' = \frac{\text{Im } A_2}{\text{Re } A_2} = -\eta\beta \quad (71)$$

and hence

$$\epsilon' = 0 \quad (72)$$

i.e., in the $K \rightarrow 2\pi$ sector this model is indistinguishable from the superweak theory, although we will see that the two theories disagree for other decay processes.

In Table III we summarize the general patterns of CP violation which have emerged in the different models. One can see that each theory has a different structure. With a nonzero measurement of ϵ'/ϵ , one could rule out two of these models. However present bounds do not distinguish between them.

Fig. 11. The LR box diagram in $SU(2)_L \times SU(2)_R \times U(1)$ theories

Model	Short Distance M_R	Dispersive M_{12}	$\Delta S=1$ CP Violation	ϵ'/ϵ
Kobayashi Maskawa	Dominant	Small	Penguin diagrams	0.002 ± 0.01
Superweak	Total effect	None	None	0
Charged Higgs	Small	Large	Important but chirally suppressed in $K \rightarrow 2\pi$	$0.006 \pm ?$
Isoconjugate LR	Dominant	Small	Present, but common phase in A_0, A_2	0

Table III. The pattern of CP violation seen in $K \rightarrow 2\pi$ for a representative set of models.

IV. CP Violation and $K \rightarrow 3\pi$

The decay process most similar to $K \rightarrow 2\pi$ is probably $K \rightarrow 3\pi$, and considerable experimental and calculational attention has also been paid to this mode. There exist several signatures of CP violation in this case. Thus, for example, considering the decay modes $K_{L,S} \rightarrow \pi^+ \pi^- \pi^0$, $3\pi^0$ one can define (at the center of the Dalitz plot)

$$\eta_{+-0} \equiv \frac{\langle \pi^+ \pi^- \pi^0 | H_W^{(-)} | K_S \rangle}{\langle \pi^+ \pi^- \pi^0 | H_W^{(+)} | K_L \rangle} = \epsilon + \epsilon'_{+-0}$$

$$\eta_{000} \equiv \frac{\langle \pi^0 \pi^0 \pi^0 | H_W^{(-)} | K_S \rangle}{\langle \pi^0 \pi^0 \pi^0 | H_W^{(+)} | K_L \rangle} = \epsilon + \epsilon'_{000}$$
(73)

[Note here that there exists also a well-known CP conserving contribution to $K_S \rightarrow \pi^+ \pi^- \pi^0$ when the three pions are in an $I=2$ state.⁴⁰ This term, however, arising from the $\Delta I = \frac{3}{2}$ component of H_W , is quite small. Also, its kinematic form requires proportionality to $E_+ - E_-$, so that there is no contribution at the center of the Dalitz plot. Thus there should be no difficulty distinguishing this term from the bona fide CP violating amplitude.]

Of course, measurement of these quantities is much more difficult than in the $K \rightarrow 2\pi$ case due to the short lifetime of the K_S meson, and the current precision with which these numbers are determined mirrors this fact:⁴¹

$$\eta_{+-0} = (0.005 \pm 0.17) + i(0.15 \pm 0.33)$$

$$\eta_{000} = (0.08 \pm 0.18) + i(-0.05 \pm 0.27)$$
(74)

A second test of CP invariance in the neutral kaon system is provided by examination of the charge asymmetry, wherein one compares the number of events in the left and right-hand sides of the Dalitz plot in the decay $K_L \rightarrow \pi^+ \pi^- \pi^0$

$$a = \frac{N_R - N_L}{N_R + N_L}$$
(75)

Experimentally, writing the decay rate in terms of

$$|M|^2 \approx 1 + g \frac{s_3 - s_0}{2} + j \frac{s_+ - s_-}{2} + \dots$$
(76)

where

$$s_i = (k - p_i)^2$$

$$s_0 = \frac{1}{3} (s_3 + s_+ + s_-)$$
(77)

one finds for the CP violating parameter j ⁴²

$$j = 0.001 \pm 0.011$$

$$j = 0.001 \pm 0.003$$
(78)

Moving to the charged kaon decay modes

$$\tau : K^+ \rightarrow \pi^+ \pi^+ \pi^-$$

$$\tau' : K^+ \rightarrow \pi^+ \pi^0 \pi^0$$

one can define the partial rate asymmetries

$$a(\tau) = \frac{d\Gamma(\tau^+) - d\Gamma(\tau^-)}{d\Gamma(\tau^+) + d\Gamma(\tau^-)}$$

$$a(\tau') = \frac{d\Gamma(\tau'^+) - d\Gamma(\tau'^-)}{d\Gamma(\tau'^+) + d\Gamma(\tau'^-)}$$
(79)

Experimental limits are available here only for $\bar{a}(\tau)$, where the bar signifies the total decay rate, summed over the Dalitz plot⁴³

$$\bar{a}(\tau) = -0.0002 \pm 0.001$$

$$\bar{a}(\tau) = -0.0025 \pm 0.005$$
(80)

One can also compare slope parameters as a probe of CP violation

$$s(\tau) = \frac{g(\tau^+) - g(\tau^-)}{g(\tau^+) + g(\tau^-)}$$
(81)

Experimentally, one finds here⁴⁴

$$s(\tau) = 0.035 \pm 0.035$$
(82)

Thus, at present no evidence has yet surfaced suggesting that CP is violated in the $K \rightarrow 3\pi$ system, although admittedly present experimental precision is rather crude. Now let's see what various models have to say.

The simplest model, of course, is the superweak, which predicts that the only source of CP noninvariance is in the mass matrix, so that

$$\eta_{+-0} = \eta_{000} = \epsilon \quad (83)$$

with all probes of direct $\Delta S=1$ CP violation being required to vanish

$$\epsilon'_{+-0} = \epsilon'_{000} = a(\tau) = a(\tau') = \dots = 0 \quad (84)$$

Moving on to consider the KM model, we again find a situation wherein recent developments have superceded earlier expectations. The conventional approach has been to apply current algebra/PCAC techniques to an expansion of the $K \rightarrow 3\pi$ decay amplitudes which is linear in the pion energy,⁴⁵ e.g.

$$\langle \pi^+ \pi^- \pi^0 | H_W | K^0 \rangle = \alpha_1 + \alpha_3 - (\beta_1 + \beta_3) \frac{s_3 - s_0}{m_\pi^2} + \gamma_3 \frac{s_+ - s_-}{m_\pi^2} \quad (85)$$

$$\langle \pi^+ \pi^- \pi^0 | H_W | K^0 \rangle = 3(\alpha_1 + \alpha_3)$$

etc.

Here the various amplitudes $\alpha_i, \beta_i, \dots$ are, in general, complex and the subscript $i=1,3$ denotes that the term arises from that component of H_W which carries $\Delta I = 1/2, 3/2$ respectively. Since the Q-value is so small for the $K \rightarrow 3\pi$ system - $Q \approx 80$ MeV - we temporarily neglect the strong interaction phases associated with the 3π final state.

Since only left-handed currents are involved, we note the useful result that

$$[F_5^i, H_W] = [F^i, H_W] \quad (86)$$

so that in taking the soft pion limit we have

$$\langle \pi^a \pi^b \pi^c | H_W | K^0 \rangle \xrightarrow{q_c \rightarrow 0} -\frac{i}{F_\pi} \langle \pi^a \pi^b | [F^i, H_W] | K^0 \rangle \quad (87)$$

However, since the F^i ($i=1,2,3$) are merely isospin generators, we find that the off shell amplitudes for $\Delta I = \frac{1}{2}, (\frac{3}{2})$ transitions in $K \rightarrow 3\pi$ are given in terms of the physical $\Delta I = \frac{1}{2}, (\frac{3}{2})$ transition amplitudes for the $K \rightarrow 2\pi$ system. Thus, for example

$$\langle \pi^a \pi^b \pi^c | H_W | K^0 \rangle \xrightarrow{q_3 \rightarrow 0} \frac{i}{2F_\pi} \langle \pi^a \pi^b | H_W^i | K^0 \rangle \quad i = \frac{1}{2}, \frac{3}{2} \quad (88)$$

yielding

$$\alpha_1 = \frac{i}{6F_\pi} A_0, \quad \beta_1 = -\frac{i}{2F_\pi} A_0 \frac{m_\pi^2}{m_K^2 - m_\pi^2} \quad (89)$$

$$\alpha_3 = -\frac{i}{6F_\pi} \sqrt{2} A_2, \quad \beta_3 = -\frac{i\delta}{4\sqrt{2}F_\pi} A_2 \frac{m_\pi^2}{m_K^2 - m_\pi^2}$$

Similarly, looking at charged kaon decay or by taking a charged pion soft we find

$$\gamma_3 = i \frac{9}{4\sqrt{2}F_\pi} A_2 \frac{m_\pi^2}{m_K^2 - m_\pi^2} \quad (90)$$

It is easy to see then that at the center of the Dalitz plot

$$\epsilon'_{000} = \epsilon'_{+-0} = i \frac{\text{Im}(\alpha_1 + \alpha_3)}{\text{Re}(\alpha_1 + \alpha_3)} - i\xi$$

$$= i\sqrt{2} \omega\xi = -2\epsilon' \quad (91)$$

which is the prediction of Li and Wolfenstein.⁴⁶ [There exists some uncertainty here with respect to dealing with final state phases, but this is not large.]

Recent developments, however, suggest that ϵ'_{000} and ϵ'_{+-0} are in fact considerably larger. There are two additional pieces which must be added to the analysis. One, of course, is the inclusion of electromagnetic and isospin breaking effects, as already done for the case of $K \rightarrow 2\pi$. A second feature, however, is the addition of higher order terms in the weak chiral Lagrangian, such as

$$L \sim \frac{g}{\Lambda_1^2} \text{Tr} \lambda_6 \partial_\mu U \partial^\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger$$

$$+ \frac{g_2}{\Lambda_2^2} \text{Tr} \lambda_6 \partial_\mu U \partial_\nu U^\dagger \partial^\mu U \partial^\nu U^\dagger + \dots \quad (92)$$

Here the chiral scale parameters Λ_1, Λ_2 are expected to be of order one GeV or so.⁴⁸ Note that such Lagrangians should then contribute to $K \rightarrow 3\pi$ at the level

$$O(m_K^2/\Lambda^2) \sim 25\%$$

and are in fact seen at this level in the data.⁴⁹ However, since the transition amplitude vanishes upon taking any of the mesons soft, there exist no restrictions to these Lagrangians coming from the $K \rightarrow 2\pi$ sector.

The effects of these additional contributions is substantial. In the case of the electromagnetic penguin and isospin breaking terms one finds⁴⁷

$$\epsilon'_{+-0} = -(2+4)\epsilon' \quad (93)$$

which is only a modest enhancement. However, the higher order chiral terms are even more important since they permit CP violation to occur in the $\Delta I = \frac{1}{2}$ sector which is unrelated to that found in $K \rightarrow 2\pi$. Since one does not have a reliable way in which to estimate $\text{Im } g_1$, $\text{Im } g_2$ the actual size of the expected effect cannot be precisely determined. Nevertheless, within the context of a specific model - vacuum saturation - one finds⁴⁷

$$\epsilon'_{+-0}(\text{higher order L}) \approx i 10 \epsilon' \quad (94)$$

Thus these additional effects increase the expected size of the direct CP violating signal by about an order of magnitude from that which had been expected.

For the charged kaon sector, it is necessary to deal with the final state interactions in the three pion final state. Thus, defining

$$\begin{aligned} \alpha_1 &\rightarrow \alpha_1 e^{i\delta_1} \\ \beta_1 &\rightarrow \beta_1 e^{i\delta_2} \\ \gamma_3 &\rightarrow \gamma_3 e^{i\delta_3} \end{aligned} \quad (95)$$

we find that

$$a(\tau) \approx \frac{27}{4} \frac{s_3 - s_0}{m_K^2 - m_\pi^2} (\sin(\delta_1 - \delta_2) + 3 \sin(\delta_1 - \delta_3)) |\epsilon'| \quad (96)$$

as derived by Avilez.⁵⁰ If one uses the final state interaction phases as given by Zeldovich⁵¹ and averages over the Dalitz plot, the asymmetry turns out to be

$$\bar{a}(\tau) \approx 0.1 |\epsilon'| \quad (97)$$

This expectation is raised somewhat by the inclusion of the higher order terms

in the $K \rightarrow 3\pi$ amplitude. In the same manner as described in the ϵ'_{+-0} section above, the higher order terms allow an interference within the $\Delta I=1/2$ sector of the theory, and can raise the expected asymmetry by an order of magnitude⁵²

$$a(\tau) \lesssim |\epsilon'| \quad (98)$$

The charge asymmetry in $K_L \rightarrow \pi^+ \pi^- \pi^0$ can be estimated by use of PCAC. Using the definition of the asymmetry in Eq. (75) and the expansion of the matrix element, Eq. (76), we find that j is given by⁵³

$$\begin{aligned} j &= \frac{2\text{Re } \gamma_3}{\text{Re}(\alpha_1 + \alpha_2)} \text{Re}\{e^{i(\delta_3 - \delta_1)} [\epsilon + i(\frac{\text{Im } \gamma_3}{\text{Re } \gamma_3} - \frac{\text{Im } A_0}{\text{Re } A_0})]\} \\ &= \frac{2\text{Re } \gamma_3}{\text{Re}(\alpha_1 + \alpha_3)} \{\text{Re}(\epsilon e^{i(\delta_3 - \delta_1)}) - \sin(\delta_3 - \delta_1) (\frac{\text{Im } \gamma_3}{\text{Re } \gamma_3} - \frac{\text{Im } A_0}{\text{Re } A_0})\} \end{aligned} \quad (99)$$

Using the PCAC relations this reduces to

$$\begin{aligned} j &= \frac{27}{2\sqrt{2}} \frac{m_\pi^2}{m_K^2} \frac{\omega}{1 - \sqrt{2}\omega} [\text{Re}(\epsilon e^{i(\delta_3 - \delta_1)}) - \sin(\delta_3 - \delta_1) (\frac{\text{Im } A_2}{\text{Re } A_2} - \frac{\text{Im } A_0}{\text{Re } A_0})] \\ &= \frac{27}{2\sqrt{2}} \frac{m_\pi^2}{m_K^2} \frac{\omega}{1 - \sqrt{2}\omega} [\text{Re}(\epsilon e^{i(\delta_3 - \delta_1)}) - \frac{\sqrt{2}\epsilon' \sin(\delta_3 - \delta_1)}{\omega}] \end{aligned} \quad (100)$$

There are several problems with use of this observable as a test of non- ϵ physics. The non- ϵ signal is small because the phase shifts are not large. Moreover the ϵ effect itself is uncertain because of the presence of the poorly known phase shifts in that term. Thus, even if j could be measured, it would be difficult to interpret.

Overall, we see then that rather than the relation

$$|\epsilon'/\epsilon| \lesssim 0.1 \omega \quad (101)$$

which sets the scale for CP violating effects in $K \rightarrow 2\pi$, a more appropriate estimate in the $K \rightarrow 3\pi$ sector is

$$|\epsilon'_{+-0}/\epsilon| \sim 10 |\epsilon'/\epsilon| \sim 0.05 \quad (102)$$

since higher order chiral terms effectively remove the $\Delta I = \frac{3}{2}$ suppression factor ω .

While this is encouraging, such direct CP violating effects in $K \rightarrow 3\pi$ seem considerably beyond the range of present experiments.

Interestingly the Higgs model analysis is remarkably similar. The point here is that in the dominant Higgs penguin diagram the light d-quarks in the CP-violating Lagrangian appear only left-handedly. The commutation relation

$$[F_i^5, H_W] = [F_i, H_W] \quad i = 1, 2, 3 \quad (103)$$

is valid for the Higgs Hamiltonian. Since our previous lowest order KM model predictions arose solely from this assumption, the same results must exist for the Higgs model - when expressed in terms of $K \rightarrow 2\pi$ experimental parameters. The models are indistinguishable.

Of course, to the extent that mixing, isospin breaking and higher order chiral effects were important in the standard model, one expects analogous corrections to the Higgs picture predictions, although these have not yet been calculated. If they are similar to the KM model, then roughly speaking, any CP violating quantity is expected to be of order

$$|\epsilon'_{+0}/\epsilon| \sim 10|\epsilon'/\epsilon| \sim 0.05 \quad (104)$$

as before. The precise size is unpredictable, however, because of uncertainties associated with both the higher order and lower order Lagrangians for this case.

We found previously that the isoconjugate left-right model was for $K \rightarrow 2\pi$ decays indistinguishable from the superweak theory. This is no longer the case in the $K \rightarrow 3\pi$ sector, since the parity conserving left- and right-handed Hamiltonians differ by not only a phase but also by an overall sign. We find then

$$\epsilon'_{+0} = \epsilon'_{000} = -2i \frac{M_L^2}{M_R^2} \sin \beta \quad (105)$$

Thus in this picture there exists a substantial direct decay signal in the parity conserving channel $K \rightarrow 3\pi$ even though the direct emission vanishes identically for the parity violating decay $K \rightarrow 2\pi$.

V. $K_L \rightarrow \gamma\gamma$ and $K_S \rightarrow \gamma\gamma$

The rare decays $K_{L,S} \rightarrow \gamma\gamma$ have proved important in restricting the existence of flavor changing neutral currents. However, these reactions can also serve as a test of CP symmetry. The general effects to be studied have been described by Sehgal and Wolfenstein,⁵⁴ while the standard model analysis has been given by Golowich⁵⁵ and Chau and Chang.⁵⁶

The final photon configuration can be in either of two states

$$|FF\rangle \equiv \frac{1}{\sqrt{2}} [|LL\rangle + |RR\rangle] \quad (106)$$

$$|F\bar{F}\rangle \equiv \frac{1}{\sqrt{2}} [|LL\rangle - |RR\rangle]$$

where the former is CP even and the latter CP odd. The symbols L(R) refer to left- (right-) handed circular polarizations for the photons. If CP were conserved, then $K_S \rightarrow FF$ and $K_L \rightarrow F\bar{F}$. The standard description of these processes involves one and two meson intermediate states, i.e. $K_S \rightarrow \pi\pi \rightarrow \gamma\gamma$ and $K_L \rightarrow \pi^0, \eta, \eta' \rightarrow \gamma\gamma$ with diagrams such as Figs. 12, 13. Possible uncertainties in the pole model analysis of $K_L \rightarrow \gamma\gamma$ have recently been discussed in Ref. 57. However, in the case of $K_S \rightarrow \gamma\gamma$, the use of chiral Lagrangians has recently been shown to produce a finite and parameter-free prediction at the one loop level.⁵⁸

The remaining two possible transitions can be produced in the presence of CP violation

$$\eta_{FF} \equiv \frac{\langle FF | H | K_L \rangle}{\langle FF | H | K_S \rangle} \equiv \epsilon + \epsilon'_{FF} \quad (107)$$

$$\eta_{F\bar{F}} \equiv \frac{\langle F\bar{F} | H | K_S \rangle}{\langle F\bar{F} | H | K_L \rangle} \equiv \epsilon + \epsilon'_{F\bar{F}}$$

As usual, the leading signal in each case is ϵ , and the possibility of new physics is contained in the ϵ'_{FF} terms which arise if there exists CP violation in the direct $K \rightarrow \gamma\gamma$ amplitude. It is perhaps easiest to analyze the magnitude of these direct terms if we work in a phase convention such that the A_0 term in $K \rightarrow 2\pi$,

Fig. 12. The dominant diagram for $K_S \rightarrow \pi \pi \gamma \gamma$ Fig. 13. The diagrams of the pole model for $K_L \rightarrow \pi \pi \gamma \gamma$

is real. This choice of convention will also require that the π^0, η poles in $K \rightarrow \pi^0, \eta \rightarrow \gamma\gamma$ be real up to small $\Delta I = 3/2$ corrections, so that these terms will not generate direct CP violating contributions to ϵ'_{FF} . However, the transitions $K \rightarrow \eta' \rightarrow \gamma\gamma$ will in general not be real, as the penguin diagram contributes with a different strength in $K \rightarrow \eta'$ compared to $K \rightarrow \pi^0$.⁵⁹ Thus the $K \rightarrow \eta' \rightarrow \gamma\gamma$ transition produces direct CP violation to the FF final state with estimated strength

$$\epsilon'_{FF} \sim \frac{22}{[1-\Omega]} \epsilon' \quad (108)$$

i.e. there is no $\Delta I = 3/2$ suppression. The CP violating transition to the CP even final state, FF , proceeds through charged pions only, so that at first sight one might suspect

$$\begin{aligned} \eta_{FF} &= \eta_{+-} \\ \epsilon'_{FF} &= \epsilon' \end{aligned} \quad (109)$$

However, similar to what was concluded in ref. 47, modifications to these speculations can be expected, because the $\gamma\gamma$ final state can be sensitive to higher momentum dependence in the weak vertices in a different amount than present in the on shell $K \rightarrow 2\pi$ amplitude. This occurs because the vertices in Fig. 13 involve the loop momentum, so that if the phase of the vertex depends on the momentum scale (as would be expected from higher order terms in the chiral Lagrangian) the phase of a loop calculation and a tree level result could well be different. This possibility has not yet been explored in detail, but on general grounds one would expect a signal of magnitude

$$\epsilon'_{FF} \sim \frac{22}{[1-\Omega]} \epsilon' \frac{m_K^2}{\Lambda_x^2} \quad (110)$$

where the factor of $m_K^2/\Lambda_x^2 \sim 1/4$ reflects the need for higher order momentum dependence (see Sec. IV) while the factor $22/[1-\Omega]$ indicates the absence of the $\Delta I = 3/2$ suppression.

These effects can be observed experimentally in two basic ways.⁵⁴ One probe

is that the photons in $K_{L,S}$ decays will have a net circular polarization if CP is violated, with a magnitude of η_{FF} in $K_L \rightarrow \gamma\gamma$ and η_{FF} for $K_S \rightarrow \gamma\gamma$. However, the measurement of such small effects is probably hopeless in the present generation of experiments. A more feasible method for observing CP violation involves the interference of K_L and K_S in the $\gamma\gamma$ final state. Thus, at a $p\bar{p}$ machine such as LEAR one can use the flavor tagging trick

$$\begin{aligned} p\bar{p} &\rightarrow K^- \pi^+ K^0 \\ &K^+ \pi^- \bar{K}^0 \end{aligned} \quad (111)$$

to identify and follow the decay of a state which initially is a K^0 or $\bar{K}^0$. For example, the time development of a K^0 would have the form

$$\begin{aligned} N(K^0(t) \rightarrow \gamma\gamma) &\sim |A(K_L \rightarrow FF)|^2 [e^{-\Gamma_L t} + |\eta_{FF}|^2 e^{-\Gamma_S t} + 2\text{Re}(\eta_{FF} e^{-i\Delta m t}) e^{-\frac{(\Gamma_L + \Gamma_S)t}{2}}] \\ &+ |A(K_S \rightarrow FF)|^2 [e^{-\Gamma_S t} + |\eta_{FF}|^2 e^{-\Gamma_L t} + 2\text{Re}(\eta_{FF} e^{i\Delta m t}) e^{-\frac{(\Gamma_L + \Gamma_S)t}{2}}] \end{aligned} \quad (112)$$

with the estimates yielding $\Gamma(K_S \rightarrow FF)/\Gamma(K_L \rightarrow FF) \approx 2$. Measurement of such interference effects is in principle feasible at LEAR. However, because of the larger branching ratio the interference in $K \rightarrow 3\pi$ is more likely to be tested first, with $K \rightarrow \gamma\gamma$ studies appearing only after the technique has been perfected in $K \rightarrow 3\pi$.

VI. $K_L \rightarrow \pi^0 e^+ e^-$

Another test of CP in rare K decays involves the decay mode $K_L \rightarrow \pi^0 e^+ e^-$. This reaction can occur even if CP is conserved. However an unusual situation occurs in that the CP violating amplitude is expected to be larger than that which conserves CP. The latter is suppressed due to the fact that the one photon exchange amplitude is forbidden by CP conservation. Thus the leading CP conserving amplitude must proceed via two photon exchange, (Fig. 14), leading to an amplitude of the form

$$A(K_L \rightarrow \pi^0 e^+ e^-) = A_{\gamma\gamma}^{\pi^0} \quad (113)$$

Dimensional analysis suggests that this amplitude has the form

$$A_{\gamma\gamma} \sim G_F m_K^2 \left(\frac{\alpha}{\pi}\right)^2 \frac{m_e}{E_e} \sim 10^{-8} G_F m_K^2 \quad (114)$$

where the final factor of m_e/E_e must occur to generate a helicity violating amplitude as in Eq. (113) from the helicity conserving photon couplings. In contrast, the CP violating process may proceed through one photon exchange, Fig. 15, with an amplitude of the form

$$A(K_L \rightarrow \pi^0 e^+ e^-) = A_{CP} \frac{(k+p)_\mu}{m_K} \bar{u} \gamma^\mu v \quad (115)$$

with $k(p)$ being the momentum of the kaon (pion). In this case dimensional analysis suggests

$$A_{CP} \sim G_F m_K^2 \left(\frac{\alpha}{\pi}\right) \epsilon \sim 10^{-6} G_F m_K^2 \quad (116)$$

It is comparisons such as this which make it plausible that CP violation may well dominate the rate.

This situation has been examined in more detail in Ref. 60, wherein the CP conserving rate was estimated by a dispersive calculation involving the two photon intermediate state $K_L \rightarrow \pi^0 \gamma\gamma$. This latter amplitude is related to that for $K_S \rightarrow \gamma\gamma$ by chiral symmetry,⁵⁸ and overall a rate of

$$\left. \frac{\Gamma(K_L \rightarrow \pi^0 e^+ e^-)}{\Gamma(K_L \rightarrow \pi\pi)} \right|_{\gamma\gamma}^{\text{CP conserving}} = 2.3 \times 10^{-13} \quad (117)$$

Fig. 14. The CP conserving two photon contribution to $K_L \rightarrow \pi^0 e^+ e^-$

Fig. 15. The CP violating one photon contribution to $K_L \rightarrow \pi^0 e^+ e^-$

was found. The CP violating amplitude proceeds through the electromagnetic penguin, and was first calculated by Gilman and Wise.⁶¹ It is interesting because both the mass matrix CP violation (i.e. ϵ) and the CP violation in the direct process (non- ϵ) proceed through the same diagram. The transition is proportional to a coefficient called c_7 by Gilman and Wise, with the property that

$$\text{Im } c_7 = 0.2 s_2 s_3 s_8 > |\epsilon| \text{Re } c_7 = 0.1 |\epsilon| \sim 0.1 s_2 s_3 s_8 \left(\frac{B}{0.33}\right) \quad (118)$$

so that the non- ϵ signal may actually be larger than the effect of ϵ . This is the only place in kaon physics where the reversal occurs. Overall it is found that

$$\frac{\Gamma(K_L \rightarrow \pi^0 e^+ e^-)}{\Gamma(K_L \rightarrow \text{all})} \Big|_{\text{CP-violation}} \sim 4 \times 10^{-12} \quad (119)$$

or roughly an order of magnitude larger than the CP conserving signal. This dominance of the CP-odd effect is not as large as the naive dimensional analysis indicated, but still appears to be the case.

The precise magnitude and relative contributions of the ϵ and non- ϵ terms depend on the QCD renormalization of the electromagnetic penguin. We have used the values calculated by Gilman and Wise.⁶¹ Recently there has been a criticism of the Gilman-Wise treatment of the real part of the penguin diagram.⁶² At the present time the resolution is not clear, but this could modify the numerical results. However, this question can be solved and should not be a long term uncertainty. Another concern could be long distance contributions to $K^0 \rightarrow \pi^0 e^+ e^-$. These have been calculated in the chiral Lagrangian framework⁶⁰ and are found to be smaller than short distance contribution by an order of magnitude. Finally, if the top quark is very massive ($m_t \gg M_W$), second order weak diagrams involving the box diagram and Z^0 exchange could become important, and would have to be added to the analysis.

The present experimental bound on this branching ratio is at the 10^{-6} level, but this should be pushed down dramatically by the current generation of rare

kaon decay experiments. In fact, there appears to be the possibility of reaching the 10^{-12} level in a dedicated experiment. The difficult question which has to be addressed is whether simply observing the mode at this level will be sufficient to demonstrate CP violation. At present the answer to that question is negative. One needs a more convincing case that the CP conserving rate must be below the 10^{-12} level. This could be helped experimentally by the observation of the $K_L \rightarrow \pi^0 \gamma \gamma$ mode (or the related $K_S \rightarrow \gamma \gamma$ process) as this would firmly pin down the normalization of the dispersive component to $K_L \rightarrow \pi^0 e^+ e^-$. Perhaps with more theoretical work a similar consensus could be reached even without the above experimental input. However, a more convincing way to determine the CP property of the transition is by a study of the Dalitz plot for the final $\pi^0 e^+ e^-$ state. The CP conserving and violating amplitudes lead to quite different Dalitz structure and, if enough events could be accumulated, this could confirm the CP violating nature of the transition.

It is also possible to have CP violation in charged particle transitions $K^\pm \rightarrow \pi^\pm e^+ e^-$. Here the signal would be a partial rate difference

$$a = \frac{\Gamma(K^+ \rightarrow \pi^+ e^+ e^-) - \Gamma(K^- \rightarrow \pi^- e^+ e^-)}{\Gamma(K^+ \rightarrow \pi^+ e^+ e^-) + \Gamma(K^- \rightarrow \pi^- e^+ e^-)} \quad (120)$$

This signal requires both CP violation in the decay amplitude (present at the 10^{-3} level from the electromagnetic penguin diagram) and final state interactions (presumably through the $K \rightarrow 3\pi$ γ intermediate state). Although no calculation has yet been done it is clear that $a < 10^{-4}$, and this is far too small to be observed.

VII. $K_L \rightarrow \mu^+ \mu^-$

It has been known for quite a while now that the rare decay $K_L \rightarrow \mu^+ \mu^-$ can be used as a probe of CP non-conservation.⁶⁴ In general, the lepton pair produced by such decay can be in a CP even or odd configuration. Direct observation of the decay rate is not useful as a probe of CP since it is not possible to calculate reliably the transition amplitude with the required accuracy. However, the interference of the two amplitudes gives rise to a CP violating observable, namely the longitudinal polarization of the leptons.

One can write a general decay amplitude as⁶⁵

$$\begin{aligned} A(K_L \rightarrow \mu^+ \mu^-) &= i\bar{u}(a_1 \gamma_5 + ib_1)v \\ A(K_2 \rightarrow \mu^+ \mu^-) &= \bar{u}(a_2 \gamma_5 + ib_2)v \end{aligned} \quad (121)$$

with a_1, b_2 being P and CP violating amplitudes. Defining the K_L decay amplitude as

$$A(K_L \rightarrow \mu^+ \mu^-) = \bar{u}(A \gamma_5 + iB)v \quad (122)$$

the longitudinal polarization of the muon is found to be:

$$P = \frac{m_K \beta_\mu^2}{4\pi\Gamma} \text{Im}(BA^*) \quad (123)$$

where $\beta_\mu^2 = 1 - 4 \frac{m_\mu^2}{m_K^2}$ and

$$\Gamma = \frac{m_K \beta_\mu^2}{8\pi} (|A|^2 + \beta_\mu^2 |B|^2) \quad (124)$$

is the total $K_L \rightarrow \mu^+ \mu^-$ decay rate.

CPT forces a_2 and b_2 to be relatively real up to "unitarity" phases arising from real intermediate states and small electromagnetic final state interactions.⁶⁶ Using Eqs. (121) - (124) and $K_L = K_2 + \epsilon K_1$ we can write (keeping terms up to first order in CP violation):

$$P = \frac{m_K \beta_\mu^2}{4\pi\Gamma} \text{Im}(b_2 a_2^* + i\epsilon b_1 a_2^*) \quad (125)$$

The contributions to the first term come from possible real intermediate states. Within the standard model it is generated in lowest order by the CP even

two photon contribution, Fig. 16. This process is expected⁶⁷ to be dominated by $K_2 \rightarrow \pi\pi + \gamma\gamma$ which is suppressed by a factor $\varepsilon'/\varepsilon \ll 1$ (see Sec. V).

The standard model prediction for $\bar{\rho}$ comes mainly from the K_1, K_2 mixing represented by the second term in Eq. (125). A reliable calculation has not yet been performed, but we can estimate its value following Herczeg⁶⁵ and considering only the absorptive contributions to b_1 and a_2 . With the numbers found in the literature⁶⁸ this turns out to be

$$|\bar{\rho}|_{\text{abs}} = 7.1 \times 10^{-4} \quad (126)$$

Values for $|\bar{\rho}|$ of order 10^{-3} are thus expected within the standard model. In theories other than the KM model larger values can be generated for the first term in Eq. (125) and this would signal the presence of new sources of CP violation. Several estimates have been done. A polarization $|\bar{\rho}| \sim 10^{-2} - 10^{-3}$ is found within left-right-symmetric models,⁶⁹ and Higgs models⁶⁵ can give values as large as allowed by present experimental constraints: $|\bar{\rho}| = 0.96$. It has also been noted that a very light Higgs could generate large values of $|\bar{\rho}|$ within the standard model.⁷⁰

Fig. 16. The two photon contribution to $K \rightarrow \mu^+ \mu^-$

VIII. $K_L \rightarrow \pi^+ \pi^- \gamma$

The decay discussed in this section, $K_L \rightarrow \pi^+ \pi^- \gamma$, is especially interesting because the CP violation is present at a level already seen by present experiments.⁷¹ However, the observed signal - inner bremsstrahlung - yields essentially the same information as gained from $K_L \rightarrow \pi^+ \pi^-$ and so we have not yet learned anything new from this process. Nevertheless, further CP violation beyond η_+ is expected to be present, and the challenge is to uncover it.

The analysis of $K_L \rightarrow \pi^+ \pi^- \gamma$ is conventionally divided^{54,72} into two pieces i) inner bremsstrahlung (IB), which is radiation off the charged pions in the decay $K_L \rightarrow \pi^+ \pi^-$, and ii) direct emission (DE) terms which encompasses all other effects. The inner bremsstrahlung contribution is absolutely normalized⁷² by $K_L \rightarrow \pi^+ \pi^-$

$$\text{Amp}(K_L \rightarrow \pi^+ \pi^- \gamma) = eA(K_L \rightarrow \pi^+ \pi^-) [(p \cdot \epsilon)(q \cdot k) - (k \cdot \epsilon)(p \cdot q)] [(p \cdot q)^2 - \frac{1}{4} (q \cdot k)^2]^{-1} \quad (127)$$

where $p = \frac{1}{2} (p_+ + p_-)$, $k = (p_+ - p_-)$, and q is the photon's momentum.

The lowest multipole photons permitted for the $K_L \rightarrow \pi^+ \pi^- \gamma$ decay are M1 (CP conserving) and E1 (CP violating) while higher multipoles are generally neglected. The most recent experiment⁷¹ was able to separate IB from DE via use of the characteristic bremsstrahlung energy spectrum and found roughly comparable amounts of each.

The CP conserving (M1) component of the direct emission has a matrix element of the form

$$A_{DE}(K_L \rightarrow \pi^+ \pi^- \gamma) = a \epsilon^{\alpha\beta\gamma\delta} K_\alpha p_{+\beta} p_{-\gamma} \epsilon_\delta^* \quad (128)$$

The pions are in a relative P-wave and the amplitude may well be dominated by the long distance terms

$$K_L \rightarrow \pi^0, \eta, \eta' \rightarrow \pi^+ \pi^- \gamma \quad (129)$$

There is also the possibility of a short distance $s \rightarrow d \gamma$ contribution but this is expected to be more than an order of magnitude smaller because of the GIM

Fig. 17. Diagrams relevant for $K_L \rightarrow \pi^+ \pi^- \gamma$

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cancellation,⁷² although this conclusion could be modified if the t quark is very heavy. Many authors have calculated the M1 amplitude a ,⁷³ yielding a result generally somewhat larger but of the same rough size as that found experimentally.

The prime question, as far as CP violation is concerned is whether there exists any E1 strength beyond the IB. Indeed direct emission CP violating terms are expected - Fig. 17c. However, in order to produce a gauge invariant amplitude, three factors of momentum are required. If then the calculation is performed using effective chiral Lagrangians, this implies that such contributions arise from the higher order Lagrangians, and hence are suppressed by a factor such as m_K^2/Λ^2 with $\Lambda \sim 1$ GeV. The possible magnitude of such terms is then expected to be

$$\frac{A_{DE}^{CP}}{A_{IB}} \sim (1 + O(\frac{22\epsilon'}{[1-\Omega]}/\epsilon)) \frac{m_K^2}{\Lambda^2} \quad (130)$$

although it is difficult to calculate the magnitude with any reliability. In principle then, additional CP violation could exist at a level not far below the IB strength.

A vexing problem is how to observe a useful signal in this decay mode. Although we expect the CP violation to be relatively large, it is not easy to extract it. The simplest signal to measure, the charge asymmetry in the $\pi^+\pi^-$ Dalitz plot, is not expected to be sizable.⁷⁴ Indeed with only E1 and M1 term, the charge asymmetry will vanish identically. This can be easily seen because an interference between E1 and M1 must be linear in the antisymmetric tensor $\epsilon^{\alpha\beta\gamma\delta}$ (see Eq. (129)) while, if the photon polarization is summed over, there do not exist enough independent momenta to contract with $\epsilon^{\alpha\beta\gamma\delta}$, i.e.

$$\epsilon^{\alpha\beta\gamma\delta} k_{\alpha} p_{+\beta} p_{-\gamma} q_{\delta} = 0 \quad .$$

It is possible to generate a charge asymmetry via higher multipoles, but such effects are expected to be small.

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An interference between $K_L \rightarrow \pi^+\pi^-\gamma$ and $K_S \rightarrow \pi^+\pi^-\gamma$ would also be a signal of CP violation.⁵⁴ Here our rough estimates given above would indicate

$$\frac{A(K_L \rightarrow \pi^+\pi^-\gamma)_{CP}}{A(K_S \rightarrow \pi^+\pi^-\gamma)} = \epsilon + \epsilon'_{\pi\pi\gamma} \quad (131)$$

with

$$\epsilon'_{\pi\pi\gamma} \sim O(\frac{22\epsilon'}{(1-\Omega)}) \frac{m_K^2}{\Lambda^2} \sim 5\epsilon' \quad (132)$$

Such an interference could be attempted using a regenerator or perhaps the flavor tagging technique described in Sec. V.

We hope to improve on these rough estimates in a future publication.

IX. CP Violation in Hyperons

Hyperon decays are characterized by three parameters:⁷⁵ the total decay rate Γ and two coefficients determining the angular distribution and polarization of the decay products. For a reaction $B_i(\vec{p}_i, \vec{s}_i) \rightarrow B_F(\vec{p}_F)\pi$ one finds the angular distribution of B_F to be given by:

$$w(\theta) = \frac{1}{4\pi} [1 + \alpha \vec{s}_i \cdot \hat{p}_F] \quad (133)$$

and its polarization

$$\langle \vec{\sigma} \rangle_F = \frac{1}{1 + \alpha \vec{s}_i \cdot \hat{p}_F} \{ (\alpha + \vec{s}_i \cdot \hat{p}_F) \hat{p}_F + \beta \vec{s}_i \times \hat{p}_F + \sqrt{1 - \alpha^2 - \beta^2} [\hat{p}_F \times (\vec{s}_i \times \hat{p}_F)] \} \quad (134)$$

The decay is therefore totally described with the three real numbers Γ , α , and β . Similarly, the corresponding anti-hyperon decay: $\bar{B}_i \rightarrow \bar{B}_F \pi$ is determined by its parameters $\bar{\alpha}$, $\bar{\beta}$ and its decay rate $\bar{\Gamma}$. Comparing the decays of a hyperon and its anti-hyperon it is possible to construct several observables that are odd under CP.

To see how these are constructed it is convenient to write the general matrix element for $B_i \rightarrow B_F \pi$ as

$$M = S + P \vec{\sigma} \cdot \hat{q} \quad (135)$$

with $S(P)$ being the S-wave (P-wave) parity violating (conserving) amplitude and $\hat{q}$ being a unit vector in the direction of the pion momentum. In terms of S and P the three interesting quantities are

$$\begin{aligned} \Gamma &\sim |S|^2 + |P|^2 \\ \alpha &= \frac{2 \operatorname{Re}(S^*P)}{|S|^2 + |P|^2} \\ \beta &= \frac{2 \operatorname{Im}(S^*P)}{|S|^2 + |P|^2} \end{aligned} \quad (136)$$

CP violation is observed through interference of amplitudes^{76,77} with different phases. In this case the different amplitudes that can interfere are the

S-wave and P-wave final states, or final states with different isospin (1/2 or 3/2). We can decompose each amplitude to show explicitly the different factors that play a role⁷⁷, for example in Λ decay we have

$$\begin{aligned} S &= S_1 e^{i(\delta_1^S + \phi_1^S)} + S_3 e^{i(\delta_3^S + \phi_3^S)} \\ P &= P_1 e^{i(\delta_1^P + \phi_1^P)} + P_3 e^{i(\delta_3^P + \phi_3^P)} \end{aligned} \quad (137)$$

Here, the subscript indicates isospin, ϕ is a CP violating phase due to the weak interaction and δ is a strong final state interaction phase. For the corresponding anti-hyperon decay $\bar{B}_i \rightarrow \bar{B}_F \pi$ one can write the matrix element in general as

$$M = \bar{S} + \bar{P} \vec{\sigma} \cdot \hat{q}$$

In this case however, some of the phases occur with different signs

$$\begin{aligned} \bar{S} &= -S_1 e^{i(\delta_1^S - \phi_1^S)} - S_3 e^{i(\delta_3^S - \phi_3^S)} \\ \bar{P} &= P_1 e^{i(\delta_1^P - \phi_1^P)} + P_3 e^{i(\delta_3^P - \phi_3^P)} \end{aligned} \quad (138)$$

It is clear then, that by comparing particle and anti-particle decays, one can extract the CP violating phases. It has been found that the most useful signals that can be constructed from these phases are⁷⁷ (to first order in small quantities)

$$\begin{aligned} B &= \frac{\beta + \bar{\beta}}{\beta - \bar{\beta}} = \frac{B'}{\tan(\delta_1^P - \delta_1^S)} \quad \text{with} \quad B' = \frac{\beta + \bar{\beta}}{\alpha \bar{\alpha}} \approx \sin(\phi_1^S - \phi_1^P) \\ A &= \frac{\alpha + \bar{\alpha}}{\alpha - \bar{\alpha}} \approx \sin(\phi_1^S - \phi_1^P) \sin(\delta_1^P - \delta_1^S) \\ \Delta &= \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} = -2 \frac{S_3}{S_1} \sin(\delta_3^S - \delta_1^S) \sin(\phi_3^S - \phi_1^S) - 2 \frac{P_3}{P_1} \sin(\delta_3^P - \delta_1^P) \sin(\phi_3^P - \phi_1^P) \end{aligned} \quad (139)$$

The optimal signal B' then involves CP violation directly. A is suppressed by an additional small final state interaction phase. and Δ is further suppressed by the $\Delta I = 1/2$ rule. Unfortunately, the ease with which these signals can be measured is in an inverse ordering to their strength.

Numerical estimates for these observables have been obtained by Donoghue et al.⁷⁷ using the different models for CP violation. Results vary from model

to model and can be as large as $B = 8.6 \times 10^{-2}$ for the $\Sigma^- \rightarrow n\pi^-$ decay in the Weinberg-Higgs model. More typical numbers, however, are of order 10^{-3} - 10^{-4} .

It is possible to estimate the size of the CP violating signals in a model independent way. Hyperon decays probe $\Delta S=1$ CP violation whose strength we can characterize with a parameter X . The CP odd signals are then roughly

$$\begin{aligned} B' &\approx X \\ A &\approx 1/10 X \\ \Delta &\approx 1/100 X \end{aligned} \quad (140)$$

It is possible⁷⁸ to have $\Delta S=1$ CP violation without generating the parameter ϵ' , with a parity conserving CP violating interaction, or with one that has the same isospin structure as the CP even interaction in $K \rightarrow \pi\pi$. What this means is that the bound on X is given by ϵ , not ϵ' . In fact, values of X up to 10^{-2} are consistent with the magnitude of ϵ .

One could argue that the comparison of measured values for hyperon and anti-hyperon decay parameters is affected by different systematic errors and backgrounds. Clearly, a signal that comes from a single experiment would be much cleaner. Realistic CP-violating signals of this type appear in low energy $p\bar{p}$ colliders.^{78,79}

The advantage of such a $p\bar{p}$ machine comes from the fact that the initial state is related to itself under CP. In particular, working in the center of mass frame

$$PC|p(\vec{k})\bar{p}(-\vec{k})\rangle = |p(\vec{k})\bar{p}(-\vec{k})\rangle \quad (141)$$

Therefore if CP is conserved, the final state must have the same CP symmetry. For example consider the reaction: $p_i + \bar{p}_i \rightarrow \Lambda + \bar{\Lambda} \rightarrow p_F + \pi^- + \bar{p}_F + \pi^+$. Using the particle names to label their momentum it is possible to generate one CP odd triple product correlation. Using momentum conservation it can be written in many equivalent forms, a useful one being⁷⁹

$$\theta = \vec{p}_i \times \vec{\Lambda} \cdot (\vec{p}_F + \vec{\bar{p}}_F) \quad (142)$$

This correlation θ probes the distribution of protons and anti-protons above and below the production plane defined by $\vec{p}_i \times \vec{\Lambda}$. Triple scalar products are often not useful as tests of T invariance because signals can be faked by uncalculable final state interaction phases. In this case, the definite CP property of the initial state prevents such phases from simulating CP violation.

The experimental measurement of the asymmetry θ requires counting the number of events above and below the production plane

$$A_{\text{exp}} = \frac{N_P(\text{up}) + N_{\bar{P}}(\text{up}) - N_P(\text{down}) - N_{\bar{P}}(\text{down})}{N_{\text{total}}} \quad (143)$$

where up (down) means $\vec{p}_i \times \vec{\Lambda} \cdot \vec{p}_F > 0$ (< 0). From (1) we know

$$w(\theta) = \frac{1}{4\pi} (1 + \alpha_\Lambda \rho_\Lambda \hat{S}_i \cdot \hat{p}_F) \quad (144)$$

where ρ_Λ is the Λ polarization, and a similar distribution holds for $\bar{\Lambda}$. A joint decay distribution is calculated, and it can be integrated in the way defined by (11) to give⁷⁹

$$A_{\text{exp}} = 1/2 \rho_\Lambda (\alpha + \bar{\alpha}) \quad (145)$$

Thus, measurement of this counting asymmetry is a measurement of the CP odd signal A in (7). One can see from looking at Figure 18 that Eqn. (142) is really a term of the form $\hat{S}_\Lambda \cdot \hat{p}_F$ and also Eqn. (144) shows how this asymmetry gives Eqn. (145), the key fact being that Λ and $\bar{\Lambda}$ are produced with a polarization perpendicular to the production plane. Measurement of the optimal signal B is more complicated since it is difficult to measure the spin of the decay product. However one can look at $p\bar{p} \rightarrow \Xi\bar{\Xi} \rightarrow \Lambda\bar{\Lambda}\pi\pi \rightarrow p\pi\pi\bar{p}\pi\pi$ where the spin of the Λ is analyzed by its own decay. Writing the Ξ polarization as $\vec{S}_\Xi = \rho_\Xi \hat{y}$, a triple product correlation

$$\tilde{B} \equiv \hat{y} \cdot (\vec{p}_F \times \vec{\Lambda} - \vec{\bar{p}}_F \times \vec{\bar{\Lambda}}) \quad (146)$$

generates a counting asymmetry⁴

$$\tilde{B} \sim \pi/8 \rho_{\Xi\Lambda} (\beta + \bar{\beta})_\Xi \quad (147)$$

63.

64.

Experiments of this sort are not easy, and they require very high statistics. Nevertheless, in view of present experimental sensitivities there is reason to be optimistic about measuring these signals in a next generation of experiments.

Fig. 18. The reaction $p\bar{p} \rightarrow \Lambda\bar{\Lambda} \rightarrow p\pi^- \bar{p}\pi^+$

X. Electric Dipole Moments

It is well-known that an interaction of the form

$$H \sim -\vec{G} \cdot \vec{E} \quad (148)$$

violates both parity and time reversal. Thus measurement of a non-zero electric dipole moment for an elementary particle is a direct probe for CP noninvariance. At the present time the most stringent limit is that placed by both American and Soviet investigators on the electric dipole moment of the neutron⁸⁰

$$d_n \leq 6 \times 10^{-25} \text{ ecm} \quad (149)$$

Experiments by both groups are ongoing and hope to be able to improve this upper bound by at least two orders of magnitude. Thus it is important to see what the various CP noninvariant models predict for this quantity.

In the case of the superweak theory, a definitive prediction is difficult to come by since a specific model is lacking. Nevertheless, assuming that the size of the parity violating $\Delta S=0$ component of the superweak interaction is comparable to its $\Delta S=2$ counterpart, Wolfenstein has given

$$d_n \sim \frac{e}{m_n} \frac{|\langle K_+ | H_{sw} | K_- \rangle|}{m_K} \sim 10^{-30} \text{ ecm} \quad (150)$$

as a crude order of magnitude estimate.⁸¹

Much work has been done on the KM model prediction for the neutron moment. Since the CP violating phase δ appears only in the heavy quark sector of the KM matrix, it is clear that a neutron EDM cannot arise to first order in the weak interaction—at least two W-exchanges are required. Many different approaches have been used, all yielding roughly the same order of magnitude.⁸² A typical calculation is that of Khriplovich and Zhitvitsky⁸³ who utilize the $n \rightarrow \Sigma^- \pi^+ \gamma \rightarrow n$ diagrams indicated in figure 19. Here one vertex is CP violating and is approximated by the penguin contribution. The other is CP conserving and taken from experiment. The size of the moment then is expected to be of order

$$d_n \sim \frac{e}{2m_n} (G_F \mu^2)^2 \text{Im } C_5 s_1^2 \quad (151)$$

$$\sim 10^{-32} \text{ ecm}$$

Fig. 19. A diagram which generates an electric dipole moment for the neutron

where μ is a typical hadronic scale, $\mu = 300$ meV. This estimate agrees roughly with the result

$$d_n \sim 2 \times 10^{-32} \text{ ecm} \quad (152)$$

calculated by these authors and is not dissimilar to that found via other approaches. Obviously such a moment is far below present capabilities of detection.

For the Higgs model, the most detailed calculations have been performed by Beall and Deshpande and by Bigi and Sanda who find⁸⁴

$$|d_n| \geq 10^{-25} \text{ ecm} \quad (153)$$

In this case the effect arises at single Higgs exchange order and thus is considerably larger than found in the KM model, though still consistent with present limits. It should be noted, however, that a significant level of heavy quark degrees of freedom in the neutron wave function would increase the theoretical prediction substantially and could rule out the model even with present limits.

A dipole moment can arise in the left-right model via two-W-exchanges, one left-handed and CP-conserving, the second right-handed and CP-violating. Thus an effect of order

$$|d_n| \sim \frac{e}{2m_n} \times (G\mu^2)^2 \frac{m_L^2}{m_R^2} \sin^2 \theta_1 \sin \beta \quad (154)$$

$$\sim 10^{-30} \text{ ecm}$$

is to be expected.

Recently a new aspect of this field has emerged in that techniques have been developed which allow the precise measurement of atomic EDM's. In fact a recent experiment involving ¹⁹⁹Hg yielded the limit⁸⁵

$$|e_{199\text{Hg}}| \leq 10^{-26} \text{ ecm} \quad (155)$$

i.e. and order of magnitude below the present limit for the neutron. Of course, the interpretation of such a limit involves a combination of electron and nuclear

moments. Even in models such as KM wherein the electron moment vanishes, detection of the nuclear moment must contend with the implications of Schiff's theorem, which requires perfect shielding of a point nucleus from the influence of an external electric field.⁸⁶ Fortunately finite nuclear size interrupts this shield and produces a nonzero result proportional to the difference between the charge radii of the ordinary charge and EDM distributions. Of course, this introduces model dependence into the interpretation.

Despite these drawbacks, however, two recent theoretical developments suggest that measurement of such atomic moments may well surpass in sensitivity the beautiful neutron measurements. The first involves calculations by Haxton and Henley,⁸⁷ confirming a suggestion by Feinberg⁸⁸ that nearby opposite parity doublets found in certain heavy nuclei may mix via the CP nonconserving NN interaction, thereby enhancing the expected size of the nuclear moment by as much as a factor of 10^4 over that expected for the nucleon. The second development involves the work of Khriplovich and collaborators who argue that the existence of a CP violating kaon exchange diagram allows nuclear EDM's in the KM model to be as much as a factor of a hundred or so larger than the corresponding neutron moment.⁸⁹ There is some question, however, whether chiral constraints on the weak interaction were properly included in this work.

Obviously additional and improved experimental and theoretical work is called for here. Nevertheless, it is clear that measurement of atomic electric dipole moments is a very promising new tool as a probe of possible CP violation.

XI. Low Energy Triple Product Correlations

There are a number of experiments performed at low energy which probe the validity of time reversal invariance (and therefore, by the CPT theorem, the validity of CP). These generally involve looking for the existence of a triple product correlation, such as

$$\vec{S} \cdot \vec{p}_1 \times \vec{p}_2 \quad \text{or} \quad \vec{S}_1 \times \vec{S}_2 \cdot \vec{p}_1$$

Since both spin and momentum are odd under the time reversal operation, the existence of such correlations would appear to signal violation of time reversal invariance. However, since T is anti-unitary this is not quite correct — final state interactions can produce nonzero correlation parameters even in a T conserving theory. Nevertheless these effects are in principle calculable and can be subtracted from the experimental signal in order to probe for true T-violation. Thus, for example, an early experiment of this type used the correlation

$$\vec{S}_A \cdot \vec{S}_p \times \vec{p}$$

in the decay

$$\Lambda \rightarrow p \pi^-$$

to measure a phase difference⁹⁰

$$\phi_{sp} = 9.0^\circ \pm 5.5^\circ \quad (156)$$

between S-wave and P-wave amplitudes to be compared with the difference⁹¹

$$\delta_s - \delta_p = 6.5^\circ \pm 1.5^\circ \quad (157)$$

determined from low energy π -p scattering data. Clearly there is no evidence for T violation. However, the potential for a precision test is limited by the accuracy to which the strong interaction phases can be measured, which is not encouraging in this regard.

A second approach uses beta decay reactions wherein final state interactions are purely electromagnetic and therefore not only precisely calculable but also very small. Thus, for example, in the decay

$$^{19}\text{Ne} \rightarrow ^{19}\text{F} + e^+ + \nu_e$$

measurement of the correlation

$$D \vec{S}_{\text{Ne}} \cdot \vec{p}_e \times \vec{p}_\nu \quad (158)$$

has yielded the very precise value⁹²

$$D = (0.7 \pm 6) \times 10^{-4} \quad (159)$$

which is still well above the value

$$D_{\text{FSI}} = 2.5 \times 10^{-4} \quad (160)$$

expected from final state Coulombic effects.⁹³ This experimental value corresponds to a phase difference

$$\phi_{VA} = (-0.8 \pm 7) \times 10^{-4} \quad (161)$$

between vector and axial vector amplitudes.

Similar experiments involving neutron beta decay have yielded⁹⁴

$$D = \begin{aligned} &(-1.1 \pm 1.7) \times 10^{-3} \\ &(2.2 \pm 3.0) \times 10^{-3} \end{aligned} \quad (162)$$

corresponding to phase differences

$$\phi_{VA} = \begin{aligned} &(-1.5 \pm 2.2) \times 10^{-3} \\ &(2.9 \pm 3.9) \times 10^{-3} \end{aligned} \quad (163)$$

respectively.

While quite precise, one does not expect to see a signal here for any but the left-right model of CP violation. Even in this case, however, the effect which arises from allowing $W_L - W_R$ mixing, is expected to be of order⁹⁴

$$D \leq 10^{-4} \quad (164)$$

A somewhat different approach is the search for transverse polarization

$$\vec{\sigma}_\mu \cdot \hat{p}_\mu \times \hat{p}_\nu$$

in the decay

$$K \rightarrow \pi \nu \bar{\nu}$$

Describing this transition in terms of the two vector form factors f_+, f_- , the result can be expressed in the form⁹⁶

$$\text{Im} \frac{f_-}{f_+} = -0.01 \pm 0.02 \quad (165)$$

Since this decay involves only the polar vector hadronic current, there are no effects expected in either the KM or left-right models. The Higgs picture could yield a nonzero signal since the Higgs coupling is proportional to the lepton mass m_μ and can interfere with the usual W-exchange. However, the anticipated size of the effect is below present experimental uncertainties.⁹⁷

Although none of the models discussed above expects any effect, we note for completeness that T noninvariance has also been carefully probed in strong and electromagnetic interactions. In the case of the former, detailed balance studies involving the reactions

have provided a limit⁹⁸

$$\left| \frac{A(\text{T-violating})}{A(\text{T-conserving})} \right| < 5 \times 10^{-4} \quad (166)$$

For the latter, the most precise experiments have looked for correlations between the alignment direction of a parent nucleus and the cross product of the momentum and electric field of a photon produced in a subsequent decay

$$\underline{J} \cdot \hat{k}_Y \times \hat{E}_Y \cdot \underline{J} \cdot \hat{k}_Y$$

Typical results are indicated below together with calculated final state interaction effects⁹⁹

Nucleus	Expt. Phase ($\times 10^4$)	FSI Phase ($\times 10^4$)	
${}^{57}\text{Fe}$	-3.1 ± 6.5	-6	
${}^{191}\text{Ir}$	-47 ± 3	-43 ± 4	(167)
${}^{181}\text{Xe}$	-12 ± 11	-1	

Obviously, there is no sign of an effect here at the level of about a part in 10^3 .

XII. CP Violation in B Meson Physics

The decays of B^0 , B^- and B_s^0 offer an extremely rich system in which to explore CP violation. There are many different types of signals which can be examined, and in most cases the tests of CP can be applied to a number of different final states. The study of B decays will be a major part of the experimental program at future accelerators. Because of the importance of finding CP violation outside of the kaon sector, the study of B meson system then deserves high support.

The standard model predicts significant CP violation in B meson decays. In order to understand the expected pattern of the signals it is useful to remember that any CP violation in the KM model is proportional to the combination of angles

$$s_1^2 s_2 s_3 s_\delta \leq 10^{-4} \quad (168)$$

This implies that if a transition were to take place unsuppressed by any small angles (as would $t \rightarrow b$ decay), then all CP odd observables would be small and at most at the level of Eq. (168). It is possible, however, to generate larger signals by choosing reactions for which the CP conserving amplitude is also suppressed by small angles. Then the relative size of the CP violating effect can be nearly comparable to its CP conserving counterpart. This strategy is extensively exploited in study of the B system, and CP odd asymmetries of order tens of percent are possible. However, the difficult feature is that since these processes are suppressed by small angles, the branching ratios are correspondingly very small. These are rare B decays. The choice between small signals in relatively common processes or large signals in rare processes is a fact of life associated with the KM model. There is some disagreement in the literature as to the number of B mesons which are needed in order to observe CP violation. It is probable that, unless one is rather lucky, about 10^7 B mesons will be needed. This is far beyond the output of present machines, but can be viewed as a design goal for future accelerators and detectors. It should, of course,

be emphasized that tests of CP should be performed at all levels regardless of the predictions of the standard model. Alternative sources of CP breaking could generate signals larger and different from those of the KM model.

The various tests of CP in the B meson system which shall be discussed are listed in Table IV and will be explained in more detail in the following subsections.

TEST	COMMENTS
1) <u>MIXING AND SEMILEPTONIC DECAY</u> $\frac{\Gamma(B^0(t) \rightarrow \ell^+ X) - \Gamma(B^0(t) \rightarrow \ell^- X)}{\Gamma(B^0(t) \rightarrow \ell^+ X) + \Gamma(\bar{B}^0(t) \rightarrow \ell^- X)}$	Mixing sizable, but CP violation $\sim 10^{-4} \rightarrow 10^{-3}$
2) <u>CHARGED SEMILEPTONIC RARE DECAYS</u> $\frac{\Gamma(B^+ \rightarrow K^+ \ell^+ \ell^-) - \Gamma(B^- \rightarrow K^- \ell^+ \ell^-)}{\Gamma(B^+ \rightarrow K^+ \ell^+ \ell^-) + \Gamma(B^- \rightarrow K^- \ell^+ \ell^-)}$	B.R. $\leq 10^{-6}$; CP $\leq 1\%$
3) <u>MIXING AND COMMON FINAL STATES</u> $\frac{\Gamma(B^0(t) \rightarrow f) - \Gamma(\bar{B}^0(t) \rightarrow \bar{f})}{\Gamma(B^0 \rightarrow f) + \Gamma(\bar{B}^0 \rightarrow \bar{f})}$	Requires mixing and $B^0 \rightarrow f$, $\bar{B}^0 \rightarrow \bar{f}$ Examples $f = \psi K_S$, $D\bar{D}$, $D^+ \pi^- \dots$ Asymmetries can be large ($\sim 10\%$) in some low B.R. modes Most promising class of tests
4) <u>CHARGED B DECAYS</u> $\frac{\Gamma(B^- \rightarrow f) - \Gamma(B^+ \rightarrow \bar{f})}{\Gamma(B^- \rightarrow f) + \Gamma(B^+ \rightarrow \bar{f})}$	Requires: Final state interactions <u>Two pathways to same state</u> Asymmetries can be large (if lucky) Expts. should be done
5) <u>$B^0 \bar{B}^0$ FINAL STATES</u> $e^+ e^- \rightarrow B^0_+ B^0_- \rightarrow (\psi K_S) (\psi K_S)$	Very small rates
6) <u>DIRECT OBSERVATION OF TIME DEPENDENCE</u>	Applied to class 3 above
7) <u>SPIN ASYMMETRIES</u> E.G. $B^0_S, \bar{B}^0_S \rightarrow \phi\phi$ $B^0, \bar{B}^0 \rightarrow \psi K_S$	Polarization in $B, \bar{B}$ decays Doesn't require mixing or final state interactions Not yet analyzed

Table IV. Tests of CP Invariance in B Meson Decays

i) $B^0 - \bar{B}^0$ Mixing

The formulation of the mixing problem of $B^0 \bar{B}^0$ is similar to the kaon analysis.

The two by two Hamiltonian matrix

$$H = M - \frac{i\Gamma}{2} \quad (169)$$

is diagonalized to find the two mass eigenstates¹⁰⁰ (hereafter called "heavy" and "light")

$$|B_H\rangle \equiv p |B^0\rangle + q |B^0\rangle_L \quad (170)$$

with

$$\frac{p}{q} = \sqrt{\frac{M_{12} - i\Gamma_{12}/2}{M_{12}^* - i\Gamma_{12}^*/2}} \quad (171)$$

The width and mass differences of the two states are then

$$\Delta m = m_H - m_L = 2\text{Re}\sqrt{(M_{12} - i\Gamma_{12}/2)(M_{12}^* - i\Gamma_{12}^*/2)} \quad (172)$$

$$\Delta\Gamma = \Gamma_H - \Gamma_L = -4\text{Im}\sqrt{(M_{12} - i\Gamma_{12}/2)(M_{12}^* - i\Gamma_{12}^*/2)}$$

It is the box diagram, Fig. 20a, which can generate M_{12} and the square of decay diagrams, Fig. 20b, which produce Γ_{12} . Note that the top quark intermediate state dominates M_{12} while Γ_{12} possesses only small KM angles and no factor of m_t^2 . This leads to the prediction $\Gamma_{12} \ll M_{12}$, $\Delta\Gamma \ll \Delta m$, which are on reasonably firm grounds.

A state which starts off at $t=0$ as a pure B^0 or $\bar{B}^0$ will evolve into a mixture¹⁰⁰

$$|B^0(t)\rangle = f_+(t) |B^0\rangle + \frac{q}{p} f_-(t) |\bar{B}^0\rangle \quad (173)$$

$$|\bar{B}^0(t)\rangle = \frac{p}{q} f_-(t) |B^0\rangle + f_+(t) |\bar{B}^0\rangle$$

at later times, with (using $\Delta\Gamma \approx 0$)

$$f_{\pm}(t) = \frac{1}{2} e^{\pm i m_H t} \{1 \pm e^{i \Delta m t}\} \quad (174)$$

The standard way to seek this mixing^{101,102} is to study the production of like sign lepton pairs in processes which produce B^0 and $\bar{B}^0$ equally

(a)

(b)

Fig. 20. a) The box diagram for $B^0 \bar{B}^0$ mixing. b) The product of decay amplitudes which generate Γ_{12} in the B meson sector.

$$r = \frac{\Gamma(B^0 \bar{B}^0 \rightarrow e^+ e^+ x) + \Gamma(B^0 \bar{B}^0 \rightarrow e^- e^- x)}{\Gamma(B^0 \bar{B}^0 \rightarrow e^+ e^- x)} = \frac{|q|^2}{|p|^2} \frac{x^2}{2+x^2} \quad (175)$$

for $B^0 \bar{B}^0$ in a P-wave, with

$$x = \frac{\Delta m}{\Gamma}$$

The prediction of Δm has several uncertainties, such as the unknown value of the top quark mass and the difficulty of calculating the appropriate hadronic matrix elements. However estimates¹⁰³ put

$$\frac{\Delta m}{\Gamma} \begin{cases} \approx 0 \rightarrow 0.1 & \text{for } B_d^0 \\ \approx 0.1 \rightarrow 1 & \text{for } B_s^0 \end{cases} \quad (176)$$

i.e. substantial mixing is expected, especially for B_s^0 . The larger mixing for B_s^0 is easy to understand because of the larger KM angles for the transitions - $b \rightarrow W(t,c,u) \rightarrow W W$ s compared to $b \rightarrow W(t,c,u) \rightarrow W W$ d in the box diagram. For CP violating mixing, one needs more than just the t intermediate state, and hence the CP odd contributions are kinematically suppressed. Typical estimates¹⁰³ are

$$\frac{p}{q} - 1 \sim \begin{cases} 10^{-3} & B_{u,d}^0 \\ 10^{-4} & B_s^0 \end{cases} \quad (177)$$

At the time of this writing there are preliminary indications from UAL of a possible dilepton signal compatible with maximal B_s mixing.¹⁰⁴

ii) Mixing and Semileptonic Decay

If mixing occurs, states which are originally B^0 or $\bar{B}^0$ will mix as they evolve in time and this will lead to semileptonic decay to the wrong sign lepton. If CP violation occurs in the mixing, the amount of wrong sign decay will be different for B^0 and $\bar{B}^0$,^{101,102}

$$\frac{\Gamma(B^0 \rightarrow e^+ x) - \Gamma(\bar{B}^0 \rightarrow e^- x)}{\Gamma(B^0 \rightarrow e^- x) + \Gamma(\bar{B}^0 \rightarrow e^+ x)} = \frac{|q|^2 - |p|^2}{|q|^2 + |p|^2} \quad (178)$$

and for pair produced $B^0 \bar{B}^0$

$$\frac{\Gamma(B^0 \bar{B}^0 \rightarrow e^+ e^+ x) - \Gamma(B^0 \bar{B}^0 \rightarrow e^- e^- x)}{\Gamma(B^0 \bar{B}^0 \rightarrow e^+ e^- x) + \Gamma(B^0 \bar{B}^0 \rightarrow e^- e^+ x)} = \frac{|q|^2 - |p|^2}{|q|^2 + |p|^2} \quad (179)$$

In the KM picture these are small quantities⁴

$$\frac{|q|^2 - |p|^2}{|q|^2 + |p|^2} \sim \begin{cases} 10^{-3} & B_d^0 \\ 10^{-4} & B_s^0 \end{cases} \quad (180)$$

which make it extremely unlikely that such probes will yield a nonzero signal in the case of the standard model. However the cleanness of the semileptonic reactions make them good tools to see if unexpected new physics enters.

iii) Semileptonic Decay $B \rightarrow K\ell^+\ell^-$

The dominant diagrams for producing $B \rightarrow K\ell^+\ell^-$ or $B \rightarrow K\ell^+\ell^- X$, at least for large $\ell^+\ell^-$ invariant masses, are those of the electromagnetic penguin process, Fig. 21 plus related Z diagrams. Because the intermediate states include, u, c, t, there exist CP violating phases in the loop amplitude. These can provide a CP-odd observable via the partial decay rate asymmetry¹⁰⁵

$$A_{K\ell^+\ell^-} = \frac{\Gamma(B^- \rightarrow K^- \ell^+ \ell^-) - \Gamma(B^+ \rightarrow K^+ \ell^+ \ell^-)}{\Gamma(B^- \rightarrow K^- \ell^+ \ell^-) + \Gamma(B^+ \rightarrow K^+ \ell^+ \ell^-)} \quad (181)$$

or the corresponding asymmetry in the inclusive final state $B \rightarrow K\ell^+\ell^- + X$. The branching ratio and asymmetries for these modes have recently been calculated,¹⁰⁵ and the results can be easily understood. In order to produce an asymmetry, one needs both the CP violating phases and also final state interactions. Significant final state interactions can be generated because of the existence of real intermediate states, which arise automatically in some of the time orderings of the electromagnetic penguin diagrams, e.g. Fig. 22. These requirements mean that the top quark, which dominates the real part of the penguin diagram, is unable to induce CP violation. Thus the CP violating asymmetry must be suppressed, as it is generated by light quarks within the loop. Detailed calculations in the standard KM model produce a branching ratio of order $10^{-6} + 10^{-5}$ and a CP-odd asymmetry of at most 1%. Even given optimistic predictions for the number of B mesons available in the future, any CP violation here appears impossible to measure. The situation with four generations is scarcely any better. In this case the extra mixing angles can generate a 10% asymmetry in some regions of parameter space with a branching ratio of order 10^{-6} but even this is difficult to observe. Despite the cleanliness of this channel, it is probably not a good CP laboratory within the standard model.

Fig. 21. Electromagnetic penguin diagrams for $b \rightarrow s e^+ e^-$. Also important are diagrams involving Z^0 's [see ref. 105]

iv) B^0 , B_S^0 Nonleptonic Decays

The clear and simple semileptonic tests of CP violation do not appear promising because of the small signals which are expected. Thus, Carter and Sanda¹⁰¹ and Bigi and Sanda¹⁰⁰ have made the case that useful tests must involve nonleptonic decay. There exist in fact then a large number of possible signals. The most carefully studied sector is that of the neutral mesons where CP violation can occur both in the B - $\bar{B}$ mixing matrix as well as in the direct transition amplitude. However, the experimental effects here generally make use of the mixing even if the CP violation is primarily in the decay amplitude.

Thus, consider a final state f for which both $B^0 \rightarrow f$ and $\bar{B}^0 \rightarrow f$ can occur. This, of course, also requires the existence of the related transitions $B^0 \rightarrow \bar{f}$ and $\bar{B}^0 \rightarrow \bar{f}$, where $\bar{f}$ is the CP conjugate of f . An example is given in Fig. 23. States that start out as pure B^0 or $\bar{B}^0$ will evolve in time into mixed states of $B^0, \bar{B}^0$, c.f. Eq. 173. For this mixed state, interference between $B^0 \rightarrow f$ and $\bar{B}^0 \rightarrow f$ will produce a CP violating signal via

$$\frac{\Gamma(B^0(t) \rightarrow f) - \Gamma(\bar{B}^0(t) \rightarrow \bar{f})}{\Gamma(B^0(t) \rightarrow f) + \Gamma(\bar{B}^0(t) \rightarrow \bar{f})} = \frac{\sqrt{2r(1-r)}}{1+r} \text{Im} \left[\frac{P}{Q} \frac{A(\bar{B}^0 \rightarrow \bar{f})}{A(B^0 \rightarrow f)} \right] \quad (182)$$

where r is the mixing parameter defined by

$$r = \frac{\Gamma(B^0 \rightarrow e^+ x)}{\Gamma(B^0 \rightarrow e^- x)} = \left| \frac{q}{p} \right|^2 \frac{x^2}{2+x^2} \quad (183)$$

with $x = \Delta m/\Gamma$. One can see that such signals vanish for either no mixing, $r=0$, or full mixing, $r=1$. For cases where $f \neq \bar{f}$, the modes $B^0 \rightarrow D^+ \pi^-$ and $B_S^0 \rightarrow F^+ K^+$, $D^0 \phi$ seem reasonably promising. The former could have a sizable branching ratio, $O(1\%)$, because the transition involves the largest KM angles possible in B decay, Fig. 23a. However, the CP violating signal only occurs due to interference with a diagram involving KM suppressed vertices, Fig. 23b. The resulting CP odd asymmetry is estimated to be less than a percent.^{100,101,103} On the other hand, processes such as $B_S^0 \rightarrow D^0 \phi$, Fig. 24, could have large asymmetries, of order tens

Fig. 22. A real intermediate state in the penguin diagram which leads to non-zero final state interaction phases

Fig. 23. Diagrams for $B^0, \bar{B}^0 \rightarrow D^+ \pi^-$ which can interfere if $B^0 \bar{B}^0$ mixing occurs

of percent, if V_{ub} is not too small, because both interfering diagrams, cf. Fig. 24 - can be comparable in magnitude. However the price to be paid is that the branching ratio is KM suppressed, and is estimated to be of order 10^{-3} or less.

These tests can also be useful when the final state f is itself a CP eigenstate, i.e. $f = \bar{f}$. Examples of such modes include $B_d^0 \rightarrow \psi K_S$, $\psi K_S \pi^0$, $D \bar{D} K_S$, $D \bar{D}$ and $B_S \rightarrow F^+ F^- \psi$. Here the diagrams for $B^0 \rightarrow f$ and $\bar{B}^0 \rightarrow f$ have necessarily the same topologies, with the KM elements involved being complex conjugate of each other. The knowledge of the hadronic matrix elements of the decay process is not important to the understanding of the asymmetry. However, the predictions are still somewhat imprecise due to the uncertainties in the KM angles, and in the $B^0 - \bar{B}^0$ mixing parameters. In B_d^0 decays asymmetries of order 10% could be reached with branching ratios of order 10^{-3} . For B_S , the predicted asymmetries are smaller.^{100,101}

CP violation in some modes can also proceed in the absence of mixing through final state interactions, as discussed in the next section. However, if mixing occurs, it would most likely be more important than the contributions due to final state interactions.

Fig. 24. Diagrams for $B_s^0, \bar{B}_s^0 \rightarrow D^0 \phi$

v) Charged B Decays - Final State Interactions

There can also be partial rate asymmetries in the decays of charge B mesons.¹⁰⁶

The physics requirement here is that there need to be two different mechanisms to reach a given final state, involving both i) different CP violating weak phases and ii) different strong interaction final state interactions.

In order to understand this requirement, consider a particular transition which can be generated via differing mechanisms 1,2 with corresponding transition amplitudes A_1, A_2

$$A(B^- \rightarrow f) = |A_1| e^{i\phi_1} e^{i\delta_1} + |A_2| e^{i\phi_2} e^{i\delta_2} \quad (184)$$

where here ϕ_1 is the CP violating phase and δ_1 is the phase associated with final state interactions. In B^+ decay the amplitude will then be given by

$$A(B^+ \rightarrow \bar{f}) = |A_1| e^{-i\phi_1} e^{i\delta_1} + |A_2| e^{-i\phi_2} e^{i\delta_2}, \quad (185)$$

that is, the weak phase has reversed sign. Squaring and subtracting yields the partial rate difference

$$\frac{\Gamma(B^- \rightarrow f) - \Gamma(B^+ \rightarrow \bar{f})}{\Gamma(B^- \rightarrow f) + \Gamma(B^+ \rightarrow \bar{f})} = \frac{2|A_1||A_2| \sin(\phi_1 - \phi_2) \sin(\delta_1 - \delta_2)}{|A_1|^2 + |A_2|^2 + 2|A_1||A_2| \cos(\phi_1 - \phi_2) \cos(\delta_1 - \delta_2)} \quad (186)$$

This formula illustrates some of the difficulties involved in producing a large asymmetry. Even if the two amplitudes have large weak phase differences, one needs in addition significant final state interactions. Estimates of the latter are quite uncertain, and it is correspondingly difficult to claim any reliability for the estimates of asymmetries.

Various mechanisms which can produce the needed interference have been identified. 1) The penguin diagram can interfere with the usual four fermion interactions, as in Fig. 25. In this case significant final state interactions can be generated by the existence of real intermediate states in the penguin diagram, as was discussed above in connection with $B \rightarrow K \ell^+ \ell^-$. 2) The annihilation diagram, if present at all, can appear with different couplings from the

Fig. 25. Interfering diagrams for $B^- \rightarrow K^- \pi^0$ involving the penguin diagram

standard diagrams generating a given final state as in Fig. 26. Here, however, the final state interactions are extremely uncertain. 3) Cascade decay chains, such as shown in Fig. 27, can also lead to common final states. One cannot say with confidence that any one mechanism will yield a significant signal. However there exist enough possible B decays that many different channels can be checked to whatever precision possible experimentally. It may happen that one channel has the required combination of comparable weak amplitudes and large final state interactions needed to produce an observable signal.

Fig. 26. Interfering diagrams for $B^- \rightarrow D^- D^{*0}$ involving the annihilation diagram

Fig. 27. Different quark cascades to reach the same final state

vi) Generation of Specific Final States by $B^0\bar{B}^0$

In e^+e^- production of $B^0\bar{B}^0$ the pair of mesons is produced in a P-wave [to the extent that the production mechanism involves one photon]. In terms of the mass eigenstates, B_L and B_H , one each of B_L and B_H must be produced. In this case, if both B's decay to an identical final state which is also an eigenstate of CP, this would indicate CP violation. Thus, Wolfenstein has suggested¹⁰⁷

$$e^+e^- \rightarrow B_H + B_L \rightarrow \psi + K_S + \psi + K_S \quad (187)$$

although other examples are clearly possible.¹⁰⁸ In the case of two identical final states, as in the above example, the rate is proportional to the mixing parameter x^2 (see Eq. 176) the square of the CP allowed branching ratio, and also the CP violating parameters. It is thus expected to be unobservably small.

vii) Decay Asymmetries

In preparing this review it was realized that there is an additional class of CP tests in B decays which has not yet been discussed in the literature. These involve asymmetries in final state $B+\bar{B}$ decay distributions similar in spirit to the decay asymmetries in hyperon decays discussed in Sec. IX. As a simple, but probably not optimal, example consider $B_S^0 \rightarrow \phi\phi$ as in Fig. 28. The $\phi\phi$ system can be produced either in a 0^{++} or 0^{-+} total state (much like the FF and $\tilde{F}\tilde{F}$ states described in the $K+\gamma\gamma$ section). The interference of these states then leads to a polarization of the ϕ 's. If CP were conserved the polarization of ϕ 's in $\bar{B}_S^0$ decay would be opposite to that in B_S^0 decay. Thus a net ϕ polarization in $e^+e^- \rightarrow B_S^0\bar{B}_S^0 \rightarrow \phi\phi + X$ would signal CP nonconservation. There are many more such examples, and we will provide a detailed analysis in a future publication.¹¹⁰

The characteristics of such asymmetries can be seen without a full analysis. Theoretically they appear promising in that neither mixing nor final state interactions are required. If $B^0 - \bar{B}^0$ mixing turns out to be small, and final state interactions are not large, such asymmetries may be the only feasible way to find CP violation in the B system. One would expect the decay asymmetries to be similar in magnitude to other types of CP nonconservation in the B system, i.e. they should be small in the dominant decay channels, and could be significant in certain of the rarer modes. The biggest disadvantage of these tests is the difficulties which they present for the experimenter. Particle reconstruction must be very good in order to be able to measure spin properties of final state particles. Such experiments will be significantly more difficult than the simple counting asymmetries in each decay mode discussed in Sec. XIII iv) above. However performing any of the B meson CP tests will require sophisticated and powerful experiments, and perhaps these decay asymmetries will prove feasible.

Fig. 28. The penguin diagram generating $B_s^0 \rightarrow \phi\phi$

viii) Prospects for B Meson CP Violation

The variety of signals of CP violation in the B meson sector cannot compensate for the fact that all of these experiments will be very difficult if the standard model estimates are correct. The main problem is in the production of enough B mesons to be able to see these small effects, in an environment clean enough that the signal can be extracted from the background. So far studies of physics at future accelerators have not been optimistic about being able to observe these effects. At neither SLC, LEP, nor the Tevatron collider will enough B mesons be generated. At the SSC, under present designs, there could be up to sufficient B mesons produced, but the detection and use of them poses problems. Nevertheless experimenters have succeeded in overcoming similar difficult problems before, and it is clear that a dedicated effort is justified in this case. In addition there are preliminary discussions of a B meson factory facility which have been started at the present time. The physics of CP violation should be a high priority in such a design. It is fair to be cautiously optimistic that such CP studies will pay off in the future.

The most promising tests are those involving the nonleptonic final states described in Secs. iv, v above. The use of specific hadronic final states is difficult experimentally, but seems to be necessary in order to find a signal of reasonable magnitude. It is, of course, possible that signals are larger than those predicted in the standard model. For example in the charged Higgs models, the CP violating coupling strength grows with the mass of the quark. Simple and rough scaling arguments suggest that the CP odd strength could be as large as 10% of the standard W exchange contribution. A detailed analysis of the application of this model to B decays has not yet been undertaken. Similarly, superweak models could produce a large CP-odd $B^0\bar{B}^0$ mixing if the flavor changing coupling grows with mass. However, experimenters need not pay attention to the details of any particular model. Testing CP violation at basically any level in

the B system is a significant contribution to our knowledge, and it is possible that large signals may be found if new physics is present.

XIII. D and T Decays

In the standard model D and T mesons appear less promising for the observation of CP violation than do B mesons. This is primarily because d,t decays can take place unsuppressed by small KM angles. With the dominant decay being of order unity, and all CP odd signal proportional to $s_1^2 s_2 s_3 s_8$, it follows that observable CP violation is either very small, or it is significant only in very rare processes. Nevertheless each system, D and T, has certain advantages which are worth exploring. In the case of D mesons, this advantage is the existence of excellent and precise experiments (primarily the Mark III at SLAC and ARGUS at DESY) which are presently exploring these decays. On the other hand, the t quarks, when they are found, have the advantage of their large mass (which should make them most sensitive to interactions such as the Higgs model wherein coupling strength grows with energy) and also are expected to have essentially electromagnetically stable vector T^* mesons, which could allow spin correlation experiments such as will be discussed later in Sec. XVI.

The generic signals in T and B decay are similar to those in B physics. However, $D^0 - \bar{D}^0$ mixing is known to be small and appears theoretically to be dominated by its dispersive component¹¹¹ due to SU(3) breaking in real intermediate states, Fig. 29. This results in very small mixing. The Mark III has data which could possibly be interpreted as indicating somewhat larger mixing, but even this signal is not nearly as large as the mixing expected in B decays. On the other hand, final state interactions are known to be significant in the D mass region, which could help in the generation of partial rate asymmetries as discussed in Sec. XII(v) above, if CP violating phases are also present. The standard model appears to predict small signals, but Bigi and Sanda¹¹² have suggested that supersymmetric theories, in particular, may yield observable CP violation effects.

T decays could provide the first real high energy test of CP. Whatever

Fig. 29. The dispersive component to $D^0 \bar{D}^0$ mixing

mechanism generates CP violation is most likely associated with physics at the weak scale or above. At high energies the CP violation could be stronger than in low energy kaon physics. One model which incorporates this enhancement is the charged Higgs model wherein the Higgs coupling is proportional to quark masses. The charged Higgs interaction in t decay has the same flavor structure as the standard W exchange process (Fig. 30) but the Dirac gamma matrix structure is different. The Hamiltonian has the form (for $t \rightarrow bc\bar{s}$)

$$H_W = G_F V_{cb}^* V_{cs} \{ \bar{c} \gamma_L (1 + \gamma_5) s \bar{b} \gamma_L (1 + \gamma_5) t + \eta \eta_c \eta_t \bar{c} (1 + \gamma_5) s \bar{b} (1 - \gamma_5) t \} \quad (188)$$

where¹¹³

$$\eta = \sum \frac{1}{2} \frac{\text{angles}}{\eta_{H_1}^2} \quad (189)$$

The KM-like angles are those associated with the Higgs sector, and if the angles are the same in t decay as in $\Delta S=1$ strange quark transitions, we could estimate¹¹⁴ $\text{Im } \eta \sim 0.03 \text{ GeV}^{-2}$ so that $\text{Im } \eta \eta_c \eta_t$ could easily be of order unity. Thus the Higgs interaction could conceivably play an important role in t decay. Exclusive final states have not yet been studied in T meson decay, but a quark level signal will be discussed in Sec. XVI.

Fig. 30. Interfering diagrams for t decay within the charged Higgs modelXIV. $Z^0 \rightarrow s\bar{b}, b\bar{s}$

One of the prime areas of experimental interest in the future will be the decays of the Z^0 . CP violation will enter Z^0 decay only through loop diagrams, as all tree diagrams conserve flavor and CP. This implies that, in any of the main decay channels of the Z^0 , CP nonconservation will be a very small effect. Only by going to rare decay modes can CP-odd effects be sizable in comparison to the CP-even amplitude. It has been suggested, for example, that flavor changing processes such as $Z^0 \rightarrow s\bar{b}$ and $Z^0 \rightarrow b\bar{s}$ may be the optimal place to look for CP violation because both the CP odd and even amplitudes are generated through the same one loop diagram, Fig. 31. The observable would be a partial rate difference

$$R_{Z^0} = \frac{\Gamma(Z^0 \rightarrow s\bar{b}) - \Gamma(Z^0 \rightarrow b\bar{s})}{\Gamma(Z^0 \rightarrow s\bar{b}) + \Gamma(Z^0 \rightarrow b\bar{s})} \quad (190)$$

The generation of this asymmetry requires the presence of final state interactions, but these are automatically included in the calculation, as Fig. 31a has an absorptive component from real $q\bar{q}$ intermediate states.

The decay rate and partial rate difference have been calculated by two groups.^{115,116} Both agree that the branching ratio is unobservably small in the case of three generations. With four generations, Ref. 115 demonstrates that, if the KM angles are given their full range allowed by known constraints, then branching ratios of order 10^{-6} can be possible with asymmetries R_{Z^0} approaching unity. On the other hand, Ref. 116 examines several suggested patterns for the KM angles in the fourth generation and finds that all of them have unobservable CP violation, although the 10^{-6} branching ratios are possible. This situation implies that the four generation model could possibly manifest this form of CP violation, although for most of the parameter space of KM angles it will be too small to be seen. Note that other theories of CP violation could have larger signals.¹¹⁶

Even if the quark level asymmetry is sizable, there remains the difficult

Fig. 31. Diagrams for $Z^0 \rightarrow b\bar{s}, s\bar{b}$

experimental task of identifying $\bar{s}b$ and $\bar{b}s$ from each other and from a background of $Z^0 \rightarrow q\bar{q}$ which is 10^6 times the strength of the desired signal. Flavor changing Z^0 decays pose interesting but very difficult questions.

XV. $p\bar{p} \rightarrow e^+ + X$

In attempting to study CP symmetry at high energy machines, new forms of signals must be found. One approach which has been suggested¹¹⁷ involves the reactions $p\bar{p} \rightarrow e^+ + X$ and $p\bar{p} \rightarrow e^- + X$, in particular the asymmetry

$$A_e(90^\circ) \equiv \frac{\frac{d\sigma}{d\Omega}(p\bar{p} \rightarrow e^+ + X) - \frac{d\sigma}{d\Omega}(p\bar{p} \rightarrow e^- + X)}{\frac{d\sigma}{d\Omega}(p\bar{p} \rightarrow e^+ + X) + \frac{d\sigma}{d\Omega}(p\bar{p} \rightarrow e^- + X)} \Big|_{\theta^* = 90^\circ} \quad (191)$$

measured in the $p\bar{p}$ center of mass at a lepton angle of 90° with respect to the beam direction. This is reasonably easy to measure, and should be able to be attempted at the CERN $p\bar{p}$ collider.

The expected signal within the standard KM model is far too small to be seen. The reaction is thus better suited for the search for new forms of CP violation. This is a potentially important function for high energy colliders, since new forms of CP violation may become more visible at high energy. The authors of Ref. 117 performed a rough estimate of this signal using an effective interaction to approximate the new physics. They found a rather small signal which would need a very large integrated luminosity in order to be observable. This seems unlikely in the near future. However tests like this are easy enough to perform that they should be studied to any available precision in an exploratory mode. It is sensitive to a type of physics which we otherwise have little opportunity to explore.

XVI. CP Violation in Jet Physics

When looking for CP violation in different systems, one is naturally led to consider high energy collisions. In this energy region, experiments looking for CP nonconservation must be different from the standard ones in meson or hyperon physics, because one does not have particle identification. Instead, the variables that can be measured, apart from the properties of the initial state, are those defining the jets.

The tests which can be constructed from these variables are in the form of triple products. If one considers reactions in which the initial state has a definite CP property, such as $e^+e^- \rightarrow \text{jets}$ or $p\bar{p} \rightarrow \text{jets}$, it is possible to identify true CP violating signals that cannot be simulated by final state interactions.¹¹⁸

The vectors that can be used to form the triple products are: the beam (p, e^-) momentum, $\vec{k}^-$; a possible total polarization of the incident particles, $\vec{S}$; and the momenta of the different jets, $\vec{p}_i$. In order to construct CP odd signals one needs events with at least three jets. There are then two types of tests that can be constructed.¹¹⁸ The first involves three jet momenta

$$\vec{J} = \vec{p}_i \times \vec{p}_j \cdot \vec{p}_k \quad (192)$$

while the second utilizes only a single jet momentum:

$$J = \vec{k}^- \times \vec{S} \cdot \vec{p}_i \quad (193)$$

To label the jet momenta one must use a "CP blind" ordering scheme. For example, $\vec{p}_i, \vec{p}_j, \vec{p}_k$ can be ordered by fastness (energy) or fatness (transverse momentum). One can then construct a CP odd observable¹¹⁸

$$A_J = \frac{N(J > 0) - N(J < 0)}{N_{\text{tot}}} \quad (194)$$

where $N(J > 0)$ means the number of events for which the triple product $\vec{J}$ is $> (<) 0$. A_J is then directly measurable as a counting asymmetry. Less useful tests using dot products or partial particle identification can also be constructed.¹¹⁸

We can give specific examples of both kinds of tests. Thus for type (1)

consider the reaction (near the Z^0 peak)

$$e^+ e^- \rightarrow t\bar{t} \rightarrow \text{jets}$$

The t-quark can decay weakly $t \rightarrow b\bar{c}s$ and in this way violate CP. In the t-quark rest frame one finds a correlation of the form¹¹⁸

$$\beta \vec{S}_t \cdot \vec{p}_b \times \vec{p}_c \quad (195)$$

where the coefficient β depends on the specific model of CP violation. This CP odd asymmetry is generated only via a spin correlation, which is unfortunate and gives rise to a few complications. The t quark must be produced with a definite polarization, which happens near the Z^0 peak. Also it must retain its spin information after hadronization. This is not the case if the quark forms a spinless meson, or a vector meson that decays strongly or electromagnetically. Using a very heavy quark (such as the t quark) we avoid this problem since a vector meson is produced a good fraction of the time and it decays weakly.¹¹⁹

When the form 195 is boosted back to the center of mass frame, one obtains a correlation of type in Eq. 192 with the replacement $\vec{S}_t \rightarrow -P\vec{p}_t$ where P is the polarization. The observable A_J is then proportional to the product βP .

Using a Higgs model of CP violation we estimate at the quark level

$$A_{\text{bare}} = 0.04 \quad (196)$$

where this is a crude estimate that must still be corrected for jet misidentification (probably through a jet Monte Carlo) and for the loss of spin information occurring during hadronization.

As a second example, consider a model with a spinless particle ϕ whose interactions violate CP. A polarized $p\bar{p}$ annihilation into three jets through the interference of W and ϕ exchanges will then produce an asymmetry of type 193¹¹⁸

$$J = (\vec{k} \times \vec{S}) \cdot \vec{p}_{\text{jet}}$$

where $\vec{p}_{\text{jet}}$ is the momentum of the fastest jet. For suitable energies, this can generate an observable of up to 0.1 the strength of the ϕ interaction. Such a test has the advantage of being less sensitive to hadronization.

These observables turn out to be quite small in the standard model, so they will be more useful as probes of new interactions that violate CP and become important at higher energies. Their main value is as a new type of tool for the study of CP violation in new physical regions.

XVII. Conclusion

It has been over two decades since the discovery of the phenomenon of CP violation. Despite heroic experimental effort and a plethora of theoretical studies, we are in 1987 not too much closer to answering the question of the ultimate source of CP nonconservation than we were in 1967. Nevertheless, as we have tried to make clear, there exist now a multitude of avenues by which an attack on the question may be mounted and a number of experiments over the next four or five years appear to have the requisite sensitivity to reveal a non-zero effect if the source of CP violation is not superweak.

We have outlined studies in the venerable kaon system, heavy quark decays, nucleon and nuclear electric dipole moments, low energy $p\bar{p}$ annihilations, high energy jet events, etc. any of which could reveal an interesting signal. It is probably not by only one such experiment but via a co-ordinated experimental attack that this remaining mystery of low energy particle dynamics will be solved. We hope that our review will stimulate work in this direction.

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Figure Captions

- Fig. 1. Superweak theories involve direct $\Delta S=2$ transitions.
- Fig. 2. Diagrams containing CP violation in theories where charge Higgs exchange is important.
- Fig. 3. The two sources of CP violation in the kaon sector a) the $K^0\bar{K}^0$ mass matrix ($\Delta S=2$), and b) direct $\Delta S=1$ CP violation in the $K \rightarrow 2\pi$ transition.
- Fig. 4. Dispersive contributions to $\text{Im } M_{12}$.
- Fig. 5. The "Siamese penguin" or "dipenguin" contribution to $K^0\bar{K}^0$ mixing.
- Fig. 6. The box diagram for $K^0\bar{K}^0$ mixing.
- Fig. 7. The $\Delta S=1$ penguin interaction.
- Fig. 8. The "electromagnetic penguin" diagram in $\Delta S=1$ transitions.
- Fig. 9. The diagram which generates isospin breaking via η - η' mixing.
- Fig. 10. Direct and tadpole diagrams which cancel for a $(3, \bar{3})$ operator.
- Fig. 11. The LR box diagram in $SU(2)_L \times SU(2)_R \times U(1)$ theories.
- Fig. 12. The dominant diagram for $K_S \rightarrow \gamma\gamma$.
- Fig. 13. The diagrams of the pole model for $K_L \rightarrow \gamma\gamma$.
- Fig. 14. The CP conserving two photon contribution to $K_L \rightarrow \pi^0 e^+ e^-$.
- Fig. 15. The CP violating one photon contribution to $K_L \rightarrow \pi^0 e^+ e^-$.
- Fig. 16. The two photon contribution to $K \rightarrow \mu^+ \mu^-$.
- Fig. 17. Diagrams relevant for $K_L \rightarrow \pi^+ \pi^- \gamma$.
- Fig. 18. The reaction $p\bar{p} \rightarrow \Lambda\bar{\Lambda} \rightarrow p\pi^- \bar{p}\pi^+$.
- Fig. 19. A diagram which generates an electric dipole moment for the neutron.
- Fig. 20. a) The box diagram for $B^0\bar{B}^0$ mixing. b) The product of decay amplitudes which generate Γ_{12} in the B meson sector.
- Fig. 21. Electromagnetic penguin diagrams for $b \rightarrow s e^+ e^-$. Also important are diagrams involving Z^0 's [see ref. 103].
- Fig. 22. A real intermediate state in the penguin diagram which leads to non-zero final state interaction phases.

Fig. 23. Diagrams for $B^0, \bar{B}^0 \rightarrow D^+ \pi^-$ which can interfere if $B^0 \bar{B}^0$ mixing occurs.

Fig. 24. Diagrams for $B_s^0, \bar{B}_s^0 \rightarrow D^0 \phi$.

Fig. 25. Interfering diagrams for $B^- \rightarrow K^- \pi^0$ involving the penguin diagram.

Fig. 26. Interfering diagrams for $B^- \rightarrow D^- D^{0*}$ involving the annihilation diagram.

Fig. 27. Different quark cascades to reach the same final state.

Fig. 28. The penguin diagram generating $B_s^0 \rightarrow \phi \phi$.

Fig. 29. The dispersive component to $D^0 \bar{D}^0$ mixing.

Fig. 30. Interfering diagrams for t decay within the charged Higgs model.

Fig. 31. Diagrams for $Z^0 \rightarrow b \bar{s}, s \bar{b}$.