The quark content of the proton

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The quark content of the proton is studied using the quark scalar bilinear, \( \bar{q}q \), as a probe. An analysis of hadron masses and the sigma term suggests an unexpectedly large signal for \( ss \). This is seen to be reasonable within both the Skyrme model and the bag model. The interpretation of the quark content is discussed in both models.

1. Introduction. The quark model has taught us to think of protons and neutrons as bound states composed of just three quarks. Of course in deep inelastic scattering one also includes gluons and sea quarks in the description of the nucleon. However, it is commonly said that these non-valence components are a consequence of the high \( Q^2 \) limit (and/or use of the infinite momentum frame) and that at low \( Q^2 \) (or at rest) the proton is basically just the three valence quarks. We will argue in this paper that sea quarks are more numerous than commonly thought, even in the rest frame.

The fermion number operator \( \psi^+ = \bar{Q} - \bar{Q} \), (1)

so that its matrix element will only give the net quark number. To isolate nonvalence contributions we need an operator which sums the quark and antiquark components, such as \( \bar{\psi} \psi \),

\[ \bar{\psi} \psi \sim Q + \bar{Q}. \] (2)

While eq. (2) is a good mnemonic to remember the effect of \( \bar{\psi} \psi \), it is only partially correct. One of the subtle features associated with this operator is that, unlike the number operator, it can have off diagonal elements. This means that when \( \bar{\psi} \psi \) acts on a state it not only counts the number of \( Q \) and \( \bar{Q} \), but also can annihilate a quark pair or create a pair. Since the real proton can be decomposed into a series of Fock components (e.g. \( QQQ, QQQ\bar{Q}, \ldots \)) the matrix element of \( \bar{\psi} \psi \) can also be generated by transitions between the Fock components. Another subtle feature is that \( \bar{\psi} \psi \) has the quantum numbers of the vacuum and has a vacuum expectation value. This means that matrix elements of \( \bar{\psi} \psi \) reflect not just the intrinsic properties of the hadron but are really the difference between the hadron and the vacuum. Nevertheless matrix elements of \( \bar{\psi} \psi \) do provide a well-defined measure of the nucleon properties and can signal the presence of strange quarks in the nucleon (i.e. if \( \langle p | \bar{s}s | p \rangle \neq 0 \)). In this paper we will adopt the scalar density as our probe of the quark content.

The remaining parts of the paper are divided into three main sections, followed by a summary. In section 2, we explore what is known experimentally about the scalar density and show that a large \( ss \) signal is indicated. After this we turn to models to convince ourselves that this is not unreasonable. Section 3 describes the quark content of the skyrmion, a chiral soliton model of the nucleon. In section 4 the matrix element of \( \bar{\psi} \psi \) is discussed within the MIT bag model. The data, the
Skyrme model and the bag model are in remarkably good agreement and we can characterize our overall results in the ratios

\[ \frac{\langle p|\bar{u}u|p\rangle}{\langle p|\bar{u}u + \bar{d}d + \bar{s}s|p\rangle} \approx 0.40 \rightarrow 0.45, \]

\[ \frac{\langle p|\bar{d}d|p\rangle}{\langle p|\bar{u}u + \bar{d}d + \bar{s}s|p\rangle} \approx 0.33 \rightarrow 0.36, \]  

\[ \frac{\langle p|\bar{s}s|p\rangle}{\langle p|\bar{u}u + \bar{d}d + \bar{s}s|p\rangle} \approx 0.21 \rightarrow 0.29. \]  

2. Experiment - hadron masses and the sigma term.

This section makes use of an observation of Cheng [1] that the \( \pi \)-nucleon sigma term is not compatible with hadron masses if the proton matrix element of \( KS \) vanishes. This information can be inverted to get an “experimental” measure of this quantity, as we do below.

The SU(3) mass splittings are due to the quark mass portion of the QCD hamiltonian

\[ H_m = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s. \]

Here we will ignore isospin splittings, and use \( \bar{m} \equiv (m_u + m_d)/2 \). In this case,

\[ H_m = \frac{1}{3}(m_s + 2\bar{m})(\bar{u}u + \bar{d}d + \bar{s}s) \]

\[ -\frac{1}{3}(m_s - \bar{m})(\bar{u}u + \bar{d}d - 2\bar{s}s). \]

The first term is an SU(3) singlet and cannot be distinguished in the spectrum but the second term, the eighth component of an SU(3) octet, can be observed in the SU(3) mass breaking pattern within an octet. A related operator is the sigma term

\[ \delta \eta = [F_i^5, \tilde{d}_\mu A_{i\mu}] \]

\[ \sigma = m_u \bar{u}u + m_d \bar{d}d = \bar{m}(\bar{u}u + \bar{d}d), \]

which can be measured in \( \pi \)-nucleon scattering [2]

\[ \langle p|\sigma|p\rangle = 57 \pm 7 \text{ MeV}. \]  

To relate these two let us first define a new operator

\[ \delta \equiv \bar{m}(\bar{u}u + \bar{d}d - 2\bar{s}s) \]

\[ = [\bar{m}/(m_s - \bar{m})](m_s - \bar{m})(\bar{u}u + \bar{d}d - 2\bar{s}s). \]  

The ratio of quark masses is known from the current-algebra analysis of the masses of pseudoscalar mesons [3]

\[ 2\bar{m}/(m_s + \bar{m}) = m^2_\pi/m^2_k = \frac{1}{12}. \]  

The use of SU(3) then lets us evaluate the proton’s matrix element of \( \delta \) in terms of hadron masses [1]

\[ \langle p|\delta|p\rangle = [3\bar{m}/(m_s - \bar{m})](M^2 - M^2_A) \approx 26 \text{ MeV}. \]

Cheng’s observation was that if the proton’s matrix element of \( \bar{s}s \) vanished in accord with naive expectation, then \( \langle \sigma \rangle = \langle \delta \rangle \). This assumption appears to be far from reality. We will take the difference as being due to the \( \bar{s}s \) matrix element and find

\[ \langle p|\frac{1}{2}(\sigma - \delta)|p\rangle = \bar{m}(\bar{s}s|p\rangle = 15 \pm 4 \text{ MeV}. \]

Combined with eq. (7), this implies

\[ \frac{\langle p|\bar{s}s|p\rangle}{\langle p|\bar{u}u + \bar{d}d + \bar{s}s|p\rangle} = 0.21 \pm 0.03, \]

\[ \frac{\langle p|\bar{u}u + \bar{d}d|p\rangle}{\langle p|\bar{u}u + \bar{d}d + \bar{s}s|p\rangle} = 0.79 \pm 0.03. \]  

The matrix element of \( \bar{s}s \) is rather large.

This is as far as we can go with the model-independent analysis. However, if we make an assumption, which can be at least partially justified in the quark model, we can obtain the \( \bar{u}u \) and \( \bar{d}d \) weights separately. We will use the general quark-model division of the matrix elements into valence (V) and sea (S) contributions,

\[ \bar{m}(\bar{u}u|p\rangle = 2V + S, \quad \bar{m}(\bar{d}d|p\rangle = V + S, \]

\[ \bar{m}(\bar{s}s|p\rangle = S \]

from which we find \( V = 9 \text{ MeV}, S = 15 \text{ MeV} \) and hence

\[ \frac{\langle p|\bar{u}u|p\rangle}{\langle p|\bar{u}u + \bar{d}d + \bar{s}s|p\rangle} = 0.45 \pm 0.02, \]

\[ \frac{\langle p|\bar{d}d|p\rangle}{\langle p|\bar{u}u + \bar{d}d + \bar{s}s|p\rangle} = 0.34 \pm 0.02. \]  

These numbers are as close as we can come to direct data on the quark content of the proton.

3. The quark content of the skyrmion. In the Skyrme mode [4] the nucleon is treated as a soliton of the nonlinear chiral lagrangian, whose field variables, in the weak field limit, are pions, kaons and etas. The
lagrangian is
\[ L = L_0 + L_m, \]

\[ L_0 = \frac{1}{4} F^2 \text{Tr}(\partial^\mu U \partial^\nu U^\dagger), \]

\[ + \frac{1}{32 \pi^2} \text{Tr}(U_{\mu\nu} U^\dagger U_{\alpha\beta} U^\dagger)^2, \]

\[ L_m = \frac{1}{4} F^2 \text{Tr}(UM + M^* U^\dagger M - M^* M^+). \]

(15)

In these formulae \( U \) is a unitary 3 X 3 matrix of the field variables
\[ U = \exp \left( 2i \lambda^A \phi^A (x) / F \right), \]

and the matrix \( M \) in the mass term is a diagonal matrix whose entries are connected with the quark masses. A baryonic solution can emerge as a soliton of the form
\[ U = A(t) U_0 A^{-1}(0), \]

(16)

where \( A(t) \) are SU(3) matrices. The baryons are described by wavefunctions depending on the collective coordinates \( A \), which have the form
\[ \psi(A) = (\text{dim } n)^{1/2} D^{(n)}_{a,b}(A), \]

(18)

where \( D^{(n)}_{a,b}(A) \) is the unitary matrix for the representation \( (n) \) of the SU(3) transformation \( A \). Each subscript is a set of three numbers: \( a = (-Y, I, -I) \) determines hypercharge and isospin quantum numbers of the given baryon and \( b = (1, S, +S) \) determines the spin quantum numbers.

The unitary matrix \( U \) in the chiral lagrangian transforms under SU(\( N_f \)) X SU(\( N_f \)) the same way as the quark bilinear \( \bar{q}(1 + \gamma_5)q \) does. A plausible, although crude, phenomenological ansatz is to identify them up to an unknown (dimensionful) coefficient
\[ U^{\dagger}_i = \bar{q}(1 + \gamma_5)(1 + \gamma_5)q. \]

(19)

This identification is also supported by the structure of the mass term where \( \text{Tr}(UM + M^* U^\dagger) \) plays the role of \( \bar{q} M q \). Consequently, if \( U \) is the rotated skyrmion, we can interpret
\[ \text{Tr}(U + U^\dagger) \sim \langle \bar{u}u + \bar{d}d + \bar{s}s + \ldots \rangle_{\text{baryon}}, \]

(20)

and
\[ \text{Tr}(U + U^\dagger - 2) \sim \langle \bar{u}u + \bar{d}d + \bar{s}s + \ldots \rangle_{\text{baryon}}, \]

\[ - \langle \bar{u}u + \bar{d}d + \bar{s}s + \ldots \rangle_{\text{vacuum}}. \]

(21)

More in general, for any matrix \( X \)
\[ \text{Tr}[XU + U^\dagger X^\dagger - X - X^\dagger] \sim \langle qXq \rangle_{\text{baryon}} \]

\[ - \langle qXq \rangle_{\text{vacuum}}. \]

(22)

This way we will be able to compute the expectation values of quark bilinears \( \langle \bar{u}u \rangle, \langle \bar{d}d \rangle, \langle \bar{s}s \rangle \) (from now on we indicate the difference \( \langle \cdot \rangle_{\text{baryon}} - \langle \cdot \rangle_{\text{vacuum}} \) simply by \( \langle \cdot \rangle \)) in the chiral soliton model. Let us consider the two flavor case first and take \( X = \text{diagonal} \ (1, -1) \). Then the left-hand side of (22) is the difference \( \bar{u}u - \bar{d}d \) and we get
\[ \langle \bar{u}u - \bar{d}d \rangle = 0. \]

(23)

Namely in the SU(2) baryon
\[ \langle \bar{u}u \rangle = \langle \bar{d}d \rangle, \]

(24)

in agreement with the observation that isospin cannot be broken.

Similarly, we can compute \( \langle qq \rangle \) in the various baryons in the three-flavor case. For instance, if we want to compute \( \langle \bar{u}u \rangle \) we take \( X = \text{diagonal} \ (1, 0, 0) \) and we get
\[ \langle \bar{u}u \rangle = k \left[ 1 + \frac{1}{\sqrt{3}} \text{Tr}(\lambda_3 A \lambda_8 A^{-1}) \right] \]

\[ + \frac{1}{4} \text{Tr}(\lambda_8 A^{-1} \lambda_8 A), \]

(25)

where \( k \) is some model dependent constant. In order to evaluate the above expression in a baryon state we write operators of collective coordinates \( A \) and baryon wavefunctions in terms of unitary matrices of the proper representation of the SU(3) transformation \( A \).

For instance
\[ \frac{1}{2} \text{Tr}(\lambda_3 A^\dagger \lambda_8 A) = D_{000,013}^8(A), \]

(26)

and
\[ \psi_{\text{proton}}(A) = 2\sqrt{2} D_{-1 1/2 -1/2}^8(A). \]

(27)

This way we can evaluate (25) between baryon states. For the proton and neutron \( P \) and \( N \) we get
\[ \langle \bar{u}u \rangle_P = k \frac{13}{10}, \]

\[ \langle \bar{d}d \rangle_P = \langle \bar{u}u \rangle_N = k \frac{11}{10}, \]

\[ \langle \bar{s}s \rangle_P = \langle \bar{s}s \rangle_N = k \frac{7}{10}. \]

(28a,b,c)

\[ \text{For a recent review see ref. [5].} \]
For the cascade $\Xi^0, \Xi^-$ we get
\[ \langle \bar{u}u \rangle_{\Xi^0} = \langle \bar{d}d \rangle_{\Xi^-} = k \tfrac{11}{10}, \quad \langle \bar{d}d \rangle_{\Xi^0} = \langle \bar{u}u \rangle_{\Xi^-} = k \tfrac{7}{10}. \]
(29a,b)
\[ \langle \bar{s}s \rangle_{\Xi^0} = \langle \bar{s}s \rangle_{\Xi^-} = k \tfrac{12}{10}. \]
(29c)
Notice that the results (29) for the $\Xi^0, \Xi^-$ system can be obtained from those for the $P, N$ system by changing $d$ quarks into $s$ quarks. Notice also that from (28) one gets
\[ \langle \bar{u}u \rangle_p = \tfrac{12}{11}\langle \bar{d}d \rangle_p, \quad \langle \bar{d}d \rangle_N = \tfrac{12}{11}\langle \bar{u}u \rangle_N, \]
(30)
which are a slight improvement of SU(2) result (24), and again seem to reflect the fact that at the SU(2) level isospin breaking cannot be incorporated in the Skyrme model.

These results transform into fractional amounts for the quark bilinears of the form
\[ \frac{\langle p | \bar{u}u | p \rangle}{\langle p | \bar{u}u + \bar{d}d + \bar{s}s | p \rangle} = 0.40, \]
\[ \frac{\langle p | \bar{d}d | p \rangle}{\langle p | \bar{u}u + \bar{d}d + \bar{s}s | p \rangle} = 0.37, \]
\[ \frac{\langle p | \bar{s}s | p \rangle}{\langle p | \bar{u}u + \bar{d}d + \bar{s}s | p \rangle} = 0.23. \]
(31)
These results are reasonably consistent with the sigma term analysis.

4. Quark content in the bag model. There are two separate effects in the bag model which can generate an expectation value for $\bar{s}s$ in a proton. One arises because the vacuum produces an $\bar{s}s$ matrix element while the bag model proton may have less $\bar{s}s$, hence a difference between the two is generated. The other mechanism is the more obvious one of quark pairs in the proton. The $\bar{s}s$ matrix element due to quark pairs enters only at $O(\alpha_s^2)$ and has not yet been calculated. Hence we will focus on the vacuum effect below.

The theoretical description of the bag model [6] involves two types of "vacuum". The true vacuum of QCD is the lowest energy state in the theory and is a complicated configuration of quark and gluon fields. In the bag model one assumes that the interior of the hadrons is the simple "perturbative vacuum" (similar to the ground state of QED) which has an energy density $\rho$ greater than the true vacuum. The true vacuum has nonvanishing matrix elements of the quark scalar density, as is known from the theory of chiral symmetry [3]
\[ m^2_{\pi} = (m_u + m_d)\langle \pi | \bar{q}q | \pi \rangle \]
\[ = -\frac{1}{F^2} \langle \pi | \bar{q}q | \pi \rangle, \]
(32)
and
\[ m^2_K = -\frac{1}{F^2} (m_s + m_u) \langle 0 | \bar{q}q | 0 \rangle. \]
(33)
In the last equation SU(3) symmetry has been used. The numerical value of the vacuum expectation value depends on the absolute size of the quark masses, not on mass ratios. For an estimate we will use the bag model value of the mass of the strange quark $m_s \approx 300$ MeV and $m_u < m_s$ to obtain
\[ \langle 0 | \bar{q}q | 0 \rangle = -\frac{m^2_F}{F^2} \frac{1}{m_s + m_u} \approx -0.007 \text{ GeV}^3. \]
(34)

On the other hand, the interior of the bag is a region of perturbative vacuum. In the limit that the quark masses go to zero an infinite volume of perturbative vacuum would have no $\bar{q}q$ matrix element, as it is chirally symmetric. Hence at the simplest level we would expect a vanishing bag matrix element
\[ \langle \text{bag} | \bar{q}q | \text{bag} \rangle = 0. \]

In principle, however, a finite region of perturbative vacuum can in fact have a $\bar{q}q$ matrix element due to boundary effects (similar to the Casimir effect [8] in QED and the zero point energy in the bag model [9]). This appears to occur in actual bag model calculations [10]. However, one would expect the bag matrix element to be reduced compared to that of the true vacuum. Let us use
\[ \langle \text{bag} | \bar{q}q | \text{bag} \rangle = \eta \langle 0 | \bar{q}q | 0 \rangle, \]
(35)
where $\eta(0 < \eta < 1)$ will in general be a function of the bag radius ($\eta \rightarrow 0$ as $R \rightarrow \infty$). Consider a proton which has only valence quarks plus the bag. We write the matrix elements as
\[ \langle P(p) | \bar{q}_i q_j | P(p) \rangle = A_j \bar{u}(p) u(p) \]
(36)
for $i = u, d, s$. The matrix element is a difference between the bag proton and the vacuum. Neglecting
center of mass corrections

\[ A_i = \text{bag}(P \int d^3x \bar{q}_i(x)q_i(x) |P)_{\text{bag}} - \langle 0 | \int d^3x \bar{q}_i(x)q_i(x) |0 \rangle \]  \tag{37}

The result divides into valence and vacuum contributions

\[ A_u = 2Z + \text{vac}, \quad A_d = Z + \text{vac}, \quad A_s = \text{vac}, \]  \tag{38}

where

\[ Z = \int d^3x(u^2 - l^2) = 0.48, \]  \tag{39}

and

\[ \text{vac} = (\eta - 1)\langle 0 \bar{q} q 10 \rangle V_p, \]  \tag{40}

where \( V_p \) is the volume of the proton \((R_p = 1 \text{ fm})\). For \( \eta < 1 \) the nonvalence contribution arises because the interior of the bag has less \( \bar{s}s \) than the true vacuum.

Due to lack of knowledge about the vacuum effects, firm predictions cannot be made, but the size of the vacuum effect is substantial. In the naive bag model limit \( (\eta = 0) \) one would find \( \text{vac} = 3.6 \)

\[ A_u/(A_u + A_d + A_s) = 0.38, \]

\[ A_u/(A_u + A_d + A_s) = 0.33, \]  \tag{41}

\[ A_s/(A_u + A_d + A_s) = 0.29, \]

while for \( \eta = 1/2 \) the result is

\[ A_u/(A_u + A_d + A_s) = 0.41, \]

\[ A_d/(A_u + A_d + A_s) = 0.33, \]  \tag{42}

\[ A_s/(A_u + A_d + A_s) = 0.26. \]

Thus even a valence bag model can accommodate a large \( \bar{s}s \) matrix element. Of more interest than the specific numbers is the possible explanation of the \( \bar{s}s \) signal as being due to the proton having less \( \bar{s}s \) than the vacuum. This may also be what occurs in the Skyrme model.

5. Summary. The size of the \( \bar{s}s \) matrix element is larger than would be expected in the naive quark model. However, both the data and the models seem to be consistent and we feel that eq. (3) is most likely accurate. The interpretation of this number may be somewhat subtle. Because the vacuum also contains a (negative) matrix element of \( \bar{s}s \), the proton's expectation value may be caused by it having less \( \bar{s}s \) than the vacuum. In any case, it appears to be a real effect and indicates a need to go beyond the simple valence quark model.

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Note added. Several of the questions raised here have been addressed in a paper by Jaffe [11]. In that paper an attempt is made at a solution in the context of a model intermediate between a bag model and what would now be recognized as a Skyrme model.

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