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Low energy weak interactions of quarks

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Abstract:

We review the progress that has been made in understanding weak interaction effects in hadrons. Our focus is on the quark model and its role in connecting hadronic processes with the underlying $SU(2)_L \times U(1)$ weak interaction theory. Open questions, new techniques and future directions are also discussed.

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1. Introduction

By the standards of high-energy physics, the subject of low-energy semileptonic and nonleptonic interactions is a rather ancient one. Indeed, by the late 1960's its experimental foundations were already well established, and important theoretical insights had emerged. These are described in the classic treatise of Marshak, Riazuddin and Ryan [1] which was printed in 1969. The book contains a wealth of information on data, kinematics, and symmetry structures of the weak interactions, much of which remains essentially valid up to the present.

However, one also notices how little was known at that time about the origins of the weak interactions. Since then, remarkable progress in both theory (especially in the area of nonabelian gauge interactions) and experiment (neutral currents, etc.) has led to the successful $SU(2)_L \times U(1)$ description of the electroweak interactions [2, 3]. With the very recent discovery of the $W^\pm$ [4] and Z^0 [5] bosons our confidence in this model is now secure. As the development of weak interaction physics occurred, substantial activity in revealing the nature of the strong interactions was also taking place. Of particular significance was the growth of the quark model [6], motivated especially by the discovery of the heavy quarks [7] and by the emergence of quantum chromodynamics (QCD) as a theory of interacting quarks and gluons [8].

We view the present report as an updating of the analyses of semileptonic and nonleptonic processes described in ref. [1]. The unifying concept is the quark degree of freedom as it contributes to the electroweak currents. Although our ability to calculate precisely within this picture remains limited, there is no question of its utility in providing a unified description of an impressively diverse collection of data. Indeed we shall consider quarks confined in low-energy mesonic, baryonic, and nuclear systems which undergo a variety of $|\Delta S| = 0, 1, 2$ weak processes, including some associated with CP-violations. We define 'low-energy' systems to be those containing just the light up, down and strange (u, d and s) quarks. The heavy charm, bottom and top (c, b and t) quarks enter the physics only through virtual processes.

Throughout, our emphasis will be unabashedly phenomenological. That is, we are primarily interested in confronting theory with experiment wherever possible. The idea is to attain an intuitive and physical understanding of these phenomena. This will of necessity involve occasionally employing model, rather than rigorous, descriptions of quark spatial wavefunctions. Despite the uncertainty this introduces into our analysis, we hope to convince the reader that the data, viewed as a whole, is in at least qualitative accord with expectations derived from the conventional six-flavor $SU(3)_c \times SU(2)_L \times U(1)$ quark model. With the *possible* exception of the KM mixing matrix description of CP-violations [9], there appears to be no need to invoke 'new physics' to explain observed processes of the type considered here.

Before embarking on our survey of weak processes involving quarks, we shall need to define our system of notations and conventions. This is done in the next section. Our usage is relatively standard, and the informed reader may turn directly to section 3, which contains a 'user's guide to quark models'. This is an attempt on our part to provide the reader with an explicit example of how a calculation involving confined quarks is actually performed. The interplay between the underlying fundamental interactions (assumed known) and the smearing effect of quark confinement (not very well known) will hopefully be clarified.

To close this Introduction, we note that the material contained in this review summarizes the experimental and theoretical situation as of December 1984.

2. General background of the standard model

2.1. Conventions

We employ the metric convention $g^{00} = -g^{11} = -g^{22} = -g^{33} = 1$, and adopt units such that $\hbar = c = 1$. The Dirac equation for a free particle of mass m is

$$(i\gamma^\mu \partial_\mu - m) \psi(x) = 0. \quad (2.1)$$

Zero mass solutions of this equation can be characterized by the chirality,

$$\psi_L \equiv \frac{1+\gamma_5}{2} \psi, \quad \psi_R \equiv \frac{1-\gamma_5}{2} \psi \quad (2.2)$$

which is maintained under Lorentz transformations. Later we shall encounter the chiral combinations

$$\Gamma_L^\mu \equiv \gamma^\mu (1 + \gamma_5), \quad \Gamma_R^\mu \equiv \gamma^\mu (1 - \gamma_5) \quad (2.3)$$

and their scalar counterparts

$$\Gamma_L \equiv 1 + \gamma_5, \quad \Gamma_R \equiv 1 - \gamma_5 \quad (2.4)$$

in our discussion of weak currents and the nonleptonic Hamiltonian. Of particular importance are the Fierz transformations

$$\begin{aligned} (\Gamma_L^\mu)_{\alpha\beta} (\Gamma_{L\mu})_{\gamma\delta} &= -(\Gamma_L^\mu)_{\alpha\delta} (\Gamma_{L\mu})_{\gamma\beta} \\ (\Gamma_L^\mu)_{\alpha\beta} (\Gamma_{R\mu})_{\gamma\delta} &= 2(\Gamma_L)_{\alpha\delta} (\Gamma_R)_{\gamma\beta}. \end{aligned} \quad (2.5)$$

2.2. Electroweak $(SU(2)_L \times U(1))$ interactions

According to the Glashow–Salam–Weinberg model, chiral (massless) quarks transform under operations of the weak isospin as doublets if left-handed

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L, \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L, \quad (2.6)$$

and singlets if right-handed

$$u_R, d_R, s_R, c_R, b_R, t_R. \quad (2.7)$$

There is mixing among left-handed quarks,

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_L = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \quad (2.8)$$

where the matrix V is unitary. In the Kobayashi–Maskawa (KM) parameterization we have [10]

$$V = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 + s_2 s_3 e^{i\delta} & c_1 c_2 s_3 - s_2 c_3 e^{i\delta} \\ -s_1 s_2 & c_1 s_2 c_3 - c_2 s_3 e^{i\delta} & c_1 s_2 s_3 + c_2 c_3 e^{i\delta} \end{pmatrix} \quad (2.9)$$

where $s_i = \sin \theta_i$ ($i = 1, 2, 3$), etc. A nonzero value for δ signals the presence of CP-violating processes.

For quarks the charged weak current, neutral weak current, and electromagnetic currents are respectively

$$J_\mu^c = (\bar{u} \bar{c} \bar{t}) \Gamma_{L\mu} V \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad (2.10)$$

$$J_\mu^N = (\bar{u} \bar{c} \bar{t}) \left[\frac{1}{2} \Gamma_{L\mu} - \frac{4}{3} \sin^2 \theta_w \gamma_\mu \right] \begin{pmatrix} u \\ c \\ t \end{pmatrix} + (\bar{d} \bar{s} \bar{b}) \left[-\frac{1}{2} \Gamma_{L\mu} + \frac{2}{3} \sin^2 \theta_w \gamma_\mu \right] \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad (2.11)$$

$$J_\mu^{\text{EM}} = \frac{2}{3} (\bar{u} \bar{c} \bar{t}) \gamma_\mu \begin{pmatrix} u \\ c \\ t \end{pmatrix} - \frac{1}{3} (\bar{d} \bar{s} \bar{b}) \gamma_\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \quad (2.12)$$

These currents couple locally to the $W^\pm$ and Z^0 fields and to the photon field (A) as

$$L_{\text{INT}} = e \left[\frac{W^{\mu+} J_\mu^c + \text{h.c.}}{2\sqrt{2} \sin \theta_w} + \frac{Z^\mu J_\mu^N}{\sin 2\theta_w} + A^\mu J_\mu^{\text{EM}} \right] \quad (2.13)$$

where e is the electric charge, normalized in terms of the fine structure constant α as $\alpha = e^2/4\pi$. In view of the large masses of the $W^\pm$, Z^0 bosons, it is useful for momentum transfer $Q^2 \ll M_w^2$ to work with the effective nonleptonic interactions

$$L_{\text{NL}} = \frac{G_F}{\sqrt{2}} [J_c^\mu J_\mu^{c\dagger} + 2\rho J_N^\mu J_\mu^N] \quad (2.14)$$

where

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M_w^2 \sin^2 \theta_w}, \quad \rho = \left(\frac{M_w}{M_z \cos \theta} \right)^2. \quad (2.15)$$

The interaction eq. (2.14) is modified in several important respects by QCD radiative corrections (see section 2.3 and section 6).

The numerical values of parameters defined in both this section and also later in this report are displayed in table 2.1. The origin of quantities such as quark masses and KM mixing angles is a subject of obvious interest, as indeed is the question of which physical mechanism underlies the breaking of $SU(2)_L \times U(1)$ symmetry. However these matters lie beyond the scope of our review. Our intent is rather to see whether the concepts described here can bring coherence to the set of available data.

2.3. Strong ($SU(3)_c$) interactions

Following some introductory remarks, we briefly comment on QCD radiative corrections to weak interaction processes and then chiral symmetry.

The basis for the strong interactions is believed to be provided by the QCD Lagrangian [11]

$$L_{\text{QCD}} = \bar{q}(i\not{D} - M)q - \frac{1}{4}F_{\mu\nu}^A F_{\mu\nu}^A \quad (2.16)$$

where q represents the quark fields u, d, s, c, b, t, and where M is the quark mass matrix, $F_{\mu\nu}^A$ is the gluon (G_{μ}^A) field tensor ($A = 1, \dots, 8$),

$$F_{\mu\nu}^A = \partial_{\mu}G_{\nu}^A - \partial_{\nu}G_{\mu}^A + gf^{ABC}G_{\mu}^B G_{\nu}^C \quad (2.17)$$

and the covariant derivative is defined by

$$\not{D}q = \left(\partial_{\mu} - ig \frac{\lambda^A}{2} G_{\mu}^A \right) \gamma^{\mu} q. \quad (2.18)$$

The matrices $\lambda^A/2$ are generators of $SU(3)_c$ in the triplet representation with $\text{Tr } \lambda^A \lambda^B = 2\delta^{AB}$. In eqs. (2.17), (2.18), g is the gauge coupling constant of $SU(3)_c$, with an associated ‘fine structure constant’ $\alpha_s = g^2/4\pi$. At the renormalized level of QCD, it is most convenient to characterize the interaction

Table 2.1.
Numerical values of parameters of the standard model

$G_F = (1.16632 \pm 0.00002) \times 10^{-5} \text{ GeV}^{-2}$	$ V_{ud} = 0.974 \pm 0.003$
$\sin^2 \theta_w = 0.217 \pm 0.014$	$ V_{us} = 0.225 \pm 0.005$
$\rho = 1.002 \pm 0.02$	$ V_{ub} < 0.01$
$M_w = 80.9 \pm 2.8 \text{ (UA1)}; 83.1 \pm 1.9 \pm 1.2 \text{ (UA2)}$	$ V_{cb} = 0.044 \pm 0.005$
$M_z = 93.9 \pm 2.9 \text{ (UA1)}; 92.7 \pm 1.7 \pm 1.4 \text{ (UA2)}$	$F_{\pi} = 93 \text{ MeV}$
$m_c \approx 1.5 \text{ GeV}$	$F_K = 117 \pm 2 \text{ MeV}$
$m_b \approx 4.9 \text{ GeV}$	$50 \text{ MeV} \leq \Lambda_{\text{QCD}} \leq 500 \text{ MeV}$
$30 \text{ GeV} \leq m_i \leq 60 \text{ GeV (UA1)}$	

between quarks and gluons at energy scale μ with an effective (or ‘running’) coupling constant [12]

$$\alpha_s(\mu^2) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda_{\text{QCD}}^2)} \quad (2.19)$$

where β_0 is related to the number of quark flavors N_f as $\beta_0 = 11 - \frac{2}{3}N_f$.

QCD radiative corrections have important effects on weak interaction transitions. The most useful theoretical tools for treating them are operator product expansions and renormalization group methods [13]. Briefly, with the operator product expansions one characterizes the generally singular product of field operators at short distances with an appropriate basis of local operators O_n , which are ordered according to dimension and each of which is multiplied by a spatially dependent coefficient function E_n . The renormalization group procedure is then used to integrate the E_n from the origin out to some distance μ (the renormalization point), typically chosen (in part) so that the QCD coupling eq. (2.19) is sufficiently small to justify use of perturbation theory. The effects of distances greater than μ^{-1} is accounted for by matrix elements of the local operators.

An outstanding example of the procedure just outlined involves effective nonleptonic Hamiltonians [14]. Here the operator product expansion takes the form (for definiteness consider the only flavor change to be $\Delta S = 1$)

$$T(J_{\Delta S=1}^\mu(x) J_\mu^{\Delta S=0}(0)) = \sum_n E_n(\mu x, g) O_n(0), \quad (2.20)$$

which is to be multiplied by the W-boson propagator and integrated over space. The presence of the W-boson propagator evidently ensures that the expansion in eq. (2.20) is a rapidly converging one. The resulting form for $H_{\Delta S=1}$ is

$$H_{\Delta S=1} = \sum_n c_n O_n, \quad (2.21)$$

where the dependence of each c_n on the scale μ is logarithmic, $c_n \sim [1 + a_n \ln(M_w/\mu)]^{b_n}$ (a_n, b_n are constants). We shall develop this subject in greater detail in section 6.

Two approximate properties manifested by the light hadrons are the ‘flavor’ symmetries $SU(N)$ ($N = 2, 3$) and the chiral symmetries $SU(N)_L \times SU(N)_R$ ($N = 2, 3$). At this point let us define our conventions regarding the relation between the physical baryon states P , Λ , etc. and the octet of states $\{B_i\}$ ($i = 1, \dots, 8$) in Cartesian basis,

$$\begin{aligned} \Sigma^\pm &= \sqrt{\frac{1}{2}}(B_1 \pm iB_2) & \Sigma^0 &= B_3 & \Lambda &= B_8 \\ P &= \sqrt{\frac{1}{2}}(B_4 + iB_5) & \Xi^- &= \sqrt{\frac{1}{2}}(B_4 - iB_5) \\ N &= \sqrt{\frac{1}{2}}(B_6 + iB_7) & \Xi^0 &= \sqrt{\frac{1}{2}}(B_6 - iB_7). \end{aligned} \quad (2.22)$$

These states can be transformed into each other via the ladder operators

$$T_\pm = F_1 \pm iF_2, \quad U_\pm = F_6 \pm iF_7, \quad V_\pm = F_4 \pm iF_5 \quad (2.23)$$

along with the algebraic relations

$$[F_i, B_j] = if_{ijk} B_k \quad (i, j, k = 1, \dots, 8), \quad (2.24)$$

where the $\{F_i\}$ are the generators of SU(3). Identical relations hold between the physical pseudoscalars π^0 , K^- , etc. and the octet of states $\{P_i\}$.

The above relations enter our discussion at several points. They underlie the quark wave functions for baryons and mesons which appear in section 3. Also, from time to time we shall find it useful to represent matrix elements of octet operators $\{O_i\}$ by their SU(3) approximations

$$\langle B_i | O_j | B_k \rangle = if_{ijk} F + d_{ijk} D \quad (2.25)$$

where F, D are form factors.

Chiral symmetry constitutes an important feature of the low-energy or threshold structure of hadronic systems [15]. The $SU(3)_L \times SU(3)_R$ chiral symmetry arises if the masses m_u, m_d, m_s are neglected in the QCD Lagrangian. It then becomes meaningful to consider SU(3) flavor transformations of left-handed and independently of right-handed quarks. The vacuum state determines how the chiral symmetry is realized. It contains a quark-antiquark condensate ($\langle \bar{q}q \rangle_{\text{vac}} \neq 0$ for $q = u, d, s$) which is invariant under the flavor transformations of SU(3) but not the full set of $SU(3)_L \times SU(3)_R$ chiral operations. Thus, associated with the chiral symmetry is a set of eight Goldstone bosons, the 0^- octet. In reality they are not massless because chiral symmetry is not exact ($m_{u,d,s} \neq 0$). Clearly the $SU(2)_L \times SU(2)_R$ symmetry with light pions is an even better approximation of Nature than is $SU(3)_L \times SU(3)_R$.

Probably the chief impact of chiral symmetry on weak interaction theory resides in soft-meson theorems such as ($a = 1, 2, 3$) [15]

$$\langle \pi_a(q) | \beta | O | \alpha \rangle \xrightarrow{q \rightarrow 0} \frac{-i}{F_\pi} \langle \beta | [F_a^5, O] | \alpha \rangle + R_a \quad (2.26)$$

where

$$R_a = \lim_{q \rightarrow 0} q_\mu M_a^\mu(q) \quad (2.27)$$

with

$$M_a^\mu(q) = \int d^4x e^{iq \cdot x} \theta(x^0) \langle \beta | [A_a^\mu(x), O] | \alpha \rangle. \quad (2.28)$$

Here $R = 0$ unless there exists a state degenerate with α or β and which contributes to $M_a^\mu(q)$. In eq. (2.26) O is typically a weak current or one of the local operators occurring in the effective nonleptonic Hamiltonian. An important test of the soft-pion technique has been to relate $K \rightarrow 2\pi$ and $K \rightarrow 3\pi$ amplitudes. The results are accurate to within 10% for both the magnitudes and slope parameters for $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$ transitions [16]. Besides affirming the validity of PCAC, this places stringent bounds in charged weak currents of right-handed admixtures [17].

To implement the constraints of chiral symmetry in actual calculations, it is often convenient to employ chiral Lagrangians [18]. Consider for definiteness the $SU(3)_L \times SU(3)_R$ chiral symmetry. Let us construct a Lagrangian depending on a 3×3 matrix M which transforms as $(3_L, 3_R^*)$. Aside from symmetry breaking terms we expect L to transform as $(1_L, 1_R)$ and $(8_L, 1_R)$ for the strong and weak interactions respectively. It can be shown that M is unitary ($M^\dagger M = I$), has meson content

$$M = \exp(i\lambda \cdot \phi / F_\pi) \quad (2.29)$$

where ϕ are the eight pseudoscalar meson fields, and is mapped into $M^\dagger, M^T$ under the operations of parity and charge conjugation respectively [19]. In fact, there exist an infinity of Lagrangian consistent with the restrictions of chiral symmetry just stated, corresponding to a hierarchy in the number of derivatives acting on M . Moreover any one Lagrangian contains an infinity of interaction terms (given the power series expansion of M) and so is nonrenormalizable in the usual sense. Given these facts, one must be careful in applying chiral Lagrangians. Two generally accepted procedures are (i) to pick a Lagrangian containing the smallest number of derivatives, and (ii) to then work in tree approximation (i.e., no loops). Doing so provides a correct chiral description of any given process at low energies. Lagrangians with more derivatives start to contribute significantly as the energy is increased.

A strong interaction illustration is provided by an $SU(2)_L \times SU(2)_R$ model of pion elastic scattering. Consider the Lagrangian $L = L_K + L_M$ where

$$L_K = \frac{F_\pi^2}{4} \text{Tr}(\partial^\mu U \partial_\mu U^\dagger) \quad (2.30)$$

$$L_M = -\frac{m_\pi^2 F_\pi^2}{2} (2 - \frac{1}{2} \text{Tr}(U + U^\dagger)) \quad (2.31)$$

with the 2×2 matrix $U = \exp(i\tau \cdot \pi / F_\pi)$. For L_K and L_M the coefficients are fixed by terms quadratic in the pion fields. The Lagrangian L_M serves to give the pions a mass and therefore is not a chiral invariant. Expanding L_K to fourth order yields the isospin-angular momentum amplitudes (A_I^J),

$$A_0^0 = \frac{2s - m_\pi^2}{F_\pi^2}, \quad A_0^2 = \frac{2m_\pi^2 - s}{F_\pi^2}, \quad A_1^1 = \frac{4k^2 \cos \theta}{F_\pi^2} \quad (2.32)$$

with all others vanishing. Thus the Weinberg scattering lengths are obtained almost effortlessly.

We are now ready to study how one describes the spatial dependence of quarks confined by the QCD interaction.

3. A users guide to the quark model

3.1. Spin-color wavefunctions

The earliest usage of the quark model was to describe the quantum numbers of the hadrons [20]. The method by which baryons and mesons are built of qqq and $q\bar{q}$ configurations is now standard

knowledge. However less well known is the procedure by which the details of spin, flavor and color are interwoven with the spatial properties to produce the total wavefunction describing a given hadron. In this section a general treatment of the quark model will be given, with an eye towards application to the weak interaction.

We find it most convenient to use the language of field theory to describe the quark model states. In any given model one can solve the quark equations of motion in the confining potential (or bag) to obtain a complete set of wavefunctions for quarks, $\psi_a(x)$, and antiquarks, $\psi_{\bar{a}}(x)$, where a and $\bar{a}$ label the different members of the complete set. The quark field operator can then be built out of these wavefunctions,

$$\psi(x) = \sum_a [\psi_a(x) \exp(-i\omega_a t) b(a) + \psi_{\bar{a}}(x) \exp(i\omega_{\bar{a}} t) d^\dagger(\bar{a})] \quad (3.1)$$

where $\omega_a, \omega_{\bar{a}}$ are the energy eigenvalues and $b^\dagger(a)$ and $d^\dagger(a)$ are creation operators for quarks and antiquarks. The creation and annihilation operators satisfy the usual anticommutation relations

$$\begin{aligned} \{b(a), b^\dagger(a')\} &= \delta_{aa'} \\ \{d(a), d^\dagger(a')\} &= \delta_{aa'} \end{aligned} \quad (3.2)$$

$$\{b(a), b(a')\} = \{d(a), d(a')\} = \{b(a), d(a')\} = \{b(a), d^\dagger(a')\} = 0$$

and the field operator will also obey its anticommutation relation if the wavefunctions are orthonormal. The sets a and $\bar{a}$ for a given flavor of quark contain information on spatial, spin and color properties.

Thus far we have been very general. However there is an assumption made in all practical quark models which greatly simplifies the analysis. This is the assumption that the spatial, spin and color degrees of freedom factorize, at least in the first approximation. This is true if the zeroth order Hamiltonian is spin and color independent. Spin dependent interactions are added subsequently and treated as a perturbation. However, all of the standard quark models start off in a spin independent approximation. This lets us write the sets a and $\bar{a}$ in terms of three quantum numbers for the spatial (n), spin (m) and color (α) degrees of freedom separately, viz., $a = (n, m, \alpha)$. All the hadrons which we will be concerned with have quarks in the spatial ground state ($n = 0$). Hence we will often suppress the spatial index n . In addition, the quark flavor symbol will be used instead of $b^\dagger$ and $d^\dagger$ for the creation operators. Thus we have

$$\begin{aligned} u^\dagger(m, \alpha) &\equiv b^\dagger(n = 0, m, \alpha) \\ \bar{u}^\dagger(m, \alpha) &\equiv d^\dagger(n = 0, m, \alpha) \end{aligned} \quad (3.3)$$

for up quarks, with similar expressions for down and strange quarks.

The hadrons can now be constructed in the Fock space defined by the creation operators for quarks and antiquarks. All hadrons are color singlets, which for baryons means being antisymmetric in color. The simplest state to construct is the Δ^{++} with $S_z = \frac{3}{2}$

$$|\Delta^{++}, S_z = \frac{3}{2}\rangle = \frac{1}{6} \epsilon_{\alpha\beta\gamma} u^\dagger(\uparrow, \alpha) u^\dagger(\uparrow, \beta) u^\dagger(\uparrow, \gamma) |0\rangle. \quad (3.4)$$

A summation over colors is implied and $\epsilon_{\alpha\beta\gamma}$ is the total antisymmetric Levi Civita tensor. The normalization constant is fixed by requiring that all states be orthonormal $\langle H_i | H_j \rangle = \delta_{ij}$.

The remaining baryons can be obtained by lowering operators in the spin and flavor variables. In this manner one constructs the spin and color wavefunctions of the $1/2^+$ octet and $3/2^+$ decuplet. We give in table 3.1 the explicit wavefunction of the $1/2^+$ baryon octet. Mesons are constructed similarly, but now with a symmetric sum over colors. For example the $\bar{K}^0$ is given by (the summation convention is employed for labels m, α)

$$|\bar{K}^0\rangle = \sqrt{\frac{1}{6}} \bar{d}^\dagger(m, \alpha) s^\dagger(m, \alpha) |0\rangle. \quad (3.5)$$

In our notation the state created by $d^\dagger(m, \alpha)$ has spin and color conjugate to that in $b^\dagger(m, \alpha)$ so that eq. (3.5) indeed describes a spin and color singlet particle. The pseudoscalar meson octet is given in table 3.2.

Spin wavefunctions essentially similar to these (although lacking color) were originally derived using nonrelativistic SU(6). However they are true in a more general context, and can be used in relativistic models such as the bag model. As previously mentioned, the essential criterion for their applicability is the spin independence of the zeroth order Hamiltonian.

Table 3.1
Baryon wavefunctions in the quark model

$ \mathbf{P}, \uparrow\rangle = \frac{\epsilon_{\alpha\beta\gamma}}{\sqrt{18}} [u^\dagger(\downarrow, \alpha) d^\dagger(\uparrow, \beta) - u^\dagger(\uparrow, \alpha) d^\dagger(\downarrow, \beta)] u^\dagger(\uparrow, \gamma) 0\rangle$
$ \mathbf{N}, \uparrow\rangle = \frac{\epsilon_{\alpha\beta\gamma}}{\sqrt{18}} [d^\dagger(\uparrow, \alpha) u^\dagger(\downarrow, \beta) - d^\dagger(\downarrow, \alpha) u^\dagger(\uparrow, \beta)] d^\dagger(\uparrow, \gamma) 0\rangle$
$ \Sigma^+, \uparrow\rangle = \frac{\epsilon_{\alpha\beta\gamma}}{\sqrt{18}} [u^\dagger(\uparrow, \alpha) s^\dagger(\downarrow, \beta) - u^\dagger(\downarrow, \alpha) s^\dagger(\uparrow, \beta)] u^\dagger(\uparrow, \gamma) 0\rangle$
$ \Sigma^0, \uparrow\rangle = \frac{\epsilon_{\alpha\beta\gamma}}{6} [u^\dagger(\uparrow, \alpha) d^\dagger(\downarrow, \beta) s^\dagger(\uparrow, \gamma) + u^\dagger(\downarrow, \alpha) d^\dagger(\uparrow, \beta) s^\dagger(\uparrow, \gamma) - 2u^\dagger(\uparrow, \alpha) d^\dagger(\uparrow, \beta) s^\dagger(\downarrow, \gamma)] 0\rangle$
$ \Sigma^-, \uparrow\rangle = \frac{\epsilon_{\alpha\beta\gamma}}{\sqrt{18}} [d^\dagger(\downarrow, \alpha) s^\dagger(\uparrow, \beta) - d^\dagger(\uparrow, \alpha) s^\dagger(\downarrow, \beta)] d^\dagger(\uparrow, \gamma) 0\rangle$
$ \Lambda, \uparrow\rangle = \frac{\epsilon_{\alpha\beta\gamma}}{\sqrt{12}} [u^\dagger(\uparrow, \alpha) d^\dagger(\downarrow, \beta) - u^\dagger(\downarrow, \alpha) d^\dagger(\uparrow, \beta)] s^\dagger(\uparrow, \gamma) 0\rangle$
$ \Xi^0, \uparrow\rangle = \frac{\epsilon_{\alpha\beta\gamma}}{\sqrt{18}} [u^\dagger(\uparrow, \alpha) s^\dagger(\downarrow, \beta) - u^\dagger(\downarrow, \alpha) s^\dagger(\uparrow, \beta)] s^\dagger(\uparrow, \gamma) 0\rangle$
$ \Xi^-, \uparrow\rangle = \frac{\epsilon_{\alpha\beta\gamma}}{\sqrt{18}} [d^\dagger(\downarrow, \alpha) s^\dagger(\uparrow, \beta) - d^\dagger(\uparrow, \alpha) s^\dagger(\downarrow, \beta)] s^\dagger(\uparrow, \gamma) 0\rangle$

Table 3.2
Meson wavefunctions in the quark model

$$\begin{aligned}
 |\pi^+\rangle &= \frac{1}{\sqrt{6}} \bar{d}^\dagger(m, \alpha) u^\dagger(m, \alpha) |0\rangle \\
 |\pi^0\rangle &= \frac{1}{\sqrt{12}} [\bar{u}^\dagger(m, \alpha) u^\dagger(m, \alpha) - \bar{d}^\dagger(m, \alpha) d^\dagger(m, \alpha)] |0\rangle \\
 |\pi^-\rangle &= \frac{1}{\sqrt{6}} \bar{u}^\dagger(m, \alpha) d^\dagger(m, \alpha) |0\rangle \\
 |K^+\rangle &= \frac{1}{\sqrt{6}} \bar{s}^\dagger(m, \alpha) u^\dagger(m, \alpha) |0\rangle \\
 |K^0\rangle &= \frac{1}{\sqrt{6}} \bar{s}^\dagger(m, \alpha) d^\dagger(m, \alpha) |0\rangle \\
 |\bar{K}^0\rangle &= \frac{1}{\sqrt{6}} \bar{d}^\dagger(m, \alpha) s^\dagger(m, \alpha) |0\rangle \\
 |K^-\rangle &= \frac{1}{\sqrt{6}} \bar{u}^\dagger(m, \alpha) s^\dagger(m, \alpha) |0\rangle \\
 |\eta_8\rangle &= \frac{1}{6} [\bar{u}^\dagger(m, \alpha) u^\dagger(m, \alpha) + \bar{d}^\dagger(m, \alpha) d^\dagger(m, \alpha) - 2\bar{s}^\dagger(m, \alpha) s^\dagger(m, \alpha)] |0\rangle
 \end{aligned}$$

3.2. Spatial wavefunctions

The use of quarks as dynamical entities requires the knowledge of the spatial wavefunctions of the bound states. It is here that the greatest variation of the different models occurs, but many general features still remain. In fact in most applications one is primarily sensitive to the general features. This will account for the agreement which is found in the application of several quark models to weak processes.

The most important general feature which governs quark model predictions is the spatial extent of hadrons. The bound states have scale sizes of order one fermi. As measured by electromagnetic form factors, two examples are [21]

$$\langle r^2 \rangle_{\text{proton}}^{1/2} = 0.9 \text{ fm}, \quad \langle r^2 \rangle_{\text{pion}}^{1/2} = 0.62 \pm 0.04 \text{ fm}. \quad (3.6)$$

These numbers are even more important than the quark mass in setting the scale of dimensional matrix elements.

Given this scale, all quark models then lead to somewhat similar spatial behavior. The ground state in all models is a spatially symmetric S-state, and the wavefunction peaks near $r = 0$ and falls off for large r . The normalization of the quark wavefunction

$$\int d^3x \psi^\dagger(x) \psi(x) = 1 \quad (3.7)$$

ensures that the magnitude of ψ will be similar in these models (again, given the same spatial extent). The major remaining differences in the quark wavefunctions concerns the lower two components of the Dirac wavefunction. Nonrelativistic models automatically set these equal to zero, while relativistic models generally find sizeable lower components.

One can show on general grounds that the quark motion in hadrons is at least somewhat relativistic, since quarks confined to a region of radius R ($R \approx 1$ fm) exhibit a momentum given by the uncertainty principle

$$p \gtrsim \sqrt{3}/R \approx 350 \text{ MeV}. \quad (3.8)$$

(The $\sqrt{3}$ comes from the 3 dimensions.) This condition is, of course, satisfied in all explicit models. Since this momentum is comparable to or larger than the quark masses, relativistic effects are unavoidable. A more direct indication of the relativistic nature of quarks comes from the hadron spectrum. Nonrelativistic systems are characterized by excitation energies which are small compared to the constituent masses. In the hadronic spectrum, typical excitation energies are 300 to 500 MeV, again comparable to or larger than quark masses. Therefore, models which have a nonrelativistic framework [22] nevertheless find that the velocity of the quarks is close to unity if they fit the spectrum and/or the charge radius. This had led to the study of relativistic corrections in such models [23], and to relativistic generalizations of the models. We will comment on some of the effects of relativistic motion later in the report.

The types of models which have been applied most extensively to the weak interactions range from the nonrelativistic harmonic oscillator model to the MIT bag model. The former is based on a quantum mechanical treatment of the Hamiltonian [24]

$$H = \sum_i p_i^2/2m_i + \sum_{i<j} V(r_{ij}) \quad (3.9)$$

with a two body potential

$$V(r_{ij}) = \frac{1}{2} k r_{ij}^2 + U(r_{ij}) \quad (3.10)$$

where $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ and $U(r_{ij})$ represents an anharmonic potential which is treated perturbatively. In studies of the spectrum, a Breit–Fermi interaction term is added perturbatively to incorporate spin–spin and spin–orbit forces. In the harmonic approximation ($U(r) = 0$), the center-of-mass motion can be separated from the internal wavefunctions and the ground state wavefunction has the form

$$\psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = (\alpha^2/\pi)^{3/2} \exp(i\mathbf{P} \cdot \mathbf{R}) \exp\{-\alpha^2(\rho^2 + \lambda^2)/2\} \quad (3.11)$$

with

$$\begin{aligned} \alpha^2 &= (3km)^{1/2}, & \rho &= (\mathbf{r}_1 - \mathbf{r}_2)/\sqrt{2} \\ \lambda &= (\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3)/\sqrt{6}, & \mathbf{R} &= (\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3)/\sqrt{3}. \end{aligned} \quad (3.12)$$

The parameters which emerge from a fit to the spectrum are [24]

$$m_u = m_d = 0.33 \text{ GeV}, \quad m_s = 0.55 \text{ GeV}, \quad \alpha^2 = 0.17 \text{ GeV}^2. \quad (3.13)$$

In view of our comments above on the importance of the spatial extent in determining the scale of matrix elements, it is relevant to note that this set of parameters leads to a charge radius for the proton which is somewhat small,

$$\langle r^2 \rangle_{\text{proton}}^{1/2} = 0.51 \text{ fm} . \quad (3.14)$$

If one desires to fit the charge radius one would have to use a smaller value of α [25]

$$\alpha^2 = 0.049 \text{ GeV}^2 . \quad (3.15)$$

The importance of this point has led some groups to use the latter scale in matrix element evaluations.

If the oscillator model would provide a good extended homework problem in a quantum mechanics course, the bag model [26–28] would play a similar role in the quantization section of a field theory course. The model is derivable from the Lagrangian [27]

$$L = (L_{\text{QCD}} - B) \theta(\bar{\psi}\psi) \quad (3.16)$$

where L_{QCD} is the Lagrangian of Quantum Chromodynamics and B is a constant positive energy density, i.e. the bag constant. The order parameter defining the interior of the bag is $\bar{\psi}\psi$, as might be expected in chiral theories, leading to $\bar{\psi}\psi = 0$ at the surface of the bag. While in principle the bag surface should be determined dynamically, the only manageable approximation for low-energy physics is one where the shape of the bag is forced to be spherical. In this case, the variation of the Lagrangian leads to the usual QCD equations of motion inside the bag, plus the following boundary conditions on the surface.

$$-i\hat{r} \cdot \boldsymbol{\gamma} \psi = \psi, \quad 2B = -\hat{r} \cdot \nabla(\bar{\psi}\psi) . \quad (3.17)$$

The ground state wavefunction has the form

$$\psi(x) = N \begin{pmatrix} i j_0(pr) \chi \\ -\left(\frac{E-m}{E+m}\right)^{1/2} j_1(pr) \boldsymbol{\sigma} \cdot \hat{r} \chi \end{pmatrix} \quad (3.18)$$

with χ being a two component Pauli spinor and the upper and lower Dirac components (j_0 and j_1) being spherical Bessel functions. For light quarks

$$E \simeq p \simeq 2.04/R + 0.48m, \quad N = 2.27/(4\pi R^3)^{1/2} . \quad (3.19)$$

The standard parameters of the model [26, 28] are

$$B \simeq (140 \text{ MeV})^4, \quad R \simeq 1 \text{ fm}, \quad (3.20)$$

$$m_u \simeq m_d \simeq 0 \rightarrow 32 \text{ MeV}, \quad m_A \simeq 280 \rightarrow 330 \text{ MeV} .$$

The model is solved first ignoring the gluons. These can be added perturbatively to generate spin

dependent forces. The resulting Hamiltonian can be expressed in terms of the field creation and annihilation operators. To give an example, we quote those terms which are relevant for the ground state baryons with massless quarks in a bag of volume V ,

$$H = BV + \frac{2.04}{R} b^\dagger(m, \alpha) b(m, \alpha) + \frac{0.75\alpha_s}{-R} b^\dagger(m', \alpha') \boldsymbol{\sigma}_{m'm} \lambda_{\alpha'\alpha}^A b(m, \alpha) b^\dagger(n', \beta') \boldsymbol{\sigma}_{n'n} \lambda_{\beta'\beta}^A b(n, \beta) \quad (3.21)$$

where the last term is the spin dependent Hamiltonian due to one gluon exchange.

The bag model wavefunction is an example of a form which is quite general. Any spin independent central potential will have a ground state of this form [29], i.e.,

$$\psi = \begin{pmatrix} i u(r) \chi \\ l(r) \boldsymbol{\sigma} \cdot \hat{\mathbf{r}} \chi \end{pmatrix} \exp(-iEt). \quad (3.22)$$

The Bessel functions in eq. (3.18) are just particular solutions appropriate for the bag model. This form has also been used in some relativized harmonic oscillator models which use a central potential instead of two body forces [23]. Because of this generality we will often quote the forms of matrix elements in terms of u and l in order to characterize the relativistic behavior. The normalization condition for the wavefunction is

$$\int d^3x \psi^\dagger(x) \psi(x) = \int d^3x (u^2 + l^2) = 1. \quad (3.23)$$

In the nonrelativistic limit, $l \rightarrow 0$.

Let us estimate the size of the lower components in the wavefunction. In the bag model we have

$$\int d^3x l^2(r) = 0.26 \quad (3.24)$$

for massless quarks. For potential models one often includes relativistic effects by working in momentum space and using the spinor for a quark in momentum eigenstate p

$$u(p) = \left(\frac{E + m_q}{2E} \right)^{1/2} \begin{pmatrix} \chi \\ \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E + m_q} \chi \end{pmatrix}. \quad (3.25)$$

In this case, one has the substitution

$$\int d^3x l^2(r) \rightarrow \left\langle \frac{p^2}{2E(E + m_q)} \right\rangle \quad (3.26)$$

where the averaging is taken over the momentum space wavefunction of the particular model. Using

our previous estimate of $\langle p^2 \rangle$ (eq. (3.8)) one would estimate in general

$$\left\langle \frac{p^2}{2E(E + m_q)} \right\rangle \approx 0.25 \rightarrow 0.30 \quad (3.27)$$

for a confinement scale of 1 fm. Larger effects are found if one uses $\alpha^2 = 0.17 \text{ GeV}^2$ in the harmonic oscillator models due to tighter binding. We see that the lower component is significant but not dominant in the quark wavefunctions.

It is not our purpose here to debate the relative merits of the various models. In fact, we have argued that all wavefunctions are very similar up to two features, 1) the spatial scale of the wavefunction, and 2) lower components in the Dirac wavefunction due to relativistic motion. However the latter can always be corrected for in nonrelativistic models and so the only real distinction is the former. To illustrate this we display in fig. 3.1 the charge density in the models discussed above. The harmonic oscillator model is given for the separate values of the oscillator strength. Note that when the charge radii are about equal, the bag and harmonic oscillator have roughly the same shape, while the curve where $\alpha^2 = 0.17 \text{ GeV}^2$ is much more sharply peaked. (A bag model with $R = 0.5 \text{ fm}$ would be comparably peaked.) This demonstrates the effect of the spatial scale, and one can easily see its importance in processes sensitive to $\psi(0)$ such as nonleptonic decays (or proton decay amplitudes).

Fig. 3.1. Quark charge density in the bag and oscillator models.

3.3. Connection with plane wave states

In all cases except for the nonrelativistic version of the harmonic oscillator model, one cannot explicitly separate out the center-of-mass motion. The result of a quark model approach to a bound state is a configuration localized in coordinate space, i.e., a position eigenstate. However, the analysis of scattering and decay deals with plane wave states, i.e., momentum eigenstates. In this section we give the general connection between the two approaches based on the formalism of ref. [28].

The basic assumption all quark models make is that the bound state with a given set of quantum numbers is related to only those momentum eigenstates of the same type. If we denote the quark model state for a given hadron centered around a point x as $|H(x)\rangle_{\text{QM}}$ and the plane wave state by $|H(p)\rangle$ then this relation is expressed as

$$|H(x)\rangle_{\text{QM}} = \int d^3p \phi(p) e^{ip \cdot x} |H(p)\rangle. \quad (3.28)$$

In words, the position eigenstate is a wave packet of momentum eigenstates. This is the minimal assumption possible, but certainly not the most general. There does not need to be a diagonal correspondence between the tower of quark model states with a given set of quantum numbers and the similar tower of observed plane wave states. In addition the relation does not have to be that of simple superposition, but could be a Bogoliubov transformation. However let us accept the wave packet superposition as a starting point.

There is a constraint on the superposition from the normalization of the states. Quark model states are universally normalized to unity

$${}_{\text{QM}}\langle H(x) | H(x) \rangle_{\text{QM}} = 1 \quad (3.29)$$

while plane wave states are often normalized differently for mesons and baryons. We use

$$\begin{aligned} \langle H(p) | H(p') \rangle &= 2\omega_p (2\pi)^3 \delta^3(p - p') \quad (\text{mesons}) \\ \langle H(p) | H(p') \rangle &= (2\pi)^3 \delta^3(p - p') \quad (\text{baryons}). \end{aligned} \quad (3.30)$$

This leads to the relations

$$\begin{aligned} 1 &= \int d^3p (2\omega_p) (2\pi)^3 |\phi(p)|^2 \quad (\text{mesons}) \\ 1 &= \int d^3p (2\pi)^3 |\phi(p)|^2 \quad (\text{baryons}). \end{aligned} \quad (3.31)$$

The details of the functional form of $\phi(p)$ are most often not given nor needed. As we shall see in the next section the standard quark model method emerges in the limit of heavy bound states using only the normalization condition. Within the bag model there have been attempts [28, 30] to calculate ‘center-of-

mass corrections' which are terms of order $\langle p^2 \rangle / m_H^2$ with m_H being the mass of the hadron and

$$\langle p^2 \rangle = \int d^3p \, 2\omega_p (2\pi)^3 |\phi(p)|^2 p^2 \quad (\text{mesons}) \quad (3.32)$$

$$\langle p^2 \rangle = \int d^3p \, (2\pi)^3 |\phi(p)|^2 p^2 \quad (\text{baryons}).$$

Estimates [28, 30] put this parameter at

$$\langle p^2 \rangle / m_H^2 \simeq 0.1 \quad (3.33)$$

for baryons. The wavepacket form is only important for very light mesons. Reference [28] gives a procedure for determining $\phi(p)$ for the bag model pion, with the result

$$\phi(p) = \frac{2\sqrt{3}}{F_\pi \omega_p} \int d^3x \, e^{ip \cdot x} (u^2 - l^2). \quad (3.34)$$

3.4. How to calculate matrix elements

We introduced the wavepacket treatment in the last section primarily in order to derive the quark model procedure for calculating a matrix element. Most matrix elements calculated are those of an operator between initial and final states each of which contains a single hadron. (See ref. [31] for an attempt to overcome this limitation.) The desired coupling constant is defined in the plane wave basis (using baryons as an example)

$$\langle B'(p') | O | B(p) \rangle = g \bar{u}(p') \Gamma_O u(p) e^{i(p'-p) \cdot x} \quad (3.35)$$

where Γ_O is some Dirac matrix appropriate for operator O . However, what can be calculated in the quark model is

$${}_{\text{QM}} \langle B' | O(x) | B \rangle_{\text{QM}} = f(x) \quad (3.36)$$

where all quark model states will from now on be assumed to be located at the origin. The wavepacket treatment then implies

$$\begin{aligned} {}_{\text{QM}} \left\langle B' \left| \int d^3x \, O(x) \right| B \right\rangle_{\text{QM}} &= g \int d^3x \int d^3p \, d^3p' \, \bar{u}(p') \Gamma_O u(p) e^{i(p'-p) \cdot x} \phi(p) \phi^*(p') \\ &= g \int d^3p \, (2\pi)^3 |\phi(p)|^2 \bar{u}(p) \Gamma_O u(p). \end{aligned} \quad (3.37)$$

For heavy enough bound states $\langle p^2 \rangle$ is small and one may expand

$$\bar{u}(p) \Gamma_O u(p) = \bar{u}(0) \Gamma_O u(0) + O(\langle p^2 \rangle / M^2). \quad (3.38)$$

The standard treatment consists of keeping only the first term to obtain

$$g \bar{u}(0) \Gamma_O u(0) = \left\langle B' \left| \int d^3x O(x) \right| B \right\rangle_{\text{QM}}. \quad (3.39)$$

It is interesting to note that this relation, often thought of as fundamental, is itself only an approximation.

As an example, we will provide the complete quark model procedure for the axial vector current. In the rest of this report we will not quote the details of the calculations which we discuss, assuming that the interested reader can reproduce them by following this guide. For this example, we define as usual

$$\langle P(p') | \bar{u}(x) \gamma_\mu \gamma_5 d(x) | N(p) \rangle = \bar{u}(p') g_A \gamma_\mu \gamma_5 u(p) e^{i(p'-p) \cdot x} + \dots \quad (3.40)$$

We note that choosing $\mu = 3$ gives

$$\bar{u}(0) \gamma_3 \gamma_5 u(0) = 1 \quad (3.41)$$

for spin up nucleons, so that our basic formula reads

$$g_A = \left\langle P \uparrow \left| \int d^3x \bar{u}(x) \gamma_3 \gamma_5 d(x) \right| N \uparrow \right\rangle_{\text{QM}}. \quad (3.42)$$

The field operators for the quarks are now expanded in the complete set of the particular model, as in eq. (3.1),

$$u_\alpha(x) = \sum_{n, m} [\psi_{n, m}(x) e^{-i\omega_n t} u(n, m, \alpha) + \psi_{\bar{n}, \bar{m}}(x) e^{i\omega_{\bar{n}} t} \bar{u}^\dagger(\bar{n}, \bar{m}, \alpha)]. \quad (3.43)$$

However because the initial and final states contain quarks just in the ground state, only this mode is needed

$$g_A = \left\langle P \uparrow \left| \int d^3x [\bar{\psi}_{0, m} \gamma_3 \gamma_5 \psi_{0, m'} u^\dagger(m, \alpha) d(m', \alpha) + \dots] \right| N \uparrow \right\rangle_{\text{QM}}. \quad (3.44)$$

At this stage one can factorize the spin and space components by using the general ground state wavefunction, eq. (3.22). This leads to

$$\begin{aligned} \int d^3x \bar{\psi}_{0m} \gamma_3 \gamma_5 \psi_{0m'} &= \int d^3x \chi_m^\dagger (u^2 \sigma_3 - l^2 \hat{\mathbf{r}}_3 \boldsymbol{\sigma} \cdot \hat{\mathbf{r}}) \chi_{m'} \\ &= \sigma_{mm'}^3 \int d^3x (u^2 - \frac{1}{3} l^2). \end{aligned} \quad (3.45)$$

Therefore

$$g_A = \int d^3x (u^2 - \frac{1}{3}l^2)_{QM} \langle P, \uparrow | u^\dagger(m, \alpha) \sigma_{mm'}^\dagger d(m', \alpha) | N, \uparrow \rangle_{QM}. \quad (3.46)$$

Finally the remaining spin sum can be performed resulting in

$$g_A = \frac{5}{3} \int d^3x (u^2 - \frac{1}{3}l^2) = \frac{5}{3} \int d^3x (u^2 + l^2 - \frac{4}{3}l^2) = \frac{5}{3} \left(1 - \frac{4}{3} \int d^3x l^2(r) \right). \quad (3.47)$$

The nonrelativistic harmonic oscillator would simply yield $g_A = \frac{5}{3}$, because there are no lower components in the wavefunction. If one desires to make relativistic corrections in such a model, the result can be read off from the general formula (eq. (3.47)) with the appropriate substitution, eq. (3.26).

3.5. Limitations to the quark model approach

While the quark model is the best tool presently available for the study of the weak interactions of quarks, we feel that it is important to keep in mind the fact that the true physics involved is always more complicated than appears in the quark model formulation. If one is to intelligently use the quark model as a tool, one should always be aware of the ways in which the quark model could fail to accurately describe the physics involved. There are limits to what can be described by quark models. Thus one must assess the level to which a given result may be trusted.

It is worth pointing out that the very existence of the quark model as an approximation to nature remains a mystery. QCD is a strongly interacting field theory and we know that very nonperturbative physics is needed for the formation of hadrons. One would expect that the description of a hadronic system would involve a complicated mixture of quarks, gluons and $q\bar{q}$ pairs. It is remarkable that a good description of the proton, for example, can be obtained by three quarks moving in simple potentials and otherwise acting independently and without correlation. There is as yet no compelling evidence for effects of valence gluons or $q\bar{q}$ pairs. Apparently nature has been kind and allowed a description of ‘effective’ quarks which somehow permits the complicated physics to be parameterized into a small number of model dependent constants which can be extracted from experiment. To a large extent these have allowed one to successfully correlate and predict much of the physics of hadrons. However, in novel applications one does not know a priori how to determine whether this parameterization should continue to be successful. It is not like perturbation theory where one can always compute the next order in the perturbation series to see if the result is converging properly. Here one does not know what the perturbation is and it is difficult to assess what “the next order” correction could be.

Some minimal physics issues can be described. One which is important for the topic of this report is the question of whether the quarks of the quark model are the same as the quarks in the weak currents. If the quarks are ‘dressed’ by the strong interactions then the weak currents may be transformed

$$\bar{\psi}_1 \gamma_\mu (1 + \gamma_5) \psi_2 \rightarrow \bar{\psi}_1 [\gamma_\mu (g_V + g_A \gamma_5) + \cdots] \psi_2 \quad (3.48)$$

and form factors may be generated. Within the context of some models this *can* be addressed perturbatively. For example in the bag model the weak currents can be obtained from the Lagrangian and radiative corrections can be computed. To the extent that these have been done [32] they lead to small modifications (10→30%) to the properties of baryons. The major effect of dressing appears to be in the formation of bound state wavefunctions. The results appear to justify the use of the usual weak currents modified primarily by the standard quark model binding. Another related issue which can

partially be addressed is that of gluonic corrections. These can be studied in specific models, and have not been found to be overly important. However, these studies are not yet sufficiently complete.

One feature which the quark model always fails to accommodate is chiral symmetry. It appears to be a requirement for the construction of bound states that the chiral symmetry not be manifest. In no phenomenological model is there any explicit indication or remnant of chiral symmetry. (We exclude hybrid models with fundamental pion fields [33a] because they cannot address the connection between the quark currents and hadronic manifestations, having already assumed part of the answer. These models may however play a role in future developments by better elucidating some of the chiral behavior of the matrix elements. Likewise we do not discuss attempts to show the breakdown of chiral symmetry [33b] as these models have not yet been adapted for phenomenological studies of weak properties.) This problem can be partially compensated for by using chiral symmetry relations early in a calculation and incorporating the results wherever possible.

Lest the above seem too pessimistic, we should point out why we feel that the quark model *does* work. Each calculation can be basically separated into two portions, the flavor and spin structure of the amplitude and the spatial overlap. The quark model was developed in the first place to explain the flavor and spin properties of the observed hadrons, and for this it does a good job. It also appears to accurately describe how flavor and spin are distributed inside the hadron (i.e., the wavefunction of section 3.1). Thus these aspects of the calculation are fairly general and are most often correct. The spatial aspect is less well tested. The most extensive studies come from matrix elements of currents. Because these are bilinears in the quark field, and because of the wavefunction normalization condition, the strength of these amplitudes will be nearly correct. Dimensional matrix elements are primarily governed by the radius of the bound state, and as long as this is fed into the calculation, the scale should come out right. Therefore it appears that weak interaction processes are governed primarily by the flavor and spin properties and the hadronic scale factor. The quark model is better than any other scheme at handling these features, and has deservedly become the primary language for describing low-energy systems.

4. Nuclear beta decay

One area in which the structure of the weak hadronic current has received a great deal of attention is that of nuclear beta decay. Although in some sense this represents simply a nuclear modification of the basic weak transition

$$N \rightarrow P + e^- + \bar{\nu}_e, \quad P \rightarrow N + e^+ + \nu_e \quad (4.1)$$

the use of nuclei allows specific features to be accented by the choice of levels possessing particular spins and/or parities. The subject is broad enough to have been covered in several textbook length treatises [34]. Thus, in order to give a reasonably coherent discussion here, we shall confine our attention to the case of *allowed* decays ($\Delta J = 0, \pm 1$, no parity change) only and will emphasize those aspects which have stressed the study of the weak current rather than of nuclear structure.

A particularly significant property of the weak currents is given by the conserved vector current (CVC) hypothesis [35] which states that since only the u, d quarks carry isotopic spin, an isotopic rotation applied to the electromagnetic current

$$J_\mu^{\text{EM}} = \frac{2}{3} \bar{u} \gamma_\mu u - \frac{1}{3} \bar{d} \gamma_\mu d + \dots \quad (2.12)$$

must yield the polar vector component of the weak hadronic current

$$[I_-, J_\mu^{\text{EM}}] = \bar{d}\gamma_\mu u. \quad (4.2)$$

A second feature has to do with transformation properties under the operator

$$G = C \exp(-i\pi I_2). \quad (4.3)$$

Since

$$\exp(-i\pi I_2) \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} -d \\ u \end{pmatrix} \quad (4.4)$$

while

$$C \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix} \quad (4.5)$$

it follows that the quark model requires

$$GV_\mu G^{-1} = V_\mu, \quad GA_\mu G^{-1} = -A_\mu. \quad (4.6)$$

Weinberg in a more general context has termed currents which satisfy this criterion ‘first class’ while currents with the opposite property

$$G\tilde{V}_\mu G^{-1} = -\tilde{V}_\mu, \quad G\tilde{A}_\mu G^{-1} = \tilde{A}_\mu \quad (4.7)$$

are assigned to the ‘second class’ [36]. Since the quark model *requires* the validity of eqs. (4.6), it is often said that there can be ‘no second class currents’. This test of the quark model has been particularly well studied in nuclei.

Finally, as mentioned in section 2, in a quark model with massless u, d quarks the weak axial current is divergenceless and the pion is massless

$$\partial^\mu A_\mu = 0, \quad m_\pi^2 = 0 \quad (m_u = m_d = 0). \quad (4.8)$$

The assumption of a partially conserved axial vector current [37] posits that in the real world, where m_u, m_d are small but nonzero, the weak axial current divergence and the pion field are related by a constant

$$\partial^\mu A_\mu^a = F_\pi m_\pi^2 \phi_\pi^a \quad (4.9)$$

where F_π is the pion decay constant, $F_\pi = 94 \text{ MeV}$. This hypothesis has also been tested in nuclear muon capture, as we shall discuss later.

In analyzing weak nuclear transitions, the existence of various spin–parity combinations makes a covariant formalism rather awkward. Nevertheless in the nonrelativistic situation, which nearly always obtains, we can define a simple form which covers all possible situations. Thus for a spin 1/2–spin 1/2 weak decay, it is common to make the relativistic form factor definitions

$$\begin{aligned}\langle\beta(p_2)|V_\mu|\alpha(p_1)\rangle &= \bar{u}(p_2) \left[\gamma_\mu f_1(q^2) - \frac{i\sigma_{\mu\nu}q^\nu f_2(q^2)}{2m_N} + \frac{q_\mu f_3(q^2)}{2m_N} \right] u(p_1) \\ \langle\beta(p_2)|A_\mu|\alpha(p_1)\rangle &= \bar{u}(p_2) \left[\gamma_\mu g_1(q^2) - \frac{i\sigma_{\mu\nu}q^\nu g_2(q^2)}{2m_N} + \frac{q_\mu g_3(q^2)}{-2m_N} \right] \gamma_5 u(p_1)\end{aligned}\quad (4.10)$$

where $q = p_1 - p_2$ is the momentum transfer, f_1, g_1 are the usual vector, axial weak couplings, f_2, g_2 are the weak magnetism, axial tensor terms, and f_3, g_3 are the induced scalar, pseudoscalar form factors. For the situation of a decay between members of a common isotopic multiplet – e.g., neutron beta decay – the absence of second class currents requires the axial tensor and induced scalar terms to vanish [38]

$$g_2(q^2) = f_3(q^2) = 0, \quad (4.11)$$

while the CVC hypothesis determines the $q^2 = 0$ values of f_1, f_2 to be [39]

$$f_1(0) = Q_\alpha - Q_\beta, \quad f_2(0) = \mu_\alpha - \mu_\beta \quad (4.12)$$

where Q_α, μ_α are the charge and anomalous magnetic moment of the state $|\alpha\rangle$. Finally, PCAC requires that g_3 be given in terms of g_1 and the pionic coupling $g_{\pi\alpha\beta}$ [39]

$$g_3(0) \simeq g_1(0) [1 + m_\pi^2 (g'_{\pi\alpha\beta}(0)/g_{\pi\alpha\beta}(0) - g'_1(0)/g_1(0))] 4m_N^2/m_\pi^2. \quad (4.13)$$

This formalism can be generalized to an *arbitrary* allowed weak transition via [39]

$$l^\mu \langle\beta(p_2)|V_\mu|\alpha(p_1)\rangle = \delta_{JJ'} \delta_{MM'} \left[a(q^2) \frac{P \cdot l}{2M} + e(q^2) \frac{q \cdot l}{2M} \right] + C_{J'1;J}^{M'k;M} \frac{i}{2M} b(q^2) (q \times l)_k + \dots \quad (4.14)$$

$$l^\mu \langle\beta(p_2)|A_\mu|\alpha(p_1)\rangle = C_{J'1;J}^{M'k;M} \varepsilon_{ijk} \varepsilon_{ij\lambda\eta} \frac{1}{4M} \left[c(q^2) l^\lambda P^\eta - d(q^2) l^\lambda q^\eta + \frac{1}{(2M)^2} h(q^2) q^\lambda P^\eta q \cdot l + \dots \right]$$

where here J, J' represent the spin of states α, β with projections M, M' along some quantization axis, $P = p_1 + p_2$, $M = (M_1 + M_2)/2$, and the ellipses represent higher order terms in recoil. For the case $J = J' = \frac{1}{2}$ the structure functions $a, \dots, h$ can be identified with the more familiar f_i, g_i

$$\begin{aligned}a &= f_1 & b &= \sqrt{3}(f_1 + f_2) & c &= \sqrt{3} g_1 \\ d &= \sqrt{3} g_2 & e &= f_3 & h &= \sqrt{3} g_3\end{aligned}\quad (4.15)$$

and the meanings are similar for other spin assignments.

4.1. Tests of CVC

We can now utilize this formalism to interpret various tests of the weak current in nuclear systems. First consider CVC. There are many cases in nuclei where one has an isotopic triplet of 0^+ states. Examples are found in $A = 10, 14, 26, 34, 42$, etc. Because Coulombic effects raise the mass of the $I_z = 1$ state with respect to that with $I_z = 0$, a positron decay will take place in general

$$A(I = 1, I_z = 1) \rightarrow A(I = 1, I_z = 0) + e^+ + \nu_e. \quad (4.16)$$

Since the transition is $0^+ \rightarrow 0^+$, only the vector current is involved, and the sole permitted form factor is $a(q^2)$ which is given by CVC as

$$a(0) = \sqrt{(I + I_3)(I - I_3 + 1)} = \sqrt{2}. \quad (4.17)$$

What is generally quoted for such decays is the so-called 'ft value' which is essentially the half-life multiplied by the kinematic phase space factor. Theoretically one expects

$$ft = \frac{2\pi^3 \ln 2}{m_e a^2(0)} \quad (4.18)$$

which should then be identical for each isotriplet transition. Actually, the analysis is complicated by electromagnetic corrections, which must be known precisely and corrected for before the equality of ft values is tested [40]. Much careful study has been given to this problem and the current situation is quoted in table 4.1. What is tabulated there is ft which is the experimental ft value with electromagnetic effects removed [41]. The agreement with the theoretical prediction $ft = 3080$ sec is outstanding over a wide range of nuclei and is a strong confirmation of CVC.

Experimental ft values have also been utilized in order to look for possible second class currents. In this case one compares the decays of mirror branches from two members of a common isotopic multiplet. An example is found in the $A = 12$ system

Table 4.1
Superaligned Fermi beta decay: Listed are ft values for most of the $0^+ \rightarrow 0^+$ Fermi transitions

Parent nucleus	ft (sec) [41]	Parent nucleus	ft (sec) [41]
${}^{14}\text{O}$	3083.4 ± 5.4	${}^{42}\text{Sc}$	3096.0 ± 3.0
${}^{26}\text{Al}^m$	3085.0 ± 2.1	${}^{46}\text{V}$	3100.2 ± 2.0
${}^{34}\text{Cl}$	3097.5 ± 2.2	${}^{50}\text{Mn}$	3094.5 ± 2.4
${}^{38}\text{K}^m$	3097.3 ± 9.4	${}^{54}\text{Co}$	3105.2 ± 2.6

However, this can be within an $I = 1$ multiplet as above with a common final state or with a $T = \frac{3}{2}$ multiplet for which one has a pair of final states forming an isodoublet. Defining

$$\delta = (ft)^{\beta^+}/(ft)^{\beta^-} - 1 \quad (4.20)$$

we expect, if there are no second class currents, that $\delta = 0$. In fact, an early analysis by Wilkinson suggested a possibly significant deviation from zero [42]. However, it is difficult to interpret this in terms of second class currents because various nuclear structure effects can produce a nonzero δ of the same size. The dominant effect here is that the decaying proton in the proton-rich nucleus is bound less strongly (because of the Coulombic interaction) than the decaying neutron in the mirror transition, whereas the final nucleon is relatively well bound in both cases. Thus the overlap integral

$$I_i = \int d^3r g_{fin}(r) \sigma_i f_{init}(r) \quad (4.21)$$

which determines the axial coupling strength is systematically smaller for the proton-rich transition, so that we expect $\delta < 0$ in eq. (4.20). This and other structure effects can be quite significant. However, reliable theoretical estimates are difficult to come by [43], so ft asymmetries have by and large been abandoned as a probe of second class currents.

Instead, symmetry tests have for the most part utilized experiments involving the so-called recoil form factors b, d . One such experiment is a simple test of CVC involving measurements of the shape factor for the decay of an unpolarized parent nucleus, as originally suggested by Gell-Mann for the $A = 12$ system [44]. The shape factor is simply the differential decay rate divided by phase space. Theoretically one expects for a transition not between isotopic analog states

$$S(E) = c^2 \pm \frac{4E}{3M} bc \quad (4.22)$$

where the $+$, $-$ refers to electron, positron decay respectively and E is the energy of the charged lepton. Here the Gamow-Teller term c is found by measurement of the decay rate, while CVC predicts b in terms of the lifetime of an analog M1 radiative decay. However there are two difficulties here. One is that the effect is small, since for a typical beta decay

$$E/M_N \lesssim 1\% . \quad (4.23)$$

Secondly, electromagnetic and other corrections modify the naive theoretical expression, yielding in a more complete analysis [45]

$$S(E) \approx c^2(0) \left[1 + \frac{2}{9} \frac{c'(0)}{c(0)} \left(11m_e^2 + 20EE_0 - 20E^2 - \frac{2m_e^2 E_0}{E} \right) - \frac{c'(0)}{c(0)} \left(\frac{9}{2} \left(\frac{\alpha Z}{R} \right)^2 + \frac{4}{3} \frac{\alpha Z}{R} E_0 \pm \frac{22}{3} \frac{\alpha Z E}{R} \right) \right. \\ \left. + \frac{\sqrt{10}}{6} \frac{\alpha Z}{MR} \left(2 \frac{b}{c} \pm \frac{d}{c} \pm 1 \right) - \frac{2}{3} \frac{E_0}{M} \left(1 + \frac{d}{c} \pm \frac{b}{c} \right) + \frac{2}{3} \frac{E}{M} \left(5 \pm 2 \frac{b}{c} \right) - \frac{m_e^2}{3ME} \left(2 + \frac{d}{c} \pm \frac{2b}{c} - \frac{h E_0 - E}{c M} \right) \right] . \quad (4.24)$$

Nevertheless the weak magnetism term b is still the dominant contribution to the measured slope, so that a CVC test is feasible.

The experiment in the $A = 12$ system has been performed by several groups, and they find the results shown in table 4.2. There we see that the magnetic spectrometer measurement of dS/dE by Lee, Mo and Wu is in good agreement with the CVC prediction both for the e^- and e^+ branches [46]. The NaI scintillation spectrometer measurements of Kaina et al. do not agree with the predicted values for either branch [47]. However, if one takes the difference

$$\left(\frac{dS}{dE}\right)^{\beta^-} - \left(\frac{dS}{dE}\right)^{\beta^+} \simeq c^2(0) \left[-\frac{c'(0)}{c(0)} \frac{44\alpha Z}{3R} + \frac{8}{3M} \frac{b}{c} + \frac{4m_e^2}{3ME^2} \frac{b}{c} \right] \quad (4.25)$$

which removes certain systematic uncertainties, we find good agreement with the CVC predicted value. Also listed are results from similar measurements in ^{20}F [48]. However, the mirror ^{20}Na shape factor has not been measured. Overall then there is agreement with the CVC expectations. Unfortunately, the consistency is not what one would hope for on a case-by-case basis.

4.2. Second class currents

One can also utilize recoil-level experiments in order to probe the possible existence of second class currents. This has been done in two ways. One procedure is to look at the correlation between nuclear spin and positron momentum for the beta decay of ^{19}Ne , which decays to its isotopic analog ^{19}F . The absence of second class currents then requires the vanishing of the tensor form factor d . The shape factor for the decay can be written as

$$S(E) \simeq 1 + A(E) \mathbf{J} \cdot \mathbf{p}/E \quad (4.26)$$

Table 4.2
Shape factor measurements in $A = 12, 20$ systems: Shown here are experimental shape factor slopes, dS/dE , as measured by various groups

Transition	$\frac{dS}{dE} (\%/MeV)^{\text{exp}}$	$\frac{dS}{dE} (\%/MeV)^{\text{theory}} [39]$
$^{12}\text{B} \rightarrow ^{12}\text{C}$	$0.48 \pm 0.10 [46]$ $0.91 \pm 0.11 [47]$	0.43
$^{20}\text{F} \rightarrow ^{20}\text{Ne}$	$0.4 \pm 0.5 [48b]$ $0.78 \pm 0.43 [48a]$	0.94
$^{12}\text{N} \rightarrow ^{12}\text{C}$	$-0.52 \pm 0.06 [46]$ $-0.07 \pm 0.09 [47]$	-0.50

Note for $A = 12$

$$\begin{aligned} (dS/dE)^- - (dS/dE)^+ &= 1.00 \pm 0.13 [46] \\ &= 0.98 \pm 0.14 [47] \end{aligned}$$

agrees well with the theoretical expectation 0.93.

where in the simplest approximation

$$\frac{dA(E)}{dE} \simeq \frac{-1}{3M} \left(\frac{(a/\sqrt{3})(b+d) + \frac{1}{3}c \cdot (5b-d)}{ac/\sqrt{3} + \frac{1}{3}c^2} - \frac{4bc}{a^2 + c^2} \right). \quad (4.27)$$

Since a, b are predicted from CVC and the Gamow–Teller term c can be measured from the observed decay rate, the only unknown in eq. (4.27) is the induced tensor d . Of course again, careful corrections must be made for electromagnetic terms and higher order recoil effects. When this is done and comparison is made with experiment, one finds [49]

$$d/Ac = -1 \pm 2, \quad (4.28)$$

consistent with the absence of second class effects. (In order to set the scale here, we note that the weak magnetism term is given by $b/Ac \simeq 4$.)

A second approach has involved use of mirror isotriplet decays in $A = 8, 12$ and 20 systems. For $A = 8$ one measures the correlation between the β -particle and the α -particle produced in the breakup of the daughter ${}^8\text{Be}$ nucleus [50]. For $A = 20$, the correlation between the β and a delayed photon is measured [51]. For $A = 12$, the correlation is between the β and the nuclear alignment of the parent nucleus [52]. In each case, the correlation is sensitive to the structure function combination

$$B(E) = (E/M)(1 \pm b/c - d/c). \quad (4.29)$$

Here, however, since the decays do *not* take place between isotopic analog states, the tensor form factor d is not required to vanish but rather can have a nonzero value even in the absence of second class currents [38]. However, for the e^+, e^- decays we have

$$e^-: d = d_I + d_{II}, \quad e^+: d = d_I - d_{II} \quad (4.30)$$

where d_I, d_{II} are produced from first, second class currents respectively. Thus, sensitivity to second class effects is provided by measurement of both branches. Provided b is known, from CVC and the measurement of the analog photon transition, we can then measure d_{II} . Again nuclear structure complicates the analysis, especially in the case of the $2^+ \rightarrow 2^+$ decays in the $A = 8, 20$ systems. However, using the best present experimental and theoretical results we find

$$\frac{d_{II}}{Ac} = \begin{cases} -0.3 \pm 0.5 & A = 8^{[50]} \\ +0.2 \pm 0.6 & A = 12^{[52]} \\ +0.4 \pm 0.6 & A = 20^{[51]} \end{cases} \quad (4.31)$$

so that second class currents are ruled out at the level of 10% of weak magnetism, in accord with quark model predictions.

4.3. Tests of PCAC

Thus nuclear beta decay has enabled tests of two basic features which are required in a quark picture

of the hadronic weak current – CVC and the absence of second class currents. One cannot easily probe the third feature, PCAC, via nuclear beta decay since effects are suppressed by

$$O(m_e^2/m_N E) \lesssim 0.1\% . \quad (4.32)$$

However, the corresponding suppression which one expects in nuclear muon capture is

$$O(m_\mu/m_N) \sim 10\% \quad (4.33)$$

so that muon capture is a feasible arena in which to examine the validity of the PCAC hypothesis. The drawback in this case is that typically one has available from the experiment a single number, the capture rate. In order to interpret this number, one needs to know the value of each nuclear form factor at $q^2 \simeq -0.9m_\mu^2$, which introduces some uncertainty, since the structure constants determined in beta decay are at $q^2 = 0$. Nevertheless, predicted and experimental capture rates are generally in good agreement provided one uses

- i) no second class currents
- ii) $q^2 = 0$ value of form factors from the analogous β -decay
- iii) q^2 dependence of form factors from CVC and electron scattering results
- iv) CVC value for weak magnetism
- v) PCAC for h .

The results are summarized in table 4.3.

Obviously agreement is good except for ${}^6\text{Li}$, for which the origin of the discrepancy is unknown, although it has been speculated that perhaps the spin mixture is not statistical.

Before proceeding we should emphasize one point which is relevant. When PCAC is applied, it is for the nucleon

$$2m_N c(q^2) + \frac{q^2}{2m_N} h(q^2) = 2\sqrt{3} F_\pi g_r(q^2) \frac{1}{1 - q^2/m_\pi^2} . \quad (4.34)$$

Table 4.3
Muon capture: Shown are measured and calculated muon capture rates for a few of the best-studied reactions

Reaction	Theory [56]	Experiment
$\mu^- \text{P} \rightarrow \nu_\mu \text{N}$ $S_{\mu^- \text{p}} = 0$ $S_{\text{pmp}} = \frac{1}{2}, \quad S_{\text{pp}} = 1$	$664 \pm 20 \text{ sec}^{-1}$ $506 \pm 20 \text{ sec}^{-1}$	$651 \pm 57 \text{ sec}^{-1}$ [57] $515 \pm 85 \text{ sec}^{-1}$ [58] $464 \pm 42 \text{ sec}^{-1}$ [59]
$\mu^- {}^3\text{He} \rightarrow \nu_\mu {}^3\text{H}$	$1510 \pm 40 \text{ sec}^{-1}$	$1410 \pm 140 \text{ sec}^{-1}$ [60] $1505 \pm 48 \text{ sec}^{-1}$ [61] $1465 \pm 67 \text{ sec}^{-1}$ [62]
$\mu^- {}^6\text{Li} \rightarrow \nu_\mu {}^6\text{He}$	$0.98 \pm 0.15 \times 10^3 \text{ sec}^{-1}$ $1.05 \pm 0.60 \times 10^3 \text{ sec}^{-1}$	$1.60^{+0.33}_{-0.12} \times 10^3 \text{ sec}^{-1}$ [63]
$\mu^- {}^{12}\text{C} \rightarrow \nu_\mu {}^{12}\text{B}$	$7.01 \pm 0.16 \times 10^3 \text{ sec}^{-1}$	$0.62 \pm 0.03 \times 10^4 \text{ sec}^{-1}$ [64] $0.67 \pm 0.09 \times 10^4 \text{ sec}^{-1}$ [65] $0.675^{+0.03}_{-0.075} \times 10^4 \text{ sec}^{-1}$ [66]

Then at $q^2 = 0$ we have

$$1.253 = \frac{1}{\sqrt{3}} c(0) \simeq \frac{F_\pi g_r(m_\pi^2)}{m_N} = 1.33 \quad (4.35)$$

which is the Goldberger–Treiman relation. On the other hand, taking the first moment of eq. (4.34), we find

$$h(0) \simeq (4m_N^2/m_\pi^2) c(0) \quad (4.36)$$

where we have assumed similar q^2 dependence for $c(q^2)$ and $g_r(q^2)$. Usually eq. (4.36) is written as

$$\frac{g_p(-m_\mu^2)}{g_A(-m_\mu^2)} = \frac{2m_N m_\mu}{m_\pi^2 + 0.88m_\mu^2} = 6.8. \quad (4.37)$$

When PCAC is applied in nuclei one generally does so by using a simple impulse approximation. It is this version of PCAC which is tested by the muon capture rates listed above. The direct application of PCAC in nuclei cannot generally be utilized since the pion couplings are unknown.

In the case of capture on ^{12}C , additional experimental data is available. One class of experiment involves measurement of the polarization of the recoiling ^{12}B nucleus [64]. Combining this measurement with that of the total capture rate yields a separate test of CVC as well as of PCAC. The results [65]

$$b^{\text{expt}}/b_{\text{CVC}} = 1.00 \pm 0.05, \quad g_p/g_A = 8.0 \pm 3.0 \quad (4.38)$$

are in good agreement with both symmetry assumptions.

In addition, a recent experiment at SIN has measured the average and longitudinal recoil polarizations in the ^{12}C muon capture, yielding a value for the induced pseudoscalar [66]

$$g_p/g_A = 9.0 \pm 1.7 \quad (4.39)$$

which is again in good agreement with PCAC.

The analysis of all of these experiments assumes the vanishing of the induced scalar, $e = 0$. If this is not the case, there can be additional effects of size

$$O(m_\mu/m_N) \sim 10\%. \quad (4.40)$$

Although the vanishing of form factor e is predicted both by CVC and by the absence of second class currents, it is reassuring to note that a direct analysis of $0^+ \rightarrow 0^+$ decays, wherein a scalar form factor would produce a systematic Z -dependence, has shown that [67]

$$\frac{e}{Aa} = \begin{cases} -3 \pm 8 & [67a] \\ 2 \pm 11 & [67b] \end{cases}. \quad (4.41)$$

Clearly any scalar term is not large. A somewhat more model dependent result can be derived from

analog muon capture [68]

$$e/Aa = 0.3 \pm 1.9. \quad (4.42)$$

We have seen then that via experiments in nuclei, it has been possible to verify some of the basic symmetry laws which underlie weak interaction theory in the quark model and which are not easy to test by other means.

5. Semileptonic weak decays

Semileptonic processes involve only one hadronic current, combined with a leptonic weak current. They constitute the simplest manifestation of the quark currents. The role of the quark model here is to provide the connection between the fundamental current involving quark fields and the hadronic form factors. For the most part, the quark model should be expected to describe such matrix elements fairly accurately, and we will see that this expectation is generally fulfilled. It remains to be seen whether the fine details uncovered by recent precision experiments can in turn provide useful information about quark structure. In this section we survey the various forms of the hadronic current and show how the associated matrix elements arise from the underlying currents of quarks.

5.1. Hyperon beta decay

This general category includes both $\Delta S = 0$ transitions such as

$$\begin{aligned} N &\rightarrow Pe\nu, \\ \Sigma^\pm &\rightarrow (\Lambda, \Sigma^0)e\nu, \end{aligned} \quad (5.1)$$

and those with $\Delta S = 1$

$$\begin{aligned} \Lambda &\rightarrow Pe\nu, \\ \Sigma &\rightarrow Ne\nu, \\ \Xi &\rightarrow (\Sigma^0 \Lambda)e\nu. \end{aligned} \quad (5.2)$$

There is also preliminary evidence for $\Omega^- \rightarrow \Xi e\nu$ [69], but this does not yet approach the precision of the reactions quoted above. The most general form of the matrix element of the hadronic weak current in the transition $B \rightarrow B' \ell \nu$ is, as previously given,

$$\begin{aligned} \langle B'(p_2) | J_\mu | B(p_1) \rangle = & \bar{u}(p_2) \left[f_1(q^2) \gamma_\mu - \frac{i f_2(q^2)}{m_1 + m_2} \sigma_{\mu\nu} q^\nu + \frac{f_3(q^2)}{m_1 + m_2} q_\mu + g_1(q^2) \gamma_\mu \gamma_5 \right. \\ & \left. - \frac{i g_2(q^2)}{m_1 + m_2} \sigma_{\mu\nu} q^\nu \gamma_5 + \frac{i g_3(q^2)}{m_1 + m_2} q_\mu \gamma_5 \right] u(p_1) \end{aligned} \quad (5.3)$$

with $q \equiv p_1 - p_2$. The six independent form factors are functions of q^2 . The phases of the form factors are chosen so that each is real if time reversal invariance is valid. Let us look at each of these and see how they arise in the quark model.

$f_1 \gamma_\mu$: The vector form factor can be derived from the electromagnetic coupling using CVC plus SU(3). The result is

$$f_1(0) = i f_{ijk} \quad (5.4)$$

where $i, j, k = 1, 2, \dots, 8$ refers to the SU(3) labels of the external baryons and the weak current (i.e., $B \rightarrow i$, $B' \rightarrow j$ and $J_\mu \rightarrow k = (1 + i2)(\Delta S = 0)$ or $k = 4 + i5$ ($\Delta S = 1$)). The q^2 behavior is predicted by CVC to be identical to that of the charge form factor.

The quark model does satisfy the CVC hypothesis and so must reproduce these results in the SU(3) limit. This is seen by using the time component of the vector current matrix element

$$f_1^{N \rightarrow P} = \left\langle P \left| \int d^3x \bar{u} \gamma_0 d \right| N \right\rangle \quad (5.5)$$

plus generalizations to other states. Explicit calculation yields

$$f_1^{N \rightarrow P} = \int d^3x (u_u u_d + l_u l_d) \quad (5.6)$$

using the notation for the wavefunctions introduced in section 3. To the extent that up and down quarks have the same wavefunctions this is identical to the wavefunction normalization condition

$$1 = \int d^3x (u^2 + l^2) \quad (5.7)$$

i.e., $f_1^{N \rightarrow P} = 1$, as required by CVC. In other words, the CVC predictions emerge as the normalization condition on the quark wavefunction. The same is true in $\Delta S = 1$ transitions to the extent that the strange quark and the up quark have the same wavefunctions. We will shortly take up the question of SU(3) breaking.

$-if_2 \sigma^{\mu\nu} q_\nu$: This is the “weak magnetism” form factor and it is also completely determined by CVC plus SU(3),

$$f_2(0) = i f_{ijk} f + d_{ijk} d$$

$$f = \kappa_P + \kappa_N/2 = 0.84$$

$$d = -3\kappa_N/2 = 2.86$$

$$f/d = 0.29. \quad (5.8)$$

Here κ_P and κ_N are the anomalous magnetic moments of the proton and the neutron respectively. Again the quark model should reproduce these results to the extent that it can be used to calculate magnetic moments and that SU(3) breaking is not large. In the studies of the electromagnetic current, the magnetic moments of the hyperons are reproduced fairly well. In some models the overall magnitude of the magnetic moment is not precisely predicted but relations among moments are well satisfied. There do exist small discrepancies in the SU(3) structure, but at the 20% level the model is in good agreement with experiment [70]. In particular the quark model does *better* than the SU(3) parameterizations in the magnetic moments, with the source of SU(3) breaking being the reduced magnetic moment of the strange quark due to its heavier mass. This raises the question of the validity of the SU(3) parameterization of the weak transition elements.

The magnetic moment term of a vector current (including the spin moment) can be projected out by the standard weighting of the current

$$\frac{f_1 + f_2}{2m_P} \langle \sigma \rangle = \left\langle P \left| \int d^3x \frac{1}{2} \mathbf{r} \times \bar{u} \gamma \mathbf{d} \right| N \right\rangle. \quad (5.9)$$

The Dirac equation can be used to show that, in the limit of weak binding and nonrelativistic motion, this yields the familiar expression

$$\int d^3x \frac{1}{2} \mathbf{r} \times \bar{u} \gamma \mathbf{d} = \sigma / 2m_q \quad (5.10)$$

while for relativistic motion

$$\int d^3x \frac{1}{2} \mathbf{r} \times \bar{u} \gamma \mathbf{d} = \left[\frac{1}{3} \int d^3x (u_u l_d + u_d l_u) \right] \sigma. \quad (5.11)$$

In either case the results will follow the SU(3) parameterization if the electromagnetic moments are correctly predicted. However, the quark model predicts a fixed f/d ratio

$$f/d = 0.31 \quad (5.12)$$

in good agreement with the CVC ratio. This prediction arises from the classic quark model results on the neutron and proton magnetic moments

$$\mu_N / \mu_P = -2/3.$$

$g_1 \gamma_\mu \gamma_5$: The axial vector form factor may be parameterized by SU(3)

$$g_1(0) = i f_{ijk} F + d_{ijk} D. \quad (5.13)$$

However, in contrast to the vector current amplitudes, its magnitude cannot be obtained by SU(3) alone. From neutron beta decay we find [21]

$$g_1^{\text{NP}} = F + D = 1.254 \quad (5.14)$$

and the most recent fits to hyperon decay rates [71] indicate

$$D/(D + F) = 0.61. \quad (5.15)$$

The standard way to obtain the magnitude of g_1 is to use the Adler–Weisberger relation [72]

$$1 - \frac{1}{g_1^2} = \frac{4m_N^2}{\pi g_{\pi NN}^2} \int_{m_N + m_\pi}^{\infty} dw \frac{w}{w^2 - m_N^2} [\sigma_{\pi^+ P}(w) - \sigma_{\pi^- P}(w)] \quad (5.16)$$

which is derived using PCAC. This approach yields the correct answer, $g_1 = 1.25$, although it does not provide a very intuitive connection with the quark currents.

The direct calculation of g_1 in the quark model provides a rather different approach. Here, study of the matrix element of the spatial components of the current projects out g_1 , as shown explicitly in section 3.4,

$$g_1^{N \rightarrow P} = \left\langle P \uparrow \left| \int d^3x \bar{u} \gamma_3 \gamma_5 d \right| N \uparrow \right\rangle = \frac{5}{3} \int d^3x (u^2 - \frac{1}{3} l^2). \quad (5.17)$$

Nonrelativistic models ($l \rightarrow 0$) always reproduce the SU(6) prediction $g_1^{NP} = \frac{5}{3}$. However in more relativistic models the lower component in the wavefunction reduces g_1 below this value, and can easily generate a result close to the experimental value. For example, the bag model yields a range $g_1 = 1.09$ – 1.3 , depending on the value of the quark mass used [73] and on whether or not center-of-mass corrections are included [28]. The relativistic versions of the harmonic oscillator models yield similar results [74]. While there in principle could be further corrections due to gluonic effects not already incorporated into the confining potential [75], g_1 can be used to provide information on the rough size of the lower component of the wave function

$$\int d^3x l^2 / \int d^3x (u^2 + l^2) \approx \frac{3}{4} (1 - \frac{3}{5} g_A) \approx \frac{3}{16} \quad (5.18)$$

in agreement with our general estimate in section 3.2. The quark model also predicts the SU(3) structure of the amplitudes,

$$D/(D + F) = \frac{3}{5} \quad (5.19)$$

a result which follows from the spin part of the wavefunction, independent of the spatial properties. Again, this is in good agreement with experiment. While the quark model approach cannot be expected to reach an accuracy of better than 10%, it does provide an explanation of the general features of the data.

It is interesting to compare the Adler–Weisberger approach with that of the quark model. At first glance they appear very different, with g_1 in the former method appearing to deviate from the value $g_1 = 1$ due to interactions, while in the latter case confinement causes the deviation from $g_1 = \frac{5}{3}$. However, there does exist a systematic relationship between the two approaches [76]. The Adler–

Weisberger approach also yields $g_1 = \frac{5}{3}$ in nonrelativistic limit. In this limit $m_\Delta \rightarrow m_N$ because the excitation energy is assumed to be small compared to the masses for nonrelativistic motion. All resonances except the Δ decouple from πp scattering due to wavefunction orthogonality, while the Δ resonance coupling is related to the πNN coupling by the quark model, cancelling $g_{\pi NN}^2$ in eq. (5.16). When the Adler–Weisberger integral is done the result is exactly $g_1 = \frac{5}{3}$. As one goes away from this nonrelativistic limit, πP scattering can couple to other resonances via the lower component in the wavefunction, with the dominant coupling in $\pi^- P$ channel, lowering the value of g_1 from 5/3. These effects occur at about the same rate in both the Adler–Weisberger and quark model approaches. While it is clear that one cannot expect the quark model to exactly yield the formulas of PCAC, the compatibility between the two methods is remarkably close.

$g_2 \sigma^{\mu\nu} \gamma_5 q_\nu$: This is the “second class axial” or “weak electricity” form factor. It is most often set equal to zero because it is required to vanish by the joint usage of G -parity plus SU(3). However, here is a case where SU(3) breaking is probably sizable, and in quark models a significant g_2 is found. It is calculated [77] in the way one would expect for an “electric” dipole moment, as a moment of the axial charge

$$\frac{g_2}{m_1 + m_2} + \frac{1}{2} \left(\frac{1}{2m_1} - \frac{1}{2m_2} \right) g_1 = \left\langle B_2 \uparrow \left| -i \int d^3x \, z \bar{u} \gamma_0 \gamma_5 s \right| B_1 \uparrow \right\rangle. \quad (5.20)$$

In a nonrelativistic model, this yields for the right hand side of eq. (5.20)

$$\text{r.h.s.} = \frac{1}{2} \left(\frac{1}{2m_2} - \frac{1}{2m_1} \right) g_1^{\text{SU}(6)} \quad (5.21)$$

where $g_1^{\text{SU}(6)}$ is the SU(6) prediction for g_1 (i.e., analogous to $g_1^{\text{NP}} = \frac{5}{3}$). In the bag model, one finds

$$\text{r.h.s.} = g_1^{\text{SU}(6)} \int d^3x \, z (u_s l_u - u_u l_s). \quad (5.22)$$

The ratio of g_2 to g_1 is almost universal for $\Delta S = 1$ transitions (i.e., further SU(3) breaking is small)

$$\frac{g_2}{g_1} = \begin{cases} 0.30 & (\text{bag model}) \\ 0.60 & (\text{nonrelativistic}) \end{cases} \quad (5.23)$$

while for $\Delta S = 0$ transitions g_2 is negligible because it would require isospin breaking.

In the analysis of most experiments, g_2 is set equal to zero in order to increase the predictive power. This general policy is primarily responsible for the fact that there is no measurement of g_2 available. Worse than that, it may mean that some numbers quoted for other quantities could in fact be incorrect. The UMASS-BNL group working on $\Lambda \rightarrow P e \bar{\nu}$ have found a strong dependence of the results on g_2 [78], i.e.

$$\left. \frac{g_1}{f_1} \right|_{\Lambda \rightarrow P e \bar{\nu}} = 0.715 + 0.25 \frac{g_2}{f_1}. \quad (5.24)$$

Similar effects have been uncovered by the CERN group [71]. Thus values of g_2 predicted by the quark model would produce modifications to measured quantities outside of the statistical errors of the high precision experiments. It will become imperative for future progress that g_2 be determined, and not be neglected in data analysis.

$f_3 q_\mu$ and $g_3 q_\mu \gamma_5$: These terms, the “scalar” and “pseudoscalar” form factors, play very little role in hyperon beta decay because their effect is proportional to the lepton mass. Like g_2 , the form factor f_3 is second class, and only is produced via SU(3) breaking. In the bag model, there exists the simple relation [79]

$$f_3/f_1 = g_2/g_1. \quad (5.25)$$

The pseudoscalar form factor is due to the pion or kaon pole. It is not reliably calculable in the quark model, but PCAC predicts its value to be

$$g_3 = \left(\frac{m_1 + m_2}{m_\pi} \right)^2 g_1. \quad (5.26)$$

q^2 dependence: All of the form factors discussed above are functions of q^2 . In practice, because q^2 is small in hyperon beta decay, only the leading dependence on q^2 has been required in the analysis of the data. This much can be predicted in quark models, because the first term in a form factor is related to the expectation value of r^2 . For the vector current

$$f_1(q^2) = 1 - q^2 \langle r^2 \rangle / 6 \quad (5.27)$$

where

$$\langle r^2 \rangle = \int d^3x \, r^2 \, \psi^\dagger(\mathbf{x}) \, \psi(\mathbf{x}) = \int d^3x \, r^2 (u^2 + l^2). \quad (5.28)$$

As discussed in the section on general features of the quark model, this number can be accommodated in quark models, although the harmonic oscillator values are often somewhat low. In a like manner the axial vector charge radius can be estimated [80], with a result $\langle r^2 \rangle_{\text{ax}} \simeq \langle r^2 \rangle_{\text{EM}}$.

SU(3) breaking: In addition to the importance of the question of SU(3) breaking in the phenomenological analysis of beta decays, there is a theoretical reason for being interested in this subject. One of the best constraints on the KM angles comes from $\Delta S = 1$ beta decay. SU(3) breaking could in principle mislead this enterprise, as an overall factor in $\Delta S = 1$ decay would be interpreted as a different value of the KM angles.

There is, in the quark model, reason to expect some SU(3) breaking. For the vector current the Ademollo–Gatto theorem [81] requires that any SU(3) breaking enters only in second order, and this is seen explicitly in quark model calculations [77]. For all others the breaking is first order, although it is not very large in the case of g_A . It appears that the largest effect should be in the moments f_2 and g_2 , although this is more difficult to observe.

There appears to be no experimental evidence for the bag model's version of SU(3) breaking [77]. In fact, its inclusion leads to a much poorer fit than the assumption of perfect SU(3) [71]. This is one area where, upon further study, we may learn more about the quark model from the new high precision experiments.

Status of experiments: There have been several good high statistics experiments in recent years. For an up to date review see [71]. There are conflicting results for the quality of Cabibbo fits. Several groups [82] have used the total world sample of data in the fits, and have concluded that the standard Cabibbo parameterization based on SU(3) does a poor job of fitting the data. On the other hand, the CERN-SPS group [71] has performed an analysis using only their own data, which is the dominant contribution in many modes, and obtains an excellent fit. This group interprets the difference with earlier results as being due to the dangers of combining separate experiments analyzed under differing sets of assumptions. We however are not convinced that this is the correct explanation. The CERN group's data and the average of the world sample (taken from the Particle Data Group's tables) is displayed in table 5.1. One notes that in all cases except perhaps $\Sigma^- \rightarrow N e \nu$ the world data set is quite consistent with that from the CERN experiment – with, however, smaller error bars. We reproduce the reasonable fit to the CERN data (reported as fit #6 in ref. [71]) with $\chi^2 = 12.7$ for 7 degrees of freedom. However changing to the world value of $\Lambda \rightarrow P e \nu$ alone leads to an unacceptable fit with $\chi^2 = 17$ for 7 degrees of freedom, despite the central values being compatible in the two data sets. From this we conclude that it is basically the smaller error bars which produce the poor SU(3) Cabibbo fit rather than an internal inconsistency of the data. The total world sample has a Cabibbo fit (using the analysis of ref. [82]) with $\chi^2 = 17.5$ for 10 degrees of freedom. Recall that we expect the assumption of perfect SU(3) invariance to break down at some level. Most often the magnitude of SU(3) breaking is placed at the 30% level. However these experiments are generally accurate to 2–5%. More work is clearly needed to decide finally if this system truly provides evidence for SU(3) breaking and, if so, to understand its pattern.

Table 5.1

Semileptonic hyperon data. (a) Although not directly measured in the SPS data, these quantities were used in the Cabibbo fit. (b) We follow the Particle Data Group in using a scale factor of two in this error estimate. The units of all decay rates except $N \rightarrow P e \nu$ are 10^6 sec^{-1}

Observable	CERN-SPS data	World averages
$\Gamma(\Lambda \rightarrow P e \nu)$	3.26 ± 0.14	3.18 ± 0.053
$\Gamma(\Sigma^- \rightarrow N e \nu)$	6.43 ± 0.34	6.90 ± 0.23
$\Gamma(\Xi^- \rightarrow \Lambda e \nu)$	3.44 ± 0.19	3.32 ± 0.36 (b)
$\Gamma(\Xi^- \rightarrow \Sigma^0 e \nu)$	0.53 ± 0.10	0.53 ± 0.10
$\Gamma(\Lambda \rightarrow P \mu \nu)$	–	0.597 ± 0.133
$\Gamma(\Sigma^- \rightarrow N \mu \nu)$	–	0.30 ± 0.27
$\Gamma(\Xi^- \rightarrow \Lambda \mu \nu)$	–	1.1 ± 1.1
$\Gamma(N \rightarrow P e \nu)$	(a)	$(1.091 \pm 0.017) \times 10^{-3}$
$\Gamma(\Sigma^- \rightarrow \Lambda e \nu)$	3.78 ± 0.21	3.88 ± 0.18
$\Gamma(\Sigma^+ \rightarrow \Lambda e \nu)$	–	0.25 ± 0.06
$(g_1/f_1)(\Lambda \rightarrow P e \nu)$	0.70 ± 0.03	0.717 ± 0.03
$(g_1/f_1)(\Sigma^- \rightarrow N e \nu)$	-0.34 ± 0.05	-0.37 ± 0.05
$(g_1/f_1)(\Xi^- \rightarrow \Lambda e \nu)$	$+0.25 \pm 0.05$	$+0.25 \pm 0.05$
$(g_1/f_1)(N \rightarrow P e \nu)$	(a)	1.254 ± 0.006
$(f_1/g_1)(\Sigma^- \rightarrow \Lambda e \nu)$	$+0.034 \pm 0.090$	0.034 ± 0.090

Until very recently, there was a conflict between the measurements of the electron symmetry in Σ^- beta decay and the predicted value of the Cabibbo model. Within the past year, this has been resolved by a new experiment in favor of the standard theoretical prediction [83]. Given the validity of the Cabibbo fit at least the 5% level and the resolution of the problem with the Σ^- asymmetry, it can be concluded that the standard model does provide a good explanation of the presently existing semileptonic hyperon data. Further work on SU(3) breaking may be fruitful.

5.2. π and K decay

The analogues of hyperon beta decay in the mesons are the processes $K^+ \rightarrow \pi^0 e \nu$ and $\pi^+ \rightarrow \pi^0 e \nu$. In these only the vector current can contribute, and the matrix element has the form, e.g.,

$$\sqrt{2} \langle \pi^0(p') | V_\mu | K^+(p) \rangle = f_+(q^2) (p + p')_\mu + f_-(q^2) (p - p')_\mu. \quad (5.29)$$

Considerably less theoretical effort has gone into the quark model evaluation of these form factors. SU(3) plus CVC requires that $f_+(0) = 1$ and $f_-(0) = 0$, with deviations in f_+ being second order in SU(3) breaking. In the quark model this result naturally emerges, as it is equivalent to the normalization condition. There have also been studies of SU(3) breaking in mesonic transitions [84]. It is found that the dominant factor f_+ is shifted only by a few percent, while f_- demonstrates significant SU(3) breaking, in agreement with the predictions of chiral symmetry [85].

In addition to beta decay, the charged weak currents are responsible for the semileptonic processes $\pi^+ \rightarrow e^+ \nu$ and $K^+ \rightarrow \mu^+ \nu$. Here the relevant matrix elements are

$$\begin{aligned} \langle 0 | A_\mu^{\Delta S=0} | \pi^+(p) \rangle &= i f_\pi p^\mu \\ \langle 0 | A_\mu^{\Delta S=1} | K^+(p) \rangle &= i f_K p^\mu \end{aligned} \quad (5.30)$$

with (here $f_\pi = \sqrt{2} F_\pi$)

$$f_\pi \approx 130 \text{ MeV}, \quad f_K \approx 160 \text{ MeV}. \quad (5.31)$$

These are also calculable in quark models, although not as simply. In the bag model (see ref. [28] for a fuller discussion)

$$f_\pi = \frac{1}{\sqrt{2} R_\pi} \approx 200 \text{ MeV}, \quad f_K \approx f_\pi, \quad (5.32)$$

while nonrelativistic models generally yield a larger value ($f_\pi \approx 400 \text{ MeV}$) and provide a worse estimate of

$$\frac{f_K}{f_\pi} = \sqrt{\frac{m_\pi}{m_K}} \frac{\psi_K(0)}{\psi_\pi(0)} \approx 0.5 \quad (5.33)$$

(i.e., the “van Royen–Weisskopf paradox” [86]). The bag model description contains an interesting

feature which resolves the van Royen–Weisskopf paradox and also will be important later in our discussion of nonleptonic decays. The amplitudes which determine f_π involve a cancellation between the upper and lower components of the wavefunction (it is proportional to $u^2 - l^2$). This lowers the result down towards the experimental value (and presumably explains why the bag f_π is lower than the nonrelativistic f_π). The origin of this cancellation is clear; it is related to the “helicity suppression” of $\pi \rightarrow e\nu$ with respect to $\pi \rightarrow \mu\nu$

$$\Gamma(\pi \rightarrow e\nu)/\Gamma(\pi \rightarrow \mu\nu) = 1.2 \times 10^{-4}. \quad (5.34)$$

The latter occurs because helicity conservation forbids the production of two free massless fermions by vector or axial vector currents if they are in a spin zero state. The ratio of eq. (5.33) is then suppressed by a factor of m_e^2/m_μ^2 over naive phase space considerations. For confined quarks helicity is not as well defined a quantity but the annihilation of $\bar{q}q$ by an axial current still shows this type of cancellation. As the quark mass increases the cancellation becomes weaker and the numerator factor in the calculation compensates for the dependence on the kaon mass in the denominator, keeping f_K roughly equal to f_π . Relativistic potential models also contain this feature and do an acceptable job of predicting f_π and f_K [87].

6. $\Delta S = 1$ hadronic decays

The literature on $\Delta S = 1$ hadronic decays is extensive. We cannot do justice to the totality of published papers in this area. In the following, we shall restrict our attention to works which have strongly influenced our own attempts to understand these decays as a consequence of quark dynamics.

6.1. Data and kinematics

The data set we consider here consists of nonleptonic kaon, hyperon and Ω^- transitions. We address each of these in turn.

6.1.1. $K\pi\pi$

By angular momentum conservation, the pions are emitted in an S-wave. Correspondingly any $K\pi\pi$ invariant amplitude f is isotropic, related to decay rate Γ and pion momentum q by

$$|f| = (8\pi\Gamma/q)^{1/2} m_K. \quad (6.1)$$

Numerical values are exhibited in table 6.1 [21]. Note that we work with K^0 (and not K_s) amplitudes, and that only $\Delta I = \frac{1}{2}, \frac{3}{2}$ transitions are assumed to contribute. We relate the isospin amplitudes f_{2I} to the mode amplitudes f_{+0}, f_{+-}, f_{00} by

$$\begin{aligned} f_{+0} &= 3f_3/\sqrt{2} \\ f_{+-} &= f_1 + f_3 \\ f_{00} &= f_1 - 2f_3. \end{aligned} \quad (6.2)$$

Table 6.1
K $\pi\pi$ data

	K _s	K [±]
Lifetimes (sec.)	0.8923 × 10 ⁻¹⁰	1.2371 × 10 ⁻⁸
Parent	K ⁰	K [±]
Mode	$\pi^+ \pi^-$	$\pi^0 \pi^0$
Branching ratio	0.6861	0.3139
Decay momentum (MeV)	206.0	209.1
Rate (MeV)	2.531 × 10 ⁻¹²	2.316 × 10 ⁻¹²
Invt. ampl. (m_K^{-1})	5.556 × 10 ⁻⁷	5.276 × 10 ⁻⁷

Unitarity determines the phases of f_1, f_3 to be δ_0^0, δ_0^2 respectively, where δ_I^I is the $\pi\pi$ S-wave phase shift for isospin I . From K_s data we can deduce the f_I , but only with some uncertainty since the δ_I^I are not precisely known. We find $|f_1| = 5.46 \times 10^{-7} m_K$ and $|f_3| = 1.07 - 1.23 \times 10^{-8} m_K$, corresponding to taking $29^\circ < \delta_0^0 - \delta_0^2 < 40^\circ$. A slightly different value, $|f_3| = 1.75 \times 10^{-8} m_K$, is obtained from K[±] lifetimes. This is attributable to electromagnetic contributions in the K_s, K[±] transitions. We shall not address such subtle effects in this review, but rather focus on the substantial magnitude of f_1/f_3 . The dominance of the $\Delta I = \frac{1}{2}$ signal in $|\Delta S| = 1$ transitions is the famous ‘ $\Delta I = \frac{1}{2}$ rule’.

6.1.2. K $\pi\pi\pi$

In all, there are four independent transitions: $K^+ \rightarrow \pi^+ \pi^+ \pi^-$, $\pi^0 \pi^0 \pi^+$ and $K^0 \rightarrow \pi^0 \pi^0 \pi^0$, $\pi^+ \pi^- \pi^0$. The main features of K $\pi\pi\pi$ data have been understood for a long time, particularly by using current algebra and PCAC to relate it to the K $\pi\pi$ amplitudes [88]. Here we shall restrict our attention to interpreting fairly recent precision measurements of terms in the $\Delta I = \frac{1}{2}$ K $\pi\pi\pi$ amplitude which are *quadratic* in the kinematic variables [89].

The $\Delta I = \frac{1}{2}$ K $\pi\pi\pi$ amplitude is parameterized with dimensionless quantities a, b, c, d ,

$$10^6 f(K\pi\pi\pi) = a + bY + c(Y^2 + X^2/3) + d(Y^2 - X^2/3) \quad (6.3)$$

where

$$X = (s_2 - s_1)/m_\pi^2, \quad Y = (s_3 - s_0)/m_\pi^2 \quad (6.4)$$

with

$$s_i = (p_K - p_i)^2, \quad s_0 = (s_1 + s_2 + s_3)/3. \quad (6.5)$$

By convention the “odd” pion in each K $\pi\pi\pi$ reaction is assigned the subscript 3 (in the $\pi^0 \pi^0 \pi^0$ mode, this distinction is irrelevant). The values of a, b, c, d are exhibited in table 6.2. One should not be deceived by the small experimental values of c and d . The kinematic variables X, Y are “large” in the sense that factors of pion masses appear inversely in their definitions. Thus c and d represent real effects and indeed have motivated recent theoretical work [90].

6.1.3. B → B'π

In the reaction

$$B(p, \lambda) \rightarrow B'(p', \lambda') + \pi(q) \quad (6.6)$$

Table 6.2
 $K\pi\pi\pi$ data. The theoretical fits are in terms of chiral Lagrangians as described in section 6.4

	a	b	c	d
Expt.	0.915 ± 0.0024	0.258 ± 0.004	-0.0037 ± 0.0011	-0.0125 ± 0.0012
Quadratic	0.75	0.18	0	0
Quadratic plus quartic	0.94	0.29	-0.034	-0.0105

where B, B' have spin $\frac{1}{2}$, conservation of angular momentum dictates that the decay products exist only in S-wave or P-wave. The two corresponding invariant amplitudes A, B are defined by

$$\bar{u}(p', \lambda')[A + B\gamma_5]u(p, \lambda) \quad (6.7)$$

with decay rate

$$\Gamma = \frac{1}{4\pi} \frac{q(E+m)}{M} (|A|^2 + |\bar{B}|^2) \quad (6.8)$$

where M is the parent mass, E and m are the decay baryon energy and mass respectively, and $\bar{B} \equiv ((E-m)/(E+m))^{1/2} B$. A and B are given in table 6.3(a) for each hyperon decay. The notation is standard – the hyperon and pion charges appear respectively as superscript and subscript. Although typically $B > A$ (except for Σ^-), the reader should not conclude that P-wave emission is dominant over S-wave. A more physically relevant quantity than B is $\bar{B}$, as is seen in the measured asymmetry parameters,

$$\alpha = \frac{2|A||\bar{B}|\cos(\delta_S - \delta_P)}{|A|^2 + |\bar{B}|^2} \quad (6.9)$$

Table 6.3(a)
 Hyperon nonleptonic decays.
 The A, B amplitudes in units of 10^{-7}

Mode	A		B
	Expt.	Unweighted SU(3) fit	
Λ^0	3.25	3.38	22.1
Λ^0_0	-2.37	-2.39	-15.8
Σ^+	0.13	0	42.2
Σ^+_0	-3.27	-3.18	26.6
Σ^-	4.27	4.50	-1.44
Ξ^0_0	3.43	3.14	-12.3
Ξ^-_0	-4.51	-4.45	16.6

where $\delta_{S,P}$ are the S, P-wave pion–baryon scattering phase shifts. In fact, S and P-wave emissions are competitive, with S-wave generally larger.

To obtain a measure of the isospin characteristics for these transitions, we perform an amplitude decomposition analogous to the one used for $K\pi\pi$ decays. We have

$$\begin{aligned} A(\Lambda^0) &= \sqrt{2} A_\Lambda^{(1)} - A_\Lambda^{(3)} & A(\Xi^0) &= -A_\Xi^{(1)} - \sqrt{2} A_\Xi^{(3)} \\ A(\Lambda^0) &= -A_\Lambda^{(1)} - \sqrt{2} A_\Lambda^{(3)} & A(\Xi^-) &= \sqrt{2} A_\Xi^{(1)} - A_\Xi^{(3)} \end{aligned} \quad (6.10)$$

where the superscripts (1), (3) refer respectively to the $\Delta I = \frac{1}{2}, \frac{3}{2}$ parts of the weak Hamiltonian. The Λ and Ξ P-wave amplitudes obey analogous relations. The $\Sigma^\pm$ amplitudes can be written as

$$\begin{aligned} A(\Sigma^-) &= A_\Sigma^{(1)} + A_\Sigma^{(3)} \\ \sqrt{2} A(\Sigma_0^+) &= -\frac{2}{3} A_\Sigma^{(1)} + \frac{4}{3} A_\Sigma^{(3)} + X_\Sigma \\ A(\Sigma_+^+) &= \frac{1}{3} A_\Sigma^{(1)} - \frac{2}{3} A_\Sigma^{(3)} + X_\Sigma. \end{aligned} \quad (6.11)$$

For S-waves, the combination $\sqrt{2} A(\Sigma_0^+) + A(\Sigma_+^+)$ allows a clean determination of $A_\Sigma^{(1)}$ and $A_\Sigma^{(3)}$. Analogous equations hold for P-waves but now $B_\Sigma^{(1)}$ is greatly suppressed and the large (presumably) $\Delta I = \frac{1}{2}$ signal appears only in X_Σ . For this case a measure of $\Delta I = \frac{3}{2}$ to $\Delta I = \frac{1}{2}$ effects is given by $B_\Sigma^{(3)}/X_\Sigma$. Table 6.3(b) illustrates that the $\Delta I = \frac{1}{2}$ rule is a feature of both S-wave and P-wave hyperon decay amplitude at about the same level observed in kaon decay.

Although we shall discuss current algebra methods in section 6.2, we note in passing that most analyses of the S-wave amplitudes stress their relation (via PCAC) to the matrix elements $\langle B' | H_w^{p.c.} | B \rangle$. The ability to characterize the latter with f, d parameters motivates one to proceed similarly with the former. Table 6.3(a) reveals that an acceptable fit is obtained. In particular we find

$$f = -0.92 \times 10^{-7}, \quad f/d = -2.4, \quad (6.12)$$

i.e., f and d have opposite phase and f is rather larger. Our phase convention can be inferred from tables 6.6, 6.7 later in this section.

As a final example of systematics manifested by the hyperon decay amplitudes, we consider the Lee–Sugawara relation [91]

$$\sqrt{3} \Sigma_0^+ = 2\Xi^- + \Lambda^0 \quad (6.13)$$

Table 6.3(b)
Hyperon nonleptonic decays.
Ratios of $\Delta I = 3/2, 1/2$ amplitudes

	S-wave	P-wave
Λ	0.014	0.006
Σ	-0.017	-0.049
Ξ	0.034	0.023

which we define to hold for either A or B amplitudes. As discussed in ref. [1], the derivation of this relation requires a somewhat stronger assumption than simply octet dominance of the nonleptonic Hamiltonian H_w . For example, it emerges for the $A(B)$ amplitudes upon assuming H_w transforms as the sixth (seventh) component of an octet. Referring to the amplitudes of table 6.3, we see that the relation (in units of 10^{-7}) is well obeyed for the A amplitudes ($-5.66 = -5.77$) and less so for the B amplitudes ($46.1 = 55.3$).

6.1.4. Ω^-

Many years after the discovery of the Ω^- , construction of hyperon beams has finally allowed detailed experimental studies of its decay properties. Excellent data now exist [92], and add a welcome component to the field of nonleptonic (and semileptonic) transitions.

As with hyperon decay, a given Ω^- transition into a $1/2^+ 0^-$ pair is describable in terms of two invariant amplitudes. We shall define these to be dimensionless so as to resemble the hyperon invariant amplitudes. Emission can proceed via P-wave or D-wave,

$$\frac{q^\mu}{m_\pi} \bar{u}(p', \lambda') (B + \gamma_5 C) u_\mu(p, \lambda) \quad (6.14)$$

where m_π is the charged pion mass and u_μ is a Rarita–Schwinger vector-spinor. The associated decay rate is

$$\Gamma = \frac{q^3}{12\pi M m_\pi^2} [(E + m) |B|^2 + (E - m) |C|^2]. \quad (6.15)$$

From angular distribution measurements, one concludes that $|C/B| \ll 1$, i.e., the Ω^- decays almost entirely into P-wave hyperon–pion final states with little or no D-wave emission. The B amplitudes are shown in table 6.4. An isospin decomposition of the $\Xi\pi$ amplitudes gives $B_\Omega^{(3)}/B_\Omega^{(1)} = -0.066$. Again the $\Delta I = \frac{1}{2}$ amplitude dominates.

Finally, the amplitudes for $\Omega^- \Xi^* \pi$ can be written as

$$\bar{u}^\mu(p', \lambda') [(A + B\gamma_5) g_{\mu\nu} + (C + D\gamma_5) q_\mu q_\nu] u^\nu(p, \lambda). \quad (6.16)$$

The only data currently available is in the branching ratio [92] $\Gamma(\Xi^{*0} \pi^-)/\Gamma(\text{all}) = 6.4_{-2.0}^{+5.1} \times 10^{-4}$. In view of this it is generally assumed that $C = D = 0$. The decay rate is then

$$\Gamma = \frac{1}{18\pi} \frac{q}{M} [k_+(E + m) |A|^2 + k_-(E - m) |B|^2] \quad (6.17)$$

Table 6.4
 Ω^- decay into $1/2^+ 0^-$ final states. The B amplitudes are expressed in units of 10^{-7}

	$\Lambda \bar{K}$	$\Xi^0 \pi^-$	$\Xi^- \pi^0$
Br. ratio	0.678	0.236	0.086
Mom. (MeV)	211.4	293.7	289.8
$ B $	5.61	1.86	1.14

where $k_{\pm} = (E/m)^2 \pm (E/m) + E/2$. Upon inserting numerical values we obtain

$$2.1 \times 10^{-15} = |A|^2 + 1.65 \times 10^{-5} |B|^2. \quad (6.18)$$

Evidently the observed $\Xi^{*0} \pi^-$ rate tells nothing about B , and that $|A| = 0.46 \times 10^{-7}$ for this mode.

6.2. Theoretical procedures

6.2.1. Operator product expansion and $\Delta S = 1$ Hamiltonian

The effective operator generally employed to analyze $\Delta S = 1$ nonleptonic interactions has the form [14, 93]

$$H_{\Delta S=1} = \frac{G_F \sin \theta_c \cos \theta_c}{2\sqrt{2}} \sum_{i=1}^6 c_i O_i \quad (6.19)$$

where the coefficient functions $\{c_i\}$ are c -numbers and the four-quark operators $\{O_i\}$ are defined by

$$\begin{aligned} O_1 &= H_A - H_B & O_4 &= H_A + H_B - H_C \\ O_2 &= H_A + H_B + 2H_C + 2H_D & O_5 &= \bar{d}\Gamma_L^\mu \lambda^\Lambda s \bar{Q} \Gamma_{R\mu} \lambda^\Lambda Q \\ O_3 &= H_A + H_B + 2H_C - 3H_D & O_6 &= \bar{d}\Gamma_L^\mu s \bar{Q} \Gamma_{R\mu} Q \end{aligned} \quad (6.20)$$

with

$$\begin{aligned} H_A &= \bar{d}\Gamma_L^\mu u \bar{u} \Gamma_{L\mu} s & H_C &= \bar{d}\Gamma_L^\mu s \bar{d} \Gamma_{L\mu} d \\ H_B &= \bar{d}\Gamma_L^\mu s \bar{u} \Gamma_{L\mu} u & H_D &= \bar{d}\Gamma_L^\mu s \bar{s} \Gamma_{L\mu} s. \end{aligned} \quad (6.21)$$

In the expressions for O_5 and O_6 we sum over $Q = u, d, s$. Of the six local operators contributing to $H_{\Delta S=1}$, each is $\Delta I = \frac{1}{2}$ except for O_4 which is $\Delta I = \frac{3}{2}$. The ‘penguin’ operators O_5, O_6 have a $(V-A)(V+A)$ chiral structure whereas O_1, O_2, O_3, O_4 are $(V-A)(V-A)$ operators. For definiteness, we choose the $\{O_i\}$ not to be normal-ordered. We shall return to this point presently.

The coefficients $\{c_i\}$ are obtained as solutions of renormalization group (RG) equations as described in section 2.3. They depend upon quark masses, the confinement scale Λ_{QCD} , and the scale μ at which matrix elements are to be determined. A representative set of values with $\alpha_s(\mu^2) = 1$ is given in table 6.5

Table 6.5
Coefficients in the $\Delta S = 1$ Hamiltonian[†]

$c_1 = 2.38 + i0.095\phi$	$c_4 = 0.42$
$c_2 = 0.10 - i0.013\phi$	$c_5 = -0.047 - i0.097\phi$
$c_3 = 0.084$	$c_6 = -0.009 - i0.046\phi$

[†] $\phi \equiv s_2 s_3 s_8$. The values quoted are from the work of Gilman and Wise [94] and correspond to $\alpha_s(\mu^2) = 1$, $m_t = 40 \text{ GeV}$.

[94]. Quark masses play a particularly crucial role for the coefficients c_5 and c_6 . In the SU(4) limit of u–c degeneracy, c_5 and c_6 vanish. More specifically, the ‘penguin’ effect turns on only at scales below the charm quark mass, where the GIM cancellation between the up-quark and charm-quark in the loop of fig. 6.1 breaks down. Although the associated coefficients grow with decreasing scale, they are still rather small at $\mu \simeq 1$ GeV.

Dependence on the renormalization scale μ is more subtle. In principle the coefficients contain information regarding short-distance ($<\mu^{-1}$) physics whereas long-distance ($>\mu^{-1}$) effects are accounted for by matrix elements of the $\{O_i\}$. In practice, $\mu \simeq 0.3\text{--}1$ GeV is generally chosen because at lower energies the magnitude of α_s causes perturbative estimates to break down. Typical matrix element calculations would appear to require somewhat lower values of μ than 1 GeV.

The $\{O_i\}$ constitute a set of dimension-six operators. With one exception, these are the lowest dimension objects appearing in the operator product expansion of $H_{\Delta S=1}$ and which cannot be renormalized away. The exception is a $\Delta I = \frac{1}{2}$ dimension-five operator which first appears at the two-loop level of QCD radiative corrections,

$$O_7 = \left(m_s \bar{s} \Gamma_L \sigma_{\mu\nu} \frac{\lambda^A}{2} \Gamma_L d - m_d \bar{s} \Gamma_R \sigma_{\mu\nu} \frac{\lambda^A}{2} \Gamma_R d \right) G_A^{\mu\nu}. \quad (6.22)$$

This operator has the structure of a color-gluon coupling to the anomalous magnetic moment of a quark bilinear. Matrix elements of O_7 have been estimated [95] and found to be generally comparable to those of the $\{O_i\}$. However the coefficient function c_7 , being of higher order in α_s , is found to be roughly an order-of-magnitude smaller than the penguin coefficient-functions c_5 , c_6 . Hence the effect of this operator on nonleptonic physics is not significant and we shall omit it in the analysis to follow.

6.2.2. Current algebra and matrix elements of $H_{\Delta S=1}$

Let us denote the transition amplitude for the nonleptonic emission of a pion as

$$T_{\alpha\beta}^{(a)}(q) = \langle \beta \pi_a(q) | H_w | \alpha \rangle. \quad (6.23)$$

Existing knowledge of hadron dynamics is insufficient to allow determination of $T_{\alpha\beta}^{(a)}(q)$ except in the context of some approximation. Accordingly, it is common practice to use PCAC to provide an estimate via the low-energy theorem [96, 97]

Fig. 6.1. Penguin operator. These diagrams contribute to the operators O_5 , O_6 . Examples with (a) one loop, (b) two loops are shown.

$$T_{\alpha\beta}^{(a)}(0) = \frac{-i}{F_\pi} \langle \beta | [F_a^5, H_w] | \alpha \rangle \quad (6.24)$$

where we temporarily suppress the additional contributions associated with degenerate states. This approach clearly emphasizes the chiral structure of the weak Hamiltonian. The $(V-A)(V-A)$ operators O_1, O_2, O_3, O_4 of eq. (6.20) obey

$$[F_a^5, O_i] = [F_a, O_i]. \quad (6.25)$$

The penguin operators O_5, O_6 appear at first to be different in that right-handed quarks appear. However since these occur in an $SU(3)$ *singlet* combination, they do not affect the chiral transformation property, and so O_5 and O_6 also obey eq. (6.25).

The question of whether the approximation $T_{\alpha\beta}^{(a)}(q) \simeq T_{\alpha\beta}^{(a)}(0)$ is valid is a thorny one for which there is at present no totally satisfactory answer. However for the $K \rightarrow \pi\pi$ transitions, important momentum dependence is expected to be present. Chiral Lagrangians can form the basis for a representation of this momentum dependence. An appropriate chiral Lagrangian for $\Delta I = \frac{1}{2}$ octet transitions is [19]

$$L_w = g_8 \text{Tr} \lambda_6 \partial_\mu M \partial^\mu M^\dagger \quad (6.26)$$

where M is an element of $SU(3)$ defined in eq. (2.19)

$$M = \exp(i\lambda \cdot \phi(x)/F_\pi)$$

with $\{\phi_i(x)\}$ ($i = 1, \dots, 8$) being field operators of the octet of light pseudoscalar mesons. In eq. (6.26) the coupling constant g_8 is fixed by fitting to $K_s \pi^+ \pi^-$ with the result $g_8 = 3.58 \times 10^{-8} m_\pi^2$. The off-shell $K_s \pi^+ \pi^-$ amplitude is thus determined

$$f(K_s(k) \pi^+(q^+) \pi^-(q^-)) = \frac{g_8}{F_\pi^3} (2k^2 - q_+^2 - q_-^2). \quad (6.27)$$

The amplitude at the physical point $k^2 = m_K^2$, $q_+^2 = q_-^2 = m_\pi^2$ and at, for example, the point $k^2 = q_+^2 = m_K^2$, $q_-^2 = 0$ differ by roughly a factor of two.

A related matter involves terms which contribute to the physical amplitude but vanish in the soft-pion limit. Belonging to this class are a well-studied set of processes called “vacuum-insertion” or “factorization” amplitudes. They are treated separately in section 6.4.1.

We shall encounter the Lagrangian of eq. (6.26) in our discussion of $K\pi\pi\pi$ decays shortly. In passing we note that an analogous Lagrangian exists for the 27-plet $K\pi\pi$ transitions,

$$g_{27} C \begin{pmatrix} 8 & 8 & 27_s \\ i & j & 6 \end{pmatrix} \text{Tr} \lambda_i X_\mu \lambda_j X^{\mu\dagger} \quad (6.28a)$$

where the C -symbol represents an $SU(3)$ Clebsch–Gordan coefficient. A representation similar to eq. (6.28) could have been used for the octet chiral Lagrangian of eq. (6.25), viz.,

$$g'_8 C \begin{pmatrix} 8 & 8 & 8_s \\ i & j & 6 \end{pmatrix} \text{Tr} \lambda_i X_\mu \lambda_j X^{\mu\dagger} \quad (6.28b)$$

where

$$X_\mu = (\partial_\mu M) M^\dagger. \quad (6.29)$$

6.2.3. The Feinberg–Kabir–Weinberg theorem and its relevance to nonleptonic decay

In the operator product expansion of the nonleptonic Hamiltonian one keeps only operators of dimension five and six, and drops those of dimension three and four such as $\bar{d}s$, $\bar{d}\not{D}s$. This is appropriate as a result of the early work of Feinberg, Kabir and Weinberg (FKW) [98] who gave a general proof that such off-diagonal operators cannot contribute to physical decay amplitudes. In this section we give a brief discussion of this result and some of its applications to weak decays.

The first part of the FKW theorem says that the dimension three and four operators $\bar{d}s$, $\bar{d}\gamma_5 s$, $\bar{d}_L \not{D} s_L$ and $\bar{d}_R \not{D} s_R$ should not lead to flavor changing processes because they can be removed by diagonalization of the kinetic energy and mass terms in the Lagrangian. Specifically the Lagrangian would have the form

$$\begin{aligned} L = & \bar{d}\not{D}d + \bar{s}\not{D}s + a(\bar{d}_L \not{D} s_L + \text{H.c.}) + b(\bar{d}_R \not{D} s_R + \text{H.c.}) \\ & + c(\bar{d}s + \text{H.c.}) + e(\bar{d}\gamma_5 s + \text{H.c.}) + m_d^0 \bar{d}d + m_s^0 \bar{s}s. \end{aligned} \quad (6.30)$$

This can be put into diagonal form by separate (non-unitary) redefinition of the left and right fields

$$\begin{pmatrix} d' \\ s' \end{pmatrix}_{L,R} = S_{L,R} \begin{pmatrix} d \\ s \end{pmatrix}_{L,R}. \quad (6.31)$$

Therefore there is a quantum number which can be called strangeness and which is absolutely conserved. The diagonalization is simply identifying the mass and momentum eigenstates. The second part of the FKW theorem is less intuitive. It says that even if one uses the states in the original basis, before diagonalization, one must obtain a vanishing transition matrix element as long as one deals with on-shell states. What typically happens is that there is a cancellation among several diagrams which becomes exact when the particles are on-shell. An example involving the P-wave hyperon decays is discussed in section 6.3.2 [96].

In the theory of nonlinear effective Lagrangians, a similar result holds for weak interactions transforming like $(3, \bar{3})$. An operator of the form $\text{Tr}(\lambda_6 M)$ can be diagonalized away by a chiral transformation. Even if one uses the undiagonalized basis, one obtains a vanishing answer, as occurs in the diagram of fig. 6.2a. That is, one obtains

$$\text{Amp } a = -iA_0 \quad (6.32)$$

while the ‘tadpole’ diagram of fig. 6.2b produces

$$\text{Amp } b = +iA_0 \quad (6.33)$$

giving exact cancellation. The point is that chiral symmetry requires both diagrams to be present and fixes their relative contributions such that the chiral generalization of the FKW is satisfied.

The physics discussed above justifies the neglect of dimension three and four terms in the

Fig. 6.2. Cancellation between (a) and (b) tadpole diagrams for $K\pi\pi$ transition induced by the off-diagonal mass term.

Lagrangian. However one can always add a dimension three or four operator to the Hamiltonian, if it simplifies the analysis, without changing any physical transition. This was used by us in our treatment of the penguin interaction in the bag model [96]. In addition, the chiral version of the FKW theorem implies that there will not be any $(3, \bar{3})$ transformation behavior introduced into $\Delta S = 1$ amplitudes through SU(3) breaking. Such components would have vanishing matrix elements.

6.3. Baryonic $|\Delta S| = 1$ decays

We shall in turn consider the following cases of $|\Delta S| = 1$ decays: (1) $B' \rightarrow B\pi$ (where B' is a hyperon) S-wave decay. (2) $B' \rightarrow B\pi$ P-wave decay, and (3) $\Omega^- \rightarrow BP$ (where B is a hyperon and P is a pseudoscalar meson). Some comments on $\Delta I = \frac{3}{2}$ transitions complete the section.

Two experimental features of these systems are especially noteworthy: (i) the $\Delta I = \frac{1}{2}$ rule, which can be roughly stated

$$\text{Amp}(\Delta I = \frac{1}{2}) \simeq 20 \text{ Amp}(\Delta I = \frac{3}{2}) \quad (6.34)$$

and (ii) the predominance of strange particle decays into purely hadronic states

$$\Gamma(\text{nonleptonic}) \simeq 400 \Gamma(\text{semileptonic}). \quad (6.35)$$

As a whole, there is no self-evident explanation for either the $\Delta I = \frac{1}{2}$ rule or nonleptonic dominance when viewed purely from the vantage of the weak interaction current–current description. Presumably an understanding of these phenomena is contained within the quark model. By the “quark model” we mean both the binding of quark configurations into color-singlet hadrons, and also the battery of relevant theoretical techniques such as chiral symmetry, the operator-product expansion, and QCD radiative corrections to weak interaction transition operators.

It is convenient to discuss models of the S-wave and P-wave amplitudes separately, since the theoretical approaches typically used to analyze them are distinct.

6.3.1. S-wave transitions

The traditional “first approximation” to the A amplitudes is given by the soft pion estimate

$$A_{B'B}^a \equiv \langle B' \pi^a(q) | H_w | B \rangle = -\frac{i}{F_\pi} \langle B' | [F_a^5, H_w] | B \rangle + \dots \quad (6.36)$$

The “+ . . .” shall be discussed shortly. From the chiral commutator eq. (6.25), we see that the soft pion estimate of any of the A -amplitudes is proportional to a baryon-to-baryon matrix element of H_W . In the usual model wherein baryons are color-singlet configurations of three quarks, several exact relations are present. First is (see eq. (6.21) for definitions of $H_A, \dots, H_D$)

$$\langle B' | H_A + H_B | B \rangle = 0, \quad (6.37)$$

which was proved by several authors [99] and is called the “Pati–Woo” theorem. Two additional null statements are derivable [96],

$$\langle B' | H_C | B \rangle = \langle B' | H_D | B \rangle = 0. \quad (6.38)$$

To gain some insight into the basis for such relations, consider the proof of $\langle B' | H_D | B \rangle = 0$. Note that after performing a Fierz transformation and realizing that the spin labels on the field operators appearing in H_D are summed over and thus are dummy indices, we can write the quark creation and annihilation operators in H_D as

$$\frac{1}{2} b_i^\dagger(d, \lambda_d) b_j^\dagger(s, \lambda_s^{(1)}) (b_j(s, \lambda_s^{(2)}) b_i(s, \lambda_s^{(3)}) + b_i(s, \lambda_s^{(2)}) b_j(s, \lambda_s^{(3)})). \quad (6.39)$$

The initial baryon state is antisymmetric in color indices whereas the bracketed operator is symmetric, and so $\langle B' | H_D | B \rangle = 0$ follows immediately.

The results when cast in terms of the $\{O_i\}$ of eq. (6.20) imply

$$\langle B' | O_i | B \rangle = 0, \quad i = 2, 3, 4. \quad (6.40)$$

The result $\langle B' | O_4 | B \rangle = 0$ is particularly significant because, at least in this approximation, it implies the absence of $\Delta I = \frac{3}{2}$ effects. Thus, part of the $\Delta I = \frac{1}{2}$ rule is an automatic consequence of color symmetry.

Yet another useful exact relation exists between matrix elements of the penguin operators O_5, O_6 . In this case the SU(3) identity

$$\lambda_{ij}^\Lambda \lambda_{kl}^\Lambda = 2\delta_{ij}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl} \quad (6.41)$$

allows us to write

$$O_5 = -\frac{2}{3}O_6 + 2O'_5 \quad (6.42a)$$

where

$$O'_5 = \bar{d}_i \Gamma_L^\mu s_j \bar{Q}_j \Gamma_{R\mu} Q_i. \quad (6.42b)$$

But using an argument analogous to the one just presented, we conclude $\langle B' | O'_5 | B \rangle = -\langle B' | O_6 | B \rangle$ from which

$$\langle B' | O_5 | B \rangle = -\frac{8}{3}\langle B' | O_6 | B \rangle \quad (6.43)$$

directly follows.

The soft pion limits of the various A-amplitudes is exhibited in table 6.6 and the SU(3) parameterization of the $\langle B' | H_w^{\text{PC}} | B \rangle$ matrix elements is given in table 6.7. The preceding discussion (especially eqs. (6.40), (6.43)) implies that we need consider only the operators O_1 and O_5 in the soft-pion limit. The quark model then gives

$$\langle O_1 \rangle: f = -d = -\sqrt{6}(I + J) \quad (6.44)$$

and

$$\langle O_5 \rangle: f = (4\sqrt{6}/27)(3I + 7J), \quad d = (4\sqrt{6}/9)(3I - J) \quad (6.45)$$

where I, J are quark wave function overlap integrals [100]

$$I = \int d^3x (u^2 - l^2)^2, \quad J = 4 \int d^3x u^2 l^2. \quad (6.46)$$

In eq. (6.46) we have for simplicity employed degenerate SU(3) kinematics, and denoted the upper and lower Dirac wave function components as u, l (see eq. (3.22)).

As we have seen earlier (eq. (6.12)), the experimental S-wave amplitudes have an f -to- d ratio of -2.4 . This represents a problem for any theoretical model of the S-wave amplitudes based on baryon-to-baryon matrix elements of four-quark operators with a $(V - A) \times (V - A)$ structure. The $f/d = -1$ SU(3) structure of $\langle O_1 \rangle$ (eq. (6.44)) does not depend on one's choice of spatial wavefunctions. It is rather an exact property of the valence quark model which can be associated with the SU(4) algebra of quark flavor. We mention three possible mechanisms for resolving this ' f/d problem': (i) penguin operators [93] have a $(V - A) \times (V + A)$ chiral structure and thus a generally distinct f/d ratio (eq. (6.45)), (ii) there exist several types of contributions [101] which vanish in the soft pion limit yet contribute to the physical amplitude, and (iii) in principle the baryon wave function has a $Q\bar{Q}$ 'sea' component [102] which affects the f/d structure. The first of these options lies within the framework of the soft-pion valence model whereas the latter two go beyond it.

At this point let us continue our study of the matrix elements $\langle B' | H_w^{\text{PC}} | B \rangle$, each proportional to the quantities I, J of eqs. (6.46). To obtain a feel for the overall scale of such overlap integrals, it is useful to

Table 6.6
Soft pion limit of S-wave amplitudes. $\Delta I = 1/2$ effects are considered

Mode	Soft pion limit
A_0^0	$(i/2F_\pi)\langle N H_w^{\text{PC}} A \rangle$
A_0^0	$(-i/\sqrt{2}F_\pi)\langle N H_w^{\text{PC}} A \rangle$
Σ_0^+	$(i/2F_\pi)\langle P H_w^{\text{PC}} \Sigma^+ \rangle$
Σ_1^+	0
Σ_0^-	$(i/F_\pi)\langle N H_w^{\text{PC}} \Sigma^0 \rangle$
Ξ_0^0	$(i/2F_\pi)\langle \Lambda H_w^{\text{PC}} \Xi^0 \rangle$
Ξ_1^-	$(-i/\sqrt{2}F_\pi)\langle \Lambda H_w^{\text{PC}} \Xi^0 \rangle$

Table 6.7
SU(3) parameterization of $\langle B' | H_w^{\text{PC}} | B \rangle$

Mode	SU(3) parameterization
$i\langle N H_w^{\text{PC}} A \rangle$	$3F + D$
$i\langle P H_w^{\text{PC}} \Sigma^+ \rangle$	$\sqrt{6}(F - D)$
$i\langle N H_w^{\text{PC}} \Sigma^0 \rangle$	$-\sqrt{3}(F - D)$
$i\langle \Lambda H_w^{\text{PC}} \Xi^0 \rangle$	$-3F + D$
$i\langle \Sigma^0 H_w^{\text{PC}} \Xi^0 \rangle$	$\sqrt{3}(F + D)$
$i\langle \Sigma^- H_w^{\text{PC}} \Xi^- \rangle$	$-\sqrt{6}(F + D)$

recall that u, l are normalized as (see eq. (3.23))

$$\int d^3x (u^2 + l^2) = 1$$

so that any integral containing some combination of u, l to the quartic power scales as R^{-3} , where R is a measure of the hadron's extent in space (e.g., bag radius). Thus the magnitude of hyperon nonleptonic matrix elements (and those of other hadrons as well) is determined by the hadron size.

There is a subtle point associated with the $(V-A)(V+A)$ operators $O_{5,6}$ worth pointing out. A typical contribution to the matrix element $\langle B' | H_w^{p.c.} | B \rangle$ is depicted in fig. 6.3(a) where $H_w^{p.c.}$ induces a transition $s + u \rightarrow d + u$. This process occurs for both $(V-A)(V-A)$ and $(V-A)(V+A)$ operators. However for the latter there is an additional contribution depicted in fig. 6.3(b). It is proportional to $\langle 0 | \bar{q}q | 0 \rangle$ ($q = d, s$). Its presence was first pointed out in ref. [96] but in an alternative context. There, a normal ordered nonleptonic Hamiltonian was employed (we remind the reader that in this review the normal ordering is *not* performed), and the process of fig. 6.3(b) appeared as an 'anomalous' contribution to the commutation relation eq. (6.25). The nature of this contribution and related matters was discussed subsequently in refs. [103–105].

In considering numerical results, it will be helpful to keep in mind the experimental values, $f = -0.92 \times 10^{-7}$, $f/d = -2.4$ (eq. (6.12)). Working in the context of the soft pion limit of a valence quark model, the authors of ref. [96] used bag model wavefunctions to infer that

$$\begin{aligned} f &= (-0.24c_1 + 0.25c_5 + 0.01c_6) \times 10^{-7} \\ d &= (0.24c_1 + 0.05c_5 - 0.02c_6) \times 10^{-7}. \end{aligned} \quad (6.47)$$

The effect of the penguin operators is seen to be beneficial in that the magnitude of f is increased (note $c_5 < 0$) and the f/d ratio moves towards the experimental value. Unfortunately the size of c_5 needed to bring the model into accord with experiment is at least a factor of five larger than the value deduced from RG arguments.

An instructive numerical analysis of $\langle B' | H_w^{p.c.} | B \rangle$ matrix elements has been carried out in ref. [106]. Both bag and oscillator wave functions were used in order to test the model dependence of the results. In addition to these 'nonseparable' contributions, the effect of 'separable' factorization amplitudes was also studied. Such amplitudes represent perhaps the most obvious of mechanisms to incorporate in going beyond the 'soft-pion' approach discussed thus far. Results taken from ref. [106] appear in table 6.8, where specific S-wave amplitudes are exhibited. The reader can refer to this reference for input

Fig. 6.3. Contributions to $\langle B' | H_w^{p.c.} | B \rangle$ in the quark model.

Table 6.8
Quark model S-wave amplitudes. The amplitudes are expressed in units of 10^{-7}

	Bag model						Oscillator model					
	Nonseparable		Separable			Total	Nonseparable		Separable			Total
	O ₁₋₄	O _{5,6}	O ₁₋₃	O ₄	O _{5,6}		O ₁₋₄	O _{5,6}	O ₁₋₃	O ₄	O _{5,6}	
$A(A^0)$	1.9	-0.1	-0.5	-0.1	0.9	2.1	1.9	-0.1	-0.5	-0.1	0.9	2.1
$A(\Sigma^0)$	-3.3	0	0.4	-0.2	-0.7	-3.8	-3.3	0	0.4	-0.2	-0.7	-3.8
$A(\Sigma^-)$	4.7	0	-0.6	-0.2	1.0	4.9	4.6	0.1	-0.6	1.0	4.9	4.2
$A(\Xi^-)$	-3.8	0.1	0.6	-0.2	-1.0	-4.3	-4.5	0	0.6	-0.2	-0.1	-4.4

parameters employed in the analysis. The results do not depend in any significant manner on whether bag or oscillator wavefunctions are used. Separable contributions are seen to be less important than their nonseparable counterparts, but still have nonnegligible impact.

We next consider a distinct type of correction to the soft pion limit, viz., the $1/2^-$ baryonic pole contributions as developed in ref. [101]. In addition to exposing this potentially significant correction to the soft pion estimate, our discussion will also serve to motivate the $1/2^+$ baryonic pole model, which is the standard approach for the P-wave $B \rightarrow B' \pi$ amplitudes.

Consider the off-shell retarded commutator

$$T_a(q) = \int d^4x e^{iq \cdot x} \theta(x^0) \langle \beta | [\partial A_a(x), H_w(0)] | \alpha \rangle. \quad (6.48)$$

Inserting a complete set $\{|n\rangle\}$ of intermediate states, it is straightforward to show that

$$T_a(q) = - \int d^3x e^{iq \cdot x} \langle \beta | [A_a^0(x, 0), H_w(0)] | \alpha \rangle - q_\mu M_a^\mu \quad (6.49)$$

where

$$M_a^\mu = (2\pi)^3 \sum_n \left[\delta(p_n - p_\beta - q) \frac{\langle \beta | A_a^\mu(0) | n \rangle \langle n | H_w(0) | \alpha \rangle}{p_\alpha^0 - p_n^0} + \delta(p_\alpha - p_n - q) \frac{\langle \beta | H_w(0) | n \rangle \langle n | A_a^\mu(0) | \alpha \rangle}{p_\alpha^0 - q^0 - p_n^0} \right]. \quad (6.50)$$

In the soft pion limit the first part of eq. (6.49) becomes the familiar commutator term. The second part contains several kinds of contributions, which depend on the nature of the intermediate states.

The $1/2^+$ ($56, 0^+$) states are singular in the SU(3) invariant soft pion limit. They constitute, via matrix elements of $H_w^{p.c.}$, perhaps the main contributors to the P-wave amplitudes, and are considered in more detail in the following section. In principle, the baryon pole terms can contribute to the S-wave amplitudes via matrix elements of the form $\langle B' | H_w^{p.v.} | B \rangle$. However such contributions are suppressed in two respects. As a consequence of the Swift-Lee theorem [107] the matrix elements $\langle B' | H_w^{p.v.} | B \rangle$ are nonzero only through SU(3) breaking effects. A numerical study using bag model wave functions

teaches us that typically [108]

$$\langle B' | H_w^{p.v.} | B \rangle / \langle B' | H_w^{p.c.} | B \rangle \sim 0.1. \quad (6.51)$$

Moreover baryon mass factors enter the S-wave pole terms in the combination

$$(M' - M)/(M' + M) \sim 0.1, \quad (6.52)$$

where M, M' denote characteristic baryon masses. Hence we find the $1/2^+$ pole terms to be only several per cent of the current algebra contributions for the A-amplitudes.

Returning to our enumeration of intermediate states, in order of ascending mass we next encounter the $1/2^-(70, 1^-)$ baryons. These vanish in the soft-pion limit but contribute to the on-shell amplitude, predominantly in the form $q_0 M_0$. We can qualitatively characterize the size of such terms relative to that of the commutator as follows. Observe that M^0 contains an energy denominator and thus is inversely proportional to the baryonic level spacing, $\omega \approx 400$ MeV. The pion energy q^0 very nearly equals the SU(6) breaking quantity $\delta M = M_\alpha - M_\beta \approx 200$ MeV. Overall then, the $1/2^-(70, 1^-)$ contribution can be expected to be of order $\delta M/\omega \approx 0.5$ relative to the commutator term. Moreover, the Pati–Woo theorem applies to matrix elements $\langle n | H_w(0) | \alpha \rangle$ as long as the intermediate states are color-singlet configurations of three quarks. Thus the $\Delta I = \frac{1}{2}$ rule is maintained.

A numerical analysis of the ‘commutator-plus-(70, 1[−])’ model has been carried out in ref. [101]. Quark wavefunctions are taken from the SU(6) oscillator model. We list several S-wave amplitudes, with the first and second terms representing respectively the commutator and (70, 1[−]) contributions:

$$\begin{aligned} A(\Lambda_-^0) &= \frac{-1}{\sqrt{2}F_\pi} (3F + D) - 9\sqrt{6} \frac{M_\Lambda - M_N}{F_\pi} C \\ A(\Sigma_0^+) &= \frac{\sqrt{6}}{2F_\pi} (F - D) + 27\sqrt{2} \frac{M_\Sigma - M_N}{F_\pi} C \\ A(\Sigma_-^-) &= -\frac{\sqrt{3}}{F_\pi} (F - D) - 54 \frac{M_\Sigma - M_N}{F_\pi} C \\ A(\Xi_-^-) &= \frac{1}{\sqrt{2}F_\pi} (3F - D) + 12 \frac{M_\Xi - M_\Lambda}{F_\pi} C \\ A(\Sigma_+^+) &= 0, \end{aligned} \quad (6.53)$$

where

$$\begin{aligned} F &= \frac{\sqrt{3}}{2} G_F \sin \theta_c \cos \theta_c \langle \psi^0 | \delta(\mathbf{r}_1 - \mathbf{r}_2) | \psi^0 \rangle \\ D &= -F, \quad C = \frac{1}{6\sqrt{3}} \frac{F}{\omega m^2 R^2}. \end{aligned} \quad (6.54)$$

Numerically $\langle \psi^0 | \delta(\mathbf{r}_1 - \mathbf{r}_2) | \psi^0 \rangle = 1.15 \times 10^{-2} \text{ GeV}$, and in the oscillator model, $R^{-2} = m\omega$. Results are exhibited in table 6.9. Although the numerical aspects are impressive, it should be noted that (i) no QCD radiative corrections appear (their effects can be qualitatively mimicked by adjusting the wavefunction at the origin (see section 3)), (ii) SU(6) breaking effects are not included, (iii) penguin operators are absent, (iv) there is sensitivity to the choice of oscillator parameters. In view of these considerations, it would be unwise to take the results of the model overly seriously. However it is nonetheless true that, as pointed out in ref. [101], it is hard to justify ignoring the $(70, 1^-)$ contributions. Their inclusion could well improve the ‘commutator’ model of S-wave amplitudes.

Let us at this point interject a comment regarding the S-wave Σ_+^+ amplitude. In the models discussed thus far, its value is predicted to be identically zero. The experimental value, $A(\Sigma_+^+) = (0.13 \pm 0.02) \times 10^{-7}$, although tiny compared to the other A-amplitudes, is unquestionably nonzero. According to a recent calculation [109], it is possible to obtain the approximate magnitude and correct phase for $A(\Sigma_+^+)$ from the pole contribution of the $1/2^-$ baryon $\Lambda'(1405)$. Experimental masses and coupling constants are employed, so the SU(6) symmetry breaking effects enter. However, it is not clear that this result persists upon taking other contributions into account such as the remaining $(70, 1^-)$ resonances [109b].

Another, rather different approach for alleviating the f/d problem has been analyzed in ref. [102]. This involves the effect of quark–antiquark pairs on hyperon amplitudes. Such pairs occur automatically as a consequence of the QCD time evolution operator (see fig. 6.4a). In addition to the light u, d, s quarks, heavier quarks also contribute to the wavefunction, although with reduced probability. Processes involving ‘sea’ quark contributions to the nonleptonic amplitudes are depicted in figs. 6.4b and 6.4c. Using bag model wave functions it is found that by far the largest contribution arises from fig. 6.4b where a $u\bar{u}$ sea component induces an almost pure f-type coupling which is comparable in magnitude to the valence amplitude (for which $f = -d$) and of the same phase as the f-type amplitude. As with the $1/2^-(70, 1^-)$ pole terms, the ‘sea’ mechanism would appear to resolve the f/d S-wave problem. However here too, life is not quite so simple. The ‘sea’ component to the baryon wavefunction is proportional to α_s , the QCD fine structure constant. The bag model value for this quantity, although determined by the experimentally observed Δ -nucleon mass splitting, is believed by many to be unrealistically large. Thus the *actual* ‘sea’ component might be significantly smaller in magnitude than the value employed in ref. [102]. However the issue is still open, and further work on this matter might be warranted (e.g., the work by Eeg in ref. [102]).

Table 6.9
S-wave amplitudes in the oscillator ‘commutator-plus- $(70, 1^-)$ ’ model. Columns (a), (b), (c) exhibit respectively the commutator, commutator-plus- $(70, 1^-)$, and experimental amplitudes in units of 10^{-7}

	(a)	(b)	(c)
Λ^0	4.21	3.10	3.25 ± 2
Σ_0^+	-7.30	3.32	-3.27 ± 11
Σ^-	10.18	4.42	4.27 ± 2
Σ_+^+	0	0	0.13 ± 4
Ξ^-	-8.41	4.87	-4.51 ± 4

Fig. 6.4. Quark-antiquark sea and its effect on weak transitions.

6.3.2. P-wave transitions

As has already been mentioned, a soft-pion analysis of nonleptonic transition amplitudes implies the presence of baryon pole terms in the P-wave amplitudes. These are depicted in fig. 6.5. Also exhibited there is a kaon pole term. The motivation for including such a term will be given shortly. The analytic content of fig. 6.5 is given by the expression

$$iB_{\beta\alpha}^a = \frac{M_\alpha + M_\beta}{M_\beta + M_\delta} \cdot \frac{g_{a\beta\delta} S_{\delta\alpha}}{M_\alpha - M_\delta} + \frac{M_\alpha + M_\beta}{M_\alpha + M_\gamma} \cdot \frac{g_{a\gamma\alpha} S_{\beta\gamma}}{M_\beta - M_\gamma} + \frac{g_{b\beta\alpha} W_{ab}}{m_\pi^2 - m_K^2} \quad (6.55)$$

Fig. 6.5. Pole model for hyperon P-wave transitions.

where

$$\langle \pi^a(q) \beta | H_w^{p.v.} | \alpha \rangle = B_{\beta\alpha}^a \bar{u}_\beta \gamma_5 u_\alpha \quad (6.56a)$$

$$\langle \beta | H_w^{p.c.} | \gamma \rangle = S_{\beta\gamma} \bar{u}_\beta u_\gamma, \quad (6.56b)$$

and M_α is the mass of baryon α , $m_{\pi, K}$ are the pion, kaon masses, and $g_{a\beta\gamma}$ is the pseudoscalar coupling constant between kaon a and baryons β, γ . To obtain eq. (6.55) we began with pseudovector coupling in order to avoid contact terms implied by current algebra [110] and then used the equivalence between pseudovector and pseudoscalar couplings to express the pole amplitudes in terms of the latter. It is common to implement a further reduction via the generalized Goldberger–Treiman relation

$$\frac{g_A(a\beta\gamma)}{\sqrt{2} F_\pi} = \frac{g_{a\beta\gamma}}{M_\beta + M_\gamma}, \quad (6.57)$$

where $g_A(a\beta\delta)$ is the axial vector coupling constant

$$\langle \beta | A_\mu^a | \delta \rangle = g_A(a\beta\delta) \bar{u}_\beta \gamma_\mu \gamma^5 u_\delta. \quad (6.58)$$

Thus we obtain

$$iB_{\beta\alpha}^a = \frac{M_\alpha + M_\beta}{\sqrt{2} F_\pi} \left[g_A(a\beta\delta) \frac{S_{\delta\alpha}}{M_\alpha - M_\delta} + g_A(a\gamma\alpha) \frac{S_{\beta\gamma}}{M_\beta - M_\gamma} + g_A(b\beta\alpha) \frac{W_{ab}}{m_\pi^2 - m_K^2} \right]. \quad (6.59)$$

Before proceeding with an analysis of eq. (6.59), it is worthwhile to pause and reflect on certain of its features. First, consider the role played by SU(3) symmetry. Unlike most relations which have an obvious SU(3) limit, the vanishing mass denominators in eq. (6.59) make such a limit difficult to interpret. On the other hand, it is common practice to employ an SU(3) F/D parameterization for the axial–vector couplings. These are measured at nearby momentum transfers in semileptonic hyperon decays and the SU(3) fit

$$F_A = 0.43, \quad D_A = 0.82 \quad (6.60)$$

is known to work well.

The relative size of S-wave and P-wave amplitudes is also a matter of some interest. Recall that experimental B-amplitudes are typically larger than the A-amplitudes. The presence in eq. (6.59) of mass factors which appear as sums in the numerator and differences in the denominator would appear to explain this fact. However it has long been known [1, 111] that in the baryon pole model insertion of A-amplitudes deduced from experimental amplitudes via the soft-pion procedure, $S_{\delta\alpha} \approx -iA_{\delta\alpha}/F_\pi$, implies B-amplitudes which are too small (by roughly a factor of two). By itself this need not remove the baryon pole model as a viable approach, because the extraction of $\langle \delta | H_w^{p.c.} | \alpha \rangle$ from data could be affected by the inclusion of contributions which vanish in the soft-pion limit. Recall this is the stance adopted in ref. [101] where it is argued that the ‘commutator-plus-(70, 1[−])’ model can alleviate the S-wave/P-wave problem within the context of the baryon pole model.

At any rate the reader might look askance at any model such as eq. (6.59) in which several interfering terms appear. How can one keep the phases straight? Actually this difficulty can be partially resolved by referring to the FKW theorem discussed in section 6.2.3. This not only addresses the question of relative phases, but also makes mandatory the presence of kaon pole contributions to the P-wave amplitudes. The argument proceeds as follows, where to aid the reader, we treat the specific case of $\Lambda \rightarrow N\pi^0$. For this mode there is precisely one term corresponding to each of the three processes depicted in fig. 6.6,

$$iB_{\Lambda N} = \frac{M_\Lambda + M_N}{\sqrt{2}F_\pi} \left[g_A(\pi^0 NN) \frac{S_{N\Lambda}}{M_\Lambda - M_N} + g_A(\pi^0 \Sigma^0 \Lambda) \frac{S_{N\Sigma^0}}{M_N - M_{\Sigma^0}} + g_A(\bar{K}^0 N \Lambda) \frac{W_{\pi^0 \bar{K}^0}}{m_\pi^2 - m_K^2} \right]. \quad (6.61)$$

In order to be able to invoke the FKW theorem, we must replace $H_w^{\text{p.c.}}$ on the right-hand side of eq. (6.61) with an operator which can be ‘rotated away’. It is convenient to use the Hermitian quark bilinear $H_6 \equiv \bar{d}s + \bar{s}d$, which is essentially an off-diagonal mass term. The FKW theorem then asserts that the combination

$$g_A(\pi^0 NN) \frac{\langle N | H_6 | \Lambda \rangle}{M_\Lambda - M_N} + g_A(\pi^0 \Sigma^0 \Lambda) \frac{\langle N | H_6 | \Sigma^0 \rangle}{M_N - M_{\Sigma^0}} + g_A(\bar{K}^0 N \Lambda) \frac{\langle \pi^0 | H_6 | \bar{K}^0 \rangle}{m_\pi^2 - m_K^2} \quad (6.62)$$

must vanish. Although the theorem holds to all orders of SU(3) breaking, we shall consider just the lowest possible order. Thus we employ SU(3) symmetric values for the axial couplings, use

$$\langle B_i | H_j | B_k \rangle = -if_{ijk} M_F + d_{ijk} M_D \quad (6.63)$$

for baryon octet mass terms, and

$$\langle M_i | H_j | M_k \rangle = -if_{ijk} m_F^2 \quad (6.64)$$

Fig. 6.6. Pole terms in Ω^- decays to $1/2^+ 0^-$ modes.

for mesons. Inserting eqs. (6.62)–(6.64) into the right-hand side of eq. (6.61) yields

$$\frac{-(F_A + D_A)}{\sqrt{2}} \cdot \left(\sqrt{\frac{1}{2}} \right) + \frac{2D_A}{\sqrt{6}} \cdot \left(\sqrt{\frac{1}{6}} \right) + \left(\frac{-D_A}{\sqrt{6}} - \sqrt{\frac{3}{2}} F_A \right) \cdot \left(-\sqrt{\frac{1}{6}} \right) \quad (6.65)$$

which is seen to vanish via direct cancellation. Thus not only is our phase convention verified, the need for kaon poles is also made explicit.

There is one qualification to this conclusion which should be kept in mind. Strictly speaking, the matter of relative phases is settled by the FKW theorem only if a reasonably firm theoretical understanding with respect to the quark model is available. This is because we are comparing amplitude relations between two-quark and then four-quark operators. Unless we have some assurance that the behavior observed in the two-quark sector is maintained in the four-quark sector, we cannot rely upon the FKW theorem. Our feeling is that at this time quark model estimates of $\langle \pi_a | H_w | K_b \rangle$ involve substantial cancellations and so we cannot be sure of the major contributor to this amplitude. Consequently our kaon-pole terms will be given a phase so as to provide the best numerical fit to the data.

Formulae for the ‘baryon-pole plus kaon-pole’ model of P-wave amplitudes appear in table 6.10. If, as in table 6.10, we employ octet estimates (F_A , D_A) for the baryon axial vector couplings and also for the parity-conserving weak Hamiltonian matrix elements (f , d) it then follows that the $\Delta I = \frac{1}{2}$ rule is valid in this model provided $W_{\pi^- K^-} = -\sqrt{2} W_{\pi^0 \bar{K}^0}$.

Thus far we have considered two contributions which ‘must’ be present, the baryon poles (by virtue of the current algebra analysis and the kaon poles (a consequence of the FKW theorem). Are there any others which might be expected to make substantial contributions? Much attention has been given to factorization amplitudes associated with the penguin Hamiltonian [93]. Here, two of the operators in the penguin four-quark operator connect the vacuum to the single pion state, $\langle \pi^0 | \bar{q} \gamma_5 q | 0 \rangle$, while the remaining two operators connect the baryon states, $\langle B' | \bar{d} \gamma_5 s | B \rangle$. One way of estimating this latter matrix element is to assume dominance of the kaon pole, thus implying for the full matrix element the structure (we omit color matrices and indices here)

$$\langle B' | \pi^0(q) | O_{\text{peng}} | B \rangle \sim \langle \pi^0(q) | \bar{q} \gamma_5 q | 0 \rangle \langle 0 | \bar{d} \gamma_5 s | \bar{K}^0 \rangle (m_K^2 - q^2)^{-1} g_{\bar{K}^0 B' B}. \quad (6.66)$$

Table 6.10

Baryon and kaon pole contributions to P-wave amplitudes. All contributions are in units of 10^{-7} . For fit a, we choose $F/D = -2.4$, $F = -0.92$, $F_A = 0.43$, $D_A = 0.82$ whereas for fit b, $F/D = -1.8$, $F = -1.44$, $F_A = 0.40$, $D_A = 0.85$

	Fit a			Fit b			Expt.
	Baryon	Kaon	Total	Baryon	Kaon	Total	
$B(\Lambda_0^0)$	-17.3	-1.0	-18.3	-10.4	-1.1	-11.4	-15.8
$B(\Lambda_1^0)$	24.5	1.6	26.1	14.7	1.6	16.3	22.1
$B(\Sigma_0^+)$	10.6	0.5	11.1	19.8	0.5	20.3	26.6
$B(\Sigma_+^+)$	4.3	0	4.3	28.4	0	28.4	42.4
$B(\Sigma_-^-)$	-10.9	-0.7	-11.6	-0.1	-0.7	-0.8	-1.44
$B(\Xi_0^0)$	2.5	0.3	2.8	-17.3	0.3	-17.0	-12.3
$B(\Xi_-^-)$	-3.6	-0.4	-4.0	24.4	-0.5	23.9	16.6

However this is but a single contribution, the one with vacuum intermediate states, to the kaon pole amplitude discussed earlier (again, details of color are suppressed)

$$\langle B' \pi^0(q) | B \rangle_{K\text{-pole}} \sim \langle \pi^0 | \bar{q}(1 - \gamma_5) q \bar{d}(1 + \gamma_5) s | \bar{K}^0 \rangle (m_K^2 - q^2)^{-1} g_{\bar{K}^0 B' B}. \quad (6.67)$$

Thus we feel that to include both the kaon pole terms *and* the factorization amplitudes would constitute double counting.

There is yet a different type of contribution to the P-wave amplitudes which has not been considered in the literature. It is a ‘direct’ coupling of the final state P-wave $B' \pi$ system to the initial hyperon B . This contribution appears in a chiral soliton representation of baryons. Because its relation to quarks is unclear, we defer discussion of this point to section 11 of this review.

We are now ready to examine numerical aspects of the P-wave model. It is most profitable to keep the analysis at a phenomenological level. In this way, it is the overall method (the pole model) which is tested and not details of quark wave functions.

Perhaps the most phenomenological approach one can adopt for the ‘baryon-plus-kaon’ pole model is to either fit or extract directly from experiment the various matrix elements of $H_w^{p.c.}$.

For example we can employ a chiral Lagrangian to provide estimates of $\langle \pi^0 | H_w^{p.c.} | K^0 \rangle$ and $\langle \pi^- | H_w^{p.c.} | K^- \rangle$. Both octet and 27-plet interactions occur,

$$L_8 = g_8 \text{Tr} \lambda_6 \partial_\mu M \partial^\mu M^\dagger$$

$$L_{27} = g_{27} C \begin{pmatrix} 8 & 8 & 27 \\ i & j & 6 \end{pmatrix} \text{Tr} \lambda_i X_\mu \lambda_j X^{\mu\dagger}$$

as in eqs. (6.26), (6.28a). The coefficients g_8 , g_{27} are extracted from $K_s \pi^+ \pi^-$ and $K^+ \pi^+ \pi^0$ decays,

$$g_8 = 0.358 \times 10^{-7} m_\pi^2, \quad g_{27} = -0.017 \times 10^{-7} m_\pi^2. \quad (6.68)$$

The K - π amplitudes are thus determined,

$$\langle \pi^-(q) | H_w^{p.c.} | K^-(k) \rangle = 2(g_8 + 2g_{27}) k \cdot q / F_\pi^2$$

$$\langle \pi^0(q) | H_w^{p.c.} | \bar{K}^0(k) \rangle = \sqrt{2} (-g_8 + 4g_{27}) k \cdot q / F_\pi^2. \quad (6.69)$$

It is instructive to consider different estimates for the baryon matrix elements, $\langle B' | H_w^{p.c.} | B \rangle$. On the one hand we can use S-wave amplitudes in the naive soft-pion model to infer $f/d = -2.4$, $f = -0.92 \times 10^{-7}$ (with F_A , D_A as in eq. (6.60)). Alternatively a fit [35] to the P-wave amplitudes has yielded $f/d = -1.8$, $f = -1.44 \times 10^{-7}$ (with slightly different axial vector parameters $F_A = 0.40$, $D_A = 0.85$).

Predictions of this phenomenological approach to the collection of P-wave amplitudes is displayed in table 6.10 for both sets of input parameters. The results are rather revealing in several respects. The baryon pole terms are seen to be the dominant contributors, typically an order of magnitude in excess of the kaon pole terms. The main reason for this can be understood as a consequence of chiral symmetry, which produces the momentum dependent factor $k \cdot q$ in the $K^- \pi^-$, $K^0 \pi^0$ amplitudes of eq. (6.69). Since both pion and kaon are on the pion mass shell, we have $k \cdot q = m_\pi^2$ which leads to an m_π^2/m_K^2 suppression of the kaon-pole contribution. Keep in mind that the phase of the K - π amplitude has been chosen to yield the most favorable numerical fit.

Even more significant is the sensitivity of the baryon pole contributions to the form chosen for $\langle B' | H_{\omega}^{p.c.} | B \rangle$. The value $f/d = -2.4$ implied by the naive soft-pion analysis of S-wave amplitudes does not yield an acceptable fit, as seen in table 6.10. The problem is particularly acute for the Σ^+ , Σ^- and Ξ decays. On the other hand, the fit to these same amplitudes improves dramatically for $f/d = -1.8$ (and a larger value for f), and indeed the overall fit is not unreasonable.

Perhaps the current state of affairs as regards the S-wave and P-wave hyperon decays can be summarized as follows. We have seen how the ‘naive current algebra’ estimate for the S-wave amplitudes and the ‘baryon-plus-kaon’ pole model for the P-waves cannot simultaneously reproduce the set of experimental amplitudes. Some modification of the simplest descriptions for the S-waves or for the P-waves (or for both) is required. Candidate modifications exist for both S-wave and P-wave sectors. Specific examples are respectively the $1/2^-$ baryon pole terms and a direct (or ‘contact’) interaction. At this time a definitive calculation has not yet been performed. However there is hope for expecting clarification of this somewhat muddled situation. In particular we see no reason to doubt that the quark model will, in the final analysis, be able to provide a successful description of hyperon decay.

6.3.3. Ω^- decays

The complex of Ω^- nonleptonic decays consists of $\Omega^- \rightarrow \Xi^* \pi$, $\Lambda \bar{K}$ and $\Xi \pi$ (there is not sufficient phase space for the ΣK final state). It is in principle an interesting set of reactions in that due to the large Ω^- mass, kaons or spin 3/2 excited baryons can appear in the final state as well as the usual pions and spin 1/2 baryons. Also, since the Ω^- spin is 3/2, more final state partial wave configurations are allowed, $l = 1, 2$ for $1/2^+ 0^-$ final states and $l = 0, 1, 2, 3$ for $3/2^+ 0^-$ final states.

However, as explained in section 6.1.4, for $\Omega^- \rightarrow 1/2^+ 0^-$ transitions the predominant partial-wave is P-wave, whereas for $3/2^+ 0^-$ emission, the data would appear to be sensitive only to the S-wave. The latter situation is somewhat of a kinematical curiosity because the $\Omega \rightarrow \Xi^*(1530) \pi$ phase space is so restricted that the $\Xi^*(1530)$ width must be taken into account when extracting amplitudes from the data.

The emergence of Ω^- nonleptonic decay data has stimulated a number of theoretical papers on the subject [112–115]. The underlying approach mirrors the treatment of hyperon decay rather closely – the P-wave dominated $\Omega \rightarrow 1/2^+ 0^-$ amplitudes are described with a pole model while the S-wave dominated amplitudes are approximated by the ‘soft-pion’ value.

(i) $\Omega \rightarrow 1/2^+ 0^-$

Before considering numerics, we make some general observations. It has been shown [108] that suppression of the matrix elements $\langle B' | H_{\omega}^{p.v.} | B \rangle$ continues to hold even for spin 3/2 baryons. Hence the baryon pole terms in $\Omega^- \rightarrow 1/2^+ 0^-$ are typically constructed to include $\langle B' | H_{\omega}^{p.c.} | B \rangle$ factors but not their $\langle B' | H_{\omega}^{p.v.} | B \rangle$ counterparts. The FKW theorem implies the existence of kaon pole terms in the $\Omega \rightarrow \Xi \pi$ amplitudes. The proof is entirely analogous to that given for hyperon decays (the only really new ingredient is the presence of vector-current matrix elements of spin 3/2 baryons). Figure 6.6 displays the $\Omega \rightarrow \Lambda K$, $\Xi \pi$ pole terms.

A thorough and relatively up-to-date theoretical analysis of the Ω^- system appears in ref. [115], which we summarize here. It is assumed that the D-wave decay amplitude is negligible (i.e., $C = 0$ in eq. (6.14)). The P-wave amplitude B is taken to be a sum of baryon pole and separable (i.e., factorization) contributions. The latter is used in place of kaon pole terms. For the baryon pole terms, the strong interaction coupling constants are estimated by taking $g(\Delta^{++} P \pi^+)$ from experiment and then employing SU(3) symmetry relations. The matrix elements of $H_{\omega}^{p.c.}$ are computed from bag model wavefunctions

with parameters $m_u = m_d = 0$, $m_s = 0.279$ GeV, $R(1/2^+) = 4.96$ GeV $^{-1}$ and $R(3/2^+) = 5.39$ GeV $^{-1}$. For the ‘separable’ amplitudes, axial–vector couplings are taken from the bag model and ‘current algebra’ quark mass values $m_u + m_d = 11.7$ MeV, $m_s = 150$ MeV are employed. The latter appear explicitly from using quark equations of motion. Conventional values are employed for the RG coefficients c_1 , c_2 and c_3 ($c_1 = -2.8$, $c_2 = 0.06$, $c_3 = 0.08$) whereas c_4 , c_5 and c_6 are taken as free parameters chosen to provide a best fit. From the $\Xi^0 \pi^-$, $\Xi^- \pi^0$ modes, the values $c_4 = 0.26$, $c_5 + \frac{3}{16}c_6 = -0.172$ are obtained. (Although the Pati–Woo theorem suppresses the contribution of the $\Delta I = \frac{3}{2}$ operator O_4 to the baryonic pole terms this is not the case for the separable terms, so these amplitudes do in fact contain c_4 .) Results of the model are given in table 6.11. The separable terms are seen to dominate the $\Xi\pi$ modes whereas the baryonic pole is the only contributor to the large ΛK^- mode. The fit is impressively in accord with the data.

Another recent calculation of the P-wave Ω^- decay amplitudes appears in ref. [113]. Quark overlap integrals are calculated in the harmonic oscillator model and separable contributions are replaced by kaon pole terms. These pole terms are interpreted in the sense of dispersion theory. This means the K-to- π transitions $W_{K\pi}$ are taken on the kaon mass shell, so that $k_K \cdot q_\pi = m_K^2$. We refer the reader to ref. [113] for numerical results, which are computed both with and without penguin terms. Agreement with experiment is generally satisfactory.

In a phenomenological sense the work of refs. [113] and [115] would seem to indicate that the subject of $\Omega \rightarrow \text{BP}$ P-wave decays is a closed one. However, it must be admitted that our theoretical grasp is not quite so secure. The problem lies with the $\Omega \rightarrow \Xi\pi$ modes. Depending on one’s approach these are dominated by either separable or kaon-pole contributions. In the former case, there is sensitivity to current-algebra quark mass values (see our discussion of this issue in section 6.4.1) and also to the value taken for the RG coefficient c_4 , which is about half that of its theoretically estimated value. In the latter case of kaon-poles, employing a “dispersion-theoretic” interpretation would appear to be at variance with the constraints of chiral symmetry. Indeed, we have pointed out in section 6.3.2 that a chiral Lagrangian description of the of the K-to- π transition implies a value of the kaon-pole terms smaller than that employed in ref. [113] by a factor of $(m_\pi/m_K)^2$. At the very least then, our understanding of the $\Omega \rightarrow \Xi\pi$ modes is on a far less firm footing than of the $\Omega \rightarrow \Lambda K^-$ transition. Additional work on this matter would be welcome.

(ii) $\Omega \rightarrow 3/2^+ 0^-$

The data base here is rather limited, consisting of four $\Xi\pi\pi$ events. These are interpreted in terms of the $\Xi^*(1530)\pi$ mode. Clearly, a detailed phenomenological analysis will not be particularly meaningful

Table 6.11
 Ω^- decays. Amplitude predictions are taken from ref. [115], but appear here in dimensionless form in units of 10^{-7} as defined in section 6.1.4

Modes	B_{sep}	B_{pole}	$ B_{\text{expt}} $	Amplitudes		
				A_{sep}	A_{pole}	$ A_{\text{expt}} $
$\Xi^0 \pi^-$	2.0	-0.2	1.9			
$\Xi^- \pi^0$	1.3	-0.2	1.1			
ΛK^-		-5.8	5.6			
$\Xi^{*0} \pi^-$	41.0	-3.4		-1.0	0.2	0.5

until more data is forthcoming. It is still of interest however to study the theoretical structure of the $\Xi^* \pi$ modes. This was done in several papers [112, 115] and we summarize the findings of ref. [115] in the following.

It is assumed that only S-wave and P-wave transitions occur, so only the A , B amplitudes of eq. (6.16) are considered. Three distinct types of contributions are employed for the S-wave, viz., separable, commutator (via PCAC), and pole (from the $3/2^-$ state $\Xi^*(1530)$ pole term) and also a separable contribution. The pole contribution to the A amplitude is found to be relatively insignificant, $A_{\text{pole}} \simeq (0.1-0.3)A_{\text{comm}}$. Results of the analysis for the $\Xi^{0*} \pi^-$ mode are shown in table 6.11. The B amplitude is dominated by the separable contribution whereas for the A amplitude, the commutator contribution is largest. In magnitude, it is seen that $|B/A| \gg 1$. Yet as pointed out in eq. (6.18), phase space limitations restrict measurement of the $\Xi^* \pi$ lifetime to information about A only. Additional $\Xi^* \pi$ data of sufficient quantity to allow symmetry measurements (and hence inference about the B amplitude) would be welcome. At present, all we can conclude is that theory predicts an S-wave $\Xi^{0*} \pi^-$ amplitude rather larger than its measured value. In view of the paucity of events, it would be premature to regard this as a defect of the theory.

6.3.4. $\Delta I = 3/2$ transitions

We shall not present a detailed analysis of the baryonic $\Delta I = \frac{3}{2}$ amplitudes. They are small and not well understood. Several mechanisms for generating them are possible but none stands out as particularly significant. To gain some feeling for these effects, we briefly review the venerable factorization approach. We obtain

$$\begin{aligned} \langle \pi_a(q) \beta | H_w^{\Delta I=3/2} | \alpha \rangle &= \sqrt{2} G_F \sin \theta_c \cos \theta_c c_4 \langle \pi_a(q) \beta | O_4 | \alpha \rangle \\ &\simeq \frac{4\sqrt{2}}{3} G_F \sin \theta_c \cos \theta_c c_4 \langle \pi_a(q) | "A^\mu" | 0 \rangle \langle \beta | "V_\mu" | \alpha \rangle \\ &\simeq \frac{8}{3} G_F F_\pi \sin \theta_c \cos \theta_c c_4 \bar{u}_\beta ((M_B - M_\alpha) g_V^{\alpha\beta a} + (M_B + M_\alpha) g_A^{\alpha\beta a} \gamma_5) u_\alpha \end{aligned} \quad (6.70)$$

where “ A^μ ”, “ V_μ ” are the weak currents relevant for the $\Delta I = \frac{3}{2}$ amplitudes. We adopt the numerical approach of ref. [116] which compares the relative phase and magnitude for various $\Delta I = \frac{3}{2}$ transitions. Results are exhibited in table 6.12. Agreement between the model and experiment is unsatisfactory in both the relative phase and magnitude of the S-wave and P-wave amplitudes. The factorization amplitudes cannot provide a comprehensive explanation of the $\Delta I = \frac{3}{2}$ amplitudes. A more insightful analysis of this area is required.

Table 6.12
 $\Delta I = 3/2$ factorization amplitudes. Here, A and B represent only the $\Delta I = 3/2$ part of the S-wave and P-wave amplitudes respectively

Channel	Sign (AB)		$ B/A $	
	Factoriz.	Expt.	Factoriz.	Expt.
$\Lambda \rightarrow N \pi$	−	+	9	6 ± 8
$\Sigma \rightarrow N \pi$	+	+	2	12 ± 9
$\Xi \rightarrow \Lambda \pi$	−	−	3	1 ± 3

6.4. Kaon decays

It is customary in considering low-energy nonleptonic interactions to work within the philosophy of PCAC and current algebra. This picture has been well tested by the relations between $K \rightarrow 3\pi$ and $K \rightarrow 2\pi$ which follow from current algebra. Those predictions in the $\Delta I = \frac{1}{2}$ sector are compactly summarized in the effective chiral Lagrangian (see eq. (6.26))

$$L^{\Delta I = 1/2} = g_8 \text{Tr}(\lambda_6 \partial_\mu M \partial^\mu M^\dagger)$$

with $M = \exp(i\lambda^a \phi^a / F_\pi)$ as explained in section 2.3. In particular this interaction predicts

$$\begin{aligned} \langle 0 | H_w^{\Delta I = 1/2} | K^0(k^2 = 0) \rangle &= 0 \\ \langle \pi^-(k) | H_w^{\Delta I = 1/2} | K^-(k) \rangle &= -2g_8 k^2 / F_\pi^2 \\ \langle \pi^+(q_+) \pi^-(q_-) | H_w^{\Delta I = 1/2} | K_s(k) \rangle &= \frac{ig_8}{F_\pi^2} (2k^2 - q_+^2 - q_-^2) \\ \langle \pi^+(q_1) \pi^-(q_-) \pi^+(q_2) | H_w^{\Delta I = 1/2} | K^+(k) \rangle &= \frac{2g_8 k^2}{3 F_\pi^4} \left(1 + \frac{3}{2k^2} (s_3 - s_0) \right). \end{aligned} \quad (6.71)$$

These provide a good fit (25% accuracy) to the magnitudes and slopes of the $\Delta I = \frac{1}{2}$ processes with the choice $g_8 = 3.58 \times 10^{-8} m_\pi^2$. Similarly good relations can be obtained for the $\Delta I = \frac{3}{2}$ components.

The data on K decays is now of quite high precision and does reveal the presence of quadratic momentum dependence in the $K \rightarrow 3\pi$ Dalitz plot (see section 6.1.2). Such variation is expected within the context of chiral symmetry, as the lowest order chiral predictions are only expected to be valid to order m_K^2/Λ_χ^2 , where Λ_χ is a scale factor expected to be of order 1 GeV. There are many higher order chiral Lagrangians which contain four derivatives. Most of these lead to the same relationship between $K \rightarrow 3\pi$ and $K \rightarrow 2\pi$ as does eq. (6.26), but two which can modify the original predictions are

$$L' = g_8 \left[\frac{1}{\Lambda_1^2} \text{Tr}(\lambda_6 [\partial_\mu M \partial^\mu M^\dagger \partial_\nu M \partial^\nu M^\dagger]) + \frac{1}{\Lambda_2^2} \text{Tr}(\lambda_6 [\partial_\mu M \partial_\nu M^\dagger \partial^\mu M \partial^\nu M^\dagger]) \right]. \quad (6.72)$$

Note that these quartic interactions contribute to $K \rightarrow 3\pi$ but not at all to $K \rightarrow 2\pi$. While the form of the higher derivative Lagrangian is not unique, the one quoted produces an excellent fit (to 10%) to the kaon data (including quadratic momentum behavior) when $\Lambda_1 = 0.51$ GeV and $\Lambda_2 = 0.76$ GeV, as displayed in table 6.2.

The use of chiral symmetry also has a very important practical significance in quark model calculations. It allows one to infer kaon nonleptonic decay amplitudes from the study of only the unphysical $K \rightarrow \pi$ transition. This matrix element is more amenable to quark model methods than are those of $K \rightarrow 2\pi$ or $K \rightarrow 3\pi$. The physical amplitudes can be obtained from those of $K \rightarrow \pi$ by use of current algebra, as in eq. (6.71).

We now discuss the estimates made of kaon matrix elements.

6.4.1. Vacuum saturation

The oldest [117] and most widely known (although not necessarily the most reliable) method of matrix element estimation follows from inserting the vacuum intermediate state between quark bilinears in all possible ways. For example one has

$$\begin{aligned} & \langle \pi^0(k) | \bar{d}_i \gamma_\mu (1 + \gamma_5) d_i \bar{d}_j \gamma^\mu (1 + \gamma_5) s_j | K^0(k) \rangle \\ &= \langle \pi^0(k) | \bar{d}_i \gamma_\mu (1 + \gamma_5) d_i | 0 \rangle \langle 0 | \bar{d}_j \gamma^\mu (1 + \gamma_5) s_j | K^0(k) \rangle + \langle \pi^0(k) | \bar{d}_j \gamma_\mu (1 + \gamma_5) d_i | 0 \rangle \langle 0 | \bar{d}_i \gamma^\mu (1 + \gamma_5) s_j | K^0(k) \rangle \\ &= \frac{2\sqrt{2}}{3} F_\pi F_K k^2. \end{aligned} \quad (6.73)$$

Use has been made of the Fierz rearrangement property as in eq. (2.5),

$$\bar{\psi}_1 \gamma_\mu (1 + \gamma_5) \psi_2 \bar{\psi}_3 \gamma^\mu (1 + \gamma_5) \psi_4 = \bar{\psi}_3 \gamma_\mu (1 + \gamma_5) \psi_2 \bar{\psi}_1 \gamma^\mu (1 + \gamma_5) \psi_4. \quad (6.74)$$

One of the advantages of this approach is that it always explicitly obeys the predictions of current algebra. The disadvantage is that other intermediate states seem to give results which are just as large [118], calling into question the accuracy of the method.

If one deals only with vector and axial vector currents as the quark bilinears, all the relevant matrix elements can be obtained either directly from experiment or can be related to experimental quantities by use of SU(3), PCAC or crossing. For example in the SU(3) limit

$$\begin{aligned} \langle \pi^+(q) | \bar{u} \gamma_\mu s | \bar{K}^0(k) \rangle &= (k + q)_\mu, & \langle \pi^-(q) | \bar{d} \gamma_\mu \gamma_5 u | 0 \rangle &= i\sqrt{2} F_\pi q_\mu \\ \langle \pi^+(q_+) \pi^0(q_0) | \bar{u} \gamma_\mu d | 0 \rangle &= \frac{1}{\sqrt{2}} (q_+ - q_0)_\mu, & \langle 0 | \bar{d} \gamma_\mu \gamma_5 s | \bar{K}^0(k) \rangle &= i\sqrt{2} F_\pi k_\mu. \end{aligned} \quad (6.75)$$

These forms are sufficient to apply vacuum saturation techniques to $O_{1,2,3,4}$.

The use of vacuum saturation for the penguin operators, $O_{5,6}$ raises extra problems because matrix elements of scalar and pseudoscalar densities are required due to the Fierz rearrangement property

$$\bar{\psi}_1 \gamma_\mu (1 + \gamma_5) \psi_2 \bar{\psi}_3 \gamma^\mu (1 + \gamma_5) \psi_4 = -2 \bar{\psi}_1 \gamma_\mu (1 + \gamma_5) \psi_4 \bar{\psi}_3 \gamma^\mu (1 + \gamma_5) \psi_2. \quad (6.76)$$

We do not have as direct a measure of the constants involved in matrix elements of scalar and pseudoscalar densities. Here if we assume that there is no momentum dependence in the amplitudes we obtain

$$\begin{aligned} \langle \pi^+ | \bar{u} s | \bar{K}^0 \rangle &= A, & \langle \pi^- | \bar{d} \gamma_5 u | 0 \rangle &= i\sqrt{2} F_\pi A \\ \langle \pi^+ \pi^0 | \bar{u} d | 0 \rangle &= -A/\sqrt{2}, & \langle 0 | \bar{d} \gamma_5 s | \bar{K}^0 \rangle &= -i\sqrt{2} F_\pi A \\ \langle 0 | \bar{d} d | 0 \rangle &= \langle 0 | \bar{s} s | 0 \rangle = -2F_\pi^2 A. \end{aligned} \quad (6.77)$$

All vertices can be written in terms of a single amplitude A by using the soft pion theorems of PCAC

and current algebra. If one treats the chiral symmetry breaking due to quark masses

$$H_{\text{mass}} = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s \quad (6.78)$$

to first order in the masses, then A gives the relationship between quark masses and meson masses

$$m_\pi^2 = \langle \pi^+ | H_{\text{mass}} | \pi^+ \rangle, \quad m_K^2 = \langle K^+ | H_{\text{mass}} | K^+ \rangle \quad (6.79)$$

and therefore

$$A = (m_K^2 - m_\pi^2)/(m_s - m_d) = m_\pi^2/(m_u + m_d) = m_K^2/(m_s + m_u) \quad (6.80)$$

with the quark masses clearly being the current algebra masses. In what follows we will see that one is forced to consider the momentum dependence of these amplitudes. For later convenience, a particular parameterization, although not the most general, is

$$\begin{aligned} \langle \pi^+(q) | \bar{u}s | \bar{K}^0(k) \rangle &= A(1 \pm (k-q)^2/\Lambda^2), & \langle \pi^-(q) | \bar{d}\gamma_5 u | 0 \rangle &= i\sqrt{2} F_\pi A(1 \pm q^2/\Lambda^2) \\ \langle \pi^+(q_+) | \pi^0(q_0) | \bar{u}d | 0 \rangle &= -\frac{A}{\sqrt{2}} \left(1 \pm \frac{(q_+ + q_0)^2}{\Lambda^2} \right), & \langle 0 | \bar{d}\gamma_5 s | \bar{K}^0(k) \rangle &= -i\sqrt{2} F_\pi A \left(1 \pm \frac{k^2}{\Lambda^2} \right) \\ \langle 0 | \bar{d}d | 0 \rangle &= \langle 0 | \bar{s}s | 0 \rangle = -2F_\pi^2 A. \end{aligned} \quad (6.81)$$

To determine the amplitude A one must evaluate the quark masses appropriate for the scale at which one chooses the renormalization point. Here we will repeat Weinberg's method [119], including observations made in ref. [96]. Weinberg ascribes $\Delta S = 1$ mass splittings to the quark masses treated to first order (here neglecting isospin violation)

$$\begin{aligned} m_A - m_N &= m_s \langle A | \bar{s}s | A \rangle - m_d \langle N | \bar{d}d | N \rangle \\ &\simeq m_s Z_m \simeq 150 \text{ MeV} \end{aligned} \quad (6.82)$$

with

$$Z_m = \langle A | \bar{s}s | A \rangle. \quad (6.83)$$

At this stage, most authors absorb Z_m into the mass, i.e., $m_s^* = Z_m m_s = 150 \text{ MeV}$. However we are after m_s , not m_s^* , and therefore this is not appropriate here. At low energies it is certainly not correct to set $Z_m = 1$. Z_m may be calculated at the hadronic scale using quark models. In the bag model [96]

$$Z_m = \int d^3x (u^2 - l^2) / \int d^3x (u^2 + l^2) \simeq 0.48 \quad (6.84)$$

with $u(l)$ being the upper (lower) component of the quark wavefunctions. In potential models

$$Z_m = \frac{\langle \bar{u}(p) u(p) \rangle}{\langle u^\dagger(p) u(p) \rangle} = 1 - \left\langle \frac{\mathbf{p}^2}{E(E+m)} \right\rangle. \quad (6.85)$$

If we use the model of Isgur and Karl [24] with $\alpha^2 = 0.17 \text{ GeV}^2$, we obtain

$$\left\langle \frac{\mathbf{p}^2}{E(E+m)} \right\rangle \approx 0.6, \quad Z_m \approx 0.4. \quad (6.86)$$

These estimates indicate that at the hadronic scale

$$m_s \approx 150 \text{ MeV} / Z_m \approx (300 \rightarrow 400) \text{ MeV} \quad (6.87)$$

and

$$A = \frac{m_K^2}{m_s + m_d} \approx \frac{0.25 \text{ GeV}^2}{0.15 \text{ GeV}} Z_m \approx (670 \rightarrow 800) \text{ MeV}. \quad (6.88)$$

A direct calculation of A in the bag model yields a somewhat smaller result [28]

$$A = 430 \text{ MeV}. \quad (6.89)$$

There is yet another complication which arises when one applies vacuum saturation to the evaluation of the penguin operator. The constant terms (independent of the momenta) in matrix elements of the scalar and pseudoscalar densities must vanish in the SU(3) limit because chiral symmetry requires that the amplitude be linear in momentum-squared. If we deal with the $K \rightarrow \pi$ matrix element this comes about from the cancellation of the two diagrams of fig. 6.7. For example, we find

Fig. 6.7. Contributions to the pion-to-kaon matrix element from (a) direct and (b) disconnected diagrams.

$$\begin{aligned}
\langle \pi^-(k) | O_5 | K^-(k) \rangle &= \frac{32}{9} (\langle \pi^- | \bar{d} \gamma_5 u | 0 \rangle \langle 0 | \bar{u} \gamma_5 s | K^- \rangle - 2 \langle \pi^- | \bar{d} s | K^- \rangle \langle 0 | \bar{d} d | 0 \rangle) \\
&= \frac{64 F_\pi^2 A^2}{9} ((1 \pm k^2/\Lambda^2)^2 - 1) = \pm \frac{128}{9} \cdot \frac{F_\pi^2 A^2}{\Lambda^2} k^2.
\end{aligned} \tag{6.90}$$

Thus one needs to know not only the overall amplitude A , but also the momentum dependence of the amplitude (as scaled by the parameter Λ). It is perhaps most reasonable to choose Λ in the range $\Lambda \simeq 700 \text{ MeV} \rightarrow 1 \text{ GeV}$.

6.4.2. Bag model matrix elements

The kaon matrix elements of the nonleptonic Hamiltonian may be calculated within the bag model or other quark models using the techniques outlined in section 6.3. We will quote the general form of the matrix elements, but in giving numerical values will only reproduce that of the bag model. Other quark models have been compared to the bag model in ref. [105]. For $O_{1,2,3,5}$ there are no complications and the result is

$$\begin{aligned}
\langle \pi^- | O_1 | K^- \rangle &= \frac{1}{2} \langle \pi^- | O_2 | K^- \rangle = \frac{1}{2} \langle \pi^- | O_3 | K^- \rangle = \frac{1}{2} \langle \pi^- | O_4 | K^- \rangle \\
&= 4 (2m_K^2)^{1/2} \int d^3x (u^2 + l^2 - 6u^2 l^3) = -0.052 m_K^4,
\end{aligned} \tag{6.91}$$

where $u(l)$ is the upper (lower) component of the Dirac wavefunction defined in eq. (3.22). The bag integral involved contains a large cancellation which results in the numerical value being a very sensitive function of the parameters of the model. (The number quoted corresponds to $R = 3.3 \text{ GeV}^{-1}$, $m_s R = 1$.) This is due to a helicity suppression for $q\bar{q}$ in the pseudoscalar state which is to be annihilated by the vector or axial vector currents. Because of this feature, we regard these matrix elements as not being reliably calculable in the bag model or in other quark models.

The penguin operators $O_{5,6}$ again present extra difficulties, in this case due to the second diagram in fig. 6.7. The first diagram (which we will call ‘direct’) can be easily evaluated in quark models

$$\begin{aligned}
\langle \pi^- | O_5 | K^- \rangle_{\text{direct}} &= \frac{16}{3} \langle \pi^- | O_6 | K^- \rangle_{\text{direct}} \\
&= \frac{64}{3} (2m_K^2)^{1/2} \int d^3x (u^2 + l^2)^2 = 1.87 m_K^4.
\end{aligned} \tag{6.92}$$

This diagram itself involves no cancellations and is a much firmer prediction within the model. However the second diagram (the disconnected one), which *subtracts* from the first in magnitude, must also be included. To evaluate it we use the constants defined in eq. (6.77),

$$\begin{aligned}
\langle \pi^- | O_5 | K^- \rangle_{\text{disconnected}} &= \frac{16}{3} \langle \pi^- | O_6 | K^- \rangle_{\text{disconnected}} \\
&= -\frac{64}{9} \langle \pi^- | \bar{d} s | K^- \rangle \langle 0 | \bar{d} d | 0 \rangle \\
&= -\frac{64}{9} F_\pi^2 A^2.
\end{aligned} \tag{6.93}$$

The cancellation of this with the direct diagram is substantial and leads to a sizable model dependence of the final answer. Let us now compare these results with experiment.

6.4.3. Comparison with data

In the previous two subsections, we have discussed the techniques of matrix element evaluation in quark models. In this section we compare the resulting amplitudes of the vacuum-saturation method and the bag model with each other and also with experiment.

Let us focus on the three most important operators, the $\Delta I = \frac{1}{2}$ operator with the largest coefficient O_1 , the dominant ‘penguin’ operator O_5 , and the only $\Delta I = \frac{3}{2}$ operator O_4 ,

$$H_w \simeq \frac{G_F}{2\sqrt{2}} \cos \theta_1 \sin \theta_1 \cos \theta_3 [c_1 O_1 + c_4 O_4 + c_5 O_5]. \quad (6.94)$$

The most basic comparison between the two methods is found by examining the ‘direct’ contribution to the $K \rightarrow \pi$ matrix element (i.e., fig. 6.7a, to be distinguished from the “disconnected” diagram of fig. 6.7b). In this case the vacuum saturation method yields

$$\begin{aligned} \langle \pi^+(k) | H_w | K^+(k) \rangle_{\text{Direct}}^{\text{Vac}} &= \frac{G_F}{2\sqrt{2}} \cos \theta_1 \sin \theta_1 \cos \theta_3 \left[-(c_1 + 2c_4) \frac{4}{3} F_\pi^2 k^2 + \frac{64}{9} F_\pi^2 A^2 c_5 \right] \\ &= [-1.1(c_1 + 2c_4) + 16(2Z)^2 c_5] \times 10^{-8} m_K^2 \end{aligned} \quad (6.95)$$

where Z and A are defined in section 6.4.1. (To briefly summarize our previous discussion, we advocate $Z \simeq \frac{1}{2}$ although many authors use $Z = 1$.) The corresponding formula in the bag model is

$$\langle \pi^+ | H_w | K^+ \rangle_{\text{Direct}}^{\text{Bag}} = [1.1(c_1 + 2c_4) + 41c_5] \times 10^{-8} m_K^2. \quad (6.96)$$

The bag model has the opposite sign on the first term and has a larger matrix for the penguin operator if $Z = \frac{1}{2}$. (The penguin matrix elements agree at $Z \simeq 0.8$.) We note that it is this matrix element which lattice Monte Carlo programs are estimating, as they have not yet been able to include the disconnected diagram, fig. 6.7b. This is described in section 11.

To deal with physical $K \rightarrow 2\pi$ amplitudes one must add in the disconnected diagrams and use PCAC to relate $K \rightarrow \pi$ and $K \rightarrow 2\pi$. When this is done, the vacuum saturation method yields

$$\begin{aligned} A(K^0 \rightarrow \pi^+ \pi^-)_{\text{vac}} &= -3.7 \times 10^{-8} m_K \left(c_1 + 2c_4 \pm \frac{32}{3} \frac{A^2}{\Lambda^2} c_5 \right) \\ &\simeq (-3.7(c_1 + 2c_4) \pm 56(2Z)^2 c_5) \times 10^{-8} m_K \end{aligned} \quad (6.97)$$

where Z , A and Λ are defined in section 6.4.1 and we have used $\Lambda = 700$ MeV. Likewise in the bag model

$$A(K^0 \rightarrow \pi^+ \pi^-)_{\text{Bag}} = (7.4(c_1 + 2c_4) + (250 - 45(2Z)^2) c_5) \times 10^{-8} m_K. \quad (6.98)$$

Fig. 6.8. Scalar-pole model of kaon decay.

For comparison we recall that the experimental result is

$$A(K^0 \rightarrow \pi^+ \pi^-) = (55 + 1.2) \times 10^{-8} m_K \quad (6.99)$$

where the first number is the $\Delta I = \frac{1}{2}$ contribution, and the second is that for $\Delta I = \frac{3}{2}$.

From this compilation, we see clearly the trouble which the valence quark model has in explaining the $\Delta I = \frac{1}{2}$ rule in kaon decay. Use of the perturbative short distance coefficients, $c_1 = 2.4$, $c_5 = -0.1$, yields an amplitude which is too small to reproduce the $\Delta I = \frac{1}{2}$ matrix element. The penguin operator does have a larger matrix element than O_1 , O_4 , but it does not appear to be large enough to overcome the smallness of the QCD coefficient c_5 . The value of c_5 is in fact quite generously large, using a renormalization scale parameter below the value ($\mu < 300$ MeV) expected to be reliable in perturbation theory. Ross and Hill [95] have argued that this scale should be chosen higher ($\mu > 1$ GeV) leading to a smaller coefficient ($c = 0.03$). In any case the conclusion seems to be that a perturbative QCD coefficient combined with valence quark model matrix elements is insufficient to completely understand kaon decay amplitudes.

Let us conclude by briefly discussing a class of models in which kaon decay occurs as a two-step process as in fig. 6.8. First the kaon mixes via the weak nonleptonic Hamiltonian into a spinless state of positive parity. This positive parity state propagates and then decays via the strong interactions into an S-wave pair of pions. In order for such a pole model to give a sufficiently large amplitude, it is important that the scalar meson be rather light. It is intriguing that $\pi\pi$ scattering data indeed suggests dynamical activity in the region $0.5 < E(\text{GeV}) < 1$. Unfortunately the relation of this behavior to the quark model is far from clear. In view of this uncertainty, it is prudent to regard the pole-model approach to kaon decay as an interesting but problematic attempt to understand kaon decay.

7. Other decay modes

In previous sections we have discussed the quark model picture of weak hadronic transitions, both nonleptonic and semileptonic, which are the predominant decay modes of the hadrons. However, there are a number of minor or 'rare' decay modes which have also received attention. Examples include $\pi^+ \rightarrow e^+ \nu_e \gamma$ which provides a test of CVC and plays a role in the tests of μ - e universality [124], $K \rightarrow \pi\pi\gamma$ which has been used to probe the origin of the $\Delta I = \frac{1}{2}$ rule [125], $K_L \rightarrow \gamma\gamma$ which is well understood in terms of a simple pole model and involves the combination of weak and electromagnetic interactions [103, 126], etc.

There are a considerable number of these 'rare' decays and a careful study could provide material for an entire review in itself. Hence, we shall not analyze these modes—with one exception. We shall discuss in some detail the nonleptonic-radiative hyperon decays, which have received a good deal of theoretical and experimental attention.

Experimentally, only one such decay, $\Sigma^+ \rightarrow p\gamma$, has been detected unambiguously [21], as shown in table 7.1. The important feature indicated there is not so much the branching ratio, but rather the

Table 7.1
Branching ratios (B.R.) and asymmetry parameters for the weak radiative
hyperon decays

Decay	Experiment		Theory	
	B.R.(10^{-3})	α	B.R.(10^{-3})	α
$\Sigma^+ \rightarrow p\gamma$	1.17 ± 0.14	$-0.70^{+0.31}_{-0.27}$	0.82	-0.86
$\Lambda^0 \rightarrow n\gamma$			1.02	-0.10
$\Sigma^0 \rightarrow n\gamma$			9.4×10^{-10}	-0.98
$\Xi^0 \rightarrow \Lambda\gamma$	5 ± 5		2.29	-0.41
$\Xi^0 \rightarrow \Sigma^0\gamma$	<70		5.87	-0.58
$\Xi^- \rightarrow \Sigma^-\gamma$	<1.2			
$\Omega^- \rightarrow \Xi^-\gamma$	<3.1			

asymmetry of the decay photons with respect to the initial Σ^+ polarization. An average of two experiments yields [127]

$$\alpha = -0.70^{+0.31}_{-0.27}. \quad (7.1)$$

The reason that this result is important and surprising lies in the Lee–Swift theorem [107, 128]. Thus the existence of these radiative nonleptonic hyperon decays is anticipated in view of the pole diagrams shown in fig. 7.1, which represent the transition amplitudes in terms of known magnetic moments and weak baryon–baryon matrix elements. However, since the Lee–Swift theorem requires the vanishing of $\langle B' | H_w^{p.v.} | B \rangle$, this picture suggests that the parity-violating component of the transition must itself vanish and along with it the asymmetry α [128]. Now this is only strictly true in the SU(3) limit and various authors have attempted to modify the baryon pole model by inclusion of SU(3) violating $\langle B' | H_w^{p.v.} | B \rangle$ amplitudes [108, 110, 129]. However, all attempts have yielded a parity violating amplitude *much* smaller than its parity-conserving counterpart and are thus unable to account for a nearly maximal asymmetry.

Other approaches have focused on short distance contributions to the nonleptonic radiative decays [131]. Thus evaluation of fig. 7.2 yields an effective two-quark operator of the form

$$O_{2q} = \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) d F^{\mu\nu} + \text{h.c.} \quad (7.2)$$

It is easy to see that such an operator alone cannot explain the hyperon nonleptonic radiative decays, since the predicted $\Sigma^+ \rightarrow p\gamma$ asymmetry has the wrong sign, and the amplitude for $\Xi^- \rightarrow \Sigma^-\gamma$ is predicted to be too large. Although one *can* fit the decays phenomenologically in terms of a combination of O_{2q} and O_{4q} effects, the only approaches which have been able to reasonably represent

Fig. 7.1. Pole model for hyperon radiative decays.

Fig. 7.2. Short distance contribution to nonleptonic radiative decay.

Fig. 7.3. Diagrams contributing to the effective quark-gluon-photon vertex.

the features of these decays without free parameters are those of references [131], who evaluate the parity-conserving amplitude via the baryon pole model and the parity-violating counterpart in terms of a $1/2^-$ excitation pole picture. This $1/2^-$ pole model has been reasonably successful in representing the parity violating $\Delta S = 0$ $NN\rho$ amplitude [132] and in resolving the S-wave/P-wave current algebra anomaly in nonleptonic hyperon decay as discussed earlier [101, 133], so it is not surprising that such a picture may well be successful in the nonleptonic-radiative arena as well. The predicted amplitudes are listed in table 7.1, and it is seen that there exists reasonable agreement with experiment. However, similar calculations by Chong-Huah and by Skovpen have not found a large enough parity violating amplitude [134]. Perhaps the effective quark-gluon-photon (fig. 7.3) operator recently studied by Gaillard, Li and Rudaz [135] will prove to be of significance. The coefficient function associated with this operator is unsuppressed by GIM cancellations, and substantial parity violation could result. We conclude that additional calculations as well as more and better data on modes other than $\Sigma^+ \rightarrow P\gamma$ are required before a definitive claim can be made that these nonleptonic radiative decays are indeed understood.

8. Physics of the $K^0\bar{K}^0$ system

8.1. Overview

Throughout the development of gauge theories, the $K^0\bar{K}^0$ system has played a fruitful role. The very tiny K_L-K_S mass difference [21]

$$m_L - m_S = \Delta m_{LS} = 3.5 \times 10^{-15} \text{ GeV} = 0.48 \Gamma_S \quad (8.1)$$

requires that there be no $\Delta S = 2$ effects until $O(G_F^2)$. This has been a strong constraint on the construction of gauge theory models. In addition, the existence of CP-violation has led to many attempts to incorporate this feature into the theory. The simplest mechanism for introducing CP-violating interactions is the possibility of nonvanishing phases in the KM matrix of the charged currents [10]. In many ways this has come to be the ‘standard’ mechanism, although there is no more evidence for it than for the other models of CP-violation. However, the alternate mechanisms (Higgs models [136], Left–Right models [137], superweak models [138]) all require new particles and interactions and are therefore less economical. In this section we will adhere to the minimal KM model.

The earliest ‘modern’ treatment of the $K^0\bar{K}^0$ system was that of Gaillard and Lee [139] who calculated, in the four quark model, the short distance diagrams responsible for $\Delta S = 2$ effects and estimated Δm by use of the vacuum saturation method. Since this seminal work, several developments have taken place. One is the discovery of the t and b quarks. Their inclusion modifies the analysis of Δm , and also allows for the possibility of CP-violation. Many phenomenological studies have been made to attempt to learn about the t quark from this system [140]. Another advance was the treatment of CP-violation by Gilman and Wise [14], who pointed out that the KM model generated a nonzero value of ε' , restimulating interest in this aspect of CP-violation. Finally, there has been a slowly growing awareness of the importance of long distance effects and the role which they play in the analysis of Δm and ε [142–144]. It will be our intent here to review these developments.

8.2. Short distance physics

To generate $\Delta S = 2$ interactions, one needs the product of four weak currents. Processes which can produce such effects are given in fig. 8.1. To the extent that these are sensitive to high mass quarks and to W bosons, they may be reliably calculated using the operator product expansion, and represent true short distance physics. To the extent that u quarks enter the diagrams (they are required for the GIM cancellations), the perturbative treatment yields only the short distance part of the amplitude. The long distance portion remains to be estimated by other means. The first two diagrams are called ‘box diagrams’. They were first calculated by Gaillard and Lee [139], who obtained in the four-quark model

$$H_{\text{Box}}^{\Delta S=2} = \frac{G_F^2}{16\pi^2} (m_c^2 - m_u^2) \cos^2 \theta_c \sin^2 \theta_c O^{\Delta S=2} \quad (8.2)$$

where

$$O^{\Delta S=2} = \bar{d} \gamma^\mu (1 + \gamma_5) s \bar{d} \gamma_\mu (1 + \gamma_5) s. \quad (8.3)$$

The third, which we will call the ‘Siamese penguin’ because it is composed of two penguin diagrams joined by a common gluon, yields a local operator (including only the leading CP even and odd coefficients)

$$H_{\text{S.P.}}^{\Delta S=2} = \frac{G_F^2 \alpha_s \cos^2 \theta_1 \sin^2 \theta_1}{288\pi^3} \left[\ln \left(\frac{m_c}{\mu} \right)^2 + 2i \sin \theta_2 \sin \theta_3 \sin \delta \ln \left(\frac{m_t}{m_c} \right)^2 \right] \tilde{O}^{\Delta S=2} \quad (8.4)$$

where

$$\tilde{O}^{\Delta S=2} = \bar{d} \gamma^\mu (1 + \gamma_5) \lambda^A s (\partial_\mu \partial_\nu - g_{\mu\nu} \square) (\bar{d} \gamma^\nu (1 + \gamma_5) \lambda^A s). \quad (8.5)$$

Fig. 8.1. Short distance contributions to $\Delta S = 2$ interactions.

The Siamese penguin diagram has not been included in previous analyses of $\Delta S = 2$ effects. However we feel that this oversight is not a serious one. To obtain a rough estimate of the amplitude corresponding to the Siamese penguin diagram, it is convenient to employ the vacuum saturation approximation. We then estimate that

$$\frac{\langle H_{\text{S.P.}} \rangle}{\langle H_{\text{Box}} \rangle} \lesssim 0.1. \quad (8.6)$$

Thus it appears reasonable to ignore the Siamese penguin, and we will do so in the following.

In obtaining physical quantities from the $\Delta S = 2$ Hamiltonian, the matrix element of $O^{\Delta S=2}$ is needed. This has been a subject of extensive discussion in the literature. It is conventional to normalize to Gaillard and Lee's vacuum-saturation calculation, defining the parameter B by

$$\langle K^0 | O^{\Delta S=2} | \bar{K}^0 \rangle = \frac{16}{3} F_K^2 m_K^2 B \quad (8.7)$$

with $B = 1$ being the result of vacuum saturation. This can in turn be related to the $\Delta I = \frac{3}{2}$ amplitude for $K \rightarrow 2\pi$ by SU(3) plus PCAC [145], leading to

$$B = 0.33 \pm 0.33 \quad (8.8)$$

where the method and the error estimate will be discussed more extensively in the next section.

With the addition of the top quark, the formulas for the box diagrams become more complicated if one keeps the full dependence on m_t for large m_t . The full formulas are given in many places [146]. However this no longer seems necessary because recent indications are that the top quark mass is near 40 GeV. For masses this large, the leading behavior is adequate, and much simpler. The box diagram matrix element, including QCD radiative corrections [147], becomes

$$2m_K M_{12}^{\text{Box}} = \langle K^0 | H_{\text{Box}}^{\Delta S=2} | \bar{K}^0 \rangle = 2m_K \text{Re } M_{12}^{\text{Box}} + i 2m_K \text{Im } M_{12}^{\text{Box}} \quad (8.9)$$

with

$$2m_K \text{Re } M_{12}^{\text{Box}} = \frac{G_F^2 \cos^2 \theta_1 \sin^2 \theta_1}{3\pi^2} F_K^2 m_K^2 B \left[\eta_1 m_c^2 + \eta_2 m_t^2 K^2 + 2\eta_3 m_c^2 K \ln \frac{m_t^2}{m_c^2} \right] \quad (8.10)$$

and

$$2m_K \text{Im } M_{12}^{\text{Box}} = \frac{G_F^2 \cos^2 \theta_1 \sin^2 \theta_1}{3\pi^2} F_K^2 m_K^2 B 2 \sin \theta_2 \sin \theta_3 \sin \delta \left[-\eta_1 m_c^2 + \eta_2 m_t^2 K + \eta_3 m_c^2 \ln \frac{m_t^2}{m_c^2} \right]. \quad (8.11)$$

Here the QCD coefficients are

$$\eta_1 \approx 0.7, \quad \eta_2 \approx 0.6, \quad \eta_3 \approx 0.5 \quad (8.12)$$

and the KM angles which enter are

$$K = \sin^2 \theta_2 + \sin \theta_2 \sin \theta_3 \cos \delta. \quad (8.13)$$

To obtain a feel for the size of the different contributions, let us substitute $m_c = 1.5$ GeV, $m_t = 40$ GeV, and the upper bound on K

$$K \begin{cases} \leq 0.005 & \text{if } \cos \delta = 1 \\ \approx 0.0025 & \text{if } \cos \delta = 0 \end{cases} \quad (8.14)$$

(coming from the B decay lifetime and the $(b \rightarrow u)/(b \rightarrow c)$ ratio), and use the numerical coincidence

$$\frac{G_F^2 \cos^2 \theta_1 \sin^2 \theta_1 F_K^2 m_K^2 m_c^2}{3\pi^2} \simeq [m_K \Delta m_{\text{LS}}]_{\text{Expt.}} \quad (8.15)$$

to obtain

$$\text{Re } M_{12}^{\text{Box}} = B \frac{\Delta m_{\text{LS}}}{2} [\eta_1 + 711K^2\eta_2 + 13\eta_3K] \simeq B \frac{\Delta m_{\text{LS}}}{2} [0.7 + 0.01 + 0.03] \quad (8.16a)$$

$$\text{Im } M_{12}^{\text{Box}} = B \frac{\Delta m_{\text{LS}}}{2} 2 \sin \theta_2 \sin \theta_3 \sin \delta [-0.7 + 427K + 3.3] \simeq B \frac{\Delta m_{\text{LS}}}{2} 2 \sin \theta_2 \sin \theta_3 \sin \delta [3.6]. \quad (8.16b)$$

The last line was evaluated with $\cos \delta = 0$ in order to maximize $\text{Im } M$. We see that the charm quark contribution is dominant in $\text{Re } M_{12}$ while the top quark is most important in $\text{Im } M_{12}$.

Before we examine the manifestation of the short distance physics, we need to include a discussion of the B parameter and of the “dispersive” or long distance effects not accounted for by the above quark model analysis.

8.3. The B parameter

Following Gaillard and Lee’s original estimate of the matrix element of $O^{\Delta S=2}$ there were many attempts to calculate this quantity in various frameworks. It was pointed out that the vacuum saturation method is subject to large corrections if one includes the one pion intermediate state [118, 148]. A bag model calculation [118] gave $B = -0.42$ (the sign quoted in the original reference is incorrect), while harmonic oscillator models [149] produced $B = 3$. (The latter calculation actually should reproduce the vacuum saturation value if it were able to correctly predict F_K , because in this model the diagrams considered are exactly those of vacuum saturation. The same is not true in the bag model because of the presence of the bag and the center-of-mass motion.) Hadronic sum rules have been used to bound B to be less than about unity [150]. The physical origin of the significant model dependence in attempts to determine B appears to lie in the “helicity suppression” for left–left operators discussed in section 6.4. It appears unlikely that a precision estimate can be made within the quark model. Here, as in other nonleptonic transitions, the quark model approach can hope at best for rough estimates of the matrix elements.

It is, however, possible to obtain an ‘empirical’ value for B by relating it to the $\Delta I = \frac{3}{2}$ amplitude in $K \rightarrow 2\pi$ [145]. Because of the increasing importance of this number in the study of the CP violation in the KM model, we wish to spell out very carefully the assumptions which go into this determination, and to do our best to assess the reliability of each aspect.

The basic observation which allows this determination is that $O^{\Delta S=2}$ belongs in the same SU(3) multiplet as the operator O_4 (given in eq. (6.20)) responsible for $\Delta I = \frac{3}{2}$ transitions in $\Delta S = 1$ processes. This implies that

$$\langle K^0 | O^{\Delta S=2} | \bar{K}^0 \rangle = \sqrt{2} \langle \pi^0 | O_4 | \bar{K}^0 \rangle. \quad (8.17)$$

The right-hand side can be obtained from the experimental $\Delta I = \frac{3}{2}$ $K \rightarrow 2\pi$ amplitude by use of PCAC. To do this, one writes the general form of the amplitude up to first order in the particle momentum, and uses the constraint that the $K \rightarrow 2\pi$ amplitude vanish in the SU(3) limit. This yields

$$\langle \pi^0 | O_4 | \bar{K}^0 \rangle = \frac{2\sqrt{2} F_\pi}{3} \cdot \frac{m_K^2}{m_K^2 - m_\pi^2} \cdot \langle \pi^- \pi^0 | O_4 | K^- \rangle \quad (8.18)$$

where the right-hand side is evaluated with all particles on-shell and the left-hand side is taken at $p_K^2 = p_\pi^2 = m_K^2$, as this is where we want to apply eq. (8.18). A final step uses the relation between O_4 and the weak Hamiltonian, eq. (8.2), to connect with experiment. Overall this yields

$$\langle K^0 | O^{\Delta S=2} | \bar{K}^0 \rangle = \frac{8\sqrt{2}}{3} \frac{m_K^2}{m_K^2 - m_\pi^2} \frac{\langle \pi^- \pi^0 | H_w^{\Delta I=3/2} | K^- \rangle}{G_F c_4 \cos \theta_c \sin \theta_c}. \quad (8.19)$$

In our paper [145], we also used PCAC and SU(3) to correct for the effects of π^0 - η mixing. This produces a 13% reduction of the amplitude, which is a small change, but we feel it is good physics to include it. The end result of this procedure is $B = 0.33$.

The whole method can be summarized more compactly by use of chiral SU(3) [151]. The weak Hamiltonian for mesons has the form (eq. (6.28a))

$$H_w^{(i)} = g C_{27 \ 8 \ 8}^{i \ j \ k} \text{Tr}(\partial_\mu M M^\dagger \lambda^j M \partial^\mu M^\dagger \lambda^k),$$

where the C -symbol is a Clebsch–Gordan coefficient and the 3×3 matrix M contains the meson fields as in eq. (2.19). All the connections implied by SU(3) and PCAC emerge if one expands M in a power series in the fields. This procedure then gives, in a single step, the same result which was previously obtained through several stages.

We note that the sign of B is not determined in the PCAC method. If the $\Delta I = \frac{1}{2}$ amplitudes in $K \rightarrow 2\pi$ have the same sign as the penguin matrix elements in the bag model, then B is positive. However it should be kept in mind in phenomenological usages that the sign of B is not reliably fixed.

The possible sources of uncertainty in this result can be identified and evaluated:

1) We have used the QCD coefficient c_4 . Perhaps an error in this would propagate through into the value of B . However, this is irrelevant. To see this, note that QCD renormalization of O_4 and $O^{\Delta S=2}$ is essentially identical because they belong to the same multiplet. The only difference arises from the mass effect of heavy quarks which enter slightly differently in the two cases. This, however, is a small part of the overall QCD factor, and is in a region where the QCD calculations should handle it reliably. As long as one uses QCD corrections to $H^{\Delta S=1}$ and $H^{\Delta S=2}$ which are performed in the same way, the QCD factors will cancel out in the final answer. Put in other words, the real relation is those of matrix elements

$$\text{Re } M_{12} \propto (\eta_1/c_4) \langle \pi^- \pi^0 | H_w^{\Delta S=1} | K^- \rangle, \quad (8.20)$$

and the QCD dependence essentially cancels out in the ratio η_1/c_4 . This source of uncertainty is therefore negligible.

2) The PCAC extrapolation was done using only linear dependence in the squared-momentum. Perhaps quadratic momentum dependence will modify the result. This issue can be addressed by studying the extrapolation from $K \rightarrow 3\pi$ to $K \rightarrow 2\pi$. Here the linear extrapolation works well and correctly predicts [88] the slopes and magnitudes of $K \rightarrow 3\pi$ amplitudes in terms of the $K \rightarrow 2\pi$ matrix elements, at accuracy of about 25%. There is some evidence for smaller quadratic dependence at the

25% level [89] in the $\Delta I = \frac{1}{2}$ amplitudes. The data is not good enough to see any quadratic dependence in the $\Delta I = \frac{3}{2}$ amplitude, but we expect it to be no larger, and possibly less, than in the $\Delta I = \frac{1}{2}$ channel. In any case, we will assign an uncertainty to B of about 25% due to this feature.

3) The third and most dangerous assumption of the B determination is that of SU(3) symmetry. We note that the most important dependence of mass differences has been accounted for in the kinematic variation of the amplitude. The difficult question is to what extent other SU(3) breaking factors may modify B . There has been a paper [152a] by Bijlens, Sonoda and Wise (BSW) which reports a calculation of one loop effects yielding possible very large SU(3) violations within chiral perturbation theory. In chiral perturbation theory at one loop level both logarithmic effects and quartic momentum dependence (described above) are of the same order

$$B_{1\text{ loop}} = B_{0\text{ loops}} \cdot \left[1 + C_1 \frac{m_K^2}{(4\pi F_\pi)^2} \ln \frac{\mu^2}{m_K^2} + C_2 \frac{m_K^2}{(4\pi F_\pi)^2} \right]. \quad (8.21)$$

Here C_1 and C_2 are dimensionless coefficients (C_2 will depend on the renormalization point μ), and we have neglected m_π^2 compared to m_K^2 . The logarithmic term can only arise through SU(3) violation. BSW calculate the coefficient of the logarithm and find $C_1 = 55/12$, a rather large value. The coefficient C_2 cannot be calculated from theory, but needs additional experimental input. However the large value of C_1 is cause for concern because it appears to signal trouble with the chiral perturbation theory expansion of B . It may also signal large modifications of B , but to be certain of this one needs to know $C_2(\mu)$. If $C_2(\mu)$ is small when $\mu \rightarrow m_K$, then overall corrections are not large, while if $C_2(\mu)$ is small at $\mu \sim 1 \text{ GeV}$ then B is strongly modified.

It is too early to provide a final assessment of the importance of the chiral calculation of BSW. It is clear that many people in the field will give more thought to this question. For example, a very recent calculation of Pich and de Rafael [152b] uses a finite energy sum rule to infer that $|B| = 0.33 \pm 0.09$. Our tentative solution, for the purpose of this review, is to quote factor-of-two error bars on B to reflect this ongoing uncertainty

$$B = 0.33 \pm 0.33$$

but these may be increased or decreased subject to future work.

8.4. Long distance physics

That the $\Delta S = 2$ interaction can also receive contributions from long distance processes not given by the box diagram can be seen in fig. 8.2. The first diagram (a) is similar to the box diagram, except for the extra quark pairs which convert the intermediate states into mesons. Figure 8.2b, the double penguin, is altogether distinct from the box diagram. It is easy to see that the box diagram, as considered above, contains little or no contribution from these intermediate states because the dominant contribution scales linearly with m_c^2 , while the low-energy physics is independent of m_c^2 . One can make a naive argument that the low-energy physics should be unimportant because it should scale with only typical hadronic masses, e.g., m_K^2 , leading to a suppression compared to the box diagram of order

$$\left. \frac{\text{L.D.}}{\text{Box}} \right|_{\text{Naive}} \sim \frac{m_K^2}{m_c^2} \sim 0.1. \quad (8.22)$$

Fig. 8.2. Long distance contributions to $\Delta S = 2$ interactions.

However this is contradicted by explicit calculations.

The very low-energy contributions to $\Delta S = 2$ processes involve the pseudoscalar octet as intermediate states

$$K^0 \rightarrow \pi^0, \eta, 2\pi, 3\pi, \dots \rightarrow \bar{K}^0. \quad (8.23)$$

The normalization of these contributions is known from the experimental $K \rightarrow 2\pi$ and $K \rightarrow 3\pi$ amplitudes. The $K \rightarrow \pi^0$ amplitude is obtained from $K \rightarrow 2\pi$ using PCAC, and that for $K \rightarrow \eta$ can be found through the use of SU(3). When these intermediate states are studied, it is found that the 2π contribution is dominant. Due to the cutoff dependence of the final answer and/or uncertainties in the procedure, the overall result is not reliably calculated. However the magnitude obtained is generally of the same order as the experimental mass differences, and with the same sign [143].

There are in addition intermediate range contributions, such as

$$K^0 \rightarrow \eta', \rho, \omega \rightarrow \bar{K}^0 \quad (8.24)$$

whose normalization is not obtainable from any known experimental information. Estimates of these, using various theoretical models to obtain the normalization, also indicate a significant contribution, comparable to that of the box diagram [143].

There may also be long distance effects in CP-violation. These will be discussed in the section dealing with this topic.

The point of the studies of the dispersive component has been basically to convince workers in the

field that the naive belief of the unimportance of long distance effects was in error. The resolution seems to be not that the long distance contributions are particularly large, but that there are numerical factors associated with the box diagram (such as the $3\pi^2$ in the denominator of eq. (8.10)) which compensate for the factor of m_c^2 . Perhaps a better and more realistic estimate would be

$$\text{L.D./Box} \sim 6\pi^2 m_K^2 / m_c^2 \sim 6. \quad (8.25)$$

In any case, one must allow for these long distance effects in any phenomenological analysis of the kaon system.

It is doubtful that the dispersive component can ever be calculated reliably. Many of the contributions require theoretical models to estimate their size. Even for the more accessible intermediate states, one is ultimately faced with the complications of strong interaction physics involving the $\pi\pi$ scattering amplitudes, which are known to insufficient precision, and the thresholds above 1 GeV. In addition the calculation has a strong sensitivity on the assumptions of SU(3) invariance made in the calculation, with small changes in the $K \rightarrow \eta$ amplitude producing large modifications of the final answer. Existing calculations should be viewed as estimates and attempts along these lines are unlikely ever to be trustworthy.

8.5. The K_L - K_S mass difference

Both short distance and long distance effects can contribute significantly to $\text{Re } M_{12}$. We follow Wolfenstein and Hill [142] and add a parameter, D , to characterize the long distance contributions

$$\text{Re } M_{12} = \Delta m_{LS}/2 = \text{Re } M_{12}^{\text{Box}} + D \Delta m_{LS}/2. \quad (8.26)$$

From our previous estimate, eq. (8.16a), we see that the short distance contribution is only a portion of the experimental mass difference,

$$\Delta m_{LS}/\Delta m_{LS}^{\text{Expt.}} = 0.7B + D \simeq \pm(0.23 \pm 0.23) + D. \quad (8.27)$$

The remainder must be dispersive (or perhaps due to other new physics), and we can invert this to obtain

$$D = \begin{cases} 0.77 \pm 0.33 & B > 0 \\ 1.23 \pm 0.33 & B < 0. \end{cases} \quad (8.28)$$

This is completely consistent with the expected size of D (which ranges from $-1 \leq D \leq 2$ in most model calculations) and cannot be used to indicate the existence of any new physics. Unfortunately, we feel that this is the extent of the phenomenological usefulness of Δm . It cannot be used to bound KM angles or new generations of quark masses because the true short distance contribution is unknown. However, we can conclude that the standard model does provide a rough understanding of K_L - K_S mixing and is consistent with the observed magnitude.

8.6. CP-violation

So far the only evidence for CP-violation is in the decays of kaons. The amplitude ratios

$$\begin{aligned}\eta_{+-} &= A(K_L \rightarrow \pi^+ \pi^-)/A(K_S \rightarrow \pi^+ \pi^-) \equiv \varepsilon + \varepsilon' \\ \eta_{00} &= A(K_L \rightarrow \pi^0 \pi^0)/A(K_S \rightarrow \pi^0 \pi^0) \equiv \varepsilon - 2\varepsilon'\end{aligned}\quad (8.29)$$

define the CP-violating parameters ε and ε' . Experimentally [21]

$$\begin{aligned}\varepsilon &= |\varepsilon| e^{i\phi}, \quad |\varepsilon| = 2.3 \times 10^{-3}, \quad \phi = 45 \pm 2 \\ \varepsilon'/\varepsilon &= (-3.8 \pm 5.1 \pm 3.8) \times 10^{-3}.\end{aligned}\quad (8.30)$$

For recent reviews of the status of CP-violation see refs. [9, 153].

These are basically two types of contributions which can arise:

1) 'Mass matrix' CP-violation occurs through a CP-odd piece in the $K^0 \bar{K}^0$ mass matrix, i.e., in $\text{Im } M_{12}$,

2) 'Direct' CP-violation is that which occurs in the $K \rightarrow 2\pi$ amplitudes. Both are expected to be present in the KM model. We have already seen how $\text{Im } M_{12}$ is generated by the box diagram. In addition Gilman and Wise [141] noted that the penguin diagram in the $\Delta S = 1$ interaction can lead to direct CP-violation. These generate ε and ε' as follows

$$\begin{aligned}2\sqrt{2} \varepsilon &= e^{i\pi/4} [\text{Im } M_{12}/\text{Re } M_{12} + 2 \text{Im } A_0/\text{Re } A_0] \\ \sqrt{2} \varepsilon' &= e^{i(\pi/2 + \delta_2 - \delta_0)} \omega [\text{Im } A_2/\text{Re } A_2 - \text{Im } A_0/\text{Re } A_0]\end{aligned}\quad (8.31)$$

where the $K \rightarrow 2\pi$ amplitudes with isospin I are defined by

$$\langle \pi\pi(I=n) | H_w | K^0 \rangle = A_n e^{i\delta_n} \quad (8.32)$$

and

$$\omega \equiv \text{Re } A_2/\text{Re } A_0 \cong \frac{1}{20}. \quad (8.33)$$

Both direct and mass matrix effects contribute to ε , while only direct CP-violation can lead to ε' . The parameterization above is independent of the phase convention for the $K \rightarrow 2\pi$ amplitudes. Often formulae are quoted in the Wu–Yang phase convention in which $\text{Im } A_0 = 0$, but this is actually cumbersome in the standard model where the natural basis of the quark model states would lead to $\text{Im } A_0 \neq 0$. In the KM model there is no mechanism to generate a CP-violating phase in the $I = 2$ amplitude A_2 (except perhaps through isospin breaking effects in A_0), and the most useful basis is that where $\text{Im } A_2 = 0$. We will continue to use the general forms, but will sometimes identify results in the latter basis.

Let us first consider how direct CP-violation arises. In the penguin diagram, the coupling to the c and t quarks introduces the CP-violating phase. The $\Delta S = 1$ interaction then develops a CP-odd term which is dominantly given by

$$H^{\text{CP}} = \frac{G_F}{2\sqrt{2}} \cos \theta_1 \sin \theta_1 i \text{Im } c_5 O_5 \quad (8.34)$$

with [148]

$$\text{Im } c_5 \simeq 0.017 \sin \theta_2 \sin \theta_3 \sin \delta \ln(m_t^2/m_c^2) \simeq -0.11 \sin \theta_2 \sin \theta_3 \sin \delta. \quad (8.35)$$

This is purely $\Delta I = \frac{1}{2}$ and can therefore only contribute to A_0 . With the definition

$$A_0 = |A_0| e^{i\xi} \quad (8.36)$$

there are two basic approaches to the calculation of the phase angle ξ .

The first, proposed by Gilman and Wise [141], is to parameterize the result by the fractional contribution of the real part of the penguin operator O_5 to $K \rightarrow 2\pi$,

$$\begin{aligned} \xi &= \frac{\text{Im } A_0}{\text{Re } A_0} = \frac{\text{Im } c_5 \langle \pi\pi(I=0) | O_5 | K^0 \rangle}{\sum_i \text{Re } c_i \langle \pi\pi(I=0) | O_i | K^0 \rangle} \\ &= \frac{\text{Im } c_5}{\text{Re } c_5} \cdot \frac{\text{Re } c_5 \langle \pi\pi(I=0) | O_5 | K^0 \rangle}{\sum_i \text{Re } c_i \langle \pi\pi(I=0) | O_i | K^0 \rangle} \equiv \frac{\text{Im } c_5}{\text{Re } c_5} f. \end{aligned} \quad (8.37)$$

The ratio involving c_5 has the form

$$\frac{\text{Im } c_5}{\text{Re } c_5} \simeq \sin \theta_2 \sin \theta_3 \sin \delta \frac{\ln(m_t^2/m_c^2)}{\ln(m_c^2/m_\mu^2)}. \quad (8.38)$$

This produces the maximum allowed value

$$\xi \lesssim 0.004f \quad (8.39)$$

if $m_t \simeq 40$ and $\mu \simeq 300$ MeV. We will see later in this section that the angles must be very close to their maximum allowed values. The disadvantage of this method is that neither $\text{Re } c_5$ nor f is reliably known. $\text{Re } c_5$ is sensitive to low-energy physics because the most important region of loop momentum in the penguin diagram is from low momentum, $k < 1$ GeV. The imaginary part of c_5 is more reliably known because it arises from purely short distance effects. Thus the ratio may not be as useful as it first seems. Likewise lack of knowledge of f lessens predictive power.

The second method, first advocated by Guberina and Peccei [154], proceeds through direct calculation of $\text{Im } A_0$ in a quark model and takes the real part of A_0 from experiment,

$$\begin{aligned} \xi &= -0.11 \sin \theta_2 \sin \theta_3 \sin \delta \frac{(G_F/2\sqrt{2}) \cos \theta_1 \sin \theta_1 \langle \pi^+ \pi^- | O_5 | K^0 \rangle}{78 \times 10^{-8} m_K} \\ &= -0.11 \sin \theta_2 \sin \theta_3 \sin \delta \frac{\langle \pi^+ \pi^- | O_5 | K^0 \rangle}{0.43 \text{ GeV}^3} = -0.11 \sin \theta_2 \sin \theta_3 \sin \delta P, \end{aligned} \quad (8.40)$$

where the last equation defines the parameter P . The estimates which we made in section 6.4 are equivalent to a range

$$0.7 < P < 2.6. \quad (8.41)$$

Using the largest values of the angles, this method predicts (for maximal $\sin \theta_2 \sin \theta_3 \sin \delta$)

$$0.0002 < |\xi| < 0.0006. \quad (8.42)$$

Thus both methods predict a small but nonzero phase angle, with the latter method producing the smaller, and probably more realistic, estimate.

Now let us turn to a possible imaginary component in the $K^0\bar{K}^0$ mass matrix, i.e., $\text{Im } M_{12}$. There are two types of contributions to be considered, the box diagram and the long distance contribution. Using the results of section 8.2, the box contribution is

$$2\sqrt{2} \tilde{\epsilon}_{\text{Box}} = \frac{\text{Im } M_{12}^{\text{Box}}}{(\text{Re } M_{12})^{\text{Expt}}} = 2B \sin \theta_2 \sin \theta_3 \sin \delta \left[-\eta_1 + \eta_2 m_t^2 K + \eta_3 \ln \frac{m_t^2}{m_c^2} \right]. \quad (8.43)$$

(Note that often in the literature one defines a ratio ϵ^{Box} through by dividing by the box diagram's contribution to $\text{Re } M_{12}$. Here we are using a different definition, now dividing by the full $\text{Re } M_{12}$.)

The long distance contributions to $\text{Im } M_{12}$ can arise due to nonzero phase ξ in the direct $\Delta S = 1$ amplitudes. All of the dispersive diagrams involving π^0 , η , 2π , 3π , would share the same phase because their amplitudes are related by PCAC and SU(3), and the PCAC extrapolation is the same for the CP-conserving and CP-violating interactions. These contributions all obey the relation

$$\frac{\text{Im } M_{12}|_{\pi, \eta, 2\pi, 3\pi}}{(\text{Re } M_{12})^{\text{Expt}}} = -D_\pi \frac{2 \text{Im } A_0}{\text{Re } A_0} \quad (8.44)$$

where D_π is the contribution to D from all of these intermediate states. There can in addition be other contributions which do not obey this relation, of which only the η' has been identified as particularly important [144]. These are also proportional to ξ but with a coefficient that depends on details of hadronic matrix elements. Defining N by

$$\frac{\text{Im } M_{12}|_{\eta', \dots}}{(\text{Re } M_{12})^{\text{Expt}}} = 2\xi N, \quad (8.45)$$

the quark model [144] has been used to indicate that $|N| \lesssim 1$.

The sources outlined above provide the complete characterization of the origins of CP-violation in the KM model. Combining up eqs. (8.43)–(8.45) the parameters ϵ and ϵ' emerge as

$$\begin{aligned} 2\sqrt{2} \epsilon &= e^{i\pi/4} [2\sqrt{2} \tilde{\epsilon}_{\text{Box}} + (1 - D_\pi) 2\xi + 2\xi N] \\ 2\sqrt{2} \epsilon' &= -e^{i(\pi/2 + \delta_2 - \delta_0)} \omega [2\xi]. \end{aligned} \quad (8.46)$$

At first sight the formula for ϵ would appear discouraging as it implies that our ability to predict ϵ

depends on the long distance physics contained in D_π , N and ζ . However one can show that these terms are not large because ζ is constrained to be small by the measured value of ε'/ε , eq. (8.30). This can be converted into a bound

$$\left| \frac{\xi}{2\sqrt{2} \tilde{\varepsilon}^{\text{Box}}} \right| < 0.1 \quad (8.47)$$

which implies that, as long as D_π and N are $O(1)$, the long distance physics contribution is of order 20% or less. Thus a very important conclusion can be drawn: *The magnitude of ε comes almost entirely from the box diagram, and is free from uncertainties due to long distance physics.*

There are two important predictions of the KM model which will be the subject of important studies in the near future. Dropping the small long distance terms, these are

$$\begin{aligned} |\varepsilon| = \tilde{\varepsilon}_{\text{Box}} &= \frac{B \sin \theta_2 \sin \theta_3 \sin \delta}{\sqrt{2}} \left[-\eta_1 + \eta_2 \frac{m_t^2}{m_c^2} K + \eta_3 \ln \frac{m_t^2}{m_c^2} \right] \\ &\simeq 4 \sin \theta_1 \sin \theta_2 \sin \theta_3 \sin \delta \frac{B}{0.33}, \end{aligned} \quad (8.48)$$

and

$$\varepsilon'/\varepsilon = -15\xi. \quad (8.49)$$

Both can serve as experimental tests of the KM model. The former is just barely compatible with experiment at present. If one uses the bound on the KM angles given in eq. (8.14) and employs $m_t = 40 \text{ GeV}$ one obtains

$$\tilde{\varepsilon}_{\text{Box}} \leq 0.005B \simeq (1.7 \pm 1.7) \times 10^{-3}. \quad (8.50)$$

One must stretch the values of B to the large side in order to match experiment. This is of course allowed, but implies that the angles $\sin \theta_2$ and $\sin \delta$ must be close to their maximum allowed values. In particular the $(b \rightarrow u)/(b \rightarrow c)$ ratio, which determines $\sin \theta_3$, must be close to the present upper limit if the KM model is correct. Allowing all uncertainties, we obtain the prediction [9]

$$\Gamma(b \rightarrow u)/\Gamma(b \rightarrow c) > 0.02. \quad (8.51)$$

This becomes a very important test of the KM model for CP-violation. The other test is that one should find a value for ε'/ε not far below the present limit. Since all the angles must be maximal, the only uncertainty lies in the hadronic matrix element of the penguin operator. Using the range of amplitudes discussed in section 6.4, one finds

$$0.002 \leq \varepsilon'/\varepsilon \leq 0.009. \quad (8.52)$$

Both of these tests are within the reach of experiment, although the latter will not be obtainable in the

very near future. If the KM mechanism of CP-violation were to fail these tests, it would be an indicator that some new physics outside the present standard model would be needed.

9. Nuclear parity violation

The weak interaction has a strength only

$$R_w = F_w/F_{ST} \sim \propto M_p^2/M_w^2 \sim 10^{-6} \quad (9.1)$$

that of the strong force. Yet its effects are generally easily detected when a change of quantum number (strangeness, charm, etc.) takes place, or when a lepton pair ($e\nu_e, \mu\nu_\mu$) is involved. However the weak interaction also occurs in the sector of purely strongly interacting particles, but now with no change in quantum number. In this case its presence is generally dwarfed by the competing strong interaction. However, the weak force can also be detected here by seeking the characteristic parity violation which it alone displays.

The earliest experiments in this field utilized low-energy nuclear interactions and looked for the very small ($\sim 10^{-6}$) parity-mixing of levels at the magnitude expected on the basis of previous studies [155]. Since we are dealing in this case with the low-energy limit of the parity nonconserving nucleon–nucleon force, the only significant parity-violating amplitudes are those which connect $l = 0$ and $l = 1$ states of the nucleon–nucleon system. There are then five independent amplitudes

$$\begin{aligned} {}^3S_1-{}^1P_1 & \quad \Delta I = 0 \\ {}^3S_1-{}^3P_1 & \quad \Delta I = 1 \\ {}^1S_0-{}^3P_0 & \quad \Delta I = 0, 1, 2. \end{aligned} \quad (9.2)$$

Now although it is possible to formulate the analysis of low-energy parity-violating phenomena in terms of these five amplitudes [156], the analysis traditionally is couched in the language of meson exchange [157].

It is well known that the parity conserving nucleon–nucleon force can be described reasonably well in terms of a coherent superposition of diagrams for π , ρ , ω exchange plus an important correction for the exchange of two pions, which is often represented in terms of an effective scalar boson exchange, the σ [158]. In a similar fashion one generally represents the parity violating nucleon–nucleon potential in terms of a sum of diagrams involving exchange of a single meson between pairs of nucleons. There is an important difference in this case, however, in that one meson–nucleon vertex is weak and one is strong. Also, CP-invariance forbids the exchange of neutral spinless bosons, so π^0 and/or σ exchange is disallowed [159]. Since the strong couplings are known empirically, one finds a form for the parity-violating potential in terms of seven a priori unknown weak coupling constants

$$f_\pi, h_\rho^{0,1,2}, h_\rho^1, h_\omega^{0,1}. \quad (9.3)$$

In this chapter, we follow historical usage in denoting f_π as a weak coupling constant, whereas elsewhere in our review it represents the charged pion decay constant. The subscripts in eq. (9.3)

indicate the meson which couples weakly to the nucleon and the superscript denotes the change in isospin ΔI (pion exchange is automatically $\Delta I = 1$ by Barton's theorem [159] and is generally not indicated in this fashion). The form of the resulting potential is

$$\begin{aligned}
 V_{12}^{p.v.} = & \frac{f_\pi g_{\pi NN}}{2^{1/2}} i \left(\frac{\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2}{2} \right)^z (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \left[\frac{\mathbf{p}_1 - \mathbf{p}_2}{2M}, f_\pi(r) \right] \\
 & - g_\rho \left(h_\rho^0 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + h_\rho^1 \left(\frac{\boldsymbol{\tau}_1 + \boldsymbol{\tau}_2}{2} \right)^z + h_\rho^2 \frac{(3\tau_1^z \tau_2^z - \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)}{2(6)^{1/2}} \right) (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \cdot \left\{ \frac{\mathbf{p}_1 - \mathbf{p}_2}{2M}, f_\rho(r) \right\} \\
 & + i(1 + \chi_v) \boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2 \cdot \left[\frac{\mathbf{p}_1 - \mathbf{p}_2}{2M}, f_\rho(r) \right] - g_\omega \left(h_\omega^0 + h_\omega^1 \left(\frac{\boldsymbol{\tau}_1 + \boldsymbol{\tau}_2}{2} \right)^z \right) (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \cdot \left\{ \frac{\mathbf{p}_1 - \mathbf{p}_2}{2M}, f_\omega(r) \right\} \\
 & + i(1 + \chi_s) \boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2 \cdot \left[\frac{\mathbf{p}_1 - \mathbf{p}_2}{2M}, f_\omega(r) \right] - (g_\omega h_\omega^1 - g_\rho h_\rho^1) \left(\frac{\boldsymbol{\tau}_1 - \boldsymbol{\tau}_2}{2} \right)^z (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \left\{ \frac{\mathbf{p}_1 - \mathbf{p}_2}{2M}, f_\rho(r) \right\} \\
 & - g_\rho h_\rho'^1 i \left(\frac{\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2}{2} \right)^z (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \left[\frac{\mathbf{p}_1 - \mathbf{p}_2}{2M}, f_\rho(r) \right], \tag{9.4a}
 \end{aligned}$$

where

$$f_\pi(r) = \frac{e^{-m_\pi r}}{4\pi r}, \quad f_\rho(r) = f_\omega(r) = \frac{e^{-m_\rho r}}{4\pi r} \tag{9.4b}$$

and

$$\frac{g_{\pi NN}^2}{4\pi} = 14.5, \quad \frac{g_\rho^2}{4\pi} = \frac{1}{9} \frac{g_\omega^2}{4\pi} \simeq 2.4 \tag{9.4c}$$

are the strong, empirically determined, vertex couplings.

The weak couplings given above *cannot* be directly measured. Nevertheless they can be estimated and a great deal of effort has been expended on this problem [160]. What is required is to calculate a matrix element of the form

$$\langle MN | V^\mu A_\mu | N \rangle \tag{9.5}$$

where M is either a vector or a pseudoscalar meson.

The original approaches to the vector meson coupling used the factorization approximation [157]

$$\langle MN | V^\mu A_\mu | N \rangle \simeq \langle M | V^\mu | 0 \rangle \langle N | A_\mu | N \rangle \tag{9.6}$$

wherein the vector meson is created by the polar current while the axial current connects the nucleon pair. It was later realized, however, that this procedure is inconsistent with the quark model. The point is that if both V_μ and A_μ are charge-carrying currents, emission of neutral mesons would appear to be forbidden. However in the quark model, a local product of two charged currents may be Fierz-rotated

into a neutral current pair which *does* allow coupling to a neutral vector meson. Thus arose the so-called modified factorization technique, which sums diagrams which connect the meson to the vacuum in all possible ways [161].

The pion coupling was not calculable with this method because of the conservation of the vector current. However, it was realized that PCAC and current algebra enabled f_π to be related to a combination of empirically determined hyperon decay amplitudes, at least for the charged current (Cabibbo) Hamiltonian [162]. Although some degree of cancellation takes place in this procedure [163], it appeared to be the most reliable way of predicting the elusive pion coupling.

At about the same time, techniques were proposed by McKellar and Pick [164] which enabled both vector meson and pion amplitudes to be calculated by similar methods. The idea here was to utilize the symmetry $SU(6)_w$ which relates the desired charged current amplitudes to empirical hyperon decay parameters. Although the predictions for f_π were identical to those obtained with the current algebra-PCAC approach, the vector meson amplitudes were quite different. Also, there existed an additional vector meson coupling present in this method which could not be calculated.

Results of both calculation approaches are indicated in table 9.1 where the discrepant values for the vector meson terms are clearly displayed. The quoted numbers are for the charged current or Cabibbo model only since the symmetry approach is not able to treat the neutral current counterpart.

The solution to this calculational dilemma was provided by quark model calculations which enumerated the ways by which a three-quark initial state could couple to a four-quark – one antiquark final state via a four-quark weak Hamiltonian [165]. The possible diagrammatic ways are displayed in fig. 9.1 and correspond precisely to the reduced amplitudes obtained via the $SU(6)_w$ analysis. Specifically, figs. 9.1a and 9.1b are those which can be related to hyperon decay amplitudes, while fig. 9.1c corresponds to the additive contribution which is undetermined via symmetry techniques. However in the quark model analysis, this “undetermined” piece is clearly seen to correspond to the usual modified factorization contribution, which should be *added* to the symmetry-related contributions in figs. 9.1a, b. A further bonus is that neutral current contributions can also be handled in a parallel fashion. Thus, for the first time a reasonably reliable technique appears to be available with which to estimate the $\Delta S = 0$ weak parity-violating potentials.

Unfortunately, things are not quite as simple as indicated above. There are two reasons for this. The first involves the usual strong interaction enhancement factors which are rather larger than those

Table 9.1

Calculated values for weak parity-violating couplings. All numbers are quoted in terms of the so-called sum rule value $g_\pi = -3.8 \times 10^{-8}$. These predictions are for the charged-current (Cabibbo) model, and the $SU(6)_w$ results are given subject to an uncalculable additive correction

	PCAC/factorization	$SU(6)_w$
f_π	1	1
h_ρ^0	15	-80
h_ρ^1	-0.7	
$h_\rho^{1'}$		
h_ρ^2	-58	0
h_ω^0	6	-30
h_ω^1	-2	2

Fig. 9.1. Coupling of a three-quark state to a four-quark one-antiquark state via a four-quark weak Hamiltonian.

appearing in the $\Delta S = 1$ Hamiltonian [166]. Associated with these more significant effects are strong cancellations which then increase the sensitivity to the precise value of the renormalization group parameters. Secondly the $SU(6)_w$ symmetry route connecting pion and vector meson amplitudes is fraught with some uncertainty due to the large π, ρ mass splitting. Quark model calculations indicate that the vector meson amplitudes should be reduced by an overall multiplicative factor whose precise value is uncertain and whose size depends on the relativistic nature of the quark wavefunctions [165]. However, even with these uncertainties it is possible to place fairly rigorous bounds on the range of values permitted for the various weak couplings, as given in table 9.2. Both charged and neutral current contributions are indicated. The only significant change introduced by the presence of neutral currents is that the pion coupling is increased tremendously. This is because $\Delta I = 1$ effects in the Cabibbo Hamiltonian are suppressed by a factor $\tan^2 \theta_c \approx 0.05$ [155].

Within the range of values indicated, various groups have estimated more precise values of the weak couplings by making assumptions about the uncertainties discussed above [167]. Typical values are given in table 9.3 for different calculational techniques. It is seen that there is rough agreement between the various estimates. However, theoretical error bars are difficult to estimate. Let us emphasize that the various quark model calculations of f_π are basically similar. The differing numerical values have to do with assumptions concerning both quark masses (which are needed to evaluate certain “penguin-

Table 9.2

Given is the range of values which can be assumed by the weak parity-violating couplings under various assumptions about strong interaction effects, quark model wavefunctions, etc. Values are quoted both for the Cabibbo (charged current) model and for the full Weinberg-Salam (charged plus neutral current) picture

	Charged current (Cabibbo) model	Charged plus neutral (Weinberg-Salam) model
f_π	$0 \rightarrow 1$	$0 \rightarrow 30$
$h_\rho^{(0)}$	$15 \rightarrow -64$	$30 \rightarrow -81$
$h_\rho^{(1)}$	$0 \rightarrow 0.7$	$-1 \rightarrow 0$
$h_\rho^{(1')}$		-2
$h_\rho^{(2)}$	-58	$-20 \rightarrow -29$
$h_\omega^{(0)}$	$0 \rightarrow -2$	$-5 \rightarrow -2$

type" matrix elements [168]) and strong interaction effects. All calculations agree that f_π should be *much* larger than its PCAC-current algebra Cabibbo model value of unity. (Throughout, we use units in multiples of $g_\pi \equiv 3.8 \times 10^{-8}$, which is the original Cabibbo model prediction.)

In the case of the ρ coupling, the calculated values were obtained by very different means. The number given by Desplanques et al. [165] is obtained using the $SU(6)_w$ symmetry scheme discussed above, while that quoted by Palle et al. [169] is obtained via use of $1/2^-$ resonance pole contributions, as shown in fig. 9.2. In principle this is only a piece of the total contribution but its good agreement with the corresponding symmetry value is gratifying.

In order to confront these weak potentials with experiment, one requires a set of independent empirically determined parameters which are sensitive to parity violation. The most direct of these involve the nucleon-nucleon system itself. A great deal of effort has involved the longitudinal asymmetry measured in proton-proton scattering (the difference in scattering cross section for protons polarized parallel and antiparallel to the beam direction)

$$A_L = (\sigma_+ - \sigma_-)/(\sigma_+ + \sigma_-) \tag{9.7}$$

which has been measured at 15 MeV at LAMPF and 45 MeV at SIN [170].

A second approach has involved radiative capture of thermal neutrons by protons

$$N + P \rightarrow d + \gamma. \tag{9.8}$$

Table 9.3

Values for weak parity-violating couplings as calculated by various theoretical groups

	f_π		$h_\rho^{(0)}$	$h_\rho^{(1)}$
Guberina, Tadic and Trampetic [167]	+25	Palle, Picek, Tadic and Trampetic [169]	-25	-0.5
Desplanques, Donoghue and Holstein [165]	+12	Desplanques, Donoghue and Holstein [165]	-30	-0.5
Korner, Kramer and Willrodt [167]	+47			

Fig. 9.2. Weak coupling of ρ -meson to nucleons via $1/2^-$ pole contributions.

In this case Lobashov et al. [171] have measured the circular polarization P_γ of the resulting gamma ray, while a Grenoble experiment has sought the asymmetry in the direction of photon emission A_γ with respect to the polarization of the neutron beam [172].

Results of these experiments are given in table 9.4, where they are compared with theoretical predictions based upon the estimates of the weak couplings given in ref. [165]. Agreement is seen to be quite satisfactory in each case. It should be noted that a number of years ago Lobashov reported a much higher value of P_γ which was much quoted. However, he has now reported that on the basis of new data his earlier number should be discounted, as it suffered from systematic errors which were not adequately controlled [171].

A second class of experiments deals with low- A targets, which are presumably amenable to reliable calculations. Three experiments in this class are measurements of the asymmetry in scattering of longitudinally polarized protons from deuterium [173] and ^4He targets [174], and the resonant capture of α particles on deuterium leading to the 1^- excited state of ^6Li , a process which is forbidden if parity is conserved [175]. Current experimental results are compared with theoretical predictions in table 9.4, where it is seen that agreement is again quite reasonable.

A fourth measurement in this class is the asymmetry in capture of thermal neutrons on deuterium

which has been performed at Grenoble [176]. Results have been reported and are compared with the approximate Faddeev calculations of Hadjimichael et al. [177] in table 9.4. Possible disagreement occurs, but it is difficult to say anything definitive due to the preliminary nature of the theoretical work and the considerable experimental uncertainty.

A third group of experiments involves medium A (p and s - d shell) nuclei for which hopefully reasonably precise shell model calculations can be performed. The most interesting of these involve a

Table 9.4

Experimental and theoretical values for various two-nucleon parity-violating processes. Here the theoretical values are for the “best value” DDH predictions

P + P	$A_L(15 \text{ MeV}) \times 10^7$	-1.7 ± 0.8 [170a]	-1.6
	$A_L(45 \text{ MeV}) \times 10^7$	-2.3 ± 0.8 [170b]	-2.9
P + d	$A_L(15 \text{ MeV}) \times 10^7$	0.3 ± 0.4 [172]	-0.5
P + α	$A_L(45 \text{ MeV}) \times 10^7$	-3.3 ± 0.9 [174]	-3.3
N + P $\rightarrow$ d + γ	$P_\gamma \times 10^7$	1.8 ± 1.8 [171]	0.6
	$A_\gamma \times 10^7$	0.3 ± 0.4 [172]	-0.5
N + d $\rightarrow$ t + γ	$A_\gamma \times 10^8$	7.8 ± 3.4 [176]	1.0
$^6\text{Li}(0^+, 2.08 \text{ MeV})$	$\Gamma_a(\text{eV}) \times 10^8$	$<10^3$ [175]	3

pair of levels with identical spin and opposite parity which are mixed by the parity-nonconserving potential,

$$\begin{aligned} |J_1\rangle &= |J^+\rangle + \varepsilon |J^-\rangle \\ |J_2\rangle &= |J^-\rangle - \varepsilon |J^+\rangle \end{aligned} \quad \varepsilon = \frac{\langle J^- | V^{\text{pv}} | J^+ \rangle}{E^+ - E^-}. \quad (9.10)$$

The level splitting can be so small, e.g., 5.7 keV in ^{21}Ne , that it is sufficient to treat the problem as a simple two-state system. An added advantage is that in eq. (9.10), since a typical matrix element of the parity-violating potential is of order of an electron volt ($R_w \times$ a typical nuclear level spacing) and $E^+ - E^-$ is of order several keV, the level mixing can be amplified by factors as large as several hundred. Another device may be utilized in order to enhance the effect even more. Since parity violation is generally observed via interference between a parity-conserving and a parity-violating amplitude, it is useful to choose a case in which the parity-allowed amplitude is suppressed for some reason. This effectively increases the strength of the parity-violating effect. Thus for example, in a situation where one has nearby 2^+ , 2^- levels which decay to a 1^+ state via emission of a photon, it is generally advisable to measure the circular polarization resulting from the $2^+ \rightarrow 1^+ \gamma$ transition rather than the $2^- \rightarrow 1^+ \gamma$ case since the parity allowed (forbidden) multipolarity is as shown below

	<u>$2^+ \rightarrow 1^+ \gamma$</u>	<u>$2^- \rightarrow 1^+ \gamma$</u>	
<i>P</i> allowed	M1, E2	E1	
<i>P</i> forbidden	E1	M1, E2	(9.11)

Thus since M1, E2 transitions are generally suppressed with respect to E1 by a factor

$$kr \ll 1 \quad (9.12)$$

one can effectively enhance the parity-violating effect even further by this choice.

Unfortunately, while careful choice of levels can indeed be used as a sort of nuclear amplifier, one also pays a price in that considerable “noise” is present in the form of wavefunction uncertainties. That this is a serious consideration is easily demonstrated. Calculations of parity-mixing amplitudes

$$\langle J^- | V^{\text{p.v.}} | J^+ \rangle \quad (9.13)$$

are, of course, performed with the best available shell model wavefunctions. In a harmonic oscillator basis, one has then

$$\begin{aligned} |J^+\rangle &= |\psi_{0\hbar\omega}\rangle + \varepsilon |\psi_{2\hbar\omega}\rangle + \cdots \\ |J^-\rangle &= |\psi_{1\hbar\omega}\rangle + \varepsilon |\psi_{3\hbar\omega}\rangle + \cdots \end{aligned} \quad (9.14)$$

where the dominant positive, negative parity configurations ($0\hbar\omega$, $1\hbar\omega$ respectively) are augmented by residual $2\hbar\omega$, $3\hbar\omega$ excitations which are hopefully small if the shell model picture is to be reasonably precise. However, these “core polarization” corrections are not necessarily as small as they might appear to be. That is, in calculations of magnetic moments or Gamow–Teller matrix elements, etc. where the one-body operator does not connect the $0\hbar\omega$ – $2\hbar\omega$ or $1\hbar\omega$ – $3\hbar\omega$ components, we have

$$\langle J^+ | 0 | J^+ \rangle = \langle \psi_{0\hbar\omega} | 0 | \psi_{0\hbar\omega} \rangle + O(\varepsilon^2) \quad (9.15)$$

so that such corrections are second order. However, in the case of weak parity mixing

$$\langle J^- | V^{\text{p.v.}} | J^+ \rangle = \langle \psi_{1\hbar\omega} | V^{\text{p.v.}} | \psi_{0\hbar\omega} \rangle + O(\varepsilon) \quad (9.16)$$

so that core polarization can be considerably more significant. That this is indeed an important correction has been confirmed experimentally by measuring beta decay rates between states which are isotopic analogs of the states which are weakly mixed. The idea here, as suggested by Bennett, Lowry and Krien [178], is that since the pion exchange contribution to the axial current is simply an isotopically rotated version of $V_{\pi}^{\text{p.v.}}$, then from measurement of the beta decay matrix element, the corresponding weak mixing matrix element due to pion exchange can be determined empirically. (Actually, of course, the two-body piece of the axial current cannot be kinematically separated from the one-body operator contribution. However, Haxton has noted that the ratio of two-body to one-body matrix elements is essentially model independent [179].) This program has been carried out both in the $A = 18$ and $A = 19$ systems [180], and one finds that

$$\frac{\langle J^- | V_{\pi}^{\text{pv}} | J^+ \rangle^{\text{Expt.}}}{\langle J^- | V_{\pi}^{\text{pv}} | J^+ \rangle_{0\hbar\omega-1\hbar\omega}^{\text{Thy.}}} \simeq 0.33 \quad (9.17)$$

for both cases. Here the theoretically predicted matrix element is calculated in a full $0\hbar\omega$ – $1\hbar\omega$ basis [181]. Thus there is a rather substantial renormalization taking place. That core polarization effects are at the root of this discrepancy has been confirmed by a $0\hbar\omega$ – $1\hbar\omega$ – $2\hbar\omega$ model space calculation for the $A = 18$ system by Haxton [1979], which gives

$$\frac{\langle J^- | V_{\pi}^{\text{pv}} | J^+ \rangle_{0\hbar\omega-1\hbar\omega-2\hbar\omega}}{\langle J^- | V_{\pi}^{\text{pv}} | J^+ \rangle_{0\hbar\omega-1\hbar\omega}} \sim \frac{1}{3}. \quad (9.18)$$

Since ρ , ω exchange matrix elements should scale similarly, we shall analyze these s–p shell experiments by applying a uniform renormalization factor of 0.33 to the $0\hbar\omega$ – $1\hbar\omega$ space predictions of Gibson, Haxton and Henley [181]. There exist three key experiments here. Two involve the measurement of circular polarization in a nuclear electromagnetic transition. The third involves the detection of

an asymmetry in the electromagnetic decay of a polarized nuclear excited state [183]. Comparison of these results with theoretical predictions is given in table 9.4. Again agreement is quite satisfactory.

An interesting test of the consistency of the meson exchange approach is provided by noting that there are only two dominant weak couplings, f_π and $h_\rho^{(0)}$. If we use these as effectively free parameters, one can fit the low A experiments on P-P and P- α to determine

$$f_\pi = 13 \pm 8, \quad h_\rho^{(0)} = -26 \pm 10 \quad (9.19)$$

or the s-p shell results in ^{21}Ne , ^{19}F yielding

$$f_\pi = 10 \pm 2, \quad h_\rho^{(0)} = -27 \pm 6. \quad (9.20)$$

The consistency here is gratifying and indicates the basic validity of this approach. One can also utilize this empirically determined value of f_π to restrict models of the weak neutral current. In a generalized $\text{SU}(2)_L \times \text{U}(1)$ picture we find [184]

$$\rho = 1.1 \pm 0.2, \quad \sin^2 \theta_w = 0.0 \pm 0.4 \quad (9.21)$$

in reasonable agreement with the canonical values

$$\rho = 1, \quad \sin^2 \theta_w = 0.223. \quad (9.22)$$

It should be noted that there exist at least three additional classes of experiments which we are not addressing. One involves f-p shell and higher nuclei in which parity-violation has been detected [185]. Examples here are

$$\begin{aligned} ^{181}\text{Ta}(5/2^+ \rightarrow 7/2^+) \quad P_\gamma &= -(5.2 \pm 0.5) \times 10^{-6} \\ ^{180}\text{Hf}(8^- \rightarrow 0^+) \quad A_\gamma &= (1.66 \pm 0.18) \times 10^{-2}. \end{aligned} \quad (9.23)$$

While the nuclear amplification effect is clearly operating well in these cases, the wavefunction “noise” is much too large at present to allow a convincing “signal” to be picked out. The second class is that of higher energy experiments. For example the asymmetry in polarized proton scattering has been measured both at 800 MeV and at 6 GeV [186],

$$\begin{aligned} P + P (800 \text{ MeV}) \quad A_L &= (1.0 \pm 1.6) \times 10^{-7} \\ P + \text{H}_2\text{O} (800 \text{ MeV}) \quad A_L &= (1.7 \pm 3.3) \times 10^{-7} \\ P + \text{H}_2\text{O} (5300 \text{ MeV}) \quad A_L &= (26.5 \pm 6.0) \times 10^{-7}. \end{aligned} \quad (9.24)$$

However, at such energies a simple meson exchange picture is clearly inadequate. An interesting attempt has been made to understand the 6 GeV measurement via a fundamental quark-gluon picture [187]. However, the problem of interpreting such high-energy measurements is not yet solved, and we shall not discuss them further. Finally, a third group of parity-violating experiments involve the

scattering of thermal neutrons from heavy nuclei [188]. However, the existence of a detectable effect is associated with the proximity to threshold of p-wave resonance, and the interpretation of such experiments in terms of weak interaction theory is far from clear [189].

We conclude that after two decades of hard work, both theoretical and experimental, we may finally be reaching the stage where nuclear parity violation is able to provide reasonable constraints on the fundamental interaction which produces it.

10. Weak decays of hypernuclei

An additional manifestation of the nonleptonic weak interaction is that of the decay of a hypernucleus. During the past decade or so, much attention has been focused on the properties of Λ -hypernuclei, the study of which has yielded rich and valuable information concerning the Λ -N force. Typically such hypernuclei are produced via a (K^-, π^-) reaction, utilizing [190] kinematics wherein the resulting Λ is produced with $q \approx 0$. The Λ is not generally left in its ground state. It rather proceeds there via emission of a series of γ rays, the study of which reveals the energy level differences of hypernuclear states.

Although considerable experimental effort has been devoted to the study of this production and cascade process, surprisingly little work has been performed analyzing what occurs once the Λ has finally reached its ground ($1S_{1/2}$) state. The answer is, of course, that the Λ decays, but there are interesting aspects of this process which we now discuss.

A free Λ hyperon decays primarily via the modes

$$\begin{array}{l} \nearrow P + \pi^- \\ \searrow N + \pi^0 \end{array} \quad (10.1)$$

However, it was realized early on by Cheston and Primakoff that when a Λ is bound in a hypernucleus, its decay properties change dramatically [191]. The point is that even for free $\Lambda \rightarrow N\pi$ decay the energy and momentum of the final state nucleon

$$T_N = \frac{(M_\Lambda - M_N)^2 - m_\pi^2}{2M_\Lambda} \sim 6 \text{ MeV} \quad (10.2)$$

$$p_N = \sqrt{(T_N + M_N)^2 - M_N^2} \sim 100 \text{ MeV}$$

are such that except for the very lightest nuclei the Pauli exclusion principle forbids the $N\pi$ decay of a Λ confined to a nuclear medium. The suppression is actually even somewhat stronger than just indicated since typically the Λ is bound by 5–25 MeV, which reduces the available free nucleon energy even further.

Thus, the dominant decay mode within a nucleus becomes the weak nonmesonic process

$$\Lambda + N \rightarrow N + N \quad (10.3)$$

which is, of course, unavailable to a free Λ . This reaction is the $\Delta S = 1$ analog of the weak

$$N + N \rightarrow N + N \quad (10.4)$$

interaction responsible for nuclear-parity mixing, but with the important difference that in the $\Delta S = 1$ case, the weak parity-conserving decay mode is also observable (in the $NN \rightarrow NN$ situation the weak parity conserving component is dwarfed by the strong interactions). Note that the energy and momentum available in hypernuclear decay are, if shared equally by both outgoing nucleons

$$T_N \approx \frac{1}{2}(M_\Lambda - M_N) \approx 90 \text{ MeV} . \quad (10.5)$$

$$p_N \approx \sqrt{(T_N + M_N)^2 - M_N^2} \approx 420 \text{ MeV}$$

which is well above the Fermi energy and momentum. Thus the exclusion effect does not act to suppress the nonmesonic mode. An indication of the significance of the Pauli suppression is shown in fig. 10.1, where the measurement of the nonmesonic ($\Lambda N \rightarrow NN$) to mesonic ($\Lambda \rightarrow N\pi$) branching ratio is depicted as a function of nuclear mass number A . It is evident that once $A \geq 10$ the mesonic decay becomes but a small fraction ($\leq 15\%$) of the dominant nonmesonic process [192].

A basic question which can be asked about the hypernuclear decay is its strength relative to the decay rate of a free Λ —is the Λ lifetime larger or smaller in heavy nuclei than in free space? Experimentally there has been but a single measurement of the hypernuclear lifetime for a nucleus with $A > 5$, viz., a Berkeley experiment on $^{16}_\Lambda\text{O}$ yielded [193]

$$\tau_\Lambda^{\text{free}} / \tau_\Lambda^{^{16}\text{O}} \approx 3 \pm 1 . \quad (10.6)$$

This is a low statistics experiment with sizable background contamination, however, and confirmation by a soon to be announced $^{12}_\Lambda\text{C}$ experiment is anxiously awaited [194].

On the theoretical side, the most comprehensive analysis was made by Adams in 1967 [195] who pictured the $\Lambda N \rightarrow NN$ interaction as taking place via exchange of a single virtual meson (π , K , ρ , K^* , ...) analogous to the theory underlying the analysis of nuclear parity violation (one mesonic vertex is strong and the other is weak). Adams' calculation, performed in the large A (nuclear matter) limit,

Fig. 10.1. Ratio of nonmesonic ($\Lambda N \rightarrow NN$) to mesonic ($\Lambda \rightarrow N\pi$) branching ratios as a function of A .

indicated that

$$\tau_A^{\text{free}} / \tau_A^{\text{Nucl. Matt.}} \approx \frac{1}{20}. \quad (10.7)$$

This very lengthened lifetime, in stark contrast to the shortened lifetime observed at LBL, is apparently due to two primary effects:

- i) exchanges other than that of the pion are strongly suppressed by short range correlation effects in both initial and final states,
- ii) for pion exchange the smallness of the decay rate appears to arise from an accidental cancellation in the dominant channel.

It is obvious that this calculation, if valid, indicates a severe difficulty in understanding the experimental picture. Thus it is of great interest to reexamine the Adams analysis, hopefully even to the extent of treating a finite nucleus.

A second experimental question which warrants theoretical scrutiny is the rate for proton-stimulated Λ decay

$$\Lambda + P \rightarrow N + P \quad (10.8)$$

compared to that of the neutron-stimulated analog

$$\Lambda + N \rightarrow N + N. \quad (10.9)$$

The experimental situation [192] is indicated in fig. 10.2, which shows that

$$R_{\text{PN}} \equiv \Gamma(\Lambda P \rightarrow NP) / \Gamma(\Lambda N \rightarrow NN) \sim 2 \quad (10.10)$$

for $A \geq 15$. Were only pion exchange operative, we might expect

$$R_{\text{PN}} \gg 1 \quad (10.11)$$

Fig. 10.2. Measurement of $R_{\text{PN}} \equiv \Gamma(\Lambda P \rightarrow NP) / \Gamma(\Lambda N \rightarrow NN)$.

since the ${}^3S_1 \rightarrow {}^3D_1$ transition, which dominates the $\Lambda P \rightarrow NP$ interaction rate, is forbidden for two neutrons. Thus again the experimental situation appears to be in contradiction to our expectation based on the Adams calculations and appears to indicate the importance of exchanges of mesons other than the pion.

There have been three recent analyses of weak hypernuclear processes. One by Cheung, Kisslinger and Heddle utilizes the so-called hybrid quark – two-baryon model of the Λ –N interaction [196]. In this approach one makes use of projection operators in coordinate space to define ΛN wavefunctions as conventional nuclear wavefunctions if $r \geq r_0$ and six quark relativistic shell model wavefunctions for $r \leq r_0$, with $r_0 \approx 0.8$ fm. The direct pion exchange contribution is found to be quite small. However, there is a large amplitude which arises from the six quark sector, which yields for nuclear matter

$$\tau_A^{\text{Free}} / \tau_A^{\text{Nucl. Matt.}} \sim 3 \quad (10.12)$$

in good agreement with the experimental result and in contradiction with the Adams prediction.

A second calculation is that of McKellar and Gibson who examined the case of pion and rho exchange only [197]. In the case of pion exchange, the $\Lambda N \pi$ weak vertices were determined experimentally, while in the case of the ρ , simple factorization estimates were employed in order to calculate the $\rho \Lambda N$ couplings. Their result is

$$\tau_A^{\text{Free}} / \tau_A^{\text{Nucl. Matt.}} \sim 1.3 \text{ or } 0.7 \quad (10.13)$$

where the bimodal result arises since the relative phase of the π -exchange and ρ -exchange contributions is not known in this semiphenomenological approach.

The most comprehensive analysis is that of de la Torre et al. [198] who utilize quark model techniques identical to those used in ref. [165] for nuclear parity-violation in order to calculate the weak parity-violating vertices. A pole model is employed in the parity-conserving case. All relevant light mesons (π , ρ , ω , K, K^* , η) are utilized and the calculation is performed in nuclear matter. The results are displayed in table 10.1 as a function of the parity conserving/parity violating contributions and of the particular meson being exchanged.

We see that the predicted lifetime of the Λ bound in nuclear matter is not much different from that of the free Λ . However, this must be regarded as purely accidental since the relevant decay mechanisms

Table 10.1
Indicated are the hypernuclear decay rates predicted by de la Torre et al. [198]. The notation $+\eta$ indicates that $\pi + \eta$ exchange is included, $+K$ means $\pi + \eta + K$, etc.

Exchanged meson	$\Gamma(\Lambda A)/\Gamma(\Lambda)$
π	1.34
$+\eta$	1.20
$+K$	0.92
$+\rho$	0.81
$+\omega$	0.86
$+K^*$	0.98

are markedly different. We note also that although inclusion of mesons other than the pion does not have a very noticeable effect on the overall decay rate, the existence of nonpionic exchange is clearly indicated in two ways. First we see that the importance of the parity-violating sector becomes much more substantial. Thus, if one looks for a correlation of the form $\mathbf{J} \cdot \mathbf{p}$ between the momentum of one of the outgoing nucleon and the spin of the Λ and of the decaying nucleus, the effect should be much larger than the pion exchange prediction of several per cent.

A second probe for the effect of nonpionic exchange is provided by measurement of R_{PN} . The pion exchange only prediction

$$R_{\text{PN}}^{\pi} \sim 1/16 \quad (10.14)$$

becomes

$$R_{\text{PN}}^{\text{Total}} \sim 1/2.2. \quad (10.15)$$

We note that the preliminary data shown in fig. 10.2 do appear to be inconsistent with the pion-only-exchange scenario but more and better data would be useful.

For completeness, we note that some of the very early experimental work is the area of hypernuclear decay involved emulsion studies and was analyzed by Block and Dalitz [199]. Here too there appeared to exist a sharp disclaimer of the pion-only-exchange hypothesis. Indicated was a dominance of transitions to $I = 1$ final states, and a measurement of the ratio of proton-stimulated to neutron-stimulated capture to the $I = 1$ channel gave

$$R_{\text{P1}}/R_{\text{N1}} \sim 0.5 \quad (10.16)$$

consistent with the validity of the $\Delta I = \frac{1}{2}$ rule. The quark calculation of de la Torre et al. also yields a $\Delta I = \frac{1}{2}$ rule, and predicts the neutron/proton capture ratio to be

$$\frac{\Gamma(\Lambda \underline{N} \rightarrow \underline{NN})}{\Gamma(\Lambda \underline{P} \rightarrow \underline{NP})} = \begin{cases} 0.056 & \pi \text{ only} \\ 0.419 & \text{all mesons.} \end{cases} \quad (10.17)$$

Since the numerator involves purely $I = 1$ final states, while the denominator is a mixture of $I = 0, 1$, we see that pion-only-exchange is in very strong contradiction to these emulsion results, while the presence of additional exchanges is in better but hardly good agreement with these experimental claims. Of course, it must be kept in mind that the emulsion data is on very light ($A = 4, 5$) hypernuclei while the calculations are for nuclear matter.

In general then, both theory and experiment in the field of hypernuclear decay are in a preliminary stage, and much additional effort is required before we develop real confidence in either.

11. Recent developments

Thus far we have discussed the weak interaction phenomenology of quarks bound in color-singlet configurations by QCD interactions. Our treatment of the quark wavefunctions and transition amplitudes has been analytic throughout, with fundamental properties like current algebra, CVC, etc.

being employed where possible, and model dependent descriptions applied when necessary. There are two topics – computation of nonleptonic amplitudes by means of lattice gauge theory and the weak interactions of chiral solitons – which lie outside the context of this approach. Both are recent developments, and hold promise for substantial research activity well into the future. For this reason we include a background description and status report of each topic in this review.

11.1. Nonleptonic interactions and lattice gauge theory

Thus far there have been lattice estimates of K^0 -to- $\bar{K}^0$ and K -to- π matrix elements of four-fermion local operators [200–203]. As we have already shown, the latter matrix elements can be related by means of current algebra methods to the $K\pi\pi$ amplitudes.

To see how much a computation can proceed, consider for definiteness $\langle \pi^+ | O | K^+ \rangle$, where O represents any of the four-fermion ‘left–left’ or ‘left–right’ local operators appearing in the operator product expansion of the $|\Delta S| = 1$ nonleptonic Hamiltonian. We start with the quantity

$$\langle 0 | \chi_{\pi^+}(x) O(y) \chi_{K^+}^\dagger(z) | 0 \rangle \quad (11.1)$$

where χ_{π^+} , χ_{K^+} are interpolating fields for the respective particles. Upon inserting complete sets of states and employing translation invariance, we obtain

$$\sum_{j,k} \langle 0 | \chi_{\pi^+}(0) | j \rangle \langle j | O(0) | k \rangle \langle k | \chi_{K^+}^\dagger(0) | 0 \rangle \\ \times \exp\{i\mathbf{p}_j \cdot (\mathbf{y} - \mathbf{x})\} \exp\{i\mathbf{p}_k \cdot (\mathbf{z} - \mathbf{y})\} \exp\{-E_j(\tau_x - \tau_y)\} \exp\{-E_k(\tau_z - \tau_y)\}, \quad (11.2)$$

where $\tau_{x,y,z}$ are Euclidean time variables. The desired matrix element can be extracted by summing over any two of x, y, z (thus rendering p_j and p_k zero), taking $\tau_x - \tau_y$ and $\tau_y - \tau_z$ large compared to the inverse mass differences between the ground and first-excited states, and dividing out the χ -factors and τ -dependent exponentials. Up to this point, the calculation has proceeded strictly according to quantum theory; lattice methods have yet to appear.

The χ -factors, being strong interaction quantities, can be computed in conjunction with the weak transition of interest. Each χ -factor is expressible as a quark bilinear, e.g., $\chi_{\pi^+} = \bar{d}\gamma_5 u$ and $\chi_{K^+}^\dagger = \bar{u}\gamma_5 s$. There are two processes which can contribute to the given amplitude. These are depicted in figs. 11.1 and 11.2. In each, quark propagators are required in order to proceed with the calculation. These are

Fig. 11.1. Lattice calculation of ‘figure-eight graph’.

Fig. 11.2. Lattice calculation of ‘eye graph’.

determined with the Monte Carlo procedure, which simulates an average over gauge configurations. For any quantity O the averaging can be expressed as

$$\langle O \rangle = \int D[U] \exp(-S_{\text{eff}}(U)) O(U) / \int D[U] \exp(-S_{\text{eff}}(U)) \quad (11.3)$$

where U is the lattice-gauge link variable. At present, only calculations using the ‘quenched approximation’, which employs a pure gauge action (i.e., no explicit fermionic degrees of freedom), have been performed. In practice the averaging is replaced by a sampling of independent gauge configurations. For example in ref. [200] fourteen gauge configurations are generated on a $10^3 \times 20$ lattice, whereas in ref. [201], six configurations on a $6^3 \times 10$ lattice are used. For the latter, initialization of the lattice is followed by five thousand thermalization sweeps, and then six hundred sweeps between each sampling.

The process depicted in fig. 11.1 (‘figure-eight graph’) is directly amenable to the lattice-theoretic approach in that it involves the propagation of quarks from a *fixed* site (the location of the four-quark operator O). Thus one need only generate one column of the inverse ‘quark matrix’ to describe the propagation of quarks from this site to all others. This is not the case for the diagram in fig. 11.2 (‘eye graph’), where not all propagators begin or end at any one point. Hence the usual approaches would appear to take prohibitively much computer time. For this reason, the ‘eye-graph’ has not yet been treated within the framework of Monte Carlo sampling. However an interesting discussion of an alternative approach is given in ref. [202], and work is in progress to implement it.

Both refs. [200] and [201] have provided numerical estimates of $\langle K^0 | O_{\Delta S=2} | \bar{K}^0 \rangle$ and π -to-K matrix elements of various $\Delta S = 1$ operators. The parameters appearing in such computations are a nonzero lattice spacing a , an input bare coupling constant g_0 , and a bare mass parameter κ . Reference [200] employs $g_0^2 = 1$, $\kappa = 0.150$ and 0.155 , $a^{-1} = 2 \text{ GeV}$ whereas in ref. [201], $g_0^2 = 1$, $0.145 \leq \kappa \leq 0.150$, and $a^{-1} = m_K$. For reference, the chiral limit $m_\pi \rightarrow 0$ corresponds to $\kappa_c \approx 0.157$ with $m_\pi \sim (\kappa_c - \kappa)^{1/2}$. Radiative corrections renormalize the bare amplitudes and parameters with the lattice spacing providing an ultraviolet cutoff. A one-loop calculation involving four-fermion operators appears in ref. [203].

The numerical results thus far are encouraging but, for technical reasons, should be regarded as preliminary. For example, Monte Carlo estimates of $\langle K^0 | O_{\Delta S=2} | \bar{K}^0 \rangle$ are not inconsistent with those appearing in the literature (see section 8). However a strong coupling calculation gives a value two orders of magnitude larger [201]. The K-to- π amplitudes exhibit enhancement of the penguin operators $O_{5,6}$ over the left-left operator O_1 . As explained earlier, this is to be expected on the basis of helicity suppression. But until the ‘eye-graphs’ can be handled within the Monte Carlo framework, the $\Delta I = \frac{1}{2}$ rule cannot be said to be ‘understood’. Moreover, fluctuations leave the statistical data less and less useful in the chiral limit ($\kappa \rightarrow \kappa_c$).

The presence of chiral symmetry in lattice calculations is interesting in several respects. First, it provides a check on the matrix elements of interest in the chiral invariant limit, $m_\pi \rightarrow 0$. Recall that in this limit, ‘left-left’ operators should vanish as m_π^2 whereas ‘left-right’ operators approach some nonzero constant. Since the $\kappa \rightarrow \kappa_c$ limit is unattainable with Monte Carlo methods at present, one can hope to obtain meaningful predictions by comparing quantities having a corresponding scaling property. For example, results are quoted in ref. [200] for $\langle \pi^+ | O_{LL} | K^+ \rangle / m_\pi^2$. The distance from the chiral limit is evidenced by the ‘pion’ mass having the value 1.2 GeV . In ref. [201], ratios of various chiral operators are plotted versus increasing κ ($0.145 < \kappa < 0.150$) in order to spot tendencies as the chiral limit is approached.

The usual approach to describing fermions on the lattice explicitly violates chiral invariance. A parameter r in the Wilson form of the lattice fermion action

$$\sum_x \left[-\frac{1}{2a} \sum_\mu [\bar{\psi}(x)(r - \gamma_\mu) U_\mu(x) \psi(x + \mu) + \bar{\psi}(x + \mu)(r + \gamma_\mu) U_\mu^\dagger(x) \psi(x)] + \left(m + \frac{4r}{a}\right) \bar{\psi}(x) \psi(x) \right] \quad (11.4)$$

introduces symmetry breaking. Both refs. [200, 201] employ $r = 1$. As discussed in ref. [203], this shows up in the mixing of operators with different chirality by QCD radiative correction.

To summarize, the first generation of lattice-theoretic calculations of nonleptonic phenomena is now underway. Given the historic difficulty of this field, any progress in calculational ability is viewed with great anticipation. Aside from the work reviewed here, much remains to be done. The baryon-to-baryon matrix elements and the full $K\pi\pi$ amplitudes have not yet been explored. It will be interesting to see whether current computer technology is equal to the task.

11.2. Nonleptonic interactions of chiral solitons

Earlier in this review, we have discussed how the chiral Lagrangian approach is useful in treating a variety of mesonic weak interaction phenomena. Recently, following the work of Witten [204], there has been considerable activity in pursuing Skyrme's original idea [205] of describing baryons as soliton solutions of chiral Lagrangians. In this scenario it is essential to employ at least quartic terms in the chiral Lagrangian in order to stabilize the soliton. Working with an $SU(2)$ chiral Lagrangian

$$\begin{aligned} L &= L_0 + L_{\text{Sk}} \\ L_0 &= \frac{F_\pi^2}{4} \text{Tr}(\partial U \cdot \partial U^\dagger) \\ L_{\text{Sk}} &= \frac{-1}{16e^2} \text{Tr}[(\partial U \cdot \partial U^\dagger)^2 - \partial_\mu U \partial_\nu U^\dagger \partial^\mu U \partial^\nu U^\dagger] \end{aligned} \quad (11.5)$$

where $U \in SU(2)$. Adkins, Nappi and Witten [206] were able to demonstrate that the properties of this soliton (called the 'Skyrmion') are in reasonable accord with those of the proton. The mathematical form taken by the Skyrmion at the classical level is

$$U_0 = \exp(iF(r)\tau \cdot x) \quad (11.6)$$

where $F(r)$ is a monotonically decreasing positive function ($F(0) = \pi$). Quantization of the model is affected by casting the theory in terms of collective coordinates $A(t)$. The substitution $U = A(t) U_0(x) A^\dagger(t)$ with $A \in SU(2)$ (i.e., $A = a_0 + i\mathbf{a} \cdot \boldsymbol{\tau}$) results in the Hamiltonian

$$H = M + \frac{1}{8\lambda} \sum_{k=0}^3 \pi_k^2 \quad (11.7)$$

where $\pi_k = -i\partial/\partial a_k$ and M, λ are complicated functionals of F . The wavefunction solutions of eq. (11.7) are products of spherical harmonics and Chebyshev polynomials, representing the spin and isospin content respectively of the quantum nucleon. In ref. [206] the parameters of the model were taken as e (see eq. (11.5)) and F_π . A fit to the nucleon and Δ masses yielded the values $e \simeq 5.5$, and $F_\pi \simeq 65$ MeV (normalized as $F_\pi^{\text{expt}} \simeq 94$). Subsequent to this, the present authors realized that an independent determination of the parameters could be obtained from pion-pion scattering data [207]. The point is that low energy D-wave phase shifts provide information on quartic terms in the effective Lagrangian. There are only two such terms in the limit $m_\pi = 0$ [208], e.g.

$$L^{(4)} = L_{\text{Sk}} + \frac{\gamma}{8e^2} [\text{Tr}(\partial U \cdot \partial U^\dagger)]^2. \quad (11.8)$$

A fit of the parameters F_π, e, γ showed that the Skyrme contribution is roughly three times the other quartic term in eq. (11.8), and implied the values $F_\pi \simeq 75$ MeV and $M_p = 0.88 \pm 0.3$ GeV. The unexpected but successful relation between pion-pion data and the proton mass constitutes a nontrivial consistency check of the soliton approach.

Additional research on the phenomenology of Skyrmions has been carried out in the area of nonleptonic decays. The ‘semiclassical approximation’, which is associated with the fact that the solitons are slowly rotating, has formed the basis of this work. In such an approach there is a hierarchy associated with the number of time derivatives acting on the soliton solution, the lowest order having no time derivatives, the next with one time derivative, and so on. To compute nonleptonic amplitudes requires extension of the SU(2) model to SU(3). The 2×2 matrix of eq. (11.6) becomes the 3×3 matrix

$$\Sigma_0(\mathbf{x}) = \begin{bmatrix} \exp(-iF(r) \boldsymbol{\tau} \cdot \mathbf{x}) & 0 \\ 0 & 1 \end{bmatrix}. \quad (11.9)$$

The wavefunction of the collective coordinates $A(t)$ for baryon α are expressed in terms of SU(3) representation matrices [210]

$$\psi_\alpha(A) = N_\alpha^{-1/2} D_{a_\alpha b_\alpha}^{(N_\alpha)}(A). \quad (11.10)$$

In eq. (11.10), the flavor-SU(3) and spin labels appear respectively in $a_\alpha = (Y_\alpha, I_\alpha, I_{3\alpha})$ and $b_\alpha = (1, S_\alpha, S_{3\alpha})$, and N_α is the dimension of the associated SU(3) representation.

It has been shown in ref. [209] that, to lowest order of the semiclassical approximation, the set of baryon-to-baryon matrix elements of any chiral Lagrangian transforming as $(8_L, 1_R)$ has SU(3) structure $f/d = -5/3$. In the absence of chiral symmetry breaking effects, this will hold for the S-wave nonleptonic amplitudes as well. The prediction $f/d = -5/3$ should be regarded as a successful prediction of the underlying approach. It provides motivation for further exploration of the soliton model. Additional problems worth addressing are the overall normalization of the S-waves and the subject of the P-wave amplitudes.

A systematic method for addressing these questions involves considering pion perturbations about the soliton solution

$$\Sigma = U_\pi A(t) \Sigma_0 A^{-1}(t) U_\pi \quad (11.11)$$

where

$$U_\pi = \exp(i\lambda \cdot \phi/2F_\pi). \quad (11.12)$$

As shown in ref. [211], the structure of eq. (11.11) obeys the dictates of chiral symmetry. A study of S-wave and P-wave amplitudes using the ansatz of eq. (11.11) along with the chiral Lagrangian,

$$L = g \text{Tr}(\lambda_6 \partial_\mu \Sigma \cdot \partial^\mu \Sigma^\dagger) + g' \text{Tr}(\lambda_6 \partial_\mu \Sigma \partial^\mu \Sigma^\dagger \partial_\nu \Sigma \partial^\nu \Sigma^\dagger) + g'' \text{Tr}(\lambda_6 \partial_\mu \Sigma \partial_\nu \Sigma^\dagger \partial^\mu \Sigma \partial^\nu \Sigma^\dagger) \quad (11.13)$$

has been carried out in ref. [212].

For the S-wave emission of a pion no derivatives of pion fields can appear in the Lagrangian, so that

$$\partial_i \Sigma \rightarrow U_\pi A(t) \partial_i \Sigma_0 A^{-1}(t) U_\pi. \quad (11.14)$$

Since only a single pion is emitted, one expands U_π to just first order. Upon carrying out the trace operation in the Lagrangian (11.13) and performing the spatial integral, we obtain for the S-wave decay amplitude of $\alpha \rightarrow \beta \pi^0$,

$$\chi^\dagger(\lambda') \chi(\lambda) A(\alpha \rightarrow \beta \pi^0) = \frac{4\pi}{\sqrt{3} F_\pi} \int_0^\infty dr r^2 I(r) \langle \beta | D_{78}(A) | \alpha \rangle \quad (11.15)$$

where

$$I(r) = g \left(F'^2 + \frac{2 \sin^2 F}{r^2} \right) - g' \left(F'^2 + \frac{2 \sin^2 F}{r^2} \right)^2 - g'' \left(F'^4 - \frac{4 F'^2 \sin^2 F}{r^2} \right). \quad (11.16)$$

Evaluation of the matrix elements in eq. (11.15) naturally reproduces the $f/d = -5/3$ structure found in ref. [209], but evaluation of the radial integrals in eq. (11.16) yield S-wave amplitudes which are smaller than the experimental values by roughly a factor of three. This latter effect is brought about by a severe cancellation between the first two terms in eq. (11.16).

As explained in section 6, the standard treatment of P-wave nonleptonic amplitudes involves baryon and kaon pole terms. The baryon pole terms are proportional to baryon-to-baryon matrix elements of the parity-conserving weak Hamiltonian. The above analysis implies that these will be a factor of three too small in our model and hence similarly for the P-wave amplitudes. Fortunately the soliton approach is not without interest in the P-wave sector because there turns out to be a ‘contact’ contribution; (see fig. 11.3). To our knowledge the contact term has not been heretofore explicitly taken into account, presumably because it vanishes in the soft pion limit.

To see how the contact term arises, we proceed analogously to the S-wave analysis except now focussing attention on terms having one derivative acting on a pion field. After a tedious calculation it is possible to express the contact P-wave amplitude for π^0 emission along the 3-axis as

$$B_{\text{cont}} = \frac{8\pi}{3} \frac{\bar{M}}{F_\pi} g R_0^2 [0.34 D_{63}(A) - 2.94 i \varepsilon_{3jk} D_{6j}(A) D_{3k}(A) + 1.31 (D_{63}(A) D_{38}(A) + D_{33}(A) D_{68}(A))] \quad (11.17)$$

Fig. 11.3. P-wave hyperon decay amplitude, including a 'contact term'.

where $\bar{M}$ is the average of initial and final state baryons, the dimensional quantity $R_0 = 1$ fm arises from the radial integration, and the baryon-to-baryon matrix elements of the $D(A)$ remain to be evaluated. Given the failure of the model to reproduce the correct magnitudes of the S-wave amplitudes, it is probably not meaningful to assign much importance to the values of the contact terms for various decays. Suffice it to say, they are nonnegligible in at least some decays (e.g., $\Lambda \rightarrow N\pi^0$) and should not be ignored in future analyses of the complex of hyperon decays.

We now briefly summarize the results of recent work on weak transitions of solitons. The best result, viz. $f/d = -5/3$ for S-wave amplitudes, is a fairly general statement about the soliton model. It holds in leading order of the semiclassical approximation but makes no assumption about the number of spatial derivatives present in the chiral Lagrangians. Unfortunately, to make additional statements regarding the content of the model, it appears to be necessary to truncate the number of derivatives contained in the chiral Lagrangian. Otherwise one is confronted with a plethora of arbitrary coupling constants. By working in a truncated model with just quadratic and a particular set of quartic couplings, it is possible to determine the coupling constants from $K \rightarrow 2\pi$ and $K \rightarrow 3\pi$ data. As we have just seen, the dynamical implications of this approach are not supported by the hyperon decay data. Yet a potentially important result does emerge, the existence of a new nonpole contribution to the P-wave amplitudes. It is possible to see from a phenomenological viewpoint that something like this is expected [121]. Consider construction of a (nonsoliton) baryonic chiral Lagrangian. It is possible to fit the S-wave amplitudes with a two-operator Lagrangian, each operator with an arbitrary coupling. Extension of this description to the P-waves, via the pole model, fails badly. It is then found possible to add four additional operators which involve a pion derivative to the chiral Lagrangian. Although such an approach contains no predictive power (aside from isospin relations) for P-wave transitions, it does anticipate the presence of nonpole terms such as the ones found here.

It remains to be seen whether further thought and study can extend the existing analysis of soliton nonleptonic decays into a successful quantitative description of hyperon decay.

12. Conclusions

Much of the work surveyed here was performed during the past decade or so and can be characterized as attempts to provide an underlying dynamical framework for the low-energy weak interactions. Comparison with the weak interaction theory described in ref. [1] clearly demonstrates that

a great deal of progress toward this goal has been achieved. The Weinberg–Salam model together with QCD and the quark picture provide a powerful framework which already includes much of the weak interaction structure seen experimentally.

We have attempted to emphasize that it is the *general* features common to most quark models, rather than the details of any specific model which are probed in successful applications to weak phenomena. However, with respect to nonleptonic weak processes, where strong interaction effects are crucial, the simple valence quark model approach might be inadequate. Thus, for example, we have yet to achieve a convincing theoretical explanation for the $\Delta I = \frac{1}{2}$ rule or for the possibly large asymmetry parameter in $\Sigma^+ \rightarrow p\gamma$, nor are present techniques able to convincingly reproduce S-wave and P-wave amplitudes for nonleptonic hyperon decay. Finally, it is not yet clear whether state of the art methods provide a satisfactory picture of weak processes in nuclei, or whether present problems are due to deficiencies in our ability to treat nuclear structure.

It is an interesting question to ask whether there remains any really *fundamental* physics which can be learned from the low energy weak interactions. Certainly CP-violation satisfies this criterion. The KM model is presently consistent with the one experimental measure of CP-nonconservation – the $K^0\bar{K}^0$ mixing parameter ϵ . However, it is quite possible, if the $(b \rightarrow u)/(b \rightarrow c)$ ratio decreases, that this may not be the case in the future. This would indicate the existence of new particles and/or interactions beyond the standard model.

The resolution of the problem of nonleptonic weak transitions *may* also yield fundamental insights into quark dynamics, though it is unlikely to illuminate any new weak interaction physics. It is presently poorly understood how the valence quark model emerges from QCD, and the work on exotic states is an attempt to go beyond the elementary quark picture. If an understanding of nonleptonic weak processes requires going beyond the valence quark picture, then when and if that understanding is achieved, it could shed light on the way in which QCD effects are manifested in nature. Perhaps nonperturbative chiral techniques, such as the Skyrme model, will be of value in this regard. At any rate, chiral perturbation theory will continue to be used as a probe of low-energy quark dynamics, as is the case for example, of the B parameter in $K^0\text{--}\bar{K}^0$ mixing.

As always, additional experimental work bearing on low-energy weak interaction effects would be welcome. This is especially true in the area of CP-violations where a richer range of experimental signals would permit more meaningful tests of competing models. Results on new studies of CP-violation in the kaon system and on tighter limits for the neutron electric dipole moment will be eagerly received. Also a better determination of the KM mixing parameters is needed. As alluded to earlier the $(b \rightarrow u)/(b \rightarrow c)$ ratio plays a central role here, and should be studied through both inclusive and exclusive decays of B mesons. In radiative hyperon decays further experimental work on the asymmetry question, especially for the $\Sigma^+ \rightarrow P\gamma$ mode, will have considerable impact. Continued work on weak effects in nuclear systems will be beneficial to our understanding of both particle physics and nuclear physics.

On the theoretical side, we can look in the future to the promise of numerical lattice methods to yield results which are closer to the fundamental level of QCD. However, given present capabilities, such results will not appear in the immediate future. We can hope for a deeper understanding of how the high-energy interactions of quarks differ from those at low energy and how heavy-quark physics emerges from its light quark counterpart. In phenomenological terms the issue might be to see how the low-energy language, dominated by the use of current algebra and chiral symmetry, eventually evolves into the large mass limit where vacuum saturation might be expected to be more successful. Only then might it be possible to truly understand the intermediate case of charmed particle decay, where neither theoretical method appears entirely adequate!

We conclude that although the field of low-energy weak interactions has matured greatly over the past decade, new and challenging problems still remain before one can ‘close the book’ on the subject.

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