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Proton decay via three-quark fusion

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The contribution to proton decay of three-quark fusion ($q + q + q \rightarrow \bar{l}$) is computed in the MIT bag model. Ambiguities in the calculation are discussed, and comparisons with other approaches are given. A suggestion for displaying uncertainties in proton-lifetime predictions is introduced.

I. INTRODUCTION

Much effort has gone into estimating the proton lifetime in grand unified theories¹ (GUT's). The calculation naturally divides into two parts: (i) the estimation of the superheavy-gauge-boson mass M_X from low-energy parameters, and (ii) the calculation of the Hamiltonian matrix element for whichever proton decay transition is of interest.

The uncertainty in the first of these lies mainly in the lack of precise knowledge of the QCD scale parameter Λ . Early estimates of Λ tended to suggest the range $0.4 \leq \Lambda \leq 0.6$ GeV. Later work implied rather smaller values, say, $0.08 \leq \Lambda \leq 0.2$ GeV.² Unfortunately very recent results have cast doubt on methods used to extract Λ from such diverse experimental data as Υ decay and deep-inelastic lepton scattering.³ Hopefully this situation will improve with additional experimental input and theoretical analysis, so we can expect a more accurate determination of M_X in the future. For now, it seems most sensible to present proton-decay predictions so that the dependence on Λ (and hence M_X) is reasonably easy to perceive. Indeed, this is one of our aims in this paper.

Calculation of proton-decay matrix elements depends on our admittedly limited understanding of low-energy dynamics. The earliest estimates considered decay into uncorrelated free quarks and a lepton [see Fig. 1(a)].⁴ Although the proton is perhaps somewhat too light for such an inclusive treatment, the associated lifetime estimates were near enough to then-existing experimental limits to motivate studies of specific decay modes. Subsequent calculations based on the reaction $q + q \rightarrow \bar{q} + \bar{l}$ [see Fig. 1(b)] yielded surprisingly consistent results, given the rather broad array of theoretical techniques employed.⁵ These direct

quark-model calculations suggested a lifetime $\tau_p \sim (0.6 - 1.0) \times 10^{30}$ yr. (For this value we use $M_X = 2.1 \times 10^{14}$ GeV, which corresponds to $\Lambda_{\overline{MS}} = 0.16$ GeV, where $\overline{MS}$ refers to the modified minimal-subtraction scheme. We will be more specific about the model dependence and $\Lambda_{\overline{MS}}$ behavior of this number later in the paper.) If one

FIG. 1. Mesons are represented as dashed lines, baryons as heavy solid lines, and quarks, antiquarks, and antileptons as light solid lines (only antilepton symbols are shown). (a) Free-quark model: the final-state quarks are uncorrelated. (b) Two-quark transition in a bound-state model. (c) Three-quark fusion in a bound-state model.

accepts the four Kolar-gold-mine events⁶ as actual proton-decay data, then $\tau_p \approx 8 \times 10^{30}$ yr. This then forms a lower limit on the proton lifetime, $\tau_p \geq 8 \times 10^{30}$ yr.

Recently several authors have argued that such calculations are incomplete in that the dominant proton-decay process is due to the three-quark fusion $q + q + q \rightarrow \bar{l}$ [see Fig. 1(c)].^{7,8} In particular Berezinsky, Ioffe, and Kogan⁷ (BIK) estimate this fusion amplitude by means of a sum rule and find $\tau_p \approx 3 \times 10^{28}$ yr for $M_X = 2.1 \times 10^{14}$ GeV. If BIK are correct, the SU(5) model is clearly ruled out.

Due to the importance of this issue, we have calculated the three-quark fusion amplitude in the MIT bag model. We obtain a smaller value than that found by BIK, yielding a lifetime comparable to that due to the two-quark process [Fig. 1(b)]. We also find that, unfortunately, both our analysis and that of BIK are plagued by the difficulty of estimating amplitudes off the mass shell (this matter is addressed later). However, we do agree with Refs. 7 and 8 that the three-quark-fusion amplitude of Fig. 1(c) should not be neglected.

In Sec. II we calculate several of the relevant baryon-antilepton matrix elements using the MIT bag model. The corresponding proton-lifetime estimate is given in Sec. III, along with a discussion of uncertainties inherent in this approach. We summarize our results in Sec. IV, and also introduce there what we feel to be an enlightened approach for presenting proton-lifetime predictions by explicitly displaying the dependence of $\Lambda_{\overline{MS}}$.

II. THE THREE-QUARK-FUSION AMPLITUDE

The Hamiltonian for proton decay in SU(5) (neglecting $\sin\theta_c$) is

$$H_{\text{GUT}} = \frac{G_u}{\sqrt{2}} [\bar{e}^+ (3 - \gamma_5) \mathcal{O}_1 - \bar{\nu}_e (1 + \gamma_5) \mathcal{O}_2 + 2\bar{\mu}^+ \mathcal{O}_3 - \bar{\nu}_\mu (1 + \gamma_5) \mathcal{O}_4], \quad (1)$$

where the operators $\mathcal{O}_{i\alpha}$ (α is a Dirac spinor index) are

$$\begin{aligned} \mathcal{O}_{1\alpha} &= \epsilon^{ijk} \bar{u}_i^c \gamma_\lambda (1 + \gamma_5) u_j [\gamma^\lambda d_k]_\alpha, \\ \mathcal{O}_{2\alpha} &= \epsilon^{ijk} \bar{u}_i^c \gamma_\lambda (1 + \gamma_5) d_j [\gamma^\lambda d_k]_\alpha, \\ \mathcal{O}_{3\alpha} &= \epsilon^{ijk} \bar{u}_i^c \gamma_\lambda (1 + \gamma_5) u_j [\gamma^\lambda s_k]_\alpha, \\ \mathcal{O}_{4\alpha} &= \epsilon^{ijk} \bar{u}_i^c \gamma_\lambda (1 + \gamma_5) d_j [\gamma^\lambda s_k]_\alpha. \end{aligned} \quad (2)$$

We are interested in calculating

$$\langle 0 | \mathcal{O}_i(0) | B(\vec{k}) \rangle$$

for $B = p, n, \Sigma^+, \Sigma^0, \Lambda^0$ and $i = 1, 2, 3$, or 4. Let us define

$$\langle 0 | \mathcal{O}_i(x) | B(\vec{k}) \rangle = (A_B + B_B \gamma_5) u(\vec{k}) e^{ik \cdot x}. \quad (3)$$

There is no need to include a notation for the operator number $i = 1, 2, 3$, or 4, because for a given baryon B only one operator can mediate the transition, viz., $p \leftrightarrow \mathcal{O}_1$; $n \leftrightarrow \mathcal{O}_2$, $\Sigma^+ \leftrightarrow \mathcal{O}_3$, $\Lambda, \Sigma^0 \leftrightarrow \mathcal{O}_4$.

We use the wave-packet method of Donoghue and Johnson,⁹ in which a bag-model proton is written as a superposition of plane-wave eigenstates

$$|B\rangle_{\text{bag}} = \int d^3k \frac{M_p}{E_k} \chi(k) |B(k)\rangle \quad (4)$$

with normalizations

$$\langle B(k) | B(k') \rangle = (2\pi)^3 \frac{E_k}{M_p} \delta^{(3)}(k - k'), \quad (5)$$

$${}_{\text{bag}} \langle B | B \rangle_{\text{bag}} = 1, \quad (6)$$

and

$$\int d^3k |\chi(k)|^2 \frac{M_p}{E_k} (2\pi)^3 = 1. \quad (7)$$

This method has been used in a related calculation of the pion-decay constant, yielding $F_\pi = 0.5 R_\pi^{-1} = 140$ MeV (the normalization is such that $F_\pi^{\text{expt}} = 94$ MeV). We should expect comparable accuracy for the proton-to-vacuum transition, although one might hope to do better because we understand the proton better than we do the pion.

Let us calculate $\langle 0 | \mathcal{O}_1 | p \rangle$. Combining Eqs. (3) and (4) and Fourier transforming the result, we find

$$\begin{aligned} (A_p + B_p \gamma_5) u(k) \chi(k) \\ = \frac{E_k}{M_p} \int d^3x e^{ik \cdot x} \langle 0 | \mathcal{O}_1(x) | p \rangle_{\text{bag}}. \end{aligned} \quad (8)$$

We can project out the A and B constants by using

$$\bar{u}(k) u(k) = 1, \quad (9)$$

$$\bar{u}(k) \gamma_5 u(k) = 0$$

and the normalization condition on $\chi(p)$. Eq. (9), to write

$$\begin{aligned} A_p^2 &= \int \frac{d^3k}{(2\pi)^3} \frac{M_p}{E_k} \\ &\times \left| \int d^3x e^{ik \cdot x} \langle 0 | \bar{u}(k) \mathcal{O}_1(x) | p \rangle_{\text{bag}} \right|^2, \end{aligned} \quad (10)$$

$$B_p^2 = \int \frac{d^3k}{(2\pi)^3} \frac{M_p}{E_k} \times \left| \int d^3x e^{ik \cdot x} \langle 0 | \bar{u}(k) \gamma_5 \mathcal{O}_i(x) | p \rangle_{\text{bag}} \right|^2. \quad (11)$$

A final simplification occurs because for the proton⁹

$$\frac{\langle k^2 \rangle}{M_p^2} \approx 0.3,$$

so that we can neglect k^2/M_p^2 terms in Eqs. (10) and (11), yielding

$$A_p^2 = \int d^3x | \langle 0 | \bar{u}(0) \mathcal{O}_1(x) | p \rangle_{\text{bag}} |^2, \quad (12)$$

$$B_p^2 = \int d^3x | \langle 0 | \bar{u}(0) \gamma_5 \mathcal{O}_1(x) | p \rangle_{\text{bag}} |^2. \quad (13)$$

These are the formulas which we use to calculate the transition element. The uncertainty due to the neglect of the k^2/M_p^2 should be smaller than other uncertainties in the calculation such as the value of M_X , et. Using bag-model wave functions, we find

$$\langle 0 | \bar{u}(0) \mathcal{O}_1(x) | p \rangle_{\text{bag}} = 0, \quad (14)$$

$$\langle 0 | \bar{u}(0) \gamma_5 \mathcal{O}_1(x) | p \rangle_{\text{bag}} = -i\sqrt{6}u(u^2 + l^2)$$

$$A_n = \frac{1}{2}B_p, \quad B_n = \frac{1}{2}B_p, \quad A_{\Sigma^+} = 0,$$

$$B_{\Sigma^+} = 6\sqrt{2} \left[\int d^3x [u'(u^2 + l^2) + \frac{2}{3}l(ul' - u'l)]^2 \right]^{1/2} = 0.014 \text{ GeV}^3, \quad (18)$$

$$A_\Lambda = 2\sqrt{3} \left[\int d^3x [u'(u^2 - l^2) + 2ull']^2 \right]^{1/2} = 0.0056 \text{ GeV}^3,$$

$$B_\Lambda = 0, \quad A_{\Sigma^0} = 0, \quad B_{\Sigma^0} = \frac{1}{\sqrt{2}}B_{\Sigma^+}.$$

Symbols with a prime indicate that these are wave functions of a strange quark, unprimed quantities are those of up and down quarks. The structures of the baryon to lepton amplitudes are then

$$\begin{aligned} \langle e^+ | H_{\text{GUT}} | p \rangle &= \frac{G_u}{\sqrt{2}} B_p \bar{u}_e(p) (3\gamma_5 - 1) u_p(p), \\ \langle \bar{\nu}_e | H_{\text{GUT}} | n \rangle &= \frac{G_u}{\sqrt{2}} B_p \bar{u}_{\bar{\nu}}(p) (1 + \gamma_5) u_n(p), \\ \langle \mu^+ | H_{\text{GUT}} | \Sigma^+ \rangle &= \frac{G_u}{\sqrt{2}} 2B_{\Sigma^+} \bar{u}_{\mu^+}(p) \gamma_5 u_{\Sigma^+}(p), \\ \langle \bar{\nu}_\mu | H_{\text{GUT}} | \Sigma^0 \rangle &= \frac{G_u}{2} B_{\Sigma^+} \bar{u}_{\bar{\nu}}(p) (1 + \gamma_5) u_{\Sigma^0}(p), \\ \langle \bar{\nu}_\mu | H_{\text{GUT}} | \Lambda^0 \rangle &= \frac{G_u}{\sqrt{2}} B_\Lambda \bar{u}_{\bar{\nu}}(p) (1 + \gamma_5) u_\Lambda(p). \end{aligned} \quad (19)$$

in a notation where the quark Dirac wave function is

$$\psi(x) = \begin{bmatrix} iu(r)\chi \\ l(r)\vec{\sigma} \cdot \hat{r}\chi \end{bmatrix} \quad (15)$$

with χ being a Pauli spinor and

$$u(r) = N j_0(qr/R), \quad (16)$$

$$l(r) = -N \left[\frac{E-m}{E+m} \right]^{1/2} j_1(qr/R).$$

In Eq. (16), q is the mode wave number of a bag-confined quark, and N is a normalization factor. Inserting numbers (we use $m_u = m_d = 33 \text{ MeV}$, $m_s = 330 \text{ MeV}$, $R_p = 5.5 \text{ GeV}^{-1}$) and integrating, we find

$$A_p = 0, \quad (17)$$

$$B_p = \frac{2.1}{R^3} = 0.0127 \text{ GeV}^3.$$

The other baryonic transitions are treated in the same way. We obtain

As noted in Ref. 9, the true answers could be smaller than these results, since we have neglected the overlap of the empty bag with the true vacuum after the three quarks have been removed. This should introduce an additional factor of magnitude less than unity. Although we suspect that this effect is not large, it does add to the uncertainty of the calculation, certainly decreasing the answer.

Berezinsky, Ioffe, and Kogan have used a parameter λ which has the same meaning as B_p . Their value is⁷

$$\lambda \equiv B_p^{(\text{BIK})} = 0.035 \text{ GeV}^3. \quad (20)$$

Our result is smaller by a factor of about 3 than that of BIK. If the rest of the calculation is done in an identical fashion, our final answers would then

be a factor of 9 below theirs. We do not feel that uncertainties in our approach are great enough to allow any compatibility with that of BIK.

Although he does not quote it directly, Tomozawa has calculated what we call B_p in his paper, using a Bethe-Salpeter treatment with a harmonic-oscillator potential.⁸ From his work one can extract the value

$$B_p^{(T)} = 2(2\alpha^3)^{1/2}/3\pi^2 = 0.025 \text{ GeV}^3, \quad (21)$$

where α , the oscillator strength, is taken to be $\alpha = 0.4 \text{ GeV}^2$. Although this number appears to disagree with us and agree with BIK, we argue that the reverse is the case when we correctly identify the oscillator strength.

These calculations are measuring the wave function at the origin for the three-quark system. Because the wave functions are normalized to unity, this reflects the spatial extent of the quarks in the nucleon (we assume that anomalous short-distance three-quark correlations are absent). If the quarks are very tightly bound the wave function at the origin will increase. Indeed as we see from Eq. (17) [and also, but less obviously, from Eq. (21)] the amplitude goes inversely with the radius to the third power,

$$B_p \propto R_p^{-3}. \quad (22)$$

A model which badly predicts the size of the proton will do very much worse for B_p . This is the case with the harmonic oscillator. For the oscillator parameter which Tomozawa uses, the electromagnetic charge radius of the proton is

$$\langle r^2 \rangle_{\text{EM}} \simeq 3\alpha^{-1} \simeq 0.27 \text{ fm}^2 \quad (23)$$

compared to the experimental value

$$\langle r^2 \rangle_{\text{EM}}^{\text{expt}} = 0.81 \text{ fm}^2 \quad (24)$$

and the bag-model result⁹

$$\langle r^2 \rangle_{\text{EM}}^{\text{bag}} = 0.67 \text{ fm}^2. \quad (25)$$

Indeed, this is a well known feature in harmonic-oscillator models,¹⁰ and follows from the fact that the oscillator strength is ordinarily determined by the energy difference between the excited states and the ground state for nonrelativistic quarks in baryons. However, in the calculation at hand, it is imperative that the size of the proton be correctly reproduced. The best way to do this is to choose the oscillator strength to give the correct charge radius. This gives $\alpha \simeq 0.147 \text{ GeV}^2$. Using this value, Tomozawa's calculation then results in

$$B_p^{(T)} = 0.006 \text{ GeV}^3, \quad (26)$$

which is smaller than our result but within our range of uncertainty.

III. PROTON LIFETIME AND BRANCHING RATIOS

Having determined the matrix element for the baryon-to-lepton transition, one must still relate it to proton decay. Such an exercise has several potential problems. Let us examine some of these. We calculate the diagram of Fig. 1(c) by using a pole model with members of the ground-state baryon octet as intermediate states. The only new ingredient which we need to consider is the strong vertex, baryon $\rightarrow$ virtual baryon + meson. There are two issues with which we shall be concerned: (i) What is the form of the vertex (e.g., the question of pseudoscalar or pseudovector coupling for pions)? (ii) What is the effect of the off-shell nature of the intermediate-state baryon? Note that the intermediate-state baryon has $k'^2 = m_e^2 \simeq 0$, i.e., it is far off shell. We shall ultimately not be able to completely resolve these issues. As a result this will lead to an uncertainty in the contribution of the three-quark-fusion diagram to proton decay.

Of course there exists some knowledge of the BBM vertices when both baryons are on shell, as obtained from scattering experiments. For example near $q^2 = 0$ (q is the meson momentum) we can define

$$\mathcal{M}(p \rightarrow p\pi^0) = g_{pp\pi^0} \bar{u}(k') \gamma_5 u(k), \quad (27)$$

$$\mathcal{M}(p \rightarrow p\rho^0, \omega)$$

$$= \frac{1}{2} g_{\rho, \omega} \epsilon_\lambda^+ \bar{u}(k') \left[\gamma^\lambda + \frac{\mu_{V,S}}{M_p} q_\nu \sigma^{\lambda\nu} \right] u(k),$$

where

$$\mu_S = \frac{1}{2}(\mu_p + \mu_n - 1) \quad (28)$$

$$\mu_V = \frac{1}{2}(\mu_p - \mu_n - 1).$$

Other couplings, related to these by isospin or more generally $SU(3)$ rotations, can be similarly defined. In the following we employ

$$\frac{g_{pp\pi^0}^2}{4\pi} = 14, \quad \frac{g_\rho^2}{4\pi} = 2.5, \quad \frac{g_\omega^2}{4\pi} = 22. \quad (29)$$

If we use these definitions without change in the calculation of the decay rates we obtain

$$\Gamma(p \rightarrow \pi^0 e^+) = \left[\frac{g_{\text{GUM}}^2 A}{8M_X} \right]^2 \left[\frac{q}{8\pi} \right] \left[\frac{M_p^2 - m_\pi^2}{M_p^2} \right] \times 10 \left[\frac{B_p g_{pp\pi}}{M_p} \right]^2, \quad (30)$$

where q is the pion-decay momentum, and A is the renormalization-group enhancement factor, and g_{GUM} is the coupling of the grand unification mass. Other modes have similar forms; e.g.,

$$\Gamma(p \rightarrow \omega e^+) = \left[\frac{g_{\text{GUM}}^2 A}{8M_X} \right]^2 \left[\frac{(M_p^2 - m_\omega^2)^2}{32\pi M_p^3} \right] 10 \left[\frac{B_p g_{p\omega}}{M_p} \right]^2 \left[1 + \frac{M_p^2}{2m_\omega^2} + 3\mu_S + \frac{2M_p^2 + m_\omega^2}{2M_p^2} \mu_S^2 \right]. \quad (31)$$

For $M_X = 2.1 \times 10^{14}$ GeV and $A = 2.9^2$ we obtain

$$\tau_p = 2.23 \times 10^{29} \text{ yr} \quad (32)$$

and the branching ratios which are given in the first column of Table I. It is this procedure which BIK used and our results differ from theirs only in the total rate and in some of the strange-particle decay modes.

To see that this procedure is not beyond question, we need only consider the seemingly innocent question of whether to use pseudoscalar (PS) or pseudovector (PV) couplings for the $NN\pi$ vertex. In the usual situation of both nucleons on shell, both couplings,

$$\begin{aligned} \mathcal{M}^{\text{PS}} &= g_{pp\pi} \bar{u}(k') \gamma_5 u(k), \\ \mathcal{M}^{\text{PV}} &= \frac{g_{pp\pi}^0}{2M_p} q_\mu \bar{u}(k') \gamma^\mu \gamma_5 u(k), \end{aligned} \quad (33)$$

with $q_\mu = (k' - k)_\mu$, yield the same result. However, when these are used to calculate the amplitude for $p \rightarrow \pi^0 e^+$ we obtain results which differ by a factor of 2,

$$\begin{aligned} \text{Amp}_{\text{PS}}(p \rightarrow \pi^0 e^+) &= \bar{u}_e(k') B_p (3\gamma_5 - 1) \frac{1}{k' - M_p} g_{pp\pi} \gamma_5 u(k) = -\frac{g_{pp\pi}^0 B_p}{M_p} \bar{u}_e(k') (3 - \gamma_5) u_p(k), \\ \text{Amp}_{\text{PV}}(p \rightarrow \pi^0 e^+) &= \bar{u}_e(k') B_p (3\gamma_5 - 1) \frac{1}{k' - M_p} \frac{g_{pp\pi}^0}{2M_p} (k' - k) \gamma_5 u(k) = -\frac{g_{pp\pi}^0 B_p}{2M_p} \bar{u}_e(k') (3 - \gamma_5) u_p(k). \end{aligned} \quad (34)$$

(We have neglected the electron's mass.) If we use pseudovector couplings for π, K, η and proceed as above we find the branching ratios listed in the second column of Table I and the lifetime

$$\tau_p = 6.2 \times 10^{29} \text{ yr}. \quad (35)$$

At present we know of no way to decide the question of whether pseudoscalar vs pseudovector coupling is preferable, although we note that pseudovector is required by PCAC (partial conservation of axial-vector current) when the momentum transfer is small.

There could, in addition, be some uncertainty in the use of the pole model when the nucleon intermediate state is so far off shell, with invariant mass $p_e^2 \approx 0$. For a nucleon composed of quarks there is no reliable way to estimate off-shell effects. Out of curiosity we have looked at the off-shell behavior in pion-nucleon field theory by calculating $g_{pp\pi}$ includ-

ing all one-loop diagrams. We find that when the final baryon momentum is at $p_e^2 = 0$ the renormalized coupling is about one-half of the value ob-

TABLE I. Proton branching ratios in the three-quark-fusion model. The columns headed PS and PV refer, respectively, to pseudoscalar and pseudovector coupling at the baryon-baryon-meson vertex.

Mode	Branching ratio	
	PS	PV
$\pi^0 e^+$	57.5	40.1
ηe^+	1.0	0.7
$\pi^+ \bar{\nu}_e$	22.9	16.0
$K^0 \mu^+$	2.2	1.5
$K^+ \bar{\nu}_\mu$	2.0	1.4
$\rho^0 e^+$	6.4	17.8
ωe^+	5.5	15.4
$\rho^+ \bar{\nu}_e$	2.6	7.1

FIG. 2. Proton lifetimes in the two-quark reaction process calculated in the bag model. The upper curve corresponds to a dipole form factor in the decay amplitude; the lower curve describes an amplitude with no form factor.

FIG. 3. Proton decay in the two-quark reaction as calculated by Gavella *et al.* (Ref. 5).

FIG. 4. Proton lifetimes in the three-quark-fusion process. The upper and lower curves correspond to pseudovector and pseudoscalar meson-baryon-baryon couplings, respectively.

tained when the final baryon is on its mass shell. This is not a realistic calculation for composite nucleons and pions, but it does indicate that use of the pole model can introduce additional uncertainties due to off-shell effects. These will not explicitly appear in our final results, but should be kept in mind.

IV. CONCLUSION

We have calculated the three-quark-fusion amplitude

$$\langle 0 | \mathcal{O}_i | p(k) \rangle = (0.0127 \text{ GeV}^3) \gamma_5 u(k) \quad (36)$$

using the MIT bag model. Our result is roughly a factor of 3 smaller than that of Berezinsky, Ioffe, and Kogan.⁷ While there is some uncertainty in our estimate, the most likely effect of corrections would be to decrease the amplitude. We remind the reader that a related calculation of F_π yields a value 40% too large. The analysis of Tomozawa⁸ roughly agrees with our result provided the (harmonic-) oscillator strength is properly identified.

When the quark-fusion amplitude, Eq. (36), is

FIG. 5. Proton decay as predicted by BIK (Ref. 7). The value of the annihilation amplitude $\lambda=0.035 \text{ GeV}^3$ was used in Ref. 7. Also included is our value of this quantity $\lambda=0.013 \text{ GeV}^3$.

FIG. 6. Proton lifetimes as predicted by Tomozawa in Ref. 8. The upper and lower curves correspond to oscillator strengths $\alpha=0.15 \text{ GeV}^2$ and $\alpha=0.40 \text{ GeV}^2$, respectively.

FIG. 7. Proton lifetimes as predicted by Fernandez de Labastida and Yndurain in Ref. 8. The upper and lower curves employ the respective values $N=1$, $|\Psi(0)|^2=0.015 \text{ GeV}^{-3}$ and $N=3$, $|\Psi(0)|^2=0.008 \text{ GeV}^{-3}$.

used in a pole-model calculation of proton decay and the value $\Lambda_{\overline{MS}}=0.16 \text{ GeV}$ is taken as input,² we find a (two-body emission) lifetime in the range $(2.2-6.2) \times 10^{29} \text{ yr}$ with branching ratios as listed in Table I. Such lifetimes are somewhat lower than those associated with the processes of Fig. 1(b). However, it should be borne in mind that the uncertainties due to off-shell effects are correspondingly greater. We can only state with confidence that lifetimes associated with Figs. 1(b) and 1(c) are roughly comparable and that neither should be neglected.¹¹ If we indeed have not overestimated the magnitude of the quark-fusion amplitude, then we reach the conclusion that the SU(5) model of proton decay is incompatible with the presently favored small values of $\Lambda_{\overline{MS}}$ and M_X .

To conclude, we suggest a modified format for presenting proton-decay predictions. Given current uncertainties regarding the magnitude of $\Lambda_{\overline{MS}}$, it is instructive to display the dependence of such calculations on the input quantities M_X and A (the renormalization-group enhancement factor). The dependence is not exactly on the fourth power of M_X (or $\Lambda_{\overline{MS}}$) due to the logarithmic variation of A

with M_X (or $\Lambda_{\overline{MS}}$). One should even go further by exhibiting the model dependence associated with significant dynamical assumptions. For example the bag-model calculations of Ref. 5, based on the processes of Fig. 1(b), can be written

$$\tau_p = 1.15 \times 10^{30} \left[\frac{M_X}{2.1 \times 10^{14}} \right]^4 \left[\frac{2.98}{A} \right]^2 \frac{f(n)}{f(2)} \text{ yr} \quad (37)$$

and plotted as in Fig. 2. In Eq. (37), the prediction is scaled relative to $\Lambda = 0.16$ GeV and the choice of a dipole form factor ($n=2$) to describe the dynamic effect of recoil corrections. The curves in Fig. 2 correspond to the choices $n=0$ and $n=2$. *The most significant aspect of a plot such as that of Fig. 2 is to place a lower limit on the values of $\Lambda_{\overline{MS}}$ which, in the context of a given calculation, are compatible with experimental constraints.* Thus the $n=2$ calculation becomes incompatible with experiment for $\Lambda < 0.26$ GeV. Similar curves are drawn as follows: (i) Fig. 3 displays the calculation of Fig. 1(b) by Gavela *et al.*⁵; (ii) Fig. 4 gives the bag-model predictions of Fig. 1(e) for both pseudoscalar and pseudovector couplings; (iii) Fig. 5 gives the BIK results for their and our choice of the "proton-decay constant" B_p ; (iv) Fig. 6 gives Tomozawa's results for two choices of oscillator strength; (v) Fig. 7 gives the results of the work of Fernandez de Labastida and Yndurain⁸ for two choices of parameters appearing in that calculation—their predictions yield

the shortest lifetimes of those the authors have seen, but also perhaps, the most difficult to estimate uncertainties in due to the presence of off-shell hadron cross sections. It is of course clear that our choices of uncertainties in the calculations of others is arguable. It is not our intent to provoke controversy, but rather to caution against premature judgments regarding the validity of the SU(5) model in proton decay.

We note that *all* calculations are seen to require $\Lambda_{\overline{MS}} > 200$ MeV for compatibility with the bound on the proton lifetime. In particular, the results of this paper suggest that if SU(5) is correct, $\Lambda_{\overline{MS}}$ must be larger than 300 MeV. There is a clear conflict between low values of $\Lambda_{\overline{MS}}$ and the minimal SU(5) theory. If in the future $\Lambda_{\overline{MS}}$ can be reliably determined to be less than 200 MeV, SU(5) would be ruled out. On the positive side, this result says that if SU(5) is correct, proton decay should definitely be seen by the present generation of detectors.

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¹A comprehensive review appears in P. Langacker, Phys. Rep. **72**, 185 (1981). A more recent review of proton decay appears in W. J. Marciano, in Proceedings of the 19th Orbis Scientiae, 1982 (unpublished).

²A. J. Buras, Rev. Mod. Phys. **52**, 199 (1982). In this work, a mean value $\Lambda_{\overline{MS}} \simeq 0.16$ GeV taken from a variety of analyses is quoted. We employ this throughout the paper as a useful benchmark.

³S. J. Brodsky (private communication).

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⁹J. F. Donoghue and K. Johnson, Phys. Rev. D **21**, 1975 (1980).

¹⁰For example, see J. F. Donoghue and G. Karl, Phys. Rev. D **24**, 230 (1981).

¹¹By this point the reader will surely have observed that amplitudes associated with Figs. 1(b) and 1(c) are coherent, so that one should not speak of a "lifetime" associated with either. However we expect interference effects to be relatively unimportant because the branching ratios of the competing processes of Figs. 1(b) and 1(c) tend to be rather different (the former favors ωe^+ emission whereas the latter is dominated by $\pi^0 e^+$).