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STRONG BOUNDS ON WEAK COUPLINGS [☆]

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Analysis of nonleptonic $\Delta S = 1$ weak decays strongly restricts the possible size of right-handed weak couplings. Limits thus obtained are competitive with those obtained via other means.

At present the Glashow–Weinberg–Salam model of the weak interactions accurately describes observed weak phenomena [1]. However, there are several reasons to suspect that the gauge structure of the electroweak interactions may well be richer than $SU(2)_L \times U(1)$, and that there may be new physics characterized by energy scales larger than M_w . One attractive possibility is that the weak interaction involves both left-handed and right-handed components [2]. If it exists, the right-handed vector boson W_R must be heavier than its left-handed counterpart W_L .

Various previous workers have attempted to place bounds on W_R . One approach is to analyze the low-energy neutral-current data using both $\sin^2\theta_w$ and $M(W_R)$ ranging from 0.23 and 190 GeV to 0.28 and 150 GeV can satisfy the neutral-current data to within 1.5σ [3].

A second approach uses the analysis of precision nuclear beta-decay and muon-decay experiments. Such analyses have yielded [2,4]

$$M(W_R) \gtrsim 250 \text{ GeV}.$$

Proposed new experiments in this area promise a significant improvement in such limits, but these are probably some time off.

Both of these approaches, however, assume the existence of a right-handed neutrino which is massless (or at least less massive than its left-handed counterpart). However, in most grand unified theories [5],

the right-handed neutrino is much heavier than ν_L , which means that right-handed effects will *not* show up in semileptonic or leptonic properties (except to the extent that W_L, W_R are mixed). In this case then, information relative to the mass of a right-handed weak boson can arise only from the nonleptonic sector. Recent work by Forcrand has, for example, analyzed the impact of right-handed interactions on the $K_L - K_S$ mass difference and has obtained a limit of [6]

$$M(W_R) \gtrsim 200 \text{ GeV}.$$

In this paper we wish to explore the bounds on the mass and coupling of W_R using weak $\Delta S = 1$ nonleptonic decays. Due to the difficulty of accurately describing nonleptonic transitions, we shall tend to be somewhat conservative in our quoted results. Nevertheless some of the limits are surprisingly stringent.

We will treat a general $SU(2)_L \times SU(2)_R$ theory with different W_L and W_R masses and mixings between the LH and RH bosons. In our formulae we display only the coupling of the light quarks, u, d, s , because that is all which we will need to use. The physical bosons (mass eigenstates) will be defined as

$$W_1 = W_L \cos \zeta - W_R \sin \zeta,$$

$$W_2 = W_L \sin \zeta + W_R \cos \zeta, \quad (1)$$

where W_L and W_R are the weak interaction eigenstates which couple to the LH and RH currents

$$J_L^\mu = 2^{-3/2} [\cos \theta_1^L (\bar{u}d)_L + \sin \theta_1^L \cos \theta_3^L (\bar{u}s)_L + \dots],$$

$$J_R^\mu = 2^{-3/2} [\cos \theta_1^R (\bar{u}d)_R + \sin \theta_1^R \cos \theta_3^R (\bar{u}s)_R + \dots], \quad (2)$$

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in the notation where

$$(\bar{u}d)_L \equiv u\gamma_\mu(1 + \gamma_5)d, \quad (\bar{u}d)_R \equiv u\gamma_\mu(1 - \gamma_5)d \quad (3)$$

and θ_1^L, θ_1^R etc. are the various KM angles. Many such left-right theories postulate an equality for the left-handed and right-handed angles. However, we shall assume arbitrary values, for generality. This leads to an effective $\Delta S = 1$ hamiltonian

$$H_w = G_F 2^{-1/2} [\alpha(\bar{d}u)_L(\bar{u}s)_L + \beta(\bar{d}u)_L(\bar{u}s)_R + \gamma(\bar{d}u)_R(\bar{u}s)_L + \delta(\bar{d}u)_R(\bar{u}s)_R + \dots], \quad (4)$$

where

$$\begin{aligned} \alpha &= \cos \theta_1^L \sin \theta_1^L \cos \theta_3^L (\cos^2 \zeta + \lambda \sin^2 \zeta), \\ \delta &= \cos \theta_1^R \sin \theta_1^R \cos \theta_3^R (\sin^2 \zeta + \lambda \cos^2 \zeta), \\ \beta &= -\cos \theta_1^L \sin \theta_1^R \cos \theta_3^R \cos \zeta \sin \zeta (1 - \lambda), \\ \gamma &= -\cos \theta_1^R \sin \theta_1^L \cos \theta_3^L \cos \zeta \sin \zeta (1 - \lambda), \end{aligned} \quad (5)$$

with

$$\lambda = M(W_L)^2 / M(W_R)^2.$$

In addition one needs to include the effects of short-distance strong-interaction corrections. For the LL and RR operators these corrections are identical and are well known; however, an explicit form will not be needed. We will need the results of the LR and RL operators which have been given by Altarelli and Maiani [7]. The result is

$$(\bar{d}u)_R(\bar{u}s)_L \rightarrow \frac{1}{3}(-K^{-0.12}O_{RL}^1 + K^{+0.96}O_{RL}^2), \quad (6)$$

where

$$\begin{aligned} O_{RL}^1 &= -\frac{8}{3}(\bar{d}u)_R(\bar{u}s)_L + \frac{1}{2}(\bar{d}t^A u)_R(\bar{u}t^A s)_L, \\ O_{RL}^2 &= \frac{1}{3}(\bar{d}u)_R(\bar{u}s)_L + \frac{1}{2}(\bar{d}t^A u)_R(\bar{u}t^A s)_L, \end{aligned} \quad (7)$$

t^A are the SU(3) color matrices normalized to $\text{tr}(t^A t^B) = 2\delta^{AB}$,

and

$$K \equiv \alpha_s(1 \text{ GeV}) / \alpha_s(M_w) \sim 4 \rightarrow 7. \quad (8)$$

A similar result holds with $RL \rightarrow LR$. We may neglect LR and RL "penguin-like" operators with virtual heavy-quark loops because they will be purely $\Delta I = 1/2$ while we use only the $\Delta I = 3/2$ pieces – note

that in O_{RL}^1 the strong interactions *enhance* both $\Delta I = 1/2$ and $3/2$ equally.

Let us begin our analysis by examining the weak decays of K mesons. While it is very difficult to directly calculate the nonleptonic transition amplitudes, one feature has a firm theoretical basis: PCAC should (and does) accurately describe a connection between the physical $K \rightarrow 3\pi$ and $K \rightarrow 2\pi$ decays [8]. The $K \rightarrow 3\pi$ amplitude and slope parameters are known to be accurately predicted in terms of measured $K \rightarrow 2\pi$ matrix elements – both for $\Delta I = 1/2$ and $\Delta I = 3/2$ transitions – to an accuracy of roughly 10%. The critical ingredient in this connection is the chiral structure of H_w , in particular

$$[F_i^5, H_w] = [F_i, H_w], \quad i = 1, 2, 3 \quad (9)$$

where F_i, F_i^5 are vector, axial charges. However, in the presence of right-handed currents eq. (9) is no longer satisfied. For example,

$$[F_3^5, H_w^{i(\text{pc})}] = [F_3, H_w^{i(\text{pv})}] \left(\frac{\alpha + \delta}{\alpha - \delta} \right) + \Delta H^{i(\text{pv})}, \quad (10)$$

where pc, pv indicates parity-conserving, -violating, respectively, and $i = 1/2, 3/2$ indicates the isotopic spin component of H_w^i such that

$$H_w = H_w^{1/2} + H_w^{3/2}. \quad (11)$$

We have then

$$H_w^{i(\text{pv})} = (G_F/3\sqrt{2}) \times [(\alpha - \delta)O_{VA+AV}^i + (\beta - \gamma)O_{AV-VA}^i], \quad (12)$$

where

$$\begin{aligned} O^{1/2} &= (2 \bar{d}u \bar{u}s - \bar{u}u \bar{d}s + \bar{d}d \bar{d}s), \\ O^{3/2} &= (\bar{d}u \bar{u}s + \bar{u}u \bar{d}s - \bar{d}d \bar{d}s), \end{aligned} \quad (13)$$

and the subscript $VA \pm AV$ implies, e.g.

$$(\bar{d}u \bar{u}s)_{VA \pm AV} \equiv \bar{d}\gamma_\mu u \bar{u}\gamma_\mu \gamma_5 s \pm \bar{d}\gamma_\mu \gamma_5 u \bar{u}\gamma_\mu s. \quad (14)$$

Also, we note that

$$\begin{aligned} \Delta H^{1/2(\text{pv})} &= -(G_F/3\sqrt{2}) \\ &\times [4(\beta + \gamma)O_{AV-VA}^{3/2} + (\beta + 4\gamma)O_{AV-VA}^{1/2}], \\ \Delta H^{3/2(\text{pv})} &= (G_F/3\sqrt{2}) \\ &\times [(\beta - 2\gamma)O_{AV-VA}^{3/2} - 2(\beta + \gamma)O_{AV-VA}^{1/2}], \end{aligned} \quad (15)$$

where we have neglected terms in $\beta\delta$ and $\gamma\delta$.

The fact that $\Delta H_w^i \neq 0$ and $\delta/\alpha \neq 0$ will lead to significant violations of the PCAC relations between the $K \rightarrow 2\pi$ and $K \rightarrow 3\pi$ amplitudes. In order to see the extent of such violation, let us, e.g., examine the $K^0 \rightarrow 3\pi^0$ decay amplitude in the limit of vanishing four-momentum for one of the pions. Then

$$\langle \pi_{q_1}^0 \pi_{q_2}^0 \pi_{q_3}^0 | H_w^{i(\text{pc})} | K_k^0 \rangle \xrightarrow{q_3 \rightarrow 0} \frac{-i}{2F_\pi} \left(\langle \pi_{q_1}^0 \pi_{q_2}^0 | H_w^{i(\text{pv})} | K_k^0 \rangle \times \left(\frac{\alpha + \delta}{\alpha - \delta} \right) + 2 \langle \pi_{q_1}^0 \pi_{q_2}^0 | \Delta H_w^{i(\text{pv})} | K_k^0 \rangle \right). \quad (16)$$

In order that PCAC work to $\sim 10\%$ for both $H_w^{1/2}$, $H_w^{3/2}$ independently we require that

$$|2\delta/\alpha| \lesssim 0.1, \quad (17)$$

and

$$2 \langle \pi_{q_1}^0 \pi_{q_2}^0 | \Delta H_w^{i(\text{pv})} | K_k^0 \rangle / \langle \pi_{q_1}^0 \pi_{q_2}^0 | H_w^{i(\text{pv})} | K_k^0 \rangle \lesssim 0.1. \quad (18)$$

(The possibility of a conspiracy between the two effects — δ and ΔH_w^i — can be removed by considering other $K \rightarrow 3\pi$ modes.)

The first constraint clearly leads to

$$|\delta/\alpha| \lesssim 0.05. \quad (19)$$

The second is more difficult to evaluate. In order to proceed, we utilize PCAC a second time in order to relate the amplitude for $\langle \pi_{q_1}^0 \pi_{q_2}^0 | O | K_k^0 \rangle$, which we wish to know, to an amplitude $\langle \pi_{q_1}^0 | \tilde{O} | K_k^0 \rangle$, which is calculable via the MIT bag model. From eq. (18), the strongest bound is obtained by using $H_w^{3/2(\text{pc})}$ because the physical $\Delta I = 3/2$ matrix element is smaller than that of $H_w^{1/2(\text{pc})}$ by a factor of twenty or so. We need then

$$\langle \pi_{q_1}^0 \pi_{q_2}^0 | \Delta H_w^{3/2(\text{pv})} | K_k^0 \rangle \xrightarrow{q_2 \rightarrow 0} -\frac{i}{F_\pi} \langle \pi_{q_1}^0 | [F_3^5, \Delta H_w^{3/2(\text{pv})}] | K_k^0 \rangle, \quad (20)$$

with

$$[F_3^5, \Delta H_w^{3/2(\text{pv})}] = G_F/6\sqrt{2} \times [(\beta + 6\gamma)O_{\text{VV-AA}}^{3/2} + (2\beta + 6\gamma)O_{\text{VV-AA}}^{1/2}]. \quad (21)$$

We can now take the bag model matrix elements [9], with the result

$$\langle \pi_{q_1}^0 \pi_{q_2}^0 | \Delta H_w^{3/2(\text{pv})} | K_k^0 \rangle \simeq (G_F/2F_\pi) \sqrt{2} m_k^2 \times [(4\beta + 6\gamma)(A + B) + 2\beta(A - B)] K^{0.96}, \quad (22)$$

where A, B are dynamical overlap integrals. (See ref. [9] for a complete definition of notation. For compactness we quote A and B with all quark masses equal to zero; however, in the numerics, we use $m_u = m_d = 0$, $m_s = 330$ MeV.)

$$A = \frac{N^4}{4\pi R^3} \int d^3r [j_0^2(kr) - j_1^2(kr)]^2, \\ B = \frac{N^4}{4\pi R^3} \int d^3r [2j_0(kr)j_1(kr)]^2. \quad (23)$$

Inserting numbers and using $K = 4$ (7), we find that eqs. (18), (22) implies

$$\beta + 1.6\gamma \lesssim 4 \times 10^{-4} \quad (2 \times 10^{-4}). \quad (24)$$

Other $K \rightarrow 3\pi$ modes will bound different combinations of β and γ . However, there is some uncertainty in this result, since eq. (18) was evaluated using a quark model *calculation* for the numerator but an *experimental* number in the denominator. To be more conservative, we feel that the bounds on β, γ should be increased by a factor of two, and we will use $K = 4$, yielding

$$\beta, \gamma \lesssim 8 \times 10^{-4}. \quad (25)$$

This bound is quite stringent, due to the combined effects of small PCAC breaking, a small physical $\Delta I = 3/2$ amplitude, and an enhanced $\Delta I = 3/2$ contribution of the LR and RL operators. Assuming the left-handed and right-handed mixings to be similar, so that

$$\cos \theta_1 \sin \theta_1 \cos \theta_3 \sim 0.2 \quad (26)$$

for both, we find that the mixing angle ξ is rather strongly constrained

$$\xi \lesssim 0.004/(1 - \lambda) \approx 0.004, \quad (27)$$

where the neglect of λ on the right-hand side of eq. (27) is a result of the constraint implied by eq. (19). Again assuming rough equality of θ_L^i, θ_R^i we have

$$(\sin^2 \xi + \cos^2 \xi \lambda)/(\cos^2 \xi + \sin^2 \xi \lambda) \approx \lambda \lesssim 0.05, \quad (28)$$

which implies

$$M(W_R) \gtrsim 4M(W_L) \gtrsim 300 \text{ GeV}. \quad (29)$$

It is rather striking then that from this simple analysis of nonleptonic kaon decay, we have been able to obtain limits on λ , ξ much more stringent than presently attainable by sensitive semileptonic experiments. In addition, eqs. (27), (28) are true *independent* of the size of the right-handed neutrino mass.

Having been so successful in the kaon sector it is natural to proceed to nonleptonic hyperon decays, in order to see if anything new can be gleaned. The case of hyperon nonleptonic decay is different from that of kaons, in that there are *not* two physical decay channels which can be related by use of PCAC. In the absence of very reliable calculational tools we therefore do not know any way to restrict the parameter δ . However, once again, the size of $\Delta I = 3/2$ contributions due to the LR and RL pieces of the hamiltonian may be used to bound β and γ . Unfortunately we will see that the bounds will be rather weak due to cancellations in the coefficients resulting from the strong-interaction corrections.

We proceed by direct calculation of the matrix elements, in the MIT bag model, which should be reliable to within a factor of two in the case of baryon transitions. For S-waves we use PCAC to relate the physical amplitudes to baryon to baryon matrix elements.

$$\langle B' \pi^i | H_w | B \rangle = (-i/F_\pi) \langle B' | [F_i^5, H_w] | B \rangle. \quad (30)$$

We evaluate the commutator and calculate the matrix element using the results of ref. [9]. We find

$$\begin{aligned} \langle N\pi^0 | H_w^{pv} | \Lambda \rangle + 2^{-1/2} \langle P\pi^- | H_w^{pv} | \Lambda \rangle \\ = -(G_F/F_\pi) \sqrt{3} (A - \frac{1}{3}B) (\beta - \gamma) \frac{1}{3} (4K^{-0.12} - K^{0.96}) \\ = (1.4 \pm 0.4) \times 10^{-8} \quad (\text{expt.}), \end{aligned}$$

$$\begin{aligned} \langle P\pi^0 | H_w^{pv} | \Sigma^+ \rangle + 2^{-1/2} \langle N\pi^- | H_w^{pv} | \Sigma^- \rangle \\ = (G_F/\sqrt{2}F_\pi) (A + \frac{8}{3}B) (\beta - \gamma) \frac{1}{3} (4K^{-0.12} - K^{0.96}) \\ = (2.6 \pm 1.1) \times 10^{-8} \quad (\text{expt.}), \end{aligned}$$

$$\begin{aligned} \langle N\pi^+ | H_w^{pv} | \Sigma^+ \rangle \\ = (G_F/F_\pi) 2(A + \frac{1}{3}B) (\beta - \gamma) \frac{1}{3} (4K^{-0.12} - K^{0.96}) \\ = (1.6 \pm 0.4) \times 10^{-8} \quad (\text{expt.}), \end{aligned} \quad (31)$$

$$\begin{aligned} \langle \Lambda\pi^0 | H_w^{pv} | \Xi^0 \rangle + 2^{-1/2} \langle \Lambda\pi^- | H_w^{pv} | \Xi^- \rangle \\ = -(G_F/F_\pi) \sqrt{3} (A - B) (\beta - \gamma) \frac{1}{3} (4K^{-0.12} - K^{0.96}) \\ = (0.4 \pm 0.9) \times 10^{-8} \quad (\text{expt.}). \end{aligned} \quad (31 \text{ con'd})$$

In the P-wave amplitudes we use the baryon pole model (see ref. [10]) to estimate the size of $\Delta I = 3/2$ effects. This involves the baryon to baryon matrix element of the weak hamiltonian. We find

$$\begin{aligned} \langle N\pi^0 | H_w^{pc} | \Lambda \rangle + 2^{-1/2} \langle P\pi^- | H_w^{pc} | \Lambda \rangle \\ = (g/\Delta m_B) (1 - f) \frac{10}{3} a_{27} \\ = (0.27 \pm 2.29) \times 10^{-7} \quad (\text{expt.}) \\ \langle P\pi^0 | H_w^{pc} | \Sigma^+ \rangle + 2^{-1/2} \langle N\pi^- | H_w^{pc} | \Sigma^- \rangle \\ - 2^{-1/2} \langle N\pi^+ | H_w^{pc} | \Sigma^+ \rangle = (-g/\Delta m_B) (20/\sqrt{6}) f a_{27} \\ = (3.10 \pm 2.14) \times 10^{-7}, \end{aligned}$$

$$\begin{aligned} \langle \Lambda\pi^0 | H_w^{pc} | \Xi^0 \rangle + 2^{-1/2} \langle \Lambda\pi^- | H_w^{pc} | \Xi^- \rangle \\ = -(g/\Delta m_B) (1 - f) \frac{10}{3} a_{27} \\ = -(4.81 \pm 2.76) \times 10^{-7}, \end{aligned}$$

where

$$a_{27} = \frac{2}{5} \sqrt{3} G_F (\beta + \gamma) (A + \frac{1}{3}B) \frac{1}{3} (4K^{-0.12} - K^{0.96}).$$

Again let us inflate these bounds (at 90% confidence level) by a factor of two to include possible uncertainties. We then find for $K = 4$ (7)

$$\beta - \gamma \leq 1.1 \quad (0.14)$$

from the S waves and

$$\beta + \gamma \leq 5.0 \quad (0.6)$$

from the P waves.

Unfortunately the strong-interaction factors renders these results possibly useless, as there is a large cancellation occurring, i.e.

$$\frac{1}{3} (4K^{-0.12} - K^{0.96}) = -0.13 \rightarrow -1.10,$$

for values of K from $4 \rightarrow 7$. Unless we know K precisely (impossible at present), the bounds are very weak.

We have discussed the constraints on the masses and

couplings of right-handed weak bosons. The most stringent bounds come from the deviations from PCAC predictions in K decays, both for $\Delta I = 1/2$ and $\Delta I = 3/2$. In the limit of no mixing between W_L and W_R ($\sin \xi = 0$) eq. (19) requires

$$m_{W_R} \geq 300 \text{ GeV}.$$

Eq. (24) is primarily a constraint on the mixing of W_L and W_R . If the right-handed quark mixing angles are equal to their left-handed counterparts, and m_{W_R} is large, then eq. (24) implies

$$\sin \xi < 0.004.$$

Note added. Following the completion of this work we received a copy of a preprint by Bigi and Frère [11] which uses similar ideas. However, their bound comes only from hyperon decay, is based on factorization, and does not include any limit on $M(W_R)$. Our work on kaon decay provides a more stringent restriction.

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