Higgs boson exchange models of CP violation and $K \rightarrow 2 \pi$

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I. INTRODUCTION

Within gauge theories of the weak interaction there exist at least three ways in which to incorporate the known violation of CP invariance: (1) With more than four quarks the couplings of $W^\pm$ to the quarks are in general complex, leading to CP violation, as was first stressed by Kobayashi and Maskawa.\(^1\) (2) If there exist more than two sets of Higgs doublets, the Higgs-boson couplings to quarks will in general break CP. Lee\(^2\) and Weinberg\(^3\) have constructed models of this sort. (3) If there exist additional gauge interactions characterized by a higher mass scale, there can emerge a pattern of CP breaking similar to the original superweak theory of Wolfenstein.\(^4\) Dynamical symmetry breaking is thought to lead most naturally to a structure similar to the second type. However, due to the limited range of experimental signals, it has proven difficult to test the various options.

Recently Deshpande\(^5\) and Sanda\(^6\) (hereafter D&S) have independently provided an analysis which appears to rule out the Weinberg Higgs-boson-based models and presumably most others of class 2. If correct, this would substantially modify the theoretical situation.

Because of the significance of this result, it is important to analyze the framework on which it is constructed. Unfortunately, there are several weak points. The conclusion of D&S depends upon "penguin" dominance in $\Delta I = \frac{1}{2}$, $\Delta S = 1$ nonleptonic weak decays\(^7\) and in addition upon a purely short-distance "box" explanation for the $K_L - K_S$ mass difference.\(^8\) Both assumptions have been challenged and debated.\(^9,10\) In addition to sizable uncertainties associated with these assumptions, it becomes necessary to estimate the matrix elements of these penguin and box operators as well. We find that such considerable uncertainties can be avoided by taking the amplitude for $K \rightarrow 2\pi$ and the $K_L - K_S$ mass difference directly from experiment. In addition, D&S limited their discussion to the four-quark theory, although Sanda appears to have included the $t$ quark in the diagrams which he considered. It is certainly likely, however, that inclusion of the $t$ quark, due to its large coupling to Higgs particles, will reshape the analysis by leading to new diagrams (the double-Higgs-boson box diagram), possibly providing an avenue of escape from this dilemma.

In this paper we attempt to ameliorate these weak points by providing a careful assessment of CP violation in Weinberg's model. In Sec. II, for completeness, we outline the arguments of D&S. Then in Sec. III, we redo their analysis, eliminating the dependence upon any particular mechanism for $\Delta I = \frac{1}{2}$ decays or the $K_L - K_S$ mass difference by taking these quantities from experiment. We thereby also reduce the number of matrix elements required from the analysis, and use the MIT bag
model to calculate those remaining. The bag model has proven useful in calculating matrix elements in the study of nonleptonic processes both for $\Delta S = 1$ decays and for $\Delta S = 2$ transition amplitudes. It is especially useful for the matrix elements encountered here. In Sec. IV, we consider the important effects of the $t$ quark. We conclude that D&S's conclusions are valid barring accidental cancellations: the Weinberg model and others like it appear to be ruled out. We present a critical summary in Sec. V.

II. CP PHENOMENOLOGY

There are, in general, two possible sources for the observed CP violation in $K \rightarrow 2\pi$:\cite{11} Either there can exist CP violation in the $K^0, \bar{K}^0$ mass matrix or CP can be broken directly in the $K \rightarrow 2\pi$ transition amplitude (or both). These lead to differing predictions for $\eta_{00}$ defined as

$$
\begin{align*}
\eta_{+} &= \frac{\langle \pi^+ \pi^- | H_W | K_L \rangle}{\langle \pi^0 \pi^0 | H_W | K_S \rangle} = \epsilon + \epsilon', \\
\eta_{00} &= \frac{\langle \pi^0 \pi^0 | H_W | K_L \rangle}{\langle \pi^0 \pi^0 | H_W | K_S \rangle} = \epsilon - 2\epsilon'.
\end{align*}
$$

CP violation which resides solely in the mass matrix ($\epsilon' = 0$) leads to $\eta_{+} = \eta_{00}$. On the other hand, in theories wherein CP is violated both directly as well as in the mass matrix, but wherein the CP-violating Hamiltonian contributing to $K \rightarrow 2\pi$ is strictly $\Delta I = \frac{1}{2}$, we find

$$
\frac{\epsilon'}{\epsilon} = \frac{1}{20} \frac{2\xi}{\epsilon_m + 2\xi},
$$

where, following Gilman and Wise,\cite{15} we have defined

$$
\epsilon_m = \frac{\text{Im} M_{12}}{\text{Re} M_{12}},
$$

where $M_{12}$ is the element of the mass matrix connecting $K^0, \bar{K}^0$ and

$$
\xi = \frac{\text{Im} \langle \pi \pi | H_W | K^0 \rangle}{\text{Re} \langle \pi \pi | H_W | K^0 \rangle}.
$$

Experimentally\cite{15,16}

$$
\frac{\epsilon'}{\epsilon} = -0.003 \pm 0.015
$$

leading to

$$
\frac{\xi}{\epsilon_m} = +0.03 \pm 0.25.
$$

It was the observation of Deshpande and Sanda that $\xi/\epsilon_m$ appears larger than unity in the Weinberg model of CP violation, in conflict with Eq. (6).

If we neglect the $t$ quark and assume $m_t << m_H$, the diagrams which generate mass-matrix CP violation are indicated in Fig. 1. The CP-nonconserving $\Delta S = 2$ interaction arises predominantly from the box diagrams involving one Higgs-boson and one $W$ exchange, which yield the effective interaction

$$
\mathcal{L}_{\Delta S=2} = \overline{\psi} \gamma_{\mu} (1 + \gamma_5) s \psi \times i \partial_{\mu} \left[ \overline{\psi} \gamma_{\mu} \gamma_{\mu} (1 - \gamma_5) s \right],
$$

where

$$
\overline{g} = \frac{G_F^2}{32 \pi^2} m_t^2 m_4 (\cos \theta_C \sin \theta_C)^2 \sum_{l=1}^{2} \frac{\text{Im} \gamma_l^*}{m_{H_l}^2}.
$$

Here we have defined, following Albright, Smith, and Tye,\cite{17} the Higgs-boson couplings as
where $K$ is the Kobayashi-Maskawa (KM) matrix and the angles $s_u^H, c_u^H$ are Higgs-boson coupling angles defined in Ref. 17. Then

$$
\gamma_1 = \frac{c_u^H c_d^H (c_s^H)^2 + s_u^H s_d^H c_s^H e^{i\phi_H}}{c_u^H c_d^H},
$$

$$
\gamma_2 = \frac{c_u^H c_d^H (c_s^H)^2 - s_u^H s_d^H c_s^H e^{i\phi_H}}{c_u^H c_d^H},
$$

and

$$
\text{Im} \gamma^* = -\sin \delta \frac{s_u^H s_d^H c_s^H}{c_u^H c_d^H} = -\text{Im} \gamma_2^*.
$$

A direct CP-violating $K \rightarrow 2\pi$ decay amplitude can be produced by the diagram in Fig. 2, which yields the effective operator

$$
\mathcal{L}_{D^S_{-} = 1} = i \bar{f} d \sigma^{\mu \nu} (1 + \gamma_5) \lambda^a F_{\mu \nu}^a
$$

with

$$
\bar{f} = \frac{G_F}{\sqrt{2}} \frac{g_5}{32\pi^2} m_c^2 m_s \cos \theta_C \sin \theta_C
$$

$$
\times \sum_{i=1}^{2} \text{Im} \gamma_i^* \left( \frac{m_{H_i}^2}{m_c^2} - \frac{3}{2} \right).
$$

Here $F_{\mu \nu}$ is the field-strength tensor and $g_5$ is the strong quark-gluon coupling $g_5^2 / 4\pi = \alpha_s$.

D&S calculate the CP-conserving contributions to the $K^0, \bar{K}^0$ mass matrix and to $K \rightarrow 2\pi$ by considering the diagrams of Figs. 3 and 4, which lead to the operators

$$
\mathcal{L}_{D^S_{+} = 1} = \frac{G_F}{\sqrt{2}} \frac{\alpha_s}{12\pi} \cos \theta_C \sin \theta_C \ln \frac{m_c^2}{\mu^2}
$$

$$
\times \bar{d}_i \gamma_{\mu} (1 + \gamma_5) \gamma_5 \lambda^a \sum_q \bar{q}_q \gamma_5 \lambda^a \Psi_q.
$$

They then evaluate the real and imaginary parts of $M_{12}$ and of the $K \rightarrow 2\pi$ amplitude by inserting only the vacuum intermediate state (in all possible ways) and conclude that

$$
5 < \frac{\varepsilon}{\epsilon_m} < 30,
$$

i.e., that direct CP violation in the decay rate is more effective than CP violation in the mass matrix, in contrast with Eq. (6).

Of course, vacuum saturation is always a treacherous undertaking, particularly for $\mathcal{L}_{D^S_{+} = 1}$ because of the gluon field. What is done by DeShpande and Sanda is to use the gluon equation of motion to obtain a nonlocal operator involving only quarks, which is then treated as local (by giving the gluon an effective mass $m_K$) in subsequent calculations. Only the rather sizable margin of error between theory and experiment lends credibility to their conclusions. Thus, in Sec. III we attempt to improve this aspect of the analysis.
III. IMPROVED MATRIX ELEMENTS

The first improvement over the method of D&S comes from noticing that one need not calculate the real parts of the mass matrix and $K \to 2\pi$ amplitude. These are known experimentally. Use of calculated rather than empirical values commits one to a particular origin for the EL-Kz mass difference and $K_{2m}$ amplitude. In fact, there has been considerable dispute in the literature as to the viability of $W^+, W^- = W^+$ and $W^+ = W^+$. Several authors have emphasized that important contributions to $m_{12}$ arise from low-mass intermediate states having nothing to do with $W^+ = W^+$. The assumption of penguin dominance in $E_{2m}$ decays has also been challenged and debated.

The matrix element of $W^+$ has in fact been calculated within the bag model previously. Here we simply review the method. The combination

$$2F_\pi \langle \pi^0, 0 \mid \mathcal{L}_{2m} = 1 \mid K^0 \rangle$$

can be related to a $K^0 \to \pi^0$ matrix element by using the current-algebra—PCAC (partial conservation of axial-vector current) techniques and assuming a smooth extrapolation to the soft-pion point:

$$\lim_{q_\pi \to 0} \langle \pi^0, 0 \mid H_W \mid K^0 \rangle = -\frac{i}{2F_\pi} \langle \pi^0 \mid H_W \mid K^0 \rangle .$$

The gluon field in $\mathcal{L}_{2m} = -1$ is obtained by using the exact gluon bag propagator and the quark-current sources

$$A^a_\mu(x) = g_s \int d^3x' G^{\text{bag}}_{ab}(x,x') \times \overline{\psi}(x') \gamma_\mu \lambda^a \psi(x') .$$

We emphasize that no locality assumption for the effective four-quark operator is being employed here. Nonlocal effects are taken into account by use of the actual gluon propagator. The calculation proceeds along standard lines obtaining [in the SU(3) limit]

$$\langle \pi^0 \mid \mathcal{L}_{2m} = -1 \mid K^0 \rangle = \frac{64}{\sqrt{2}} \frac{g_s N^4}{R^2 (2m_{K^0})^2} I_M ,$$

where

$$I_M = \frac{1}{4\pi} \int_0^1 dx \left[ j_0(\omega x) + j_1(\omega x) \right] y$$

$$- \frac{4}{3} j_1(\omega x z) ,$$

$$y = \frac{1}{3\omega^3} \left[ 1 + \frac{1}{\omega^3} \cos 2\omega x - \frac{3}{2} \frac{\sin 2\omega x}{\omega x} \right] ,$$

$$z = \frac{1}{3\omega^3} \left[ 1 + \frac{1}{\omega^3} \cos 2\omega x - \frac{3}{4} \frac{\sin 2\omega x}{\omega x} \right] .$$

$\omega = 2.04$ is the wave number for quarks in the bag ground state and $N = 2.27$ is a normalization constant. Numerical integration reveals

$$I_M = 1.5 \times 10^{-3} .$$

For $\mathcal{L}_{2m} = -2$ we can use the equations of motion to write

$$\langle K^0 \mid \mathcal{L}_{2m} = -2 \mid K^0 \rangle = \frac{8N^4}{R^4} 2m_K (m_3 R L + \omega) K .$$

The first of these operators has been studied by Shrock and Treiman (and for $\Delta S = 1$ in Ref. 12). The second may be neglected due to the smallness of $m_d$ (both in the bag model and in current algebra). The last piece must be calculated directly. Adding these contributions, we find

$$\langle K^0 \mid \mathcal{L}_{2m} = -2 \mid K^0 \rangle = \frac{8N^4}{R^4} 2m_K (m_3 R L + \omega) K .$$
with

\[ I_K = \frac{1}{4\pi} \int_0^1 x^2 dx \left[ j_0^4(\alpha x) - j_1^4(\alpha x) - 4 \frac{j_0(\alpha x)j_1(\alpha x)}{\alpha x} \right], \]

\[ I_L = \frac{1}{4\pi} \int_0^1 x^2 dx \left[ j_0^4(\alpha x) + j_1^4(\alpha x) - 6 j_0^2(\alpha x)j_1^2(\alpha x) \right]. \]  

(25)

Numerically

\[ I_K = 2.8 \times 10^{-2}, \quad I_L = -2.5 \times 10^{-3}. \]  

(26)

For the conventional value of \( m_\tau \sim 300 \text{ MeV}, \ m_\tau R \sim 1 \) and the matrix element of the third operator in Eq. (23) dominates although some cancellation does occur between the first and third matrix elements.

Combining these results yields the final form

\[ \frac{\epsilon}{\epsilon_m} = 2.1(m_K R)^2 \alpha_s \left[ \frac{1}{m_{H_2}^2 - m_{H_1}^2} \left( m_{H_2}^2 \ln \frac{m_{H_2}^2}{m_c^2} - m_{H_1}^2 \ln \frac{m_{H_1}^2}{m_c^2} \right) - \frac{3}{2} \right], \]

\[ \approx 2.6 \alpha_s \left[ \frac{1}{m_{H_2}^2 - m_{H_1}^2} \left( m_{H_2}^2 \ln \frac{m_{H_2}^2}{m_c^2} - m_{H_1}^2 \ln \frac{m_{H_1}^2}{m_c^2} \right) - \frac{3}{2} \right]. \]  

(27)

Choosing \( \alpha_s \sim 1, \ m_{H_2} \gg m_{H_1}, \) and \( m_\tau \sim 1.5 \text{ GeV}, \) one obtains

\[ \frac{\epsilon}{\epsilon_m} \approx 2.6 \left( \frac{m_{H_1}^2}{m_c^2} - \frac{1}{2} \right) \sim 7, \ \text{for} \ m_{H_1} \sim 10 \text{ GeV}, \]  

(28)

which is a factor of more than 20 from the experimentally allowed value. Thus, our alternate and independent calculation—using the MIT bag model and taking the amplitudes for \( K \rightarrow 2\pi \) and \( \Delta m_K \) from experiment—agrees with the conclusions of D&S.

Given the importance of this result, we need to assess the reliability of the calculation. Could we have erred by a factor of 20? Certainly such errors do not occur in the estimates of the matrix elements. The partial cancellation between \( I_K \) and \( I_L \) in Eq. (24) can yield a factor-of-two or so uncertainty, and the value of \( \alpha_s \) might contain another factor of two. There are other uncertainties which cancel in the ratio of matrix elements. There appears to be little freedom to reduce the result to an acceptable value.

Another uncertainty of possibly far greater significance is the effect of the \( t \) quark, which we have thus far ignored. We consider the important role of the \( t \) quark in Sec. IV.

### IV. EFFECTS OF THE \( t \) QUARK

The previous analysis was carried out in the four-quark model without the \( t \) and \( b \) quarks. One might feel that the neglect of the \( t \) quark is reasonable as the KM mixing angles which connect the \( t \) to \( d \) and \( s \) are small. However, Higgs-boson couplings increase with the particle mass and diagrams with a \( t \) quark are enhanced by a factor of \( m_t^2/m_c^2 \geq 140 \) over those with a \( c \) quark. For much of the possible range of the KM angles, the \( t \)-quark processes dominate and cannot be ignored. The modifications would be fairly simple except that it is quite possible that the mass of the \( t \) quark may be comparable to the mass of the Higgs. One must therefore include the full dependence upon quark masses in the Feynman amplitudes, which makes a presentation of the results more complicated. We will first present the result with \( m_t \ll m_{H_1}, m_{H_2} \) in order to display the im-
portant new features, and then generalize to the case of arbitrary masses.

The reason that \( m_t \ll m_{H_{1}}, m_{H_{2}} \) is simple is that the form of the \( \Delta S = 1 \) and \( \Delta S = 2 \) operators remains unchanged—only the coefficients are different. For \( \Delta S = 1 \) in place of Eq. (8), we have

\[
\tilde{g} = \frac{G_F^2}{32 \pi^2} m_s \sum_{i=1}^{2} \text{Im} \gamma_T^i \left[ \rho_t m_t^2 + \rho_c m_c^2 \right]
\]

\[
+ 2 \rho_t \rho_c m_c^2 \ln \frac{m_t^2}{m_c^2},
\]

(29)

The effect of the \( t \) quark depends on the size of the KM parameters in \( \rho_t \). A useful constraint comes from the \( K_L - K_S \) mass difference. Since the charmed quark by itself can account for (approximately) the observed mass difference, the \( t \) quark's contribution should not be very much larger than that of the \( c \) quark. This may be written

\[
\rho_t m_t^2 \gg \rho_c m_c^2.
\]

(32)

Since \( m_t > 18 \text{ GeV} \) and \( \rho_c \sim \sin \theta_C \), this yields

\[
\rho_t \gg 0.08 \sin \theta_C.
\]

(33)

In addition, the rate for \( K_L \rightarrow \mu^+ \mu^- \) constrains \( \rho_t \)

\[
\rho_t < |s_1 c_3| \left( \frac{61 \text{ GeV}^2}{m_t^2} \right)
\]

(34)

or

\[
\rho_t \lesssim 0.19 \sin \theta_C
\]

for \( m_t > 18 \text{ GeV} \). Note that these bounds become more powerful if \( m_t \) is taken larger. We shall primarily employ the latter constraint [Eq. (34)] in the analysis which follows.

The \( t \) quark dominates in the numerator of Eq. (31) as long as \( \rho_t m_t^2 \gg \rho_c m_c^2 \), which is much of the allowed range of KM angles. In the denominator of Eq. (31), the \( t \) quark never dominates by a large factor since the KM angles enter precisely the same form as in the \( K_L - K_S \) mass difference. Therefore the ratio \( \frac{\xi}{\epsilon_m} [\text{Eq. (31)}] \) is in general made larger if the \( t \)-quark contribution becomes important. In particular, if \( m_t \) and \( \rho_t \) are sufficiently large, Eq. (31) becomes

\[
\frac{\xi}{\epsilon_m} \sim 0.58 \frac{\alpha_s}{\rho_t} \left[ \frac{m_{H_{1}}^2}{m_t^2} - \frac{1}{2} \right] > 14 \left[ \frac{m_{H_{2}}^2}{m_c^2} - \frac{1}{2} \right],
\]

(35)

which is well above the experimental bound.\(^{24}\)

However, the possibility for a cancellation between the \( t \)- and \( c \)-quark contributions also exists. We do not know with certainty the relative sign of \( \rho_t \) and \( \rho_c \) and their expected magnitudes are such that a cancellation is possible. This is a conceivable way to save the model, at least until the parameters \( \rho_t, m_t \) are measured. However, it is rather unnatural in that the cancellation would have to be extremely precise (one part in 20).

The above comments indicate some of the changes introduced by the six-quark model. To be completely general, we must consider the case with no restriction on the \( t \) and Higgs-boson masses. The \( \Delta S = 1 \) effective operator can still be written as in Eq. (12), but with

\[
\tilde{f} = \frac{G_F g_s}{\sqrt{2} 32 \pi^2} m_s \sum_{i=1}^{2} \text{Im} \gamma_T^i \left[ \rho_t m_t^2 \left( \ln \frac{m_{H_{1}}^2}{m_t^2} - \frac{1}{2} \right) + \rho_c m_c^2 \left( \ln \frac{m_{H_{2}}^2}{m_c^2} - \frac{1}{2} \right) \right].
\]

(30)
\[ f = \frac{G_F}{\sqrt{2}} \frac{g_5}{32\pi^2} m_s \sum_{i=1}^{2} \left[ \rho_i m_i^2 \frac{\text{Im} \gamma_i^s}{m_{H_i}^2 - m_i^2} \left( -\frac{m_{H_i}^2}{m_{H_i}^2 - m_i^2} - \frac{1}{2} + \frac{m_{H_i}^4}{(m_{H_i}^2 - m_i^2)^2} \ln \frac{m_{H_i}^2}{m_i^2} \right) \right] + \rho_c m_c^2 \frac{\text{Im} \gamma_c^s}{m_{H_i}^2} \left( \frac{m_{H_i}^2}{m_{H_i}^2 - m_c^2} - \frac{1}{2} \right) \].

(36)

The \( \Delta S = 2 \) interaction, however, is somewhat modified. From the diagrams in Fig. 1 we now find

\[ \mathcal{L}_{\Delta S=2}^{(WH)} = \frac{G_F^2}{32\pi^2} m_s \sum_{i=1}^{2} \left[ \rho_i m_i^2 \left( \alpha_{ij} \gamma^\mu (1 + \gamma_5) s_i \bar{d}_j \gamma^\nu (1 - \gamma_5) s_j + G d_j \gamma^\mu \gamma^\nu (1 - \gamma_5) i \partial \gamma s_i \right) \right] \]

(37)

with

\[
F = \sum_{i=1}^{2} \frac{\text{Im} \gamma_i^s}{m_{H_i}^2} \left[ \rho_i m_i^2 \left( \frac{m_{H_i}^4 (m_{H_i}^2 + m_i^2)}{m_{H_i}^2 + m_i^2)^2} - 2 \frac{m_{H_i}^4 m_i^2}{(m_{H_i}^2 - m_i^2)^2} \ln \frac{m_{H_i}^2}{m_i^2} \right) \right]
+ 2 \rho_i \rho_c m_c^2 \left[ \frac{m_i^2}{m_{H_i}^2 - m_i^2} \ln \frac{m_i^2}{m_{H_i}^2} + \frac{m_i^4}{(m_{H_i}^2 - m_i^2)^2} \ln \frac{m_i^2}{m_{H_i}^2} \right] + \rho_c^2 m_c^2 \]

(38)

\[
G = \sum_{i=1}^{2} \frac{\text{Im} \gamma_c^s}{m_{H_i}^2} \left[ \rho_i m_i^2 \left( \frac{m_{H_i}^4 (m_{H_i}^2 - 3m_i^2)}{m_{H_i}^2 - m_i^2)^2} - 2 \frac{m_{H_i}^4 m_i^2}{(m_{H_i}^2 - m_i^2)^2} \ln \frac{m_i^2}{m_{H_i}^2} \right) \right]
+ 2 \rho_i \rho_c m_c^2 \left( \frac{m_i^2}{m_{H_i}^2 - m_i^2} \ln \frac{m_i^2}{m_{H_i}^2} \right) - \frac{m_i^4}{(m_{H_i}^2 - m_i^2)^2} \ln \frac{m_i^2}{m_{H_i}^2} + \rho_c^2 m_c^2 \right] \]

However, we also must in this case include the contributions from the diagrams of Fig. 5 wherein two Higgs bosons are exchanged. The effective \( \Delta S = 2 \) Lagrangian in this case takes the form

\[ \mathcal{L}_{\Delta S=2}^{(HH)} = \frac{G_F^2}{16\pi^2} m_s \left[ \alpha_{ij} (1 - \gamma_5) s_i \bar{d}_j (1 - \gamma_5) s_j - \bar{c} (i \partial \gamma s_i) (1 - \gamma_5) s_i \bar{d}_j \gamma^\mu (1 + \gamma_5) s_j \right] \]

(39)

where \( \bar{h} \) and \( \bar{c} \) are calculated in the Appendix in terms of \( m_s, m_{H_1} \) and the various mixing angles.

Taking matrix elements we find

\[
\langle K^0 | \mathcal{L}_{\Delta S=2}^{(WH)} | K^0 \rangle = \frac{G_F^2}{32\pi^2} m_s \sum_{i=1}^{2} \left[ \rho_i m_i^2 \left( 2 m_K (8 m_5 R G_{L} + 8 \omega F I_{K}) \right) \right]
\]

\[
\langle K^0 | \mathcal{L}_{\Delta S=2}^{(HH)} | K^0 \rangle = \frac{G_F^2}{16\pi^2} m_s \sum_{i=1}^{2} \left[ \rho_i m_i^2 \left( -10 m_5 R \bar{h} I_{S} + 12 \omega \bar{c} I_{K} \right) \right]
\]

(40)

where

\[
I_S = \frac{1}{4\pi} \int_0^1 dx x^2 [j_0^2(\omega x) + j_1^2(\omega x)]^2 = 9.6 \times 10^{-3}
\]

(41)

In order to study the range of \( \xi / \epsilon_m \) we have many variables at our disposal—\( m_s, m_{H_1} \), and \( m_{H_2} \) as well as the various mixing angles. To simplify this investigation, we consider various ranges. The limit

\[
\begin{array}{c|c|c|c|c|c|c|c}
\hline
\text{sl}(p_1) & H & d(p_2) & \text{sl}(p_1) & t & d(p_2) \\
\hline
\text{sl}(p_1) & H & d(p_2) & \text{sl}(p_1) & t & d(p_2) \\
\hline
\end{array}
\]

FIG. 5. Double-Higgs-boson box diagrams contribute significantly to \( \mathcal{L}_{\Delta S=2}^{(HH)} \) when \( m_s \geq m_{H_1} \).
\[ m_i^2 < m_{H_1}^2, m_{H_2}^2 \]

has been considered already. The double-Higgs-boson box diagram becomes important when \( m_i \) becomes comparable to or greater than the Higgs-boson mass. We can study this by considering the special case

\[ m_i^2 = m_{H_1}^2 < < m_{H_2}^2. \]

The relevant parameters in this limit are (neglecting \( m_c^2/m_i^2 \))

\[
\bar{f} = \frac{G_F}{\sqrt{2}} \frac{g_s}{32\pi^2} m_i \frac{\rho_i \text{Im} \gamma_i}{3},
\]

\[ F = \frac{G}{2} = \frac{\rho_i^2 \text{Im} \gamma_i^*}{3}, \quad (42) \]

\[ \bar{h} = -\frac{\rho_i^2}{6} [\text{Im} \gamma_i^* |V_1|^2 + \text{Im} \gamma_i^* |V_2|^2], \]

\[ \bar{c} = \frac{\rho_i^2}{12} \text{Im} \gamma_i^* |V_1|^2. \]

Combined with Eqs. (18), (19), (21), and (40), this produces for the ratio

\[
\frac{\xi}{\epsilon_m} = -\frac{0.26\alpha_s}{\rho_i \left[ 0.1 + 1.7 \frac{\text{Im} \gamma_i^* |V_1|^2}{\text{Im} \gamma_i^* |V_1|^2 |V_2|^2} + 2.3 |V_1|^2 \right]}, \quad (43)
\]

The constraint that \( \rho_i < 0.19 \sin \theta_C \) results in

\[
\frac{\xi}{\epsilon_m} \gtrsim \frac{6.1\alpha_s}{\left[ 0.1 + 1.7 \frac{\text{Im} \gamma_i^* |V_1|^2}{\text{Im} \gamma_i^* |V_1|^2 |V_2|^2} + 2.3 |V_1|^2 \right]}, \quad (44)
\]

which is much too large for acceptable values of the mixing angles \( \gamma_i \) and \( |V_1|^2 \).

As the final case we consider \( m_i^2 >> m_{H_1}^2, m_{H_2}^2 \).

In computing the coefficients it is important to keep in mind the Glashow-Iliopoulos-Maiani-type cancellation in the imaginary part of the Higgs couplings (\( \text{Im} \gamma_i^* = -\text{Im} \gamma_j^* \)). Thus, the leading contribution to all coefficients except \( \bar{h} \) will cancel.

We find, neglecting \( \mathcal{O}(m_{H_1}^2/m_i^2) \) and retaining the \( \mathcal{O}(m_c^2/m_{H_1}^2) \) terms only in \( \bar{f} \),

\[
\bar{f} = \frac{G_F}{\sqrt{2}} \frac{g_s}{32\pi^2} m_i \sum_{i=1}^{2} \frac{\text{Im} \gamma_i^*}{m_{H_i}^2} p_c m_c^2 \left[ \ln \left( \frac{m_{H_1}^2}{m_c^2} \right) - \frac{3}{2} \right],
\]

\[ F = G = 0, \quad (45) \]

\[ \bar{h} = \sum_{i,j=1}^{2} \text{Im} h^{ij} (I_{ij} + A_{ij} - A_{ij}) + \text{Im} X^{ij} I_{ij}, \]

\[ \bar{c} = 0, \]

where

\[ I_{ij} = 1 + \frac{1}{m_{H_i}^2 - m_{H_j}^2} \left[ m_{H_i}^2 \ln \frac{m_{H_i}^2}{m_i^2} - m_{H_j}^2 \ln \frac{m_{H_j}^2}{m_i^2} \right], \]

\[ A_{21} - A_{12} = \frac{m_{H_1}^2 m_{H_2}^2}{(m_{H_1}^2 - m_{H_2}^2)^2} \ln \frac{m_{H_1}^2}{m_{H_2}^2} - \frac{m_{H_1}^2 + m_{H_2}^2}{2(m_{H_1}^2 - m_{H_2}^2)} \]

Due to the large number of mixing angles and masses remaining, it is difficult to quote any precise value for \( \xi/\epsilon_m \). However, this limit is the only one which we have found in which \( \xi/\epsilon_m \) can be made small "naturally." \( \xi \) receives only vanishing contributions when \( m_i^2 >> m_{H_1}^2, m_{H_2}^2, m_c^2 \) while \( \epsilon_m \), due to the double-Higgs-boson-exchange diagrams, approaches a constant value. In order to determine what scale of masses will reduce \( \xi/\epsilon_m \) to an acceptable value, we may crudely characterize \( \bar{h} \) in Eq. (45) by \( \bar{h} = \rho_i^2 \times (\text{CP-violating angles}). \) This approximation yields

\[
\frac{\xi}{\epsilon_m} \sim 0.08\alpha_s \frac{\rho_i m_c^2}{\rho_i m_{H_i}^2} \ln \frac{m_{H_i}^2}{m_c^2} \quad (47)
\]

If \( \rho_i \) were fairly large (e.g., \( \rho_i \sim 0.19 \sin \theta_C \)), then a large enough \( m_H \) (e.g., \( m_H \geq 20 \text{ GeV} \)) could reduce \( \xi/\epsilon_m \) to an acceptable value. However, in order to satisfy the condition \( m_i^2 >> m_{H_1}^2 \) this also requires a heavy \( t \) quark, which precludes the possibility that \( \rho_i \) is large. We may incorporate this constraint upon \( \rho_i \) [Eq. (34)] into Eq. (47) and write

\[
\frac{\xi}{\epsilon_m} \gtrsim 9.6 \frac{m_c^2}{m_{H_i}^2} \frac{m_{H_i}^2}{m_{H_2}^2} \ln \frac{m_{H_i}^2}{m_{H_2}^2} \left[ \frac{m_i^2}{18 \text{ GeV}} \right]^4. \quad (48)
\]

Equation (48) in fact has no solution for which \( \xi/\epsilon_m \) is acceptable and for which the condition \( m_i^2 >> m_{H_1}^2 \) is satisfied. The limit \( m_i^2 >> m_{H_1}^2, m_{H_2}^2 \) does not provide an escape from the difficulty.
V. DISCUSSION

We have studied the sources of CP violation in Weinberg's model in which the couplings to the gauge bosons are real and CP conserving, while Higgs particles mediate the CP violation. The CP-noninvariant piece of the mass matrix, $\epsilon_m$, has been calculated from the diagrams of Figs. 1 and 5 while $\Delta I = \frac{1}{2}$ CP violation in $K \rightarrow 2\pi$ occurs through the process of Fig. 2. As previous authors have suggested, the model gives rise to an unacceptable large value of $\epsilon/\epsilon_m$ [or equivalently $\epsilon'/\epsilon = \frac{1}{2}(\eta_{+-}/\eta_{00}-1)$]. We have improved on the previous work by (1) using experimental rather than calculated numbers where possible, (2) providing a more realistic calculation of the matrix elements by using the MIT bag model, and (3) including a detailed discussion of the possible role of the $t$ quark. All of these considerations only reinforce the previous conclusion. The only exception which we have found possible is if a large (factor of 20) cancellation between the $c$- and $t$-quark contributions occurs in $\xi$. However, this would be a rather unnatural coincidence. Barring this case, we conclude that this type of Higgs-boson model of CP violation appears to be incompatible with experiment.

Note added. After submission of this paper we received a related paper on the Higgs-boson model by D. Chang [Carnegie Mellon Report No. C00-3066-172 (unpublished)], which also contains evaluations of the double-Higgs-boson box diagrams and a phenomenological analysis.

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APPENDIX

In order to calculate the double-Higgs-boson-exchange box diagrams (Fig. 5) we define the Higgs-boson couplings as

$$\mathcal{L}_H = 2^{3/2}G_F^{1/2} K \sum_{i=1}^{2} \left[ \frac{1 - \gamma_5}{2} U_i H_i^+ + \frac{1 + \gamma_5}{2} V_i H_i^+ \right] \bar{\nu} + \text{h.c.},$$

where $U$, $V$ are defined in Eq. (9) in terms of various Higgs-boson-coupling angles. Figure 5 then yields

$$\text{Amp}_5 = 2G_F^2 m_t^4 m_s \text{Im} \sum_{i,j=1}^{2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_{H_i}^2} \frac{1}{(k+p_3-p_1)^2 - m_{H_j}^2} \frac{1}{(k+p_4)^2 - m_t^2} \frac{1}{(k+p_4)^2 - m_t^2}$$

$$\times \left[ p_i^2 \left| V_j \right|^2 \left| V_i \right|^2 U_i \bar{U}(p_2)(1 - \gamma_5)\bar{s}(p_4)\bar{d}(p_3)(1 + \gamma_3)s(p_1) \right. + p_i^2 \left| V_j \right|^2 V_i U_j \bar{d}(p_2)(p_4 + k)(1 + \gamma_3)s(p_4)\bar{d}(p_3)(1 - \gamma_3)s(p_1) \right.$$

$$\left. + p_i^2 V_i^* V_j U_j m_t \bar{d}(p_2)(1 - \gamma_5)s(p_4)\bar{d}(p_3)(1 - \gamma_3)s(p_1) \right] .$$

Then defining

$$\int \frac{d^4k}{(2\pi)^4} \left[ k^2 - m_{H_i}^2 \right] \left[ (k+q)^2-m_{H_j}^2 \right] \left[ (k+p_3)^2-m_t^2 \right] \left[ (k+p_4)^2-m_t^2 \right] = \frac{1}{16\pi^2 m_t^2} \epsilon \left( A_{ij}, A_{ij} q_\mu + B_{ij} (p_3 + p_4)_\mu \right),$$

(A2)

$$h_{ij} = \rho_i^2 \left| V_j \right|^2 \left| V_i \right|^2 \gamma_i \gamma_j \gamma_i^* \gamma_j^* \left. \right. \quad ,$$

(A3)

we have for the CP-violating piece
\[ \text{Amp}_5 = - \frac{G_F^2}{8\pi} \sum_{i,j=1}^{2} \text{Im} h^{ij} \bar{d}(p_2)(1-\gamma_5) s(p_4) \bar{d}(p_3)[s_5(I + A + B) + s_4 B - s_1 A]_y (1 + \gamma_5) s(p_1) \]
\[ + \text{Im} h^{i*} \bar{d}(p_2)[s_4(I + A + B) + s_3 B - s_2 A]_y (1 + \gamma_5) s(p_4) \bar{d}(p_3)(1 - \gamma_5) s(p_1) \]
\[ + \text{Im} \chi_{ij} m_i \bar{I}_0 \bar{d}(p_2)(1-\gamma_5) s(p_4) \bar{d}(p_3)(1-\gamma_5) s(p_1), \] (A4)

which yields the result given in the text with

\[ I_{ij} = \frac{m_i^4}{m_j^2 - m_i^2} \left[ \frac{m_j^2}{m_j^2 - m_i^2} \ln \frac{m_j^2}{m_i^2} - \frac{m_H^2}{m_H^2 - m_i^2} \ln \frac{m_H^2}{m_i^2} + \frac{m_H^2 - m_i^2}{m_H^2 - m_i^2} \right] \]
\[ A_{ij} = \frac{1}{2} \frac{m_i^4}{m_j^2 - m_i^2} \left[ \frac{m_j^2}{m_j^2 - m_i^2} \ln \frac{m_j^2}{m_i^2} + \frac{m_H^2}{m_H^2 - m_i^2} \ln \frac{m_H^2}{m_i^2} + \frac{m_H^2 - m_i^2}{m_H^2 - m_i^2} \right] \]
\[ B_{ij} = -\frac{1}{2} \frac{m_i^4}{m_j^2 - m_i^2} \left[ \frac{m_H^2}{m_H^2 - m_i^2} \ln \frac{m_H^2}{m_i^2} - \frac{m_i^2}{m_H^2 - m_i^2} \ln \frac{m_H^2}{m_i^2} + \frac{m_H^2 - m_i^2}{m_H^2 - m_i^2} \right] \]
\[ + \frac{1}{2} \frac{m_i^4}{m_j^2 - m_i^2} \left[ \frac{m_H^2}{m_H^2 - m_i^2} \ln \frac{m_H^2}{m_i^2} - \frac{m_j^2}{m_H^2 - m_j^2} \ln \frac{m_H^2}{m_j^2} + \frac{m_H^2 - m_j^2}{m_H^2 - m_j^2} \right] \]  
(A5)

This gives rise to the effective Lagrangian Eq. (39), with

\[ \tilde{c} = \sum_{i,j=1}^{2} 2 \text{Im} h^{ij} B_{ij}, \quad \tilde{h} = \sum_{i,j=1}^{2} \text{Im} \chi_{ij} I_{ij} + \text{Im} h^{ij}(I_{ij} + A_{ij} - A_{ij}) \]  
(A7)

9See, e.g., J. F. Donoghue, E. Golowich, W. Ponce, and
By vacuum insertion we mean here that one or more currents are used to connect a single particle to the vacuum. Sometimes such diagrams are classed as single-particle intermediate states, cf. Ref. 5.

Particle Data Group, Rev. Mod. Phys. 52, 51 (1980).


Here it is assumed that a cancellation between $\ln m_H^2 / m_i^2$ and $\frac{1}{2}$ does not occur. In fact, such a cancellation is not possible. Equations (31) and (35) are valid only for $m_H^2 >> m_i^2$. When the full dependence on both masses is maintained [cf. Eq. (36)], one finds the minimum of this function (where $m_i^2 = m_H^2$) is $\frac{1}{2}$. Even in this extreme case Eq. (35) is still well above the experimental limit.