University of Massachusetts Amherst

From the Selected Works of John Donoghue

1981

Low mass glueballs in the meson spectrum

John Donoghue, *University of Massachusetts - Amherst*

K. Johnson, *Massachusetts Institute of Technology*

Available at: https://works.bepress.com/john_donoghue/25/

LOW MASS GLUEBALLS IN THE MESON SPECTRUM[☆]

John F. DONOGHUE

*Department of Physics and Astronomy, University of Massachusetts, Amherst, MA 01003, USA
and Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics,
Massachusetts Institute of Technology, Cambridge, MA 02139, USA*

K. JOHNSON

*Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics,
Massachusetts Institute of Technology, Cambridge, MA 02139, USA*

and

Bing An LI

*Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94305, USA
and Institute of High Energy Physics, Academia Sinica, Beijing, China*

Received 27 October 1980

The spectrum of bound states of gluons is discussed within the MIT bag model. We argue that (1) contrary to previous analysis there is no light exotic 1^{-+} state in the two gluon sector, (2) experiments on the three gluon sector can clearly differentiate the bag model from other models, (3) the state seen in J/ψ radiative decays at 1.4 GeV is most likely a pseudo-scalar glueball, and (4) there should be a second 2^{++} state underneath the $f(1270)$ resonance.

Bound states composed primarily of gluons — glueballs — are expected within QCD [1–4]. We wish to discuss the spectrum of glueballs, using the MIT bag model^{‡1} and comment on the role of glueballs in the known meson spectrum. Several of our conclusions will disagree with those of previous works [2,3] and we will discuss these differences.

In the simplest version of the bag model [2], one considers free gluon fields in the static spherical bag, with the confining boundary condition that no gluon flux pass through the surface

$$n_{\mu} F^{\mu\nu} = 0, \quad (1)$$

where n_{μ} is the normal to the surface. The eigenmodes

can be simply solved for. There are two families of solutions, transverse electric (TE) with parity $-(-1)^l$ and transverse magnetic (TM) with parity $(-1)^l$. There are no transverse $l = 0$ solutions and the lowest state is the $l = 1$ TE mode (1^{+}), with energy $E = 2.744/R$. The next excited states are the $l = 2$ TE state (2^{-}) with $3.96/R$, and the $l = 1$ TM mode (1^{-}) with $E = 4.49/R$.

We should note here that we are assuming that the more stable shape for a bag which contains vector gluons, is spherical. This is not true in the most naive model with free fields confined by a volume energy, BV , alone as has been pointed out by Giles [5]. This can be seen by considering the modes in a rectangular box with cross section b^2 and length a . There is a mode with energy $(\pi/b) \times \sqrt{2}$ and hence a “pancake” shaped glueball with mass $\propto a^{1/3} B^{1/3}$ which vanishes as $a \rightarrow 0$. Here we assume that when the gluon–gluon interactions are added that the bags with minimum mass will be roughly symmetrical. This question is presently being studied [6].

[☆] This work is supported in part through funds provided by the US Department of Energy (DOE) under contract DE-AC02-76ERO3069.

^{‡1} The bag model work of ref. [2] is superseded by the present paper.

Color singlet glueballs can be formed from two or three gluons. For the ground state of two gluons, a totally symmetric combination yields $J^{PC} = 0^{++}, 2^{++}$. For the first excited state we combine the 1^+ lower energy configuration with 2^- and 1^- modes to obtain, $3^{-+}, 2^{-+}, 1^{-+}, 2^{-+}, 1^{-+}, 0^{-+}$. However due to the static cavity approximation some of these states are spurious, corresponding to the translation of the ground state. The method for counting these states is known [7]. The spurious states correspond to a gradient (1^-) acting on the ground states ($0^{++}, 2^{++}$); i.e., $1^{-+}, 3^{-+}, 2^{-+}, 1^{-+}$ should be removed, leaving 2^{-+} and 0^{-+} . The same techniques can be used in the three gluon sector. Using the antisymmetric color factor f_{ABC} one can form a 0^{++} out of three 1^+ modes, while with d_{ABC} 1^{+-} and 3^{+-} can be obtained. For the excited two 1^+ modes and a 2^- or 1^- can form $4^{--}, 3^{--}, 2^{--}, 1^{--}, 0^{--}, 3^{--}, 2^{--}, 1^{--}, 3^{--}, 2^{--}, 1^{--}, 2^{-+}, 1^{-+}, 2^{-+}, 1^{-+}, 0^{-+}$ but many of these are spurious. The ones which remain are listed in table 1. Also included in table 1 are simple estimates of the masses of these states using the bag model with no interactions between the gluons. (With quark states this approximation produces degenerate 0^{-+} and 1^{-+} meson octets and $1/2^-$ and $3/2^-$ baryon multiplets.)

This spectrum differs from most other authors in two regards: (1) there is no exotic 1^{-+} among the first excited states of two gluons, and (2) the three gluon spectrum is of opposite parity from that found in many models. Some of the competing models generally consider gluons as massive vector fields (just like the ρ or ω) in a potential. It is our treatment of gluons as massless fields, plus the confining boundary condition, which leads to the above differences. Let us discuss each of these in turn.

The absence of the exotic 1^{-+} state arises because one cannot combine two transverse gluons into such quantum numbers. Years ago Yang [8] analysed the possible combinations of two photons, restricted by gauge and Lorentz invariance. He found that the familiar ($0^{++}, 2^{++}, 4^{++}, \dots$), ($0^{-+}, 2^{-+}, 4^{-+}, \dots$) and ($3^{++}, 5^{++}, 7^{++}, \dots$) were allowed. The absence of 1^{-+} in our work is related to the lack of 1^{-+} in Yang's analysis.

We feel this result is more general than the bag model. It can also be seen by considering interpolating fields for glueballs. With quark states one can form the appropriate spin and angular momentum combinations by considering quark bilinears. For example

$$\bar{\psi}\gamma_5\psi, \quad \text{or} \quad p_\mu\bar{\psi}\gamma^\mu\gamma^5\psi \quad (2)$$

and

$$\epsilon_\mu^{\alpha}\bar{\psi}\gamma^\mu\psi, \quad \text{or} \quad V_{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\psi,$$

with $V_{\mu\nu} = p_\mu\epsilon_\nu^* - p_\nu\epsilon_\mu^*$, project out the ground states of two quark (0^{-+} and 1^{-+}). High spin states require derivatives in the bilinears. For glueballs gauge invariance requires that we use the field strength tensors $F_{\mu\nu}^A$. The ground state can be formed from

$$F_{\mu\nu}^A F^{\mu\nu A} \sim 0^{++}, \quad \epsilon_{\mu\nu}^* F^A{}^{\mu\lambda} F^A{}_{\lambda}{}^\nu \sim 2^{++}, \quad (3)$$

where $\epsilon_{\mu\nu}^*$ is the spin two polarization tensor. The next multiplet is

$$F_{\mu\nu}^A \tilde{F}^A{}^{\mu\nu} \sim 0^{-+}, \quad \epsilon_{\mu\nu}^* F^A{}^{\mu\lambda} \tilde{F}^A{}_{\lambda}{}^\nu \sim 2^{-+}, \quad (4)$$

where $\tilde{F}^A{}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} F^A{}_{\alpha\beta}$. The 1^{-+} candidate ($V_{\mu\nu} F^A{}^{\mu\lambda} F^A{}_{\lambda}{}^\nu$) vanishes by the symmetry of the Lorentz indices. The candidate with one derivative [$\epsilon_\mu^* F^A{}^{\nu\lambda} D_\lambda \tilde{F}^A{}_{\nu}{}^\mu$] also can be shown to vanish. These arguments suggest that the 1^{-+} probably does not be-

Table 1
The low lying glueball states.

	J^{PC}	"Naive" mass estimate (GeV)
Two gluon, ground states	$0^{++}, 2^{++}$	0.96
Two gluon, first excited states	$0^{-+}, 2^{-+}$	1.3
Three gluon, ground states	$0^{++}, 1^{+-}, 3^{+-}$	1.45
Three gluon, first excited states	$3^{-+}, 2 \times 2^{-+}, 1^{-+},$ $0^{-+}, 3^{--}, 2^{--},$ $2 \times 1^{--}$	1.8

long in the same multiplet as 0^{-+} and 2^{-+} . All three approaches (our construction, Yang's analysis and the absence of simple interpolating fields) rely heavily on both gauge invariance and Lorentz invariance, and it is the consideration of these which we feel other models have over simplified, or omitted altogether. This does not mean that 1^{-+} quantum numbers cannot occur in glueballs. (E.g., Yang's theorem does not prevent three transverse spin one particles from combining to form a $J = 1$ state.) In the bag model the first such state occurs in the three gluon system in the first excited state with an estimated mass of 1.8 GeV. Such a state would presumably be more difficult to find than others lower in the spectrum. We emphasize that low mass glueballs with exotic quantum numbers are not expected and that if searches for them are negative, this is probably not very good evidence against the existence of low mass glueballs of other quantum numbers.

In the three gluon spectrum, the parity difference arises because the lowest mode in the potential models is an $l = 0$, 1^- state, while in the bag it is $l = 1$, 1^+ . We feel that the use of the $l = 0$ state is incorrect. For massless fields the $l = 0$ mode is the Coulomb mode and does not exist in the absence of sources. The lowest transverse modes in a cavity are either the $l = 1$ TE or TM solutions. The confining boundary condition, eq. (1), favors TE^{‡2}. The two different approaches can be differentiated experimentally by the observation of the three gluon states in the glueball spectrum.

The mass estimates given above do not include the spin interaction between gluons. Thorn [9] has calculated the effect of this for the 0^{++} and 2^{++} ground state. He finds that the 2^{++} shifts upwards to $M = 1.30$ GeV while the 0^{++} drops to $M = 0.10$ GeV. (With the center of mass correction of ref. [5] this state has $M^2 < 0$.) The result for the scalar state suggests that this glueball configuration is probably a component of the vacuum wave function. That is, the vacuum would contain a Bose condensate of the 0^{++} glueballs [10,6]. In this case, the physical scalar will

be a state orthogonal to the vacuum and would be at a much higher mass. As we do not yet know how to carry out this procedure, we have no firm predictions for the mass of the 0^{++} glueball, but a mass in the region of one GeV would be expected.

Since the 2^{++} state is orthogonal to the vacuum, we expect that the prediction for the 2^{++} glueball is reasonable and we have no reason to suspect more than moderate uncertainty in this estimate. However this leads to a serious problem when confronting the meson spectrum, which we will discuss later.

There is experimental data coming from the decays $J/\psi \rightarrow \gamma + \bar{X}$ which we would like to suggest as possible evidence for a glueball spectrum similar to that which we are proposing. The radiative decays of heavy vector mesons are expected in QCD to proceed through a hadronic final state with two gluons in a color singlet state. The mesons produced should map out the resonances which couple to two gluons – that is, they should be states which are either glueballs or which have a significant glueball component.

Seen in radiative J/ψ decay are the η , η' , $f(1270)$ and a state near 1.4 GeV [11]. We propose that the latter, called G(1400) below, is a glueball with $J^{PC} = 0^{-+}$, naively predicted in the bag to be near 1.3 GeV, rather than the $E(1420) J^{PC} = 1^{++}$ quark state^{‡3}. The final state which is seen is $(\delta\pi)$ which requires that it be 0^{-+} , 1^{++} , 2^{-+} etc. However, the 1^{++} configuration does not couple to two massless gluons by Yang's analysis. A detailed study of the QCD matrix element for $J/\psi \rightarrow \gamma gg$ shows that it has strong signals only in the 0^{++} , 0^{-+} , and 2^{++} channels with all other spins being negligible [12]. Combined with the observed final state this requires that G be 0^{-+} . It is of course possible that off shell gluons couple somewhat to 1^{++} , but we would expect such production to be suppressed rather than being the largest radiative branching fraction observed. In addition if the observed state were the E then its partner, the D(1285) would also be expected to be seen. (With ideal ("magic") mixing $J/\psi \rightarrow \gamma D$ would be twice as strong as $J/\psi \rightarrow \gamma E$.) Given that the bag model predicts a 0^{-+} state nearby, we feel that these arguments favor identifying the G(1400) with a pseudoscalar glueball. At such a mass the G could mix with the known pseudoscalars, η and η' , in

^{‡2} Our list differs somewhat from Bjorken [3] who considered the states which could be made from "S" wave (in our notation, " $l = 1$ " modes of the vector potential), even and odd parity operators. In the bag the mode energies of even and odd parity " $l = 1$ " modes are rather different ($2.74/R$ versus $4.49/R$). In addition there is the ($l = 2$) mode with energy ($3.96/R$) in the low mass region, which was excluded in Bjorken's analysis.

^{‡3} For a different view about the E(1440) see Carlson et al. [1].

order to produce their signals in J/ψ radiative decay. Clearly the best way to test this picture is to measure the spin of the G seen in radiative decay.

The 2^{++} glueball is more difficult to reconcile with the meson spectrum. At least in the first approximation, it is expected to be lighter than the 0^{-+} state. Like other tensor states it can couple to $\pi\pi$. However the partial wave analysis in the D wave $\pi\pi$ system appears good and, outside the $f(1270)$ and the $f'(1520)$ there does not seem to be any room for another 2^{++} resonance, even if fairly inelastic, below 1.6 GeV. As the bag model prediction is not flexible enough to push the state above 1.6 GeV, the only available option appears to be that the glueball lie exactly where predicted, i.e., underneath the $f(1270)$ peak.

This suggestion is supported by the evidence coming from J/ψ radiative processes. The decay $J/\psi \rightarrow \gamma$ is seen quite clearly while the channel $J/\psi \rightarrow \gamma f'(1520)$ is not yet seen. Due to the flavor independence of the gluon coupling the $f[= 2^{-1/2} (u\bar{u} + d\bar{d})]$ and the $f' (= s\bar{s})$ would appear in the ratio $\gamma f : \gamma f' = 2 : 1$. This is not observed, although the limit is not too far beyond this at present. If the glueball were near the f it would mix with it strongly, leading to a sizeable γf signal. The f' signal would be smaller (though not zero) because it is further from the glueball. Note that if the glueball were heavier than 1.6 GeV it would be expected to mix more strongly with the f' than the f , contrary to indications.

In a partial wave analysis with two resonances close to each other one could expect to see two separate resonances. However there is a simple mechanism which allows a particle to hide underneath another. The physics of scattering in the presence of two poles was extensively studied in the days of the split A_2 , and the mechanism was uncovered then [13]; it appears particularly well suited for what might be expected to occur in an f-glueball system.

Consider first two states which both couple to essentially only one two body channel (e.g. $\pi\pi$). Let us use a basis where the real part of the mass matrix is diagonal. The two states will mix through a width matrix $\Gamma_{\alpha\beta}$ where

$$\Gamma_{\alpha\beta} = 2\pi \langle \alpha | V | \pi\pi \rangle \langle \pi\pi | V | \beta \rangle \delta(E_{\pi\pi} - E). \quad (5)$$

The S -matrix for $\pi\pi \rightarrow \pi\pi$ is given by

$$S = 1 + \frac{i\sqrt{s}}{8\pi} \langle \pi\pi | V | \alpha \rangle \left[\frac{1}{m^2 - im\Gamma - s} \right]_{\alpha\beta} \langle \beta | V | \pi\pi \rangle, \quad (6)$$

plus a small non-resonant background term needed to maintain unitarity away from resonance ^{†4}. When the masses are well separated this gives the usual two resonance picture but as the masses become equal one state decouples from $\pi\pi$ and only one resonance is seen. This occurs because $m^2 - im\Gamma$ can be diagonalized in the limit $m_1 \rightarrow m_2$ into two states, one of which has $\Gamma = 0$ — i.e. no coupling to $\pi\pi$. This can be seen from the fact that the width matrix has a zero eigenvalue. In the complex s plane, what occurs is that of the two poles and two zeros of the S -matrix for the well separated case, one pole and one zero coalesce.

The more physically interesting situation occurs with more than one open channel. In this case if the mixing of the two states is dominantly through a particular two body channel, the particles will mix such that one state will decouple from that channel. However two resonances should be seen in other less dominant channels. This situation may be applicable to the f-glueball system. The $\pi\pi$ mode is strong (83%) for the observed resonance, and other two pseudoscalar configurations are suppressed strongly (greater than a factor of 10) by D wave phase space. The two states could mix primarily through $\pi\pi$, effectively decoupling one from this channel.

To see how well this works, consider a state A with coupling to $(\pi\pi; K\bar{K}; \text{other})$ of (83; 3; 14)% and a state B with branching fractions (50; 5; 45)% and total width $\Gamma_B = \frac{1}{3} \Gamma_A$. It is assumed that no mixing occurs for the channel "other". We have given the state B a sizeable "other" component because a glueball might have an enhanced decay to $\eta\eta$ [as the η has a sizeable gluonic component (Novikov et al. [1])] and also may have multipion final states (Roy and Walsh [1]). The factor of 3 in total width is motivated by counting in the large N_c limit [4]. For $\Delta m \ll \Gamma_1$, one obtains two states with couplings to $(\pi\pi; K\bar{K}; \text{other})$ of (95, 4, 1)% and (0.1, 1, 99)% with $\Gamma_2 = 0.27 \Gamma_1$. Such small coupling to $\pi\pi$ makes it essentially invisible in $\pi\pi \rightarrow \pi\pi$. For $\Delta m \approx \Gamma_i$ the result is not as dramatic, but the effect remains.

The way to check our proposal is to study the spin two system using channels other than $\pi\pi$. Two photon physics, J/ψ radiative decays and KK scattering should produce both states. Using $\pi\pi$ to observe the states will favor the stronger channel considerably, although

^{†4} This analysis can also be done with the K matrix where unitarity is manifest.

not as much as in $\pi\pi \rightarrow \pi\pi$. Perhaps the difficulty in fitting $\gamma\gamma \rightarrow \pi\pi$ with a single Breit–Wigner shape [16] is an indication of a second state. At present sophistication, $\pi\pi \rightarrow KK$ studies [17] need not see two states. A careful analysis which does not assume only a two channel S -matrix is needed. $\gamma\gamma \rightarrow KK$, $J/\psi \rightarrow \gamma KK$ or $KK \rightarrow KK$ are processes which would certainly need to reveal two states although the inelasticities should be large, and better precision is needed than present experiments have.

This prediction of a second state near the f mass is quite serious. If it is disproven, the bag model description of glueballs is most likely incorrect, at least if the bag model is implemented for gluons as has been successful for quarks. We have found that glueball masses start fairly low in the meson spectrum. We have made several clear testable predictions. Hopefully they will be confirmed or refuted in the near future.

We would like to thank R. Jaffe, J.L. Petersen and L. Stodolsky for useful conversations. One of us (K.J.) would like to thank SLAC and Nordita where some of this work was completed, for their support and hospitality.

References

- [1] H. Fritzsch and P. Minkowski, *Nuovo Cimento* 30A (1975) 393;
P.G.O. Freund and Y. Nambu, *Phys. Rev. Lett.* 34 (1975) 1645;
J. Willemsen, *Phys. Rev.* D13 (1976) 1327;
J. Kogut, D. Sinclair and L. Susskind, *Nucl. Phys.* B114 (1976) 199;
P. Roy and T. Walsh, *Nucl. Phys.* B140 (1978) 449;
K. Ishikawa, *Phys. Rev.* D20 (1979) 731; UCLA preprint UCLA/80/TEP/6 (1980);
V. Novikov, M. Shifman, A. Vainshtein and V. Zakharov, *Nucl. Phys.* B165 (1980) 55, 67;
R.H. Capps, Purdue preprint (1980);
A. Salomone, J. Schechter and T. Tudron, Syracuse preprint, SU 4217-174 (1980);
H. Suura, *Phys. Rev. Lett.* 44 (1980) 1319;
C.E. Carlson, Invited talk at XXth Intern. Conf. on High energy physics (Madison, WI, July 1980);
C.E. Carlson, J. Coyne, P. Fishbane, F. Gross and S. Meshkov, *Phys. Lett.* 98B (1981) 110.
- [2] R.L. Jaffe and K. Johnson, *Phys. Rev. Lett.* 34 (1976) 1645.
- [3] D. Robson, *Nucl. Phys.* B130 (1977) 328;
J.D. Bjorken, SLAC Summer Institute on Particle Physics (1979);
J.J. Coyne, P.M. Fishbane and S. Meshkov, *Phys. Lett.* 91B (1980) 259.
- [4] J.F. Donoghue, Invited talk at the VIth Intern. Conf. on Experimental meson spectroscopy (Brookhaven, April 1980), to be published (MIT CTP #854);
J.F. Donoghue, Invited talk at the XXth Intern. Conf. on High energy physics (Madison, WI, July 1980), to be published (MIT CTP #874);
K. Johnson, Invited talk QCD Symp. (Copenhagen, Denmark, June 1980), to be published.
- [5] R. Giles, to be published.
- [6] H. Hansson, K. Johnson and C. Peterson, in preparation.
- [7] C. Rebbi, *Phys. Rev.* D12 (1975) 2407; D14 (1976) 2362;
T. De Grand and R. Jaffe, *Ann. Phys. (New York)* 100 (1976) 425;
T. De Grand, *Ann. Phys. (New York)* 101 (1976) 396.
- [8] C.N. Yang, *Phys. Rev.* 77 (1950) 242.
- [9] C. Thorn, unpublished.
- [10] R. Fukuda, *Phys. Rev.* D21 (1980) 485, and earlier references contained therein.
- [11] D. Scharre, Invited talk at the VIth Intern. Conf. on Experimental meson spectroscopy (Brookhaven, April 1980), to be published (SLAC-PUB-2519);
D. Scharre, Invited talk at XXth Intern. Conf. on High energy physics (Madison, WI, July 1980), to be published.
- [12] A. Billoire, R. Lacaze, A. Morel and H. Navelet, *Phys. Lett.* 80B (1979) 381.
- [13] K.E. Lassila and V. Ruuskanen, *Phys. Rev. Lett.* 19 (1966) 762;
L. Stodolsky, in: *Proc. Intern. Conf. on Experimental meson spectroscopy* (1970), eds. C. Baltay and L. Rosenfield.
- [14] R. Hollebeck, Talk at XVth Rencontre de Moriond (March 1980), SLAC-PUB-2517.
- [15] D. Cohen et al., Argonne preprint ANL-HEP-PR-80-23.