University of Massachusetts Amherst

From the Selected Works of John Donoghue

1980

Proton lifetime and branching ratios in $SU(5)$

John Donoghue, *University of Massachusetts - Amherst*

PROTON LIFETIME AND BRANCHING RATIOS IN SU(5)[☆]

John F. DONOGHUE

*Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics,
Massachusetts Institute of Technology, Cambridge, MA 02139, USA*

Received 13 December 1979

Revised manuscript received 19 February 1980

The decay rate of the proton into two-body channels is calculated in the SU(5) grand unified theory using the hadronic wavefunctions of the bag model. Summing only the two-body modes leads to a lifetime of $\tau_p = 8 \times 10^{30}$ yr, for a heavy boson mass of 3.8×10^{14} GeV.

In 1974, Georgi and Glashow [1] suggested that the strong, electromagnetic and weak forces could be unified into a fundamental gauge interaction based on group SU(5). The SU(5) symmetry is broken down at some high energy ($\approx 10^{14}$ GeV) to the low-energy gauge group $SU(3)_{\text{QCD}} \times [SU(2) \times U(1)]_{\text{Weinberg-Salam}}$. Since quarks and leptons occur in the same SU(5) multiplets, there are interactions, mediated by heavy vector bosons, which violate baryon number (B) and lepton number (L) but leaving $B - L$ conserved. In such a theory, the proton is unstable and will decay. Estimates of the proton's lifetime [2,3] are very close to the present experimental limits [4]. Stimulated by this theory, several experimental groups are considering projects to measure or considerably reduce the limit on the proton lifetime. Given the magnitude of the effort involved, it is important that the theoretical aspects be refined. The original lifetime estimate of ref. [2] (hereafter referred to as BEGN) calculated the inclusive decay into free quarks. A more realistic but more difficult method is to calculate and sum the decay rates into specific final states. Machacek [5] has made a step in this direction by studying the relative branching fractions of the two-body modes, using non-relativistic SU(6), but did not consider the absolute magnitudes. In this paper we use the MIT bag model

to calculate the magnitude of the decay rate into all of the two-body modes, for a proton and for a bound neutron.

The baryon number violating interaction [2] has the form

$$\mathcal{H}^{\text{bv}} = (G_u/\sqrt{2}) \{ \bar{e}^+ [2(1 - \gamma_5) - (1 + \gamma_5)] O_1 + \bar{\nu}(1 + \gamma_5) O_2 \}, \quad (1)$$

with G_u given by

$$\frac{G_u}{\sqrt{2}} = \frac{g^2}{8M_x^2} = \frac{g^2}{8M_y^2} = \frac{g^2}{8M^2}, \quad (2)$$

and where g is the unified coupling constant and M is the heavy boson mass^{†1}. The operators $O_{i\alpha}$ (α is a Dirac spinor index, $\alpha = 1, 2, 3, 4$) contain three quark fields:

$$O_{1\alpha} = \epsilon^{ijk} \bar{u}_i^c \gamma_\mu (1 + \gamma_5) u_j [\gamma^\mu d_k]_\alpha, \quad (3)$$

$$O_{2\alpha} = \epsilon^{ijk} \bar{u}_i^c \gamma_\mu (1 + \gamma_5) d_j [\gamma^\mu d_k]_\alpha. \quad (4)$$

The field u^c is the charge conjugate quark field $u^c = i\gamma_2 u^*$. At low energies this interaction gets renormalized by the strong interactions, leading to an enhancement factor due to renormalization by strong, weak

[☆] This work is supported in part through funds provided by the US Department of Energy (DOE) under Contract EY-76-C 3069.

^{†1} A detailed analysis to determine the unification mass has been performed by Goldman and Ross [3] and we use their value, $M_{x,y} = 3.8 \times 10^{14}$ GeV.

and electromagnetic interaction [2,3]:

$$A \approx 3.7. \quad (5)$$

We will consider matrix elements of O_i between nucleons and mesons. The general form for a transition to a pseudoscalar meson P is

$$\langle P(q) | O_i | N(p) \rangle = (A_i + B_i \gamma_5) u(p), \quad (6)$$

where $A(B)$ is the parity violating (parity conserving) amplitude. It is understood in eq. (6) that the matrix element will be multiplied from the right by an electron or neutrino spinor. For vector mesons V we write

$$\langle V(q) | O_i | N(p) \rangle = \epsilon_\mu^* (A_i \gamma^\mu \gamma_5 + B_i \gamma^\mu) u(p), \quad (7)$$

with the same parity convention as above. More general terms such as $\epsilon_\mu^* \sigma^{\mu\nu} q_\nu u(p)$ do not appear in our static calculation. These matrices lead to the following decay rates:

$$\begin{aligned} \Gamma(N \rightarrow Pe^+) &= \frac{1}{8\pi} (4|A_1 + B_1|^2 + |A_1 - B_1|^2) \\ &\times \frac{(m_N^2 - m_P^2)^2}{m_N^3}, \\ \Gamma(N \rightarrow P\nu) &= \frac{1}{8\pi} |A_2 + B_2|^2 \frac{(m_N^2 - m_P^2)^2}{m_N^3}, \\ \Gamma(N \rightarrow Ve^+) &= \frac{1}{8\pi} (4|A_1 - B_1|^2 + |A_1 + B_1|^2) \\ &\times \frac{(m_N^2 - m_V^2)^2}{m_N^3} \frac{2m_V^2 + m_N^2}{m_V^2}, \\ \Gamma(N \rightarrow V\nu) &= \frac{1}{8\pi} |A_2 - B_2|^2 \frac{(m_N^2 - m_V^2)^2}{m_N^3} \frac{2m_V^2 + m_N^2}{m_V^2}. \end{aligned} \quad (8)$$

Symmetry properties reduce the number of these matrix elements. Machacek [5] has shown that the $\Delta I = 1/2$ character of the operators requires

$$\sqrt{2} A_1(p \rightarrow \pi^0 \text{ (or } \rho^0)) = B_1(n \rightarrow \pi^- \text{ (or } \rho^-)),$$

$$\sqrt{2} B_1(p \rightarrow \pi^0 \text{ (or } \rho^0)) = B_1(n \rightarrow \pi^- \text{ (or } \rho^-)),$$

$$A_2(p \rightarrow \pi^+ \text{ (or } \rho^+)) = \sqrt{2} A_2(n \rightarrow \pi^0 \text{ (or } \rho^0)),$$

$$B_2(p \rightarrow \pi^+ \text{ (or } \rho^+)) = \sqrt{2} B_2(n \rightarrow \pi^0 \text{ (or } \rho^0)). \quad (9)$$

Isospin also relates the parity violating amplitudes of

different operators. Using $[F_-, O_1^{\text{PV}}] = 2O_2^{\text{PV}}$,

$$A_1(p \rightarrow \pi^0 \text{ (or } \rho^0)) = \sqrt{2} A_2(p \rightarrow \pi^+ \text{ (or } \rho^+)),$$

$$A_1(p \rightarrow \eta \text{ (or } \omega)) = 2A_2(n \rightarrow \eta \text{ (or } \omega)), \quad (10)$$

and other relations equivalent to these through use of eq. (9). Finally the symmetry properties of the Dirac matrices in the vector current $\bar{u}_\alpha \gamma_\mu \gamma_5 u_\beta$ force all parity conserving matrix elements of O_1 to vanish.

The calculation of the transition matrix elements proceeds quite simply in a quark model if one knows the quark wavefunction. The bag model wavefunctions have been successfully applied to the calculation of two-quark matrix elements (currents) [6] and four-quark matrix elements (the $\Delta S = 1$ nonleptonic weak hamiltonian) [7]. These suggest that it should also provide a reasonable estimate of the baryon number violating interaction, involving three quark fields.

The bag model calculation proceeds as follows. Momentum space eigenstates are superposed to form bag states, using the wavepackets derived by Donoghue and Johnson [8]:

$$\begin{aligned} \text{bag} \langle M, k | \int d^3x O_i(x) | N \rangle_{\text{bag}} \\ = \int d^3p \frac{m_N}{E_N} \frac{1}{2E_M} \phi_M^*(p) \chi_N(p) (2\pi)^3 \\ \times \langle M(p-k) | O_i(0) | N(p) \rangle. \end{aligned} \quad (11)$$

$\phi_M(p)$ and $\chi_N(p)$ are the meson and nucleon wavepackets. To a good approximation it suffices to perform a static calculation ignoring the momentum dependence in the bag state and in the Dirac spinors. This is typically good to 20% [8]. The exception is the pion state, which will be considered later. The static approximation leads to the simple expression

$$\begin{aligned} (A + B\gamma_5) u(0) \\ = (2\tilde{E}_M)^{1/2} \text{bag} \langle M | \int d^3x O_i(x) | N \rangle_{\text{bag}}, \end{aligned} \quad (12)$$

where $(2\tilde{E}_M)^{1/2}$ is a normalization factor defined by the transformation eq. (11). For $M = \rho, \omega, \eta, \tilde{E}_M \approx 1 \text{ GeV}$ while for $M = \pi, \tilde{E}_M \approx 700 \text{ MeV}$. The matrix element of $O_i(x)$ is then calculated using quark wavefunctions of the bag model^{‡2}. For example, replacing

^{‡2} The procedure is standard and can be found in ref. [7] and references therein.

the fields in eq. (4) by bag wavefunctions leads to

$$\begin{aligned} \int d^3x O_{1m}(x) &= 2U_1 S_{1m} - U_2 S_{2m} + 2U_3 S_{3m}, \\ U_1 &= \int d^3x \tilde{u}(u^2 + l^2), \quad U_2 = \int d^3x, \\ U_3 &= \frac{2}{3} \int d^3x (u\tilde{l} + l\tilde{u})l, \\ S_{1m} &= \epsilon^{ijk} \sigma_{nn}^2 u_{in} \tilde{u}_{jn}^+ d_{km}, \\ S_{2m} &= \epsilon^{ijk} (\sigma^2 \sigma^i)_{nn} \sigma_{mm}^i u_{in} u_{jn} \tilde{d}_{km}^+, \\ S_{3m} &= \epsilon^{ijk} (\sigma^2 \sigma^i)_{nn} \sigma_{mm}^i u_{in} \tilde{u}_{jn}^+ d_{km}, \end{aligned} \quad (13)$$

for $\alpha = m = 1, 2$ and $O_{1\alpha} = 0$ for $\alpha = 3, 4$. The notation of the spin functions S_i is that $u_{im}^+ (\tilde{u}_{im}^+)$ is the creation operator for an up quark (antiquark) of color i and spin m . (The antiquark spin convention is somewhat unconventional but is consistently treated throughout the calculation.) The wavefunction overlaps U_i are expressed in terms of the upper (u) and lower (l) components of the bag wavefunction [6,7] with the tilde signifying that it is the meson's wavefunction. Performing the color and spin sums leads directly to the desired amplitudes. Since all wavefunctions are normalized by

$$\int d^3x \psi^+(x) \psi(x) = \int d^3x (u^2 + l^2) = 1,$$

the scale of the wavefunction is set by the confinement radius R . The matrix element therefore has dimensions $R^{-3/2}$.

An exception to this simple picture is the pion state. The pion in proton decay carries off a momentum of $q \approx M_N/2$, and is quite relativistic ($\beta = 0.96$). The static bag calculation is inappropriate and unfortunately no reliable method of handling this situation exists in the bag model. It is clear that it will lead to a suppression of $B \rightarrow \pi$ amplitudes. Phrased another way, the transition is sensitive to high momentum components of the quark wavefunction. Our way of handling this problem is to modify the static result by a form factor to describe the high momentum suppression. Typical guesses of this form factor, such as $F(k) \sim [1 + k^2/m^2]^n$ or $F(k) \sim e^{-Bk^2}$ lead to suppression by factors of 2–5. We will arbitrarily choose a suppression factor of 3^{†3}. This weak point in the calculation is unfortunately very difficult to resolve in the bag framework

Table 1

The branching fractions for the various two-body decay modes.

Proton decay		Neutron decay	
mode	branching ratio	mode	branching ratio
$\pi^0 e^+$	9%	$\pi^0 \nu$	2%
$\rho^0 e^+$	21%	$\rho^0 \nu$	5%
$\eta^0 e^+$	3%	$\eta^0 \nu$	1%
$\omega^0 e^+$	56%	$\omega^0 \nu$	14%
$\pi^+ \nu$	3%	$\pi^- e^+$	23%
$\rho^+ \nu$	8%	$\rho^- e^+$	55%

without considerable further work.

The results of the calculation of the branching fractions are presented in table 1. They are normalized such that the sum of all two-body branching fractions is equal to 100%. The results are similar to those of Machacek [5], with the exception that the pion mode is somewhat more suppressed in this work. Note that there is a strong dominance of modes with e^+ emission over those with $\bar{\nu}$:

$$\Gamma(p \rightarrow e^+)/[\Gamma(p \rightarrow \nu) + \Gamma(p \rightarrow e^+)] = 0.89, \quad (14)$$

$$\Gamma(n \rightarrow e^+)/[\Gamma(n \rightarrow \nu) + \Gamma(n \rightarrow e^+)] = 0.78. \quad (15)$$

This is highly desirable experimentally as these modes are most easily observed. The optimal decay $n \rightarrow \pi^- e^+$ has a reasonably large branching fraction.

To calculate the total two-body rate we sum all the modes. This yields

$$\Gamma(p \rightarrow \text{two-body}) = \frac{1}{32\pi^2} \frac{G^2 m_N^2}{R_N^3} A^2 \times 63.9, \quad (16)$$

$$\Gamma(n \rightarrow \text{two-body})/\Gamma(p \rightarrow \text{two-body}) = 0.77. \quad (17)$$

This is somewhat smaller than the inclusive estimate of BEGN [2] (who use $R = \frac{3}{4} R_N$)^{†4}:

$$\Gamma(p \rightarrow \text{all}) = \frac{307}{36\pi^2} \frac{G^2 m_N^2}{R^3} A^2. \quad (18)$$

^{†3} This is close to the suppression found in a calculation in progress using the harmonic oscillator model [9].

^{†4} The value of $|\psi(0)|^2$ used by BEGN would yield hyperon nonleptonic matrix elements which are too large by a factor of 3 when one includes the QCD renormalization of the weak hamiltonian. Despite the differences in principle, the proton lifetime estimates would be closer if a more realistic $|\psi(0)|^2$ were used.

To provide a full estimate of the proton's decay rate one needs to know the fraction of decay, ρ , which are two-body or quasi-two-body (i.e. multi-body decays proceeding through a resonance such as ρ or ω). Hopefully ρ will be close to unity. In terms of this parameter the proton lifetime is, using Goldman and Ross's estimate of the unification mass ($M = 3.8 \times 10^{14}$ GeV)

$$\tau_p = \rho \times 8 \times 10^{30} \text{ yr} . \quad (19)$$

This is in the range where future experiments should be sensitive. Note that there are significant uncertainties in this estimate due to factors external to the bag calculation. The dominant ones are the uncertainties in the value of the heavy boson mass, and, to a lesser extent, possible changes in the factor A . The variation in M is especially significant as it enters to the fourth power in the decay rate (through G_n^2) and small changes in M could have an important effect on τ_p .

If an experiment measures proton decay into a set of specific two-body final states, the value of ρ is not important, since it is only $\Gamma_{\text{tot}} \Sigma_i (\text{BR})_i = \Sigma \Gamma_i$ which is measured. For this case one can define an "effective lifetime" disregarding unobserved decay modes:

$$\tau_{\text{eff}} = \tau_p / \sum_i (\text{BR})_i , \quad (20)$$

where i runs over all the detected modes. For example, for detecting all two-body e^+ modes the effective life-

time is $\tau_{\text{eff}} = 1 \times 10^{31}$ yr independent of ρ .

We have provided an estimate of the proton's lifetime and branching ratios in SU(5) using the hadronic wavefunctions of the MIT bag model. If current estimates of the mass of the bosons which violate baryon number are correct, the proton decay should be observable by the coming generation of experiments.

I would like to thank T. Goldman, G. Kane, G. Karl, P. Langacker, M. Machacek and D. Ross for helpful conversations.

References

- [1] H. Georgi and S. Glashow, Phys. Rev. Lett. 32 (1974) 438.
- [2] A. Buras, J. Ellis, M. Gaillard and D. Nanopoulos, Nucl. Phys. B135 (1978) 66;
J. Ellis, M. Gaillard and D. Nanopoulos, Phys. Lett. 88B (1979) 320.
- [3] T. Goldman and D. Ross, private communication.
- [4] F. Reines and M. Crouch, Phys. Rev. Lett. 32 (1974) 493.
- [5] M. Machacek, Harvard preprint HUTP-79/A021.
- [6] T. DeGrand, R. Jaffe, K. Johnson and J. Kiskis, Phys. Rev. D12 (1975) 2060;
J.F. Donoghue, E. Golowich and B.R. Holstein, Phys. Rev. D12 (1975) 2875.
- [7] J.F. Donoghue, E. Golowich, B.R. Holstein and W. Ponce, MIT preprints CTP #798 and 799.
- [8] J.F. Donoghue and K. Johnson, MIT preprint CTP #802.
- [9] G. Karl and G. Kane, private communication.