1975

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(Received 27 May 1975)

We perform a phenomenological analysis of baryon matrix elements in a fixed-sphere MIT bag model. The model consists of massive, noninteracting quarks which carry the usual SU(3) quantum numbers as well as an SU(3) of color. Two-quark matrix elements investigated include those of the σ operator and axial-vector and vector currents and the magnetic moment, charge radius, and dipole moments of the electromagnetic current. Among our results are the masses 44 MeV for nonstrange quarks and 298 MeV for strange quarks. The model is successful in predicting static properties of the lowest-mass baryons, but has a problem in describing the radiative decay of the nucleon resonance, $N^*_{13}(1520)$. Four-quark matrix elements, parity-violating nonleptonic decay amplitudes of the $\frac{1}{2}^+$ baryons, are investigated and are found to be poorly described by the model.

I. INTRODUCTION

Although a decade has passed since the suggestion was made that the quark degree of freedom[1] underlies the spectrum of hadrons, dynamical calculations involving the strong interactions have met with only limited success. This is primarily due to an inability to describe extended objects in the context of quantum field theory. Briefly put, theorists have been unable to construct realistic hadron wave functions.

Recently, a framework for dealing with the strong interactions in which hadrons occur from the beginning as extended objects has been put forth.[2,3] This "MIT bag" theory has the additional virtue of sustaining models which are simple enough to be calculable. For those who have labored for so long with strong interaction dynamics, this development promises to provide a great deal of stimulation.

Several different types of interesting problems immediately come to mind. A basic parameter in the MIT bag picture is a universal pressure B, which serves to confine quark and/or gluon fields within hadrons. Progress must be made in better understanding the physical nature of this quantity. There is also the challenge of constructing increasingly sophisticated, and thus, we hope, more realistic bag models. In the work done thus far, baryon rest states are taken as noninteracting massless quarks confined to a spherical region of fixed radius.[3] The generalization to nonzero quark mass has been performed,[4] and work has been done to take quark-quark interactions into account.[5] The construction of a more realistic bag boundary also deserves serious attention.

While the pursuit of increasingly complex bag models is an interesting and worthwhile field of investigation, we feel that it would be a mistake to aviod studying the consequences of existing models. Indeed, only by comparing the predictions of new bag models with those of the older and simpler variety can it be ascertained whether real progress has been made. In addition, a phenomenological study of such models is of interest simply to compare their content with various beliefs involving the strong interactions which have been built up over a period of time, such as the Ademollo-Gatto theorem,[6] the effect of SU (3) symmetry breaking on masses and coupling constants, and so on. Finally, even with the simplest bag models, it is possible to attempt calculations which have heretofore resisted solution. In particular, we have in mind the amplitudes which describe the nonleptonic decays of hyperons (see Sec. III).

The above reasoning has motivated us to perform a systematic study of two-quark and four-quark baryon matrix elements in the following bag model. We assume that low-lying baryon states contain three massive but noninteracting quarks indentified by means of the following quantum numbers[7]: $\mathcal{P}(T=T_z=\frac{1}{2},\ Y=\frac{1}{3})$, $\mathfrak{N}(T=-T_z=\frac{1}{2},\ Y=\frac{1}{3})$, $\lambda(T=T_z=0,\ Y=-\frac{2}{3})$. Each quark is assumed to be capable of carrying any of three possible "colors," such that all physical baryon states are color singlets. The collection of all internal quantum numbers carried by a given quark is denoted by the symbol α. The equation of motion obeyed by each quark field within the bag is

$$(-i\gamma\cdot\partial+m_\alpha)\psi_\alpha(\vec{x},t)=0\,. \tag{1}$$

Permanent confinement of each quark within the baryon bag is guaranteed by means of the two surface boundary conditions,

$$in\cdot\gamma\,\psi_\alpha(x)=\psi_\alpha(x) \tag{2}$$

and

$$\sum_\alpha n\cdot\partial(\bar{\psi}_\alpha(x)\psi_\alpha(x))=2B\,. \tag{3}$$

where $n_\mu(\vec{x}, t)$ is the interior unit four-normal to the bag's surface, and B is the confinement pressure mentioned earlier. We shall assume in this paper that the boundary consists of a fixed sphere of radius R, so that the unit four-vector is given by $n^\mu = (0, \hat{r})$. Only angular momentum $j = \frac{1}{2}$ solutions can occur within such a boundary. The space-time dependence of a mode labeled by index n, parity index κ, and z component of angular momentum λ is given by

$$\psi_{n-1\lambda}(\vec{x}, t) = [4\pi(E+m)]^{-1/2} \begin{bmatrix} i(E+m)^{1/2} j_0((E^2-m^2)^{1/2} r) u_\lambda \\ -(E-m)^{1/2} j_1((E^2-m^2)^{1/2} r) \vec{\sigma}\cdot\hat{r} u_\lambda \end{bmatrix} e^{-iEt} \tag{4a}$$

for $\kappa = -1$, and

$$\psi_{n1\lambda}(\vec{x}, t) = [4\pi(E-m)]^{-1/2} \begin{bmatrix} i(E+m)^{1/2} j_1((E^2-m^2)^{1/2} r) \vec{\sigma}\cdot\hat{r} u_\lambda \\ (E-m)^{1/2} j_0((E^2-m^2)^{1/2} r) u_\lambda \end{bmatrix} e^{-iEt} \tag{4b}$$

for $\kappa = +1$. The mass and energy of a quark are denoted by m and E, respectively, in Eqs. (4a) and (4b). The mode solutions can be linearly super posed to obtain

$$\psi_\alpha(\vec{x}, t) = \sum_{n\kappa\lambda} N(\omega_{n\kappa}, m_\alpha R) a_\alpha(n\kappa\lambda) \psi_{n\kappa\lambda}(\vec{x}, t), \tag{5}$$

where N is a normalization factor[4]

$$N(\omega, mR) = \left(\frac{(\omega^2 - m^2R^2)^2}{R^3(2\omega^2 + 2\kappa\omega + mR)\sin^2[(\omega^2 - m^2R^2)^{1/2}]} \right)^{1/2}. \tag{6}$$

The a_α becomes fermion creation or annihilation operators upon quantization,

$$\begin{aligned} a_\alpha(n\kappa\lambda) &= b_\alpha(n\kappa\lambda) \quad (n>0) \\ &= d^\dagger_\alpha(-n, -\kappa, \lambda) \quad (n<0), \end{aligned} \tag{7}$$

which are assumed to obey the usual anticommutation relations,

$$[b_\alpha(n\kappa\lambda), b^\dagger_\alpha(n\kappa\lambda)]_+ = 1, \tag{8}$$

etc. Particle states are constructed by the action of creation operators upon the zero-quark state, defined from

$$b_\alpha(n\kappa\lambda)|0> = d_\alpha(n\kappa\lambda)|0> = 0. \tag{9}$$

Equations (8) and (9) ensure that the fermion number operator has only integer eigenvalues, i.e., that only integral numbers of quarks or antiquarks can be found within the bag.

In our interpretation of this model, the linear and nonlinear boundary condition equations (2) and (3) serve to fix certain properties of each baryon state under consideration. The linear boundary condition (2) relates the value of $m_\alpha R$ to the mode frequency $\omega_{n\kappa}$,

$$\tan[(\omega^2_{n\kappa} - m^2_\alpha R^2)^{1/2}] = \frac{\kappa(\omega^2_{n\kappa} - m^2_\alpha R^2)^{1/2}}{\omega_{n\kappa} - \kappa m_\alpha R + \kappa} \tag{10}$$

for a quark of mass m_α and energy

$$\omega_{n\kappa} = E_{n\kappa} R \tag{11}$$

in the mode(n, κ). The bag radius R depends upon the particular baryon state under discussion and must be determined dynamically. The nonlinear boundary condition (3) is more subtle and it is correspondingly important to understand the constraints implied by this relation. Since the right-hand side of Eq. (3) is time independent, only terms bilinear in the indices (n, κ), (n', κ') with $n = n'$, $\kappa = \kappa'$ or $n = -n'$, $\kappa = -\kappa'$ can contribute to the left-hand side. Thus, we find

$$\sum_{n\alpha\kappa\lambda} \frac{(\omega_\kappa + \kappa)(\omega^2_{n\kappa} - m^2_\alpha R^2)}{2\omega^2_{n\kappa} + 2\kappa\omega_{n\kappa} + m_\alpha R} b^\dagger_\alpha(n\kappa\lambda) b_\alpha(n\kappa\lambda) = 2\pi B R^4, \tag{12}$$

where we suppress the contribution of antiquark operators in view of our intent to study only baryon states in this paper. How does one interpret the absence of cross terms in Eq. (12)? At the classical level, it simply means that for each internal degree of freedom α, only one normal mode $a_\alpha(n\kappa\lambda)$ can be excited.[3] All the others must vanish. At the quantum level, where the coefficients $a_\alpha(n\kappa\lambda)$ become operators as in Eq. (7), the nonlinear boundary condition is seen to be a restriction on the allowed particle states, which is exactly obeyed in the model under discussion where baryons exist as color singlets.

There are two ways in which certain of our applications go beyond previous calculations involving the MIT bag model. In parts of the following two sections, we shall (i) describe a calculation of electromagnetic transitions from an excited state down to the nucleon, and (ii) evaluate commutators such as the one defining the σ term. Each of these points is sufficiently subtle to warrant discussion here. Regarding the occurrence of "inelastic" matrix elements, it is conceivable the Eq. (3) could be interpreted as forbidding such transitions in this model on the grounds that the

bilinears in creation and annihilation operators which appear in the electromagnetic current operator may connect only modes with $n' = n$ and $\kappa' = \kappa$. In our opinion, such an interpretation is not correct. The transition amplitude is simply the overlap of two quantum states via a local operator. While it is true that Eq. (3) forces such states to be orthogonal in the absence of a transition operator, it does not force all possible matrix elements to vanish. We shall return to this point in Sec. II.

At several points in our calculation, it is necessary to evaluate operator commutation relations. In Sec. II, we use the pion-nucleon σ term as input to a phenomenological estimate of the nonstrange quark mass, while in Sec. III, we shall employ a commutation relation involving the isospin charge in order to express the hyperon nonleptonic decay amplitude in terms of a matrix element of the parity-conserving (p.c.) energy density operator associated with the weak Hamiltonian. The commutators of interest are, respectively,

$$[F_a^5(x^0=0), \partial A_b(0)] \quad (a, b = 1, 2, 3) \tag{13}$$

and, for example,

$$[F_+^5(x^0=0), H_w^{\text{p.v.}}(0)]\,, \tag{14}$$

where p.v. means parity violating. Since our dynamical model contains field operators which are explicitly known, we can in principle perform the operations indicated in Eqs. (13) and (14). However, if we directly employ the field operator of Eq. (5), we do not obtain the "standard" results for the commutators (13) and (14). This is because the field operator $\psi_\alpha(x, t)$, defined by Eqs. (4)–(8), does *not* obey the usual anticommutation relation

$$[\psi_\alpha(\vec{x}, t), \psi_\alpha^\dagger(\vec{x}', t)]_+^{\text{usual}} = i\,\delta^3(\vec{x} - \vec{x}')\,. \tag{15}$$

The reason for this is easy to understand. The fields $\psi_\alpha(x, t)$ in our bag model carry only $j = \frac{1}{2}$, and the class of $j = \frac{1}{2}$ modes does not constitute a complete set. However, this poses the problem as to which way to proceed in calculating various commutation relations. We feel that in calculating the transition matrix elements discussed in the following sections, it is correct to use the "standard" commutation relations,[8]

$$[F_\alpha^5(0), \partial A_b(0)] = \delta_{ab}\,\sigma \quad (a, b = 1, 2, 3) \tag{16}$$

with

$$\sigma = m_{\mathcal{P}} \bar{\psi}\, Q_\sigma\, \psi \tag{17}$$

and

$$[F_3^5(x^0=0), H_w^{\text{p.v.}}(0)] = \tfrac{1}{2} H_w^{\text{p.c.}}(0)\,. \tag{18}$$

In Eq. (17), Q_σ is the matrix

$$Q_\sigma = \begin{pmatrix} 100 \\ 010 \\ 000 \end{pmatrix},$$

in the internal symmetry space defined by $(\mathcal{P}, \mathcal{N}, \lambda)$. Our reasoning is that the commutation relations, which describe certain attributes of fundamental interactions, should not depend upon the structure of a given hadronic state. Perhaps it is simplest to view the problem from the vantage point of a picture where the hadronic structure appears explicitly in the wave functions. Note that if we were to employ the field of Eq. (5) in evaluating commutators, not even the charge-density commutation relations of SU(3) would be valid, in which case it would be hard to see how to ascribe meaningfully to each quark the concepts of isospin or hypercharge. Moreover, the commutator would then be frame dependent, since the field (σ) would pick up $j \neq \frac{1}{2}$ modes because of the Lorentz-contracted boundary.

We conclude this section with a summary of the contents appearing in the remainder of this paper. In Sec. II, we explore various phenomenological consequences of the bag model for baryon matrix elements of two-quark operators. We begin by fixing the nonstrange quark mass and the pressure B by means of a fix to both the nucleon axial-vector coupling g_A and the nucleon σ term $\langle N|\sigma|N\rangle$. The strange quark mass is determined from the mass of the $J^P = \frac{3}{2}^+$ baryon, Ω^-. A variety of applications, which follow once these basic parameters have been determined, are then described. In Sec. III, we present a calculation of the parity-conserving nonleptonic hyperon decay amplitudes. We summarize our results in Sec. IV, and in Appendix A discuss alternative means of attaining the nonrelativistic limit of this model. In Appendix B, we present certain excited-state wave functions pertinent to our discussion of photon transitions.

II. ANALYSIS OF TWO-QUARK OPERATORS

Before we can calculate the various matrix elements of two-quark operators which form the subject matter of this Section, we must first determine certain basic parameters, such as quark mass. We shall assume that the masses of all nonstrange quarks are equal, so that there exist two independent quark masses, which we call $m_{\mathcal{P}}$ and m_λ.

We determine $m_{\mathcal{P}}$ simultaneously with the bag pressure B and the bag radius R for the nucleon by fitting the model to the degenerate nucleon-

$\Delta(1232)$ mass of 1180 MeV

$$1180 = \frac{4\pi R^3 B}{3} + \frac{3\omega_{1,-1}}{R}\,, \tag{19}$$

the nucleon axial-vector coupling constant

$$g_A = \frac{5}{9}\left(\frac{2\omega_{1,-1}^2 + 4m_{\mathcal{P}}R\omega_{1,-1} - 3m_{\mathcal{P}}R}{2\omega_{1,-1}^2 - 2\omega_{1,-1} + m_{\mathcal{P}}R}\right), \tag{20}$$

and the nucleon σ term. In our model, the σ operator, defined by Eqs. (16) and (17), may be written as

$$\sigma = m_{\mathcal{P}} \sum_{\alpha n\kappa\lambda} r(m_{\mathcal{P}}R, \omega_{n\kappa}) b_\alpha^*(n\kappa\lambda) Q_\sigma b_\alpha(n\kappa\lambda)\,, \tag{21}$$

where for $\kappa = -1$, the only physically interesting case,

$$r(mR, \omega) = \frac{\omega^2(1+2mR) + \omega mR(2mR-1) - 2(mR)^2}{(\omega + mR)(2\omega^2 - 2\omega + mR)}\,. \tag{22}$$

There are four unknowns, $m_{\mathcal{P}}, R, B, \omega_{1,-1}$ and five relations: the boundary conditions (10) and (12), nucleon mass (19), axial-vector coupling constant (20), and σ term (21). We demand that the first three relations be maintained exactly, and use the latter two to predict a wide range of axial-vector couplings and σ terms. Results are given in Table I.

At this point, we have a somewhat arbitrary choice to make as to which quark mass gives a good fit to g_A and σ. In Ref. 4, the value $m_{\mathcal{P}} = 122$ MeV was chosen to satisfy the criterion of fitting the axial-vector coupling g_A exactly. Here our decision is somewhat complicated by the uncertainty regarding the value of the nucleon σ term. Phenomenological estimates have ranged from 25 MeV to 110 MeV.[9,10] We have decided to employ parameters which give $g_A = 1.143$ and $\langle N|\sigma|N\rangle = 70$ MeV. To insist on a smaller σ term would worsen the bag model prediction for g_A, whereas a larger predicted value of g_A would give too large a predicted value for $\langle N|\sigma|N\rangle$. Our solution for the nucleon is then

$$m_{\mathcal{P}} = 44.1 \text{ MeV}, \quad B^{1/4} = 113.9 \text{ MeV}\,, \tag{23}$$

and values for the frequency $\omega_{1,-1}$ and bag inverse radius R^{-1} are presented in Table II. The quantities $m_{\mathcal{P}}$ and B are thus determined for the rest of this paper, whereas the mode frequency $\omega_{1,-1}$ and bag inverse radius R^{-1} describe only the degenerate nucleon-$\Delta(1232)$ state. Also given in Table I are the dependences of the nucleon gyromagnetic ratio g_p and charge radius $\langle r^2\rangle^{1/2}$ upon quark mass (formulas for these quantities will be given shortly). Our choice of $m_{\mathcal{P}} = 44$ MeV implies $g_p = 2.63$ and $\langle r^2\rangle^{1/2} = 1.03$ F, both reasonable approximations to the experimental values.[11]

There are several ways in which one can perform a phenomenological estimate of m_λ. We have chosen to fit exactly the mass of the Ω^- baryon, $M_{\Omega^-} = 1672$ MeV. This state contains only quarks with mass m_λ, so the energy equation to be solved is the same as Eq. (19) except that 1672 replaces 1180 and $\omega_{1,-1}$ and R refer specifically to the Ω^- channel. Therefore, the three unknowns m_λ, $\omega_{1,-1}$, and R can be determined from the three equations (10), (12), and (24). We find

$$m_\lambda = 297.8 \text{ MeV}\,, \tag{24}$$

and list $\omega_{1,-1}, R$ in Table II.

Having obtained estimates for the quark masses $m_{\mathcal{P}}$ and m_λ, we can easily calculate the strangeness -1, -2 baryon masses from the boundary conditions Eqs. (10) and (12), which determine the two unknowns $\omega_{1,-1}$ and R in a given channel. The energy equation is a generalization of Eq. (19)

$$E = \frac{4\pi R^3 B}{3} + \frac{n_{\mathcal{P}}\omega_{1,-1}(p) + n_\lambda \omega_{1,-1}(\lambda)}{R}\,, \tag{25}$$

where $n_{\mathcal{P}}$ and n_λ are the number of $\mathcal{P}$- or $\mathcal{N}$-type and λ-type quarks, respectively. The predicted masses for the baryons Λ, Σ, $Y_1^*(1385)$ and Ξ, $\Xi^*(1530)$ are compared with the corresponding experimental values in Table II. The latter are averages of the physical masses weighted according to multiplicity of states. Agreement is reasonable, given the assumption of a rigid-sphere boundary which underlies the entire model.

It could possibly be argued that instead of following procedure we have just described, it would have been preferable to determine $m_{\mathcal{P}}$ and m_λ from some sort of least-squares fit to the (weighted-average) $\frac{1}{2}^+$-$\frac{3}{2}^+$ mass spectrum. While the values of $m_{\mathcal{P}}$ and m_λ thus found would probably not precisely equal ours, it seems to us just as arbitrary a manner to adopt. The real point is that as long

TABLE I. Dependence of nucleon parameters upon nonstrange quark mass. The quark mass ($m_{\mathcal{P}}$) and σ term (σ) are given in MeV. The axial-vector coupling (g_A) and gyromagnetic ratio ($2M_p\mu$), where M_p is the proton mass, are dimensionless, and the charge radius ($\langle r^2\rangle^{1/2}$) is expressed in fermis.

$m_{\mathcal{P}}$	σ	g_A	$2M_p\mu$	$\langle r^2\rangle^{1/2}$
20	30	1.11	2.63	1.01
32	50	1.13	2.63	1.02
44	70	1.14	2.63	1.03
55	90	1.16	2.63	1.04
71	120	1.18	2.63	1.06
91	160	1.21	2.62	1.08
110	200	1.23	2.62	1.10
127	240	1.26	2.61	1.13

TABLE II. Masses of the $\frac{1}{2}^+$-$\frac{3}{2}^+$ baryons. States with the same strangeness quantum number are degenerate in this bag model. The masses and the various R^{-1} values are given in MeV. The experimental masses are averages of the physical masses weighted according to their multiplicity.

States	Mass		Mode frequency		R^{-1}
	Bag model	Experiment	$\omega_{1,-1}(\mathcal{P})$	$\omega_{1,-1}(\lambda)$	
N, Δ	1180	1180	2.206	···	136.4
Λ, Σ, Y_1	1344	1300	2.204	3.396	137.8
Ξ, Ξ^*	1508	1462	2.202	3.379	139.3
Ω^-	1672	1672	···	3.362	140.8

as we work without spin-dependent forces, degeneracy exists between the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ states and no such model can lay claim to a completely satisfactory description of the physical states. At best, one can attempt to obtain a reasonable estimate for the underlying parameters so as to gain sound insights regarding the bag model content when employed to calculate a variety of amplitudes.

A collection of static properties of the $\frac{1}{2}^+$ baryons is exhibited in Table III. Let us first discuss the axial-vector couplings. The effective operator whose diagonal matrix elements are given in the column of Table III labeled g_A has the structure

$$\sum_{\alpha,\lambda} f(m_{\mathcal{P}}R, \omega_{1,-1}) u_\lambda^\dagger \sigma_z u_\lambda b_\alpha^\dagger(1,-1,\lambda)\tau_3 b_\alpha(1,-1,\lambda), \tag{26}$$

where $n=1$, $\kappa=-1$, and the function f is given by

$$f(mR, \omega) = \frac{1}{3}\,\frac{2\omega^2 + 4mR\omega - 3mR}{2\omega^2 - 2\omega + mR}\,. \tag{27}$$

This $\Delta S=0$ operator is sensitive to the presence of nonstrange quarks only. Although we defer detailed discussion of our results to the conclusion, it seems worthwhile to point out here that the axial-vector couplings in Table III are essentially the SU(6) values multiplied by a factor 0.686 (with very minor variations) as we sweep across the baryons. The bag structure has served to renormalize the axial-vector couplings predicted by the SU(6) of the naive quark model. If the zero momentum transfer axial-vector matrix elements are parametrized in terms of SU(3),

$$\langle B_k | A_i | B_j \rangle = i f_{ijk} F + d_{ijk} D, \tag{28}$$

where we suppress all space-time dependent notation, we find $F/D = 2/3$, as expected. This aspect of the bag model appears to agree rather well with recent phenomenological determinations of the F/D ratio.[12]

A direct extension of the $\Delta S=0$ calculation just discussed is consideration of the $\Delta S=1$ axial-vector transitions connecting the $S=-1$, -2 channels to the $S=0$, -1 channels, respectively. These matrix elements are interesting because our bag model allows for breaking of SU(3) symmetry. Conceivably, there might be differences between the $\Delta S=0$ and $\Delta S=1$ amplitudes. Furthermore, the $\Delta S=+1$ amplitudes represent a departure from previous bag model calculations. These matrix elements involve "inelastic" processes, which give rise to certain subtle technical points which require interpretation. For example, in a $\Delta S=1$ axial-vector transition, a λ quark is converted to a $\mathcal{P}$ quark. The other quarks are spectators. Yet, there is even some effect of the transition on the spectators because, as is seen in Table II, the frequency of a proton quark (for example) changes slightly as we scan across the $S=0$, -1, -2 channels. Moreover, the integrals which occur involve the overlap of baryon wave functions with differing bag radii (again, see Table II). In each case, it turns out that the numerical consequence of these effects is quite small, but in principle, there is some arbitrariness in how we proceed. For the case of the overlap integrals, we chose as a cutoff the smaller of the two bag radii. For the $\Delta S=+1$ axial-vector transitions, the dynamical factors which are generalizations of the $\Delta S=0$ expression Eq. (27) are quite complicated and are not given here. The result of our calculation is that the SU(6) quark model predictions are renormalized by baryon structure with a numerical factor 0.745 or about 8% less than the $\Delta S=0$ amplitudes. This value is common to the $S=-2$ to $S=-1$ and $S=-1$ to $S=0$ transitions.

We have also performed a calculation of the analogous $\Delta S=1$ vector current transitions. According to the Ademollo-Gatto theorem, effects of SU(3) breaking should be "small" in such amplitudes. Again, because of their complexity, the amplitudes are not reproduced here. We find for the $S=-2$ to $S=-1$ and $S=-1$ to $S=0$ transitions a renormalizing factor of 0.971, consistent with the qualitative expectations following from the Ademollo-Gatto theorem.

Although data on baryon magnetic moments are not abundant, there do exist enough of sufficient quality to make some preliminary judgment regarding success of the bag model. For $n=1$, $\kappa=-1$ states, the magnetic moment operator has

TABLE III. Static properties of the $\frac{1}{2}^+$ baryons. The axial-vector coupling, gyromagnetic ratio, and charge radius (in fermis) is given for each baryon. The gyromagnetic ratio is found by multiplying the magnetic moment of each baryon by twice the proton mass M_p.

State	g_A	$2M_p\mu$	$\langle r^2\rangle^{1/2}$
p	1.14	2.63	1.03
n	−1.14	−1.76	0.0
Σ^+	0.91	2.52	1.05
Σ^-	−0.91	−0.96	0.99
Σ^0	0.0	0.78	0.25
Λ	0.0	−0.61	0.25
Ξ^0	−0.23	−1.39	0.35
Ξ^-	0.23	−0.53	0.95

the form

$$\sum_{\alpha,\lambda} g(m_\alpha R, \omega_{1,-1}) u_\lambda^\dagger \sigma_z u_\lambda b_\alpha^+(1,-1,\lambda) Q b_\alpha(1,-1,\lambda), \tag{29}$$

where Q is the quark matrix for electric charge and

$$g(mR,\omega) = \frac{R}{6}\frac{4\omega + 2mR - 3}{2\omega^2 - 2\omega + mR}. \tag{30}$$

To obtain the gyromagnetic ratio for a given baryon, we multiply the bag model magnetic moment by twice the proton mass. In Ref. 11 are listed the following gyromagnetic ratios: $p(2.79)$, $n(-1.91)$, $\Lambda(-0.67\pm0.06)$, $\Sigma^+(2.62\pm0.41)$, $\Xi^-(-1.93\pm0.75)$. It has already been mentioned[3] in the context of a zero-quark-mass calculation that the scale of these magnetic moments is not determined primarily by quark mass, but instead by the structure afforded by a finite bag radius. Indeed, this is seen clearly in Table I, where the proton gyromagnetic ratio is rather insensitive to changes in quark mass. That the bag gyromagnetic ratios do so well in fitting the experimental values is motivation for additional study of these models. Note that the magnetic moment operator of Eq. (29) is sensitive to the presence of both strange and nonstrange quarks in the baryon wave function. Although the SU(3) prediction that $\mu(\Sigma^+) = \mu(p)$ holds in our model to a high degree of accuracy, the SU(3) prediction $\mu(\Lambda) = \mu(n)/2$ exhibits the effects of symmetry breaking to a greater degree. The model does rather well in these two cases. In view of the large error bars associated with the existing Ξ^- experimental value, it seems premature to rule this case as a defect of the model.

The final column of Table III contains values of the charge radius for each $\frac{1}{2}^+$ baryon. The operator from which these numbers were obtained in part is

$$\sum_{\alpha,\lambda} h(m_\alpha R, \omega_{1,-1}) b_\alpha^*(1,-1,\lambda) Q b_\alpha(1,-1,\lambda), \tag{31}$$

where

$$h(mR,\omega) = \frac{R^2}{6}\frac{A}{(\omega^2 - m^2R^2)(2\omega^2 - 2\omega + mR)} \tag{32}$$

and

$$A = 4\omega^4 - 4\omega^3 + \omega^2(8 + 6mR - 4m^2R^2) + \omega(-6 - 8mR + 4m^2R^2) + (9mR - 6m^2R^2 - 6m^3R^3). \tag{33}$$

The charge radius operator, like that for the magnetic moment, probes the presence of both strange and nonstrange quarks. Our results for the proton bag in Table I indicate that charge radius increases as the quark mass increases, with roughly the sensitivity exhibited by the axial-vector coupling g_A. To our knowledge, there is virtually no experimental information on the charge radii of the strange baryons, so it is hard to judge the validity of the values for $\langle r^2\rangle^{1/2}$ given in Table III. One theoretical analysis[13] of the vanishing of the neutron charge radius suggests that the term linear in q^2 of the form factor $F_1(q^2)$, defined by

$$\langle \beta | V_\mu^i(0) | \alpha\rangle = \bar{u}(p_\beta)[F_1^{i\,\alpha\beta}(q^2)\gamma_\mu + \ldots]u(p_\alpha), \tag{34}$$

is pure d type. Thus, along with the vanishing of the neutron charge radius, one has simultaneously $\langle r^2\rangle = 0$ for the Λ, Σ^0, and Ξ^0. From Table III, our model is evidently not of this type. We obtain the vanishing of $\langle r^2\rangle$ for Λ, Σ^0, and Ξ^0 only in the limit of SU(3) symmetry, where the strange and nonstrange contributions to the function h of Eq. (32) cancel. However, for broken SU(3), the cancellation is incomplete, and we obtain the values given in Table III.

For the remainder of this section, we consider transition matrix elements, first of the type $\frac{3}{2}^+ \to \frac{1}{2}^+$ via pion emission, then $\frac{3}{2}^\pm \to \frac{1}{2}^+$ via photon emission.

The pion-emission amplitudes for $\frac{3}{2}^+ \to \frac{1}{2}^+$ transitions are determined from axial-vector current matrix elements of the type described in Eqs. (26) and (27). Pion pole dominance is then used to obtain a bag model prediction for the coupling constant $g[B^*(\frac{3}{2}^+) \to B(\frac{1}{2}^+)\pi]$. Upon squaring the coupling constant and inserting appropriate phase-space factors, we obtain an expression for the decay width $\Gamma(B^*B\pi)$,

$$\Gamma(B^*B\pi) = \frac{g^2 p^3 (E + M_B)}{12\pi M_{B'} m_\pi^2}, \tag{35}$$

where p is the magnitude of decay momentum, $M_{B'}$ is the decaying $\frac{3}{2}^+$ baryon mass, and E, M_B are the $\frac{1}{2}^+$ baryon energy and mass, respectively. The results, shown in Table IV, depend upon two sa-

TABLE IV. Decuplet decay widths. All widths are given in units of MeV.

Mode	Γ (bag model)	Γ (experiment)
$\Delta(1232)\to N\pi$	103	120
$Y_1(1385)\to\Lambda\pi$	41.3	30.8
$Y_1(1385)\to\Sigma\pi$	5.2	4.2
$\Xi^*(1530)\to\Xi\pi$	14.5	9.1

lient points. The first is that the bag structure which reduced the nucleon axial-vector coupling from $\frac{5}{3}$ to 1.14 plays the same role here, and it is clearly just as important in bringing the scale of the SU(6) predictions into agreement with experiment. The second noteworthy feature of our results is that, despite the SU(3) breaking contained explicitly in this bag model, the $B^*B\pi$ coupling constants turn out to be very nearly SU(3) invariant. As has been shown in the phenomenological analysis of Ref. 14, empirical $B^*B\pi$ coupling constants exhibit about 15% symmetry breaking. This explains why, as good as the results in Table IV are, they are no better.

Finally, we consider photon transitions to the ground state of charged and neutral excited nucleon states. This study is significant in providing a test of the excited-state bag-model wave functions. The first excited nucleon state has the configuration $(1S_{1/2})^2\,(1P_{1/2})$. Its energy can be determined from the solution of two linear boundary equations of the type given by Eq. (10) (one each for $n=1$ with $\kappa=+1$ and -1) and one nonlinear boundary condition. The energy equation is analogous to Eq. (25), except now the distinction is between $\kappa=+1$ and -1 quarks, not between λ and $\mathcal{P}$ quarks. We find

$$\omega_{1,-1}=2.217,\quad E=1398\text{ MeV}$$
$$\omega_{1,1}=3.862,\quad R^{-1}=128.1\text{ MeV} \tag{36}$$

Now that we have a complete description of the lowest-lying $\kappa=-1$, $+1$ nucleon states, we can proceed to consider the photon transition amplitudes. The experimental values presented in Ref. 11 are those of transition amplitudes defined as follows. Suppose a nucleon traveling along the z axis with spin component $-\frac{1}{2}$ and a photon traveling in the opposite direction but with spin component $+1$ combine to form a $J=\frac{1}{2}$ excited state at rest. The appropriate matrix element is

$$A_1=\frac{1}{\sqrt{2q}}\langle N^*(\vec{0},\tfrac{1}{2})\,|\,\vec{\epsilon}\cdot\vec{J}\,|\,N(\vec{q},-\tfrac{1}{2})\rangle\ , \tag{37}$$

where q is the initial state momentum, $\vec{J}$ is the electromagnetic current, and

$$\vec{\epsilon}=-\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\\ 0\end{pmatrix}. \tag{38}$$

For the excitation of a $J=\frac{3}{2}$ excited state, there is this amplitude and an additional one,

$$A_3=\frac{1}{\sqrt{2q}}\langle N^*(\vec{0},\tfrac{3}{2})\,|\,\vec{\epsilon}\cdot\vec{J}\,|\,N(\vec{q},\tfrac{1}{2})\rangle\ . \tag{39}$$

If the N^* has isospin $\frac{1}{2}$, then the amplitudes for the neutral and for the charged decaying states are independent. If the isospin of N^* is $\frac{3}{2}$, it suffices to consider just the proton amplitude because the electromagnetic current is then isovector. In the following, we shall denote the various nucleon resonances by the notation $L_{2T,2J}$ where L is the orbital angular momentum with which a πN com-

TABLE V. Electromagnetic transition amplitudes. All numerical values are in units of $\text{GeV}^{-1/2}$. The experimental numbers are taken from Ref. 11. The columns labeled p and n refer to decays of nucleon resonances with charge +1 and 0, respectively. The S'_{11} and S''_{11} bag model amplitudes correspond to the choice $\alpha=+1$, where α is the parameter defined in Appendix B.

		Bag model		Experiment	
		p	n	p	n
$P_{33}(1232)$	A_1	−0.102	···	−0.141 ± 0.003	···
	A_3	−0.176	···	−0.259 ± 0.005	···
$D_{13}(1520)$	A_1	0.0	−0.044	0	−0.076 ± 0.013
	A_3	0.0	−0.076	−0.177 ± 0.015	−0.124 ± 0.010
$S'_{11}(1535)$	A_1	0.131	−0.044	0.058 ± 0.025	−0.042 ± 0.013
$S_{31}(1650)$	A_1	0.031	···	0.057 ± 0.035	···
$D_{33}(1670)$	A_1	0.057	···	0.064 ± 0.028	···
	A_3	0.098	···	0.083 ± 0.050	···
$S''_{11}(1700)$	A_1	0.118	−0.079	0.036 ± 0.030	−0.027 ± 0.040

posite couples to the resonance, and T, J are the resonance's isospin and spin.

In Table V, we exhibit the amplitude for decay of the $\Delta(1232)$ state P_{33}^{+} in magnetic dipole ($M1$) approximation. Numerically, it should be a good approximation to ignore electric quadrupole radiation. The characteristic dipole ratio $(A_3/A_1)_{\text{dipole}} = \sqrt{3}$ is satisfied to a reasonable degree by the data.[15] The calculation proceeds by determining the matrix element between P_{33}^{+} and the proton of the magnetic moment operator (29). The correct normalization is obtained by means of comparison with the nucleon magnetic moment calculation. The value given in Table V has the correct phase, but is somewhat small in magnitude.

Transitions from the negative-parity states $(1S_{1/2})^2$ $(1P_{1/2})$ are not quite so straightforward because we must first construct the correct quark-state vectors. There are five independent types of states—D_{33}, S_{31}, D_{13}, S_{11A}, S_{11B}. They are all degenerate in this model. Contributing multipoles are $E1$ for the $J=\frac{1}{2}$ states and both $E1$, $M2$ for the $J=\frac{3}{2}$ states. For the latter, there is no *a priori* reason to expect dominance of any one multiple over the other. Indeed, for the D_{13}^{+} state, considerable destructive interference is clearly present in the A_1 amplitude.[11] However, we have performed our calculation in $E1$ approximation partly for simplicity, but also in response to a poor result regarding the D_{13}^{+} state which cannot be resolved by adding a higher multipole. The state vectors are written down in Appendix B, and results are exhibited in Table V. The parameter α, defined in Appendix B, is taken to be $\alpha=+1$. Although not quantitatively impressive, the bag model amplitudes are seen to have the correct phases. One might argue that by adding the $M2$ amplitudes to these results, it is conceivable that the fit might be improved, e.g., in reducing the generally large values of the S_{11} amplitudes. However, this will not patch up the $(D_{13})^{+}$ decay amplitude, for which we obtain zero in $E1$ approximation. Evidently, the D_{13} state which occurs in nature is not the state being described by the fixed-sphere bag model.

III. ANALYSIS OF FOUR-QUARK OPERATORS

In a renormalizable gauge theory, strangeness-changing nonleptonic decay is dominated by matrix elements of the time-ordered product of two weak currents mediated by a W-boson propagator.[16] Since the W-boson is presumably quite massive, the amplitude is sensitive only to the small-x region of the time-ordered product, so that a Wilson expansion may be performed.[27] Also, in an asymptotically free model wherein the strong quark-quark interactions are mediated by massless, non-Abelian neutral gauge fields, renormalization-group techniques may be utilized to give the effective Hamiltonian in the presence of the strong interactions as[18]

$$H_{\text{eff}} \cong c_+ H_+ + c_- H_- , \tag{40}$$

where

$$H_{\pm} = \frac{G_V \sin\theta\cos\theta}{2\sqrt{2}} \sum_{i,j=1}^{3} [\bar{\mathfrak{N}}_i \gamma_\mu (1+\gamma_5)\mathcal{P}_i \bar{\mathcal{P}}_j \gamma^\mu (1+\gamma_5)\lambda_j \pm \bar{\mathfrak{N}}_i \gamma_\sigma (1+\gamma_5)\lambda_i \bar{\mathcal{P}}_j \gamma^\sigma (1+\gamma_5)\mathcal{P}_j] , \tag{41}$$

and $c_- \cong 10^{0.48}$, $c_+ \cong 10^{-0.24}$ for a three-color model such as ours. In the conventional model wherein strong interaction effects are neglected, c_+ and c_- are both unity. Here H_- is a purely $\Delta I=\frac{1}{2}$ term while H_+ involves a mixture of $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ components. Within this approach then, there is an effective enhancement of $\Delta I=\frac{1}{2}$ over $\Delta I=\frac{3}{2}$ of about $10^{0.48}/10^{-0.24} \cong 5$, which is suggestive, but still too small to explain the experimentally observed ratio of about 20.[19] It is therefore of interest to compute matrix elements of this Hamiltonian as a test both of our hadronic model and of the renormalization-group ideas.

Since we are not as yet equipped to take three-hadron matrix elements within the bag scheme, we first utilize current algebra together with partial conservation of axial-vector current (PCAC) in order to reduce nonleptonic amplitudes $\langle B'\pi | H_w | B\rangle$ to those involving matrix elements of the weak Hamiltonian between two baryons. Because of the appearance of baryon pole terms, the procedure is strictly legitimate only for the S-wave (parity-violating) piece of the nonleptonic amplitude, and we find[20]

$$\langle B'\pi^0 | H_w^{\text{p.v.}} | B\rangle \cong \frac{-i}{F_\pi} \langle B' | [F_3^5, H_w^{\text{p.v.}}] | B\rangle$$

$$= \frac{i}{2F_\pi} \langle B' | H_w^{\text{p.c.}} | B\rangle . \tag{42}$$

In our model, there are guaranteed to be no $\Delta I=\frac{3}{2}$ effects since

$$\langle B' | H_+ | B\rangle = 0 \tag{43}$$

according to the Pati-Woo theorem.[21] Thus, the octet operator H_- is the only one which contributes. The calculation proceeds as before, however, now

using the 4-quark operators

$$\int d^3r\, \psi^\dagger(\vec{r}, t)\gamma_0\gamma_\rho(1+\gamma_5)\psi(\vec{r}, t) \times \psi^\dagger(\vec{r}, t)\gamma_0\gamma^\rho(1+\gamma_5)\psi(\vec{r}, t).$$

If we neglect the small shift in radius and frequency between the p, Λ, Ξ states, we may characterize our results most simply by means of

$$\begin{aligned}
&\langle n\pi^0|H_w^{\text{p.v.}}|\Lambda\rangle = \sqrt{\tfrac{1}{2}}\,\langle p\pi^-|H_w^{\text{p.v.}}|\Lambda\rangle = i(d+3f)\\
&\langle p\pi^0|H_w^{\text{p.v.}}|\Sigma^+\rangle = \sqrt{\tfrac{1}{2}}\,\langle n\pi^-|H_w^{\text{p.v.}}|\Sigma\rangle = i\sqrt{6}\,(d-f),\\
&\langle n\pi^+|H_w^{\text{p.v.}}|\Sigma^+\rangle = 0,\\
&\langle \Lambda\pi^0|H_w^{\text{p.v.}}|\Xi^0\rangle = \sqrt{\tfrac{1}{2}}\,\langle \Lambda\pi^-|H_w^{\text{p.v.}}|\Xi^-\rangle = i(d-3f),
\end{aligned} \tag{44}$$

where

$$d=-f=-c_-\frac{G_v\sqrt{3}}{4F_\pi}\cos\theta\sin\theta\prod_{i=1}^{4}N_i\int d^3r\left(\frac{1}{4\pi}\right)^2\left\{j_0(p_1r)j_0(p_2r)+\left[\frac{(E_1-m_1)(E_2-m_2)}{(E_1+m_1)(E_2+m_2)}\right]^{1/2}j_1(p_1r)j_1(p_2r)\right\}$$
$$\times\left\{j_0(p_3r)j_0(p_4r)+\left[\frac{(E_3-m_3)(E_4-m_4)}{(E_3+m_3)(E_4+m_4)}\right]^{1/2}j_1(p_3r)j_1(p_4r)\right\}, \tag{45}$$

where $p_i=(\omega_i^2-m^2R^2)^{1/2}/R$ and $F_\pi\simeq 94$ MeV is the pion decay constant. Performing the integration numerically, we find

$$d=-f\cong -8.5\times 10^{-9}c_-. \tag{46}$$

How does this result compare with experiment? Unfortunately, the parameters f, d cannot be extracted unambiguously from the S-wave hyperon decay amplitudes because in general there can be other contributions, e.g., K^* pole diagrams. Although these other contributions are probably non-negligible, none of them is expected to exceed the commutator term evaluated here. A numerical comparison of Eqs. (44)–(46) with the experimental amplitudes is presented in Table VI. Even with the use of the enhancement factor $c_-\cong 3.1$, predicted decay amplitudes are too small by factors of 3 to 5. The ratio $f/d=-1$ given by our bag model [or any other model employing SU(6) wave functions for the $\frac{1}{2}^+$ baryons] is not in disagreement with the value $f/d\cong -1.2$ obtained in Ref. 20 on the basis of fitting both the S- and P-wave amplitudes with an SU(3) parameterization.

IV. CONCLUSION

In the previous sections, we have utilized the MIT bag model in order to examine a variety of transition moments. The present analysis extends previous work in that (i) non-diagonal matrix elements are handled and (ii) commutators are utilized in order to reduce uncalculable three-hadron matrix elements to tractable two-body expectation values. The results of these calculations are generally encouraging. Calculated results for masses, magnetic moments, axial-vector and vector decay constants, etc., agree reasonably well with experimental values except in the case of the D_{13} electromagnetic transition moments and of the nonleptonic decay amplitudes. There are two possibilities here. Recent work by Aaron and Amado[22] has indicated that substantial unitarity corrections must be applied to the electromagnetic transition moments, suggesting that the experimental analysis for the D_{13} states should be redone. Also, the weak Hamiltonian used to calculate the nonleptonic decays is by no means assured to be

TABLE VI. Hyperon nonleptonic decay amplitudes. Column 2 gives the value predicted in the bag model [Eq. (46)] in terms of an arbitrary enhancement factor c_-, whereas the numbers in column 3 correspond to $c_-=3.1$.

Mode	$10^7\times$ (experimental value)	$10^7\times$ (theoretical value) (see footnote a)	$10^7\times$ (theoretical value)
$\langle n\pi^0\|H_w^{\text{p.v.}}\|\Lambda\rangle$	2.39 ± 0.05	$c_-\times 0.17$	0.53
$\langle p\pi^0\|H_w^{\text{p.v.}}\|\Sigma^+\rangle$	-3.28 ± 0.11	$-c_-\times 0.42$	-1.28
$\langle n\pi^+\|H_w^{\text{p.v.}}\|\Sigma^+\rangle$	0	0	0
$\langle \Lambda\pi^0\|H_w^{\text{p.v.}}\|\Xi^0\rangle$	-3.39 ± 0.07	$-c_-\times 0.34$	-1.05

[a] c_- is the enhancement factor defined in Eq. (40).

correct.[23] Thus one can take the point of view that the bag wave functions are substantially right, but they cannot be easily applied to the cases in question. Alternatively, and probably more likely, one can adopt the attitude that the fault lies in the nature of the model itself. The simplified version of the bag that we are employing may be too naive. Indeed, spin-spin interactions between the quarks can and should be added. Such interactions could substantially modify the wave functions we have been using and could lead to a resolution of the problems discussed above. In addition, alternatives to the fixed-sphere bag solutions should be sought. Excited states could well be better described by such wave functions than by those we have utilized.

Two other extensions of the model should be examined. In the present version of the model hadrons are always at rest—matrix elements always involve momentum transfer $\vec{q}=0$. Ways to allow the bag to move, consistent with the boundary condition, must be found in order to deal with arbitrary kinematical situations.[24] Finally, the implications of the existence of a fourth —charmed—quark can be studied especially with respect to the $\psi(3105)$. Because of certain technical subtleties and because of the speculative nature of the charm hypothesis, we have not included this work here, but will present such work in a separate publication.

APPENDIX A

It is interesting to study the relationship between this model and the familiar nonrelativistic quark model. In Ref. 4, it has already been shown that by letting the quarks become infinitely heavy, our numerical results attain the values given by the SU(6) symmetry which underlies the nonrelativistic quark model, e.g., $g_A=\frac{5}{3}$.

In this Appendix, we wish to point out the existence of a second manner in which a nonrelativistic limit can be attained. An alternative to taking quark mass to infinity is to let the bag radius R become arbitrarily large. The basic point is that the standard quark model is recovered from the bag model in any limit in which B, the confinement energy per volume, goes to zero. Of course, we must continue to satisfy the bag boundary conditions as the limit is being taken. For example, the linear boundary condition (10) implies $\omega_{1,-1}^2-m^2R^2 \to \pi^2$ as either m or R becomes infinite. It is amusing to note that in taking $R\to\infty$, all masses remain finite, and indeed,

$$m_{\mathcal{P}} \to \frac{M_p}{3}, \quad m_\lambda \to \frac{M_\Omega}{3}, \tag{A1}$$

where M_p, M_Ω are the proton and Ω^- masses. This is in distinction to the infinite quark-mass limit, for which in this model, the hadron states also become infinitely massive.

APPENDIX B

In this Appendix, we write down the quark wave functions employed in the calculation of radiative decays of nucleon resonances described in Sec. II. The state of highest weight in the $(1S_{1/2})^2(1P_{1/2})$ configuration is

$$(D_{33})^{++}_{3/2}=\frac{1}{\sqrt{3}}\,(b_R^\dagger(\tilde{\mathcal{P}}\uparrow)b_W^\dagger(\mathcal{P}\uparrow)b_B^\dagger(\mathcal{P}\uparrow)+\text{c.p.})\,|0\rangle,$$

where a quark in the $1P_{1/2}$ configuration is denoted by a tilde ($\tilde{\mathcal{P}}$ or $\tilde{\mathfrak{N}}$), the subscripts R, W, B are color indices, and c.p. means cyclic permutation of all variables aside from the color indices, whose position remains fixed. The remaining states of the $(1S_{1/2})^2(1P_{1/2})$ configuration can be reached by means of lowering operators, orthogonality conditions, and unit normalization of the states. Our phase convention here is that the lowerings are accompanied by positive signs only. The states with $T_z=J_z=\frac{1}{2}$ can be written in terms of the classes of quark creation operators

$$A=b_R^\dagger(\tilde{\mathfrak{N}}\downarrow)b_W^\dagger(\mathcal{P}\uparrow)b_B^\dagger(\mathcal{P}\uparrow),\ \ldots,$$

$$B=b_R^\dagger(\tilde{\mathcal{P}}\uparrow)b_W^\dagger(\mathfrak{N}\uparrow)b_B^\dagger(\tilde{\mathcal{P}}\downarrow),\ \ldots,$$

$$C=b_R^\dagger(\tilde{\mathfrak{N}}\uparrow)b_W^\dagger(\mathcal{P}\downarrow)b_B^\dagger(\mathcal{P}\uparrow),\ \ldots,$$

$$D=b_R^\dagger(\tilde{\mathcal{P}}\uparrow)b_W^\dagger(\mathfrak{N}\downarrow)b_B^\dagger(\mathcal{P}\uparrow),\ \ldots,$$

$$E=b_R^\dagger(\tilde{\mathcal{P}}\uparrow)b_W^\dagger(\mathcal{P}\downarrow)b_B^\dagger(\mathfrak{N}\uparrow),\ \ldots,$$

as

$$(D_{33})^+_{1/2}=\frac{1}{(27)^{1/2}}\,(A+B+C+D+E),$$

$$(S_{31})^+_{1/2}=\frac{1}{(54)^{1/2}}\,(2A+2B-C-D-E),$$

$$(D_{13})^+_{1/2}=\frac{1}{(54)^{1/2}}\,(2A-B+2C-D-E),$$

$$(S_{11A})^+_{1/2}=\frac{-2A+B+C+\alpha D-(1+\alpha)E}{\sqrt{6}(5+2\alpha^2+2\alpha)^{1/2}},$$

$$(S_{11B})^+_{1/2}=\frac{2A-B-C-\gamma D+(1+\gamma)E}{\sqrt{6}\,(5+2\gamma^2+2\gamma)^{1/2}},$$

where $-\infty<\alpha<\infty$, $\alpha\neq-\frac{1}{2}$, and $\gamma=-(5+\alpha)/(1+2\alpha)$. That is, the S_{11A} and S_{11B} states are not uniquely determined by orthogonality and normalization.

There remains a degree of freedom, the parameter α, which we fix by fitting the bag model transition amplitudes to the data.

The neutral members of this configuration can be constructed from the above states by means of isospin lowering. With our phase convention, this amounts to simply interchanging "proton"- and "neutron"-type quarks.

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*Work supported in part by the National Science Foundation.

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[24]Indeed, the experimental photon transition amplitudes in Table V refer to zero four-momentum transfer, whereas we are forced by the constraints of our model to evaluate these amplitudes at $q^2 = (M^* - M)^2$, where M^*, M are the resonance and nucleon mass, respectively.