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Unified Treatment of the Parity Violating Nuclear Force

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We discuss the weak amplitudes which determine the parity violating nuclear force. By use of the quark model and the $SU(6)_w$ symmetry, we unite the treatment of pion and vector meson vertices, and describe the interrelation of past techniques which have been applied to this problem. This allows us to catalog the uncertainties in the amplitudes, and to provide reasonable bounds on their values. The connection of our results with experiment is also discussed.

I. INTRODUCTION

Although many properties of the weak interaction are presently under scrutiny via studies at the very highest-energy accelerators, there is also a great deal of interest in experiments which probe the weak force via low-energy, parity violating nuclear processes [1]. Although a substantial experimental as well as theoretical effort has been exerted in this field for well over a decade, we are still far from a trenchant understanding of such reactions.

The reasons for this are simple. On the experimental side, effects, such as detection of circular polarization in an electromagnetic decay of a nuclear state, are expected to be very small—roughly at the level $\lesssim 10^{-p/M}$, where p is a typical nucleon momentum and M is the nucleon mass. Thus experiments are extraordinarily difficult and must be carefully screened for systematic biases. On the theoretical end the

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analysis is beset with two difficulties. Typically the experiments are performed on complex nuclei and the weak force is introduced via effective two-body operators. Evaluation of matrix elements of such operators is hampered by the usual nuclear wavefunction and short-range correlation uncertainties. In addition, since all particles involved are strongly interacting, this is a nonleptonic weak process, for which theoretical analysis is notoriously treacherous.

Nevertheless it has been and continues to be of great interest to attempt to analyze such experiments theoretically, although the original motivation has been somewhat modified. The usual assumption is that the effective weak force can be viewed as originating from a process wherein one nucleon emits a meson $M(\pi, \rho, \omega, \phi, \text{etc.})$ through a weak interaction, with the meson being subsequently absorbed by a second nucleon via a strong interaction. In such a picture the Cabibbo (charged current) piece of the weak Hamiltonian contributes to $\Delta I = 1$ processes only via $\pi^\pm$ exchange, which is suppressed by the factor $\sin^2 \theta_c \sim 1/25$, where θ_c is the Cabibbo angle. Thus it was thought that measurement of the weak $NN\pi$ vertex should provide a sensitive indication of possible neutralcurrent effects and hopefully discriminate among various competing weak interaction models. Realistic assessment of the theoretical difficulties alluded to above has dampened this hope considerably, and the availability of high-energy, high-intensity neutrino and electron beams has made it much easier to probe the neutral weak current via ν_μ, e^- experiments. However, there exists at present only a limited amount of experimental information about weak nonleptonic matrix elements, all of it coming from $\Delta S = 1$ and/or $\Delta C = 1$ decays. Thus reliable values of the $\bar{N}NM$ $\Delta S = 0$ amplitudes would be very useful additional tools with which to verify our calculational techniques.

In most of the early work on this subject, authors started directly from a model weak Hamiltonian and calculated experimental predictions with which measured results were compared. More recently it has become apparent that a more sensible approach is to break the calculation into two parts. One part involves analyzing the various experimental numbers to produce reliable values for the weak $\bar{N}NM$ vertices. This approach, requiring nuclear physics expertise and detailed nucleon wavefunction information, has been pioneered by Desplanques and Missimer [2] and Box *et al.* [3] and is still in progress. The second part is an exercise in elementary particle physics and attempts to calculate the matrix elements

$$\langle MN | \mathcal{H}_w | N \rangle$$

starting from the structure of the weak Hamiltonian $\mathcal{H}_w$. It is this latter task which is the major subject of the present paper.

In previous work the π^- emission amplitude n_-^0 was generally related to experimentally determined $\Delta S = 1$ decay amplitudes A_-^0, E_-^- via $SU(3)$ symmetry arguments, some times supplemented by current algebraic considerations [4],

$$n_-^0 = -\frac{2A}{3^{1/2}}(2A_-^0 - E_-^-). \quad (1)$$

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Here A is a numerical constant which characterizes each model of $\mathcal{H}_w$, and which gauges the relative strength of the $\Delta S = 0$ and $\Delta S = 1$ components. In the Cabibbo model, for example, we have

$$A^{\text{Cabibbo}} = \frac{1}{2^{1/2}} \tan \theta_c \quad (2)$$

while in models containing neutral currents A is typically of order $\cot \theta_c$. It had been thought that this value of n_-^0 was on relatively firm ground, since the $\Delta S = 1$ amplitudes are reliably measured. However, in most models with neutral currents the sum rule does not follow from symmetry considerations, and a dynamical model is necessary to obtain a prediction. Moreover, it has recently been emphasized that $SU(3)$ breaking effects may be considerable in this sum rule, and a detailed dynamical model may be required in order to produce a reliable theoretical prediction [5].

Most previous work on the vector meson amplitudes

$$\langle VN | \mathcal{H}_w | N \rangle$$

utilized the so-called factorization approximation first developed by Michel [6], which corresponds to writing, e.g.,

$$\begin{aligned} \langle \rho^- p | \mathcal{H}_w^{\text{Cabibbo}} | n \rangle &= \frac{G}{2^{1/2}} \cos^2 \theta_c \langle \rho^- p | A_{\pi^+}^\lambda(0) V_{\lambda}^{\pi^-}(0) | n \rangle \\ &\cong \frac{G}{2^{1/2}} \cos^2 \theta_c \langle \rho^- | V_{\lambda}^{\pi^-}(0) | 0 \rangle \langle p | A_{\pi^+}^\lambda(0) | n \rangle. \end{aligned} \quad (3)$$

It was subsequently shown that this was an exact consequence of field algebra together with the current field identity [7]. However, it has recently been noted that quark model ideas require some modification of these predictions [5]. Another approach which was suggested previously [8] was to relate $\bar{N}NV$ $\Delta S = 0$ and $\Delta S = 1$ hyperon decay amplitudes via the symmetry $SU(6)_w$. Results obtained in this scheme were apparently incompatible with factorization ideas, however.

In this paper we shall reexamine these attempts to evaluate

$$\langle MN | \mathcal{H}_w | N \rangle$$

matrix elements within the framework of a specific dynamical approach—the quark model. In the case of the vector meson amplitudes we shall examine the relation to previous— $SU(6)_w$ and factorization—approaches and show how they fit together. In the case of the $\pi^\pm$ amplitude we shall also demonstrate the consistency between quark model and previous— $SU(3)$, current algebra, $SU(6)_w$ —approaches. However, for both V and π emission we shall see that present state-of-the-art techniques unfortunately preclude making a definitive numerical prediction. Rather we must be content with providing a set of bounds within which the correctly calculated number

is assured to be. Although this conclusion is perhaps a discouraging one, we feel that it *is* of great interest

(i) to see how the various previous theoretical approaches are weaved together within the fabric of the quark model,

(ii) to provide such bounds as a challenge for further theoretical work to narrow.

Our paper is organized as follows. In Section II we describe our quark model approach and discuss its relation to previous work. Section III provides an estimate of corrections generated by strong interaction effects. Sections IV and V discuss the generation of bounds for $\pi^\pm$ and V emission, respectively. Section VI assesses the implication of these bounds on present experimental numbers, and finally we present our conclusions in Section VII.

II. $SU(6)_w$, THE QUARK MODEL, AND THEIR INTERRELATION

Since we are attempting to discuss both pion and vector meson interactions simultaneously it is advantageous to exploit a symmetry that unites them. $SU(6)_w$ is a generalization [9] of $SU(6)$ that allows a relativistically correct theory for arbitrary boosts along one direction ($\hat{z}$). This is sufficient for a full treatment of weak nonleptonic vertices. Our $SU(6)_w$ work follows that of McKellar and Pick [8], and of Balachandran *et al.* [10]. We shall in the first part of this section give the $SU(6)_w$ parameterization of nonleptonic weak vertices and display the relation to the non-relativistic quark model. This will permit us to elucidate the interrelation of the other techniques which have been applied to this problem, e.g., PCAC, field algebra, and the various factorization methods.

We shall at first deal solely with the Cabibbo (charged current) contribution to the nonleptonic parity violating Hamiltonian for both $\Delta S = 0$ and $\Delta S = 1$ processes. Once our procedures and techniques have been explained and justified, we shall expand our approach to include general neutral current $\Delta S = 0$ forms.

We begin by writing in $SU(3)$ tensor notation the $\Delta S = 1$ weak nonleptonic Hamiltonian

$${}^c\mathcal{H}_w(\Delta S = 1) = \frac{G}{2(2^{1/2})} \cos \theta_c \sin \theta_c \{J_{\mu 1}^2, J_3^{\mu 1}\} + \text{h.c.}, \quad (4)$$

where

$$J_{\mu j}^i = (V_\mu + A_\mu)_j^i \quad (5)$$

is the weak hadronic current with $SU(3)$ indices i, j , $G \approx 10^{-5} m_p^{-2}$ is the weak coupling constant, and θ_c is the Cabibbo angle. The superscript c on H_w indicates that this is the simple Cabibbo form.

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Now following the procedure of Balachandran *et al.* [10] we express the vector and axial currents in terms of $SU(6)_w$ currents P, Q, R, S

$$\begin{aligned}
 A_{+b}^a &= -R_{2b-1}^{2a}, & A_{-b}^a &= -R_{2b}^{2a-1}, \\
 V_{+b}^b &= iQ_{2b-1}^{2a}, & V_{-b}^a &= -iQ_{2b}^{2a-1}, \\
 V_{3b}^a &= \frac{1}{2}(P_{2b}^{2a} + P_{2b-1}^{2a-1}), & A_{0b}^a &= \frac{1}{2}(P_{2b}^{2a} - P_{2b-1}^{2a-1}), \\
 V_{0b}^a &= \frac{1}{2}(S_{2b}^{2a} + S_{2b-1}^{2a-1}), & A_{3b}^a &= \frac{1}{2}(S_{2b}^{2a} - S_{2b-1}^{2a-1}).
 \end{aligned} \tag{6}$$

The indices a, b are $SU(3)$ indices while on P, Q, R, S they represent $SU(6)_w$ indices. For the weak parity violating nonleptonic Hamiltonian this gives

$${}^c\mathcal{H}_w^{(-)}(\Delta S = 1) = \frac{G}{2^{1/2}} \cos \theta_c \sin \theta_c {}^{(-)}\mathcal{O}_{43}^{56} \tag{7}$$

where the $(-)$ indicates that this is the parity violating component and

$$\begin{aligned}
 {}^{(-)}\mathcal{O}_{CD}^{AB} &= T_{\{C,1\}}^{[2,A]} + T_{[C,1]}^{\{2,A\}} - T_{\{D,2\}}^{[1,B]} - T_{[D,2]}^{\{1,B\}} \\
 &+ V_{\{D,2\}}^{[2,A]} + V_{[D,2]}^{\{2,A\}} - V_{\{C,1\}}^{[1,B]} - V_{[C,1]}^{\{1,B\}} + V_C^B - V_D^A.
 \end{aligned} \tag{8}$$

Here

$$\begin{aligned}
 T_{CD}^{AB} &= P_C^A S_D^B - S_C^A P_D^B, \\
 V_{CD}^{AB} &= -iR_C^A Q_D^B + iQ_C^A R_D^B,
 \end{aligned} \tag{9}$$

and $[,]$ and $\{, \}$ represent antisymmetrization and symmetrization of the $SU(6)_w$ indices. Note that T describes the effects of currents along the boost direction $(0, 3)$, while V includes currents perpendicular to this direction $(1, 2 \text{ or } +, -)$.

We can then define, in an obvious notation, the various ways in which to couple the two baryons and the meson together in a CP invariant fashion.

$$\begin{aligned}
 a_t, a_v &: [(\bar{B}B)_{35} \otimes M_{35}]_{280, \overline{280}}, \\
 b_t, b_v &: [(\bar{B}B)_{405} \otimes M_{35}]_{280, \overline{280}}, \\
 c_v &: [(\bar{B}B)_{35} \otimes M_{35}]_{35}.
 \end{aligned} \tag{10}$$

We find then for the $\Delta S = 1$ pion decay amplitudes¹

$$\begin{aligned}
 + \frac{1}{2^{1/2}} A_-^0 &= -A_0^0 = \frac{1}{6^{1/2}} \left(\frac{1}{6} b_v - \frac{1}{12} b_t - \frac{1}{2} c_v \right), \\
 - \frac{1}{2^{1/2}} E_-^- &= E_0^0 = \frac{1}{6^{1/2}} \left(-\frac{1}{4} b_v + \frac{1}{4} b_t + \frac{1}{2} c_v \right),
 \end{aligned}$$

¹ Our phase conventions are defined in Appendix B.

$$\begin{aligned}
 -\frac{1}{2^{1/2}} \Sigma_-^- &= \frac{1}{9} b_v - \frac{5}{36} b_t - \frac{1}{6} c_v, \\
 \frac{1}{2^{1/2}} \Sigma_+^+ &= \frac{1}{36} b_v + \frac{1}{36} b_t, \\
 \Sigma_0^+ &= -\frac{5}{36} b_v + \frac{1}{9} b_t + \frac{1}{6} c_v,
 \end{aligned} \tag{11}$$

where, in standard notation,

$$\Sigma_0^+ = \langle p\pi^0 | {}^e\mathcal{H}_w^{(-)} | \Sigma^+ \rangle \tag{12}$$

is the parity violating contribution to the appropriate pion decay.

In a similar fashion we may use $SU(6)_w$ in order to list the (unphysical) $\Delta S = 1$ rho emission amplitudes. In this case we can calculate the amplitude for ρ emission either with $J_z = 0$ or with $J_z = 1$. We list here only the $J_z = 1$ amplitudes. The reasons for this are twofold. Firstly, we note that one can write the invariant amplitude for the parity violating process

$$\alpha \rightarrow \beta + V^\gamma$$

as $\epsilon^\mu M_\mu^\gamma$ where γ is an $SU(3)$ index and ϵ^μ is the polarization vector of the meson V^γ . Expanding M_μ in terms of invariants we have

$$M_\mu^\gamma = \bar{u}_\beta \gamma_5 [g_1^\gamma \gamma_\mu + g_2^\gamma k_\mu - ig_0^\gamma \sigma_{\mu\nu} k^\nu] u_\alpha. \tag{13}$$

For our later application to nuclear parity violation it is g_1^γ which we need. McKellar and Pick [8] have argued that $SU(6)_w$ relates amplitudes with the same B spin and since

$$\bar{\psi}\psi, \bar{\psi}\gamma_5\gamma_1\psi, \bar{\psi}\gamma_5\gamma_2\psi, \bar{\psi}\sigma_3\psi \tag{14}$$

all have B -spin zero, it is the transverse amplitude which gives g_1 , the longitudinal amplitude which yields g_3 (g_2^γ does not contribute to parity-violating processes with both nucleons on the mass shell and is neglected hereafter). However, this argument assumes B -spin is a good symmetry, which goes outside the assumption of $SU(6)_w$. The second reason, and one which we expouse, is that for $J_z = 1$ amplitudes the quark model approach, to be discussed shortly, yields results consistent with the $SU(6)_w$ predictions. We find then for the transverse ρ emission amplitudes

$$\begin{aligned}
 {}^\rho A_-^0 &= -\left(\frac{2}{3}\right)^{1/2} \left(\frac{1}{12} b_t - \frac{1}{3} b_v + \frac{1}{2} c_v - a_v\right), \\
 {}^\rho A_0^0 &= \frac{1}{3^{1/2}} \left(\frac{1}{4} b_t - \frac{1}{6} b_v + \frac{1}{2} c_v + a_t\right), \\
 {}^\rho E_-^- &= -\left(\frac{2}{3}\right)^{1/2} \left(\frac{1}{12} b_t - \frac{1}{12} b_v + \frac{1}{6} c_v - \frac{1}{3} a_v\right),
 \end{aligned}$$

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$$\begin{aligned}
 {}^\rho\mathcal{E}_0^0 &= \frac{1}{3^{1/2}} \left(\frac{1}{12} b_t - \frac{1}{12} b_v + \frac{1}{6} c_v + \frac{1}{3} a_t \right), \\
 {}^\rho\mathcal{E}_-^- &= \left(-\frac{1}{9} b_t + \frac{1}{18} b_v - \frac{1}{9} c_v + \frac{2}{9} a_v \right), \\
 {}^\rho\mathcal{E}_+^+ &= \left(-\frac{1}{9} b_t + \frac{2}{9} b_v \right), \\
 {}^\rho\mathcal{E}_0^+ &= \frac{1}{2^{1/2}} \left(-\frac{2}{9} b_t + \frac{5}{18} b_v - \frac{1}{9} c_v - \frac{2}{9} a_t \right),
 \end{aligned} \tag{15}$$

where ${}^\rho\mathcal{E}_0^+ = \langle p\rho^0(J_2=1) | \mathcal{H}_w^{(-)} | \Sigma^+ \rangle$ is the parity violating contribution to the appropriate ρ amplitude. These $\Delta S = 1$ rho emission amplitudes are, of course, not of physical interest. However, they will prove useful in aiding our understanding of the relationship between the $SU(6)_w$ symmetry scheme and the quark model. We note the interesting feature that the ρ -emission amplitudes involve the parameters a_t , a_v which are not present in the pion case. This was pointed out by McKellar and Pick and prevented them from being able to determine the ρ emission amplitudes in terms of those for pion emission. We shall return to this at a later stage.

The pion decay amplitudes are, of course, known experimentally and it is possible to determine the $SU(6)_w$ parameters, b_v , b_t , c_v phenomenologically. Thus from the result

$$\mathcal{E}_+^+ \approx 0 \tag{16}$$

we find

$$b_v \approx -b_t \tag{17}$$

while from the experimental value

$$\frac{\mathcal{E}_-^-}{\mathcal{A}_-^0} = 1.39 \pm 0.02 \tag{18}$$

we determine

$$c_v \approx -0.78 b_v. \tag{19}$$

These choices provide a rather good three-parameter fit to the parity violating hyperon decay amplitudes, as indicated in Table I.

For the Cabibbo (charged current) Hamiltonian, the above steps can be repeated for the strangeness conserving operator $H_w^{\Delta S=0}$. Thus in terms of $SU(3)$ tensor notation we have

$${}^c\mathcal{H}_w(\Delta S = 0) = \frac{G}{2(2)^{1/2}} [\cos^2 \theta_c \{J_{\mu 2}^1, J_1^{\mu 2}\} + \sin^2 \theta_c \{J_{\mu 3}^1, J_1^{\mu 3}\}]. \tag{20}$$

The corresponding $SU(6)_w$ transformation properties are then

$$\begin{aligned}
 {}^c\mathcal{H}_w^{(-)}(\Delta S = 0) &= \frac{G}{2^{1/2}} \{ \cos^2 \theta_c [{}^{(-)}\theta_{43}^{34} + V_2^2 - V_1^1] \\
 &\quad + \sin^2 \theta_c [{}^{(-)}\theta_{65}^{56} + V_2^2 - V_1^1] \}.
 \end{aligned} \tag{21}$$

TABLE I
Hyperon Decay Amplitudes and the $SU(6)_w$ Fit

Mode	Expt $\times 10^7$	Fit $\times 10^7$
Λ^-	$-3.27 \pm .02$	-3.27 (input)
Λ_0^0	$2.40 \pm .04$	2.36
Ξ_-^-	$-4.52 \pm .04$	-4.54
Ξ_0^0	$3.40 \pm .06$	3.22
Σ_-^-	$4.29 \pm .02$	4.51
Σ_+^+	$0.13 \pm .04$	0
Σ_0^+	$3.29 \pm .10$	3.29 (input)

We can then, as before, evaluate the appropriate pion and (transverse) vector meson emission amplitudes. Since we shall later need them, we also list the ω^0 and ϕ^0 emission amplitudes

$$\begin{aligned}
n_-^0 &= \frac{1}{3(2)^{1/2}} c_v \tan \theta_c, \\
{}^o n_-^0 &= \frac{1}{9} (4b_v - 2b_t - 10c_v + 10a_v) \cot \theta_c - \frac{5}{9} \tan \theta_c c_v, \\
{}^o p_0^+ &= \frac{1}{9(2)^{1/2}} (3b_v - 3b_t - 10c_v - 10a_t) \cot \theta_c - \frac{8}{9(2)^{1/2}} \tan \theta_c c_v, \\
\omega p_0^+ &= \frac{1}{9(2)^{1/2}} (b_v + b_t - 6c_v + 6a_t) \cot \theta_c - \frac{8}{9(2)^{1/2}} \tan \theta_c c_v, \\
\phi p_0^+ &= -\frac{8}{9} a_t \tan \theta_c.
\end{aligned} \tag{22}$$

Here we have used the quark model value for the ω - ϕ mixing angle—i.e., the assumption that ϕ is purely $\bar{s}s$.

It is useful at this point to make the connection between $SU(6)_w$ and quark model techniques. In order to do so, we note that the $SU(6)_w$ indices $A = 1, 2, \dots, 6$ correspond to quark flavor and spin states $1 = u\uparrow, 2 = u\downarrow, \dots, 6 = s\downarrow$. Then the $280 + \bar{280}$ tensors T_{CD}^{AB} contain four external indices that can be expressed by their action on four quark fields. In particular, the combination of tensors that appears in the Hamiltonian is the same as the nonrelativistic reduction of

$$H_T = V_0 A_0 - V_3 A_3. \tag{23}$$

For example, one term in this reduction is

$$H_T = (d^+ \bar{u})(u^+ s) - (d^+ \sigma^3 \bar{u})(u^+ \sigma^3 s) + \dots, \tag{24}$$

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where, u, d, s are Pauli spinors for quarks (e.g., $u = \begin{pmatrix} u \\ \downarrow \end{pmatrix}$) and $\bar{u}$ is the Pauli spinor of an antiquark. The V portion follows likewise from the $+, -$ currents.

In forming the tensors from baryon and meson fields with the same transformation properties as a given tensor T_{CD}^{AB} we must contract the $SU(6)_w$ indices that are not fixed equal to A, B, C, D . For example, for the b_v or b_t parameters we sum over indices α, β ,

$$[(\bar{B}B)_{405} \otimes M_{35}]_{280+\bar{280}} \sim \bar{B}^{\alpha\beta A} B_{\alpha CD} M_{\beta}^B \sim T_{CD}^{AB}. \quad (25)$$

Several terms of this form must be symmetrized in order to form the required Hamiltonian. These two observations can be combined to give a pictorial way in which to do the group theory. Instead of using $SU(6)_w$ we can use quark indices throughout. Performing the sums for the b_v and b_t portions is equivalent to the quark spin sums depicted in Fig. 1a. The baryons and meson are described by the $SU(6)$ wavefunctions (which are equivalent to the tensors $B_{\alpha\beta\gamma}$ and M_{β}^{α}) and the wavy line describes the $SU(6)_w$ Hamiltonian (7). A study of the indices in Eq. (7) and Fig. 1a shows that the two approaches are equivalent. Figure 1b is the graphical form for the c_v spin sum and Fig. 1c is that for a_t and a_v .

At this stage, these diagrams may be regarded as merely simple ways to obtain the $SU(6)_w$ transformation properties. However in a quark model they have a well-defined dynamical meaning, and, in some approximation, can be calculated. Since

FIG. 1. Quark model amplitudes corresponding to $SU(6)_w$ parameters.

recent progress has shown that quark models are considerably more powerful than symmetry approaches, we shall give weight to this quark model interpretation. In fact, some interesting results follow quickly. In a quark model calculation the only difference between the longitudinal and transverse current products (T and V) occurs when the final $B'\pi$ move relativistically along the Z direction in the initial baryon's rest frame. In the static limit, with little motion, it follows simply that $b_v = -b_t$, as found experimentally. This seems a reasonable limit since there is little available energy in the final state in physical hyperon decay. In fact we find, with $b_v = -b_t$ the relations (cf. Eq. (11))

$$\frac{1}{2^{1/2}} A_-^0 = -A_0^0,$$

$$\frac{1}{2^{1/2}} E_-^- = -E_0^0, \quad (26)$$

$$2^{1/2} \Sigma_0^+ = \Sigma_+^+ + \Sigma_-^-, \quad (27)$$

which are equivalent to the well-known $\Delta I = \frac{1}{2}$ rule, and

$$-3^{1/2} \Sigma_0^+ + A_-^0 - 2E_-^- = 0 \quad (28)$$

which is the Lee-Sugawara sum rule [12] and is satisfied quite well experimentally.

If we take (as we will be required in the next section) the Hamiltonian to be the normal-ordered (as opposed to time-ordered) product of two currents, then b_v and b_t give the only contributions to pion amplitudes in a naive quark model. The diagram in Fig. 1b would appear to be forbidden by the normal ordering prescription. But this is naive in that these diagrams are meant to be evaluated inside of hadrons, where virtual gluons and quark pairs (the "sea") are also present. The interaction of the Hamiltonian with a virtual quark pair could effectively produce Fig. 1b. This has lead in the past to some controversy over the size of the c_v parameter, as will be discussed in detail in Section IV.

Finally, we note that Fig. 1c yields the well-known factorization contribution to the vector meson emission amplitudes. However, there is one subtlety associated with such diagrams. Taking the lowest-order term in a short-distance expansion (cf. Section III) we can represent $H_w^{(-)}$ as a local four quark operator

$${}^c \mathcal{H}_w^{(-)}(\Delta S = 0) \cong \frac{G}{2^{1/2}} \cos^2 \theta_c \bar{d} \gamma_\mu (1 + \gamma_5) u \bar{u} \gamma^\mu (1 + \gamma_5) d. \quad (29)$$

Such an operator clearly contributes to say $n \rightarrow p \rho^-$ in the factorization approximation

$$\langle \rho^-(J_2 = 1) p | {}^c \mathcal{H}_w^{(-)} | n \rangle = \frac{G}{2^{1/2}} \cos^2 \theta_c \langle \rho^- | \bar{d} \gamma_\mu u | 0 \rangle \langle p | \bar{u} \gamma^\mu \gamma_5 d | n \rangle \quad (30)$$

but the factorization contribution to a ρ^0 amplitude is not so clear. Many previous authors have simply ignored such a contribution since a neutral particle can obviously

not connect to the vacuum via a charged current. However, this approach is inconsistent with the quark model, in which we may perform a Fierz rearrangement on the local four quark form

$${}^c\mathcal{H}_w^{(-)}(\Delta S = 0) \cong \frac{G}{2^{1/2}} \cos^2 \theta_c \bar{d}_i \gamma_\mu (1 + \gamma_5) d_j \bar{u}_j \gamma^\mu (1 + \gamma_5) u_i \quad (31)$$

so that it appears to be the local product of neutral currents. (Here the i, j represent color indices which were suppressed earlier.) We may then write

$$\begin{aligned} \langle \rho^0 p | {}^c\mathcal{H}_w^{(-)} | p \rangle &= \frac{G}{2^{1/2}} \cos \theta_c [\langle \rho^0 | \bar{u}_i \gamma_\mu u_j | 0 \rangle \langle p | \bar{d}_j \gamma^\mu \gamma_5 d_i | p \rangle \\ &+ \langle \rho^0 | \bar{d}_i \gamma_\mu d_j | 0 \rangle \langle p | \bar{u}_j \gamma_\mu \gamma_5 u_i | p \rangle]. \end{aligned} \quad (32)$$

Since physical states are color singlets, only the diagonal pieces of the currents can produce a transition. Thus

$$\begin{aligned} \langle \rho^0 | \bar{u}_i \gamma_\mu u_j | 0 \rangle &= \frac{1}{3} \delta_{ij} \sum_{k=1}^3 \langle \rho^0 | \bar{u}_k \gamma_\mu u_k | 0 \rangle, \\ \langle p | \bar{d}_j \gamma_\mu \gamma_5 d_i | p \rangle &= \frac{1}{3} \delta_{ij} \sum_{k=1}^3 \langle p | \bar{d}_k \gamma_\mu \gamma_5 d_k | p \rangle \end{aligned} \quad (33)$$

We have then, suppressing color indices so that all currents are color singlet,

$$\begin{aligned} \langle \rho^0 p | {}^c\mathcal{H}_w^{(-)}(\Delta S = 0) | p \rangle &= \frac{G}{2^{1/2}} \cos^2 \theta_c \{ \langle \rho^0 | \bar{u} \gamma_\mu u | 0 \rangle \langle p | \bar{d} \gamma^\mu \gamma_5 d | p \rangle \\ &+ \langle \rho^0 | \bar{d} \gamma_\mu d | 0 \rangle \langle p | \bar{u} \gamma^\mu \gamma_5 u | p \rangle \} \times \frac{1}{3}. \end{aligned} \quad (34)$$

Using the relation

$$\langle \rho^0 | \bar{u} \gamma_\mu u | 0 \rangle = -\langle \rho^0 | \bar{d} \gamma_\mu d | 0 \rangle \quad (35)$$

we find

$$\langle \rho^0 p | {}^c\mathcal{H}_w^{(-)} | p \rangle \approx -\frac{2}{3} \frac{G}{2^{1/2}} \cos^2 \theta_c \langle \rho^0 | V_\mu^3 | 0 \rangle \langle p | A_\mu^3 | p \rangle. \quad (36)$$

Thus we identify

$$\begin{aligned} a_v &= \frac{9}{10} \frac{G}{2^{1/2}} \cos \theta_c \sin \theta_c \langle \rho^- | V_\mu^{1-i2} | 0 \rangle \langle p | A_{1+i2}^\mu | n \rangle, \\ a_t &= \frac{3}{5} G \cos \theta_c \sin \theta_c \langle \rho^0 | V_\mu^3 | 0 \rangle \langle p | A_\mu^3 | p \rangle \\ &= \frac{1}{3} a_v. \end{aligned} \quad (37)$$

Note that the a_t , a_v contributions to the ω , ϕ couplings given in Eq. (22) are also consistent with these definitions. The ϕ coupling obviously vanishes since no strange quarks appear in ${}^c\mathcal{H}_w^{(-)}$ ($\Delta S = 0$) in order $\cos^2 \theta_c$. For the ω^0 coupling we note that just as we derived Eq. (37) we can show that

$$a_t = \frac{3}{2} G \cos \theta_c \sin \theta_c \langle \omega^0 | V_\mu^0 | 0 \rangle \langle p | A_\mu^0 | p \rangle \quad (38)$$

But using the quark model relations

$$\begin{aligned} \langle \omega^0 | V_\mu^0 | 0 \rangle &= (\frac{3}{2})^{1/2} \langle \rho^0 | V_\mu^3 | 0 \rangle, \\ \langle p | A_\mu^0 | p \rangle &= \frac{3}{5} (\frac{2}{3})^{1/2} \langle p | A_\mu^3 | p \rangle \end{aligned} \quad (39)$$

the evaluations of a_t using either ρ^0 or ω^0 emissions are seen to be identical. As we noted earlier, in the $SU(6)_w$ picture a_t , a_v do not contribute to pionic decays. This is consistent with our assumption relating them to the factorization diagrams since, although corresponding factorization diagrams exist for pionic modes, e.g.,

$$\langle \pi^- p | \mathcal{H}_w^{(-)} | A \rangle = \frac{G}{2^{1/2}} \cos \theta_c \sin \theta_c \langle \pi^- | A_\mu^{1+i2} | 0 \rangle \langle p | V_\mu^{4-i5} | A \rangle \quad (40)$$

we have

$$\langle \pi^- | A_\mu^{1+i2} | 0 \rangle = i(2)^{1/2} F_\pi q_\mu \quad (41)$$

and hence

$$\langle \pi^- p | {}^c\mathcal{H}_w^{(-)} | A \rangle \approx iG \cos \theta_c \sin \theta_c F_\pi \langle p | \partial^\mu V_\mu^{4+i5} | A \rangle \quad (42)$$

which vanishes in the $SU(3)$ limit. This is the origin of the vanishing of the a_v , a_t contribution for the pionic modes in the $SU(6)_w$ approach. (Of course, in the real world $SU(3)$ breaking effects do allow a factorization contribution to the pion amplitudes, which should be included for correctness.) Thus the problem that beset McKellar and Pick—not knowing how to evaluate a_v and a_t —is resolved by our procedure. The coefficients b_v , b_t , and c_v are derivable from hyperon decay, while a_v and a_t are calculable via the factorization approximation. This identification assumes $SU(6)_w$ and/or the nonrelativistic quark model. A separate question is how to estimate corrections to this. We will see in Section V that corrections may decrease vector meson amplitudes relative to pionic vertices, while keeping the same relations among the vector mesons.

We should mention here one subtlety in relating quark model and $SU(6)_w$ predictions—in quark model calculations it is necessary to include both “final state” and “initial state” diagrams, as shown in Fig. 2. The “final state” diagram (Fig. 2b) is that which one would typically write down. The “initial state” diagram (Fig. 2a) is in some sense a mirror image of the “final state” picture. Strictly speaking it appears to be a calculation of the pion absorption amplitude

$$\langle B' | \mathcal{H}_w | B \pi^a \rangle.$$

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FIG. 2. (a) "Final state" and (b) "initial state" quark diagrams.

However, it is related via crossing to the matrix element

$$\langle B' \pi^a | \mathcal{H}_w | B \rangle$$

for pion emission and *must* be included for correctness. Indeed if such diagrams are omitted, a spurious inconsistency develops between the quark model and $SU(6)_w$ predictions. If one attempts to draw an "initial state" diagram in "final state" form as in fig. 2b it is seen that a virtual color gluon is required to create the $q\bar{q}$ pair in the final state. This gluon is, of course, also present in the crossed diagram but its presence is not indicated for simplicity.

In order to demonstrate the point more clearly, we perform a quark model calculation of Figs. 2a, b, corresponding to b_v , b_t . We utilize simple $SU(6)_w$ wavefunctions for the baryons and the meson. The Hamiltonian is decomposed into two pieces, as before

$$\begin{aligned} {}^T \mathcal{H}_w^{(-)} &\propto \{V_0, A_0\} - \{V_3, A_3\}, \\ {}^V \mathcal{H}_w^{(-)} &\propto -\frac{1}{2}\{V_-, A_+\} - \frac{1}{2}\{V_+, A_-\}. \end{aligned} \quad (43)$$

Then one performs appropriate color and spin sums for "final state" and "initial state" diagrams—figs. 2a and b, respectively. We do not even attempt to calculate the overall normalization—evaluating such a three hadron amplitude is impossible to do reliably at the present time. Thus we have unknown parameters q_t , q_v associated with Hamiltonians ${}^t H_w^{(-)}$, ${}^r H_w^{(-)}$. The results are quoted in Table II for selected $\Delta S = 1$ amplitudes.

We note that there is complete equivalence between quark model and $SU(6)_w$ predictions *provided we add "initial state" and "final state" contributions* and make the identification

$$\begin{aligned} q_v &= \frac{1}{24} b_v, \\ q_t &= -\frac{1}{24} b_t. \end{aligned} \quad (44)$$

TABLE II
Quark Model Calculation for "Initial State" and "Final State" Amplitudes

Mode	"Initial state"	"Final state"
A_0^0	0	$\left(\frac{2}{3}\right)^{1/2} (2q_t + q_v)$
Σ_0^+	0	$-\frac{8}{3} q_t - \frac{10}{3} q_v$
${}^p A_-^0$	$-\frac{2}{3^{1/2}} (q_t + q_v)$	$-\frac{2}{3^{1/2}} (2q_t + q_v)$
${}^p \Sigma_0^+$	$-2(2)^{1/2} (q_t + q_v)$	$-\frac{2(2)^{1/2}}{3} (q_t + 2q_v)$

In a similar fashion we can demonstrate equivalence between a quark model evaluation of Fig. 1b and the $SU(6)_w$ predictions for c_v contributions to the various decays, and we have already observed the $SU(6)_w$ -quark model equivalence in the case of the factorization diagrams—Fig. 1c. In one sense then it is really a matter of taste whether one uses the $SU(6)_w$ route, as advocated by McKellar and Pick, or the quark model picture described above—they are completely equivalent. On the other hand we prefer the quark model approach since it offers some hope of actual numerical predictions for the various expansion coefficients a_v , a_t , b_v , b_t , c_v . In the case of a_t , a_v this can be done at the present time; in the case of b_v , b_t , c_v this may be possible in the future.

There is another point concerning the quark model calculation which should be noted. There is an additional class of quark diagrams—cf. Fig. 3—which we are neglecting at present. These vanish in the $SU(6)_w$ approach because of the symmetry of the baryon wavefunctions. It is possible to check that this vanishing also occurs in the simplest quark models. However if one were to describe the intermediate state, between the action of $\mathcal{H}_w$ and the emission of the meson, as an odd parity baryon resonance the diagrams need not vanish. This will enter our discussion again in Section V.

One of the most common techniques for handling hyperon decay is current algebra and PCAC. One does not deal directly with the pion field, but removes it from the problem by use of the PCAC assumption. This appears at first sight to be unrelated

FIG. 3. A diagram absent from both the $SU(6)_w$ and quark model approaches.

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to our methods, since we have explicitly used the pion field, and in fact assume it to be built up out of quarks, an assumption that is not obviously compatible with PCAC. The $SU(6)_w$ symmetry also appears unusual from the viewpoint of PCAC. The PCAC pion is almost a Goldstone boson with a very small mass and is not assumed to have a connection with the vector mesons. However there is a relation between our approach and the PCAC calculations that makes them to a large extent equivalent. We will devote much of the remainder of this section to this problem.

Using standard techniques [13] and assuming PCAC one can reduce the amplitude $B \rightarrow B' \pi$ to an effective two-body transition $B \rightarrow B'$ in the limit $q_\pi^\mu \rightarrow 0$

$$\langle \pi^a(q) B' | \mathcal{H}_w^{(-)} | B \rangle \approx -\frac{i}{F_\pi} \langle B' | [F_5^a, \mathcal{H}_w^{(-)}] | B \rangle, \quad (45)$$

where $F_\pi = 94$ MeV is the pion decay constant, and

$$F_5^a = \int d^3x A_0^a(x, t) \quad (46)$$

is the axial charge. For the standard Cabibbo Hamiltonian we have been using the $V - A$ properties of the weak hadronic current give the relation

$$[F_5^a, {}^c\mathcal{H}_w^{(-)}] = [F_5^a, {}^c\mathcal{H}_w^{(+)}], \quad (47)$$

where $H_w^{(+)}$ is the parity conserving weak Hamiltonian and

$$F^a = \int d^3x V_0^a(x, t) = I^a \quad (48)$$

is the isotopic spin operator. Thus, e.g.,

$$\langle B' \pi^0 | {}^c\mathcal{H}_w^{(-)}(\Delta S = 1) | B \rangle \approx \frac{i}{2F_\pi} \langle B' | {}^c\mathcal{H}_w^{(+)}(\Delta S = 1) | B \rangle \quad (49)$$

for the standard $\Delta S = 1$ decays. One can now evaluate the

$$\langle B' | {}^c\mathcal{H}_w^{(+)}(\Delta S = 1) | B \rangle \quad (50)$$

matrix element using various quark model wavefunctions such as the *MIT* bag model. In any such approach one can characterize the results in terms of $SU(3)$ octet couplings f, d

$$\begin{aligned} -\frac{1}{2^{1/2}} \Lambda_-^0 &= \Lambda_0^0 = \frac{1}{6^{1/2}} (d + 3f), \\ \frac{1}{2^{1/2}} \Xi_-^- &= -\Xi_0^0 = \frac{1}{6^{1/2}} (d - 3f), \\ \frac{1}{2^{1/2}} \Sigma_-^- &= \Sigma_0^+ = -d + f, \\ \Sigma_+^+ &= 0 \end{aligned} \quad (51)$$

with $f = -d$. As usual we have assumed the baryons to be at rest—a static limit as used above. The fact that in this limit only the octet components of $\mathcal{H}_w^{(-)}$ contribute is a result of a Theorem [14] most often attributed to Pati and Woo, but obtained earlier by K. Miura and T. Minamakawa.

The specific magnitude of the amplitude depends upon assumptions about the radial dependence of the wavefunctions. We do not at this stage wish to commit ourselves to a specific model, so we do not attempt a numerical prediction.

Experimentally the situation is not extremely clear since there exist additional contributions $\propto q_\mu$, the pion four-momentum, to these amplitudes, but it seems that a reasonable fit is provided to the experimental numbers with

$$f = -2.2d \quad (52)$$

which is somewhat removed from the simple result $f = -d$ given by the naive quark model. However if one allows for quark sea effects, then in addition to the usual valence quark contribution to these matrix elements, Donoghue and Golowich [15] have shown that this additional sea contribution is purely f type coupling, and a prediction much closer to the “experimental” result—Eq. (52)—is found.

It is not surprising that one can achieve similar results via use of the symmetry $SU(6)_w$. The parity violating Hamiltonian is as before (Eq. (7)) while the parity conserving one can be written as [10]

$${}^c\mathcal{H}_w^{(+)}(\Delta S = 1) = \frac{G}{2^{1/2}} \cos \theta_c \sin \theta_c {}^{(+)}\mathcal{O}_{43}^{56} \quad (53)$$

with

$$\begin{aligned} {}^{(+)}\mathcal{O}_{CD}^{AB} = & U_{\{D,2\}}^{\{1,B\}} + U_{\{C,1\}}^{\{2,A\}} + U_{[D,2]}^{\{1,B\}} + U_{[C,1]}^{\{2,A\}} + W_{\{D,2\}}^{\{2,A\}} + W_{\{C,1\}}^{\{1,B\}} \\ & + W_{[D,2]}^{\{2,A\}} + W_{[C,1]}^{\{1,B\}} + W_D^A + W_C^B, \end{aligned} \quad (54)$$

$$U_{CD}^{AB} = S_C^A S_D^B - P_C^A P_D^B, \quad (55)$$

$$W_{CD}^{AB} = -R_C^A R_D^B - Q_C^A Q_D^B.$$

Thus while ${}^c\mathcal{H}_w^{(+)}$ transforms as $280 \oplus \overline{280} \oplus 35$, ${}^c\mathcal{H}_w^{(-)}$ transforms like $405 \oplus 189 \oplus 35$. Calculating

$$\langle B' | \mathcal{H}_w^{(+)} | B \rangle$$

only $405 \oplus 35$ contribute since $\overline{56} \otimes 56$ does not contain 189. We find then

$$\begin{aligned} \frac{1}{2^{1/2}} \mathcal{A}_-^0 &= -\mathcal{A}_0^0 = \frac{1}{6^{1/2}} \left(\frac{1}{6} b_u - \frac{1}{3} b_w - c_w \right), \\ -\frac{1}{2^{1/2}} \mathcal{E}_-^- &= \mathcal{E}_0^0 = \frac{1}{6^{1/2}} \left(-\frac{1}{2} b_u + \frac{1}{2} b_w + c_w \right), \end{aligned}$$

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$$\begin{aligned}
 \frac{1}{2^{1/2}} \Sigma_-^- &= -\frac{5}{18} b_u + \frac{2}{9} b_w + c_w, \\
 \Sigma_0^+ &= -\frac{2}{9} b_u + \frac{5}{18} b_w + c_w, \\
 \frac{1}{2^{1/2}} \Sigma_+^+ &= -\frac{1}{18} (b_u + b_w),
 \end{aligned} \tag{56}$$

where b_u , b_w , c_w are the couplings

$$\begin{aligned}
 b_u, b_w &: (\bar{B}B)_{405}/2F_\pi, \\
 c_w &: (\bar{B}B)_{35}/2F_\pi.
 \end{aligned} \tag{57}$$

Now in the static limit, which is appropriate for the soft pion limit and for our previous considerations the quark model, the contributions to b_u and b_w ,

$$\begin{aligned}
 b_u &\sim \langle B' | V_0 V_0 - A_3 A_3 | B \rangle, \\
 b_w &\sim \frac{1}{2} \langle B' | A_+ A_- + V_+ V_- + A_- A_+ + V_- V_+ | B \rangle,
 \end{aligned} \tag{58}$$

are equal and opposite in sign, $b_u = -b_w$. Thus the $SU(6)_w$ predictions become

$$\begin{aligned}
 \frac{1}{2^{1/2}} \Lambda_-^0 &= -\Lambda_0^0 = \frac{1}{6^{1/2}} \left(\frac{1}{2} b_u - c_w \right), \\
 \frac{1}{2^{1/2}} \Sigma_-^- &= \Sigma_0^+ = -\frac{1}{2} b_u + \frac{1}{3} c_w, \\
 \frac{1}{2^{1/2}} \Xi_-^- &= -\Xi_0^0 = \frac{1}{6^{1/2}} (b_u - c_w), \\
 \Sigma_+^+ &= 0,
 \end{aligned} \tag{59}$$

wherein the contribution from b_u corresponds to the valence quark contribution with $f = -d$, while c_w represents the quark sea result with $d = 0$.

The present technique can also be applied to the $\Delta S = 0$ component of the weak Hamiltonian. As before we include at present only the Cabibbo (charged current) interaction. We find, using notation as before,

$$\begin{aligned}
 {}^c \mathcal{H}_w^{(-)}(\Delta S = 0) &= \frac{G}{2^{1/2}} \{ \cos^2 \theta_c [{}^c \mathcal{O}_{43}^{34} + V_2^2 - V_1^1] + \sin^2 \theta_c [{}^c \mathcal{O}_{65}^{56} + V_2^2 - V_1^1] \}, \\
 {}^c \mathcal{H}_w^{(+)}(\Delta S = 0) &= \frac{G}{2^{1/2}} \{ \cos^2 \theta_c [{}^c \mathcal{O}_{43}^{34} + W_2^2 - W_1^1] + \sin^2 \theta_c [{}^c \mathcal{O}_{65}^{56} + W_2^2 - W_1^1] \}.
 \end{aligned} \tag{60}$$

In this case since $\mathcal{H}_w$ carries $I_z = 0$ we have

$$\langle \pi^0 p | \mathcal{H}_w^{(-)} | p \rangle = \frac{-i}{F_\pi} \langle p | [F_5^3, \mathcal{H}_w^{(-)}] | p \rangle = \frac{-i}{F_\pi} \langle p | [I_3, \mathcal{H}_w^{(+)}] | p \rangle = 0, \quad (61)$$

$$\begin{aligned} \langle \pi^- p | \mathcal{H}_w^{(-)} | n \rangle &= \frac{-i}{2^{1/2} F_\pi} \langle p | [F_5^{1+i2}, \mathcal{H}_w^{(w)}] | n \rangle = \frac{-i}{2^{1/2} F_\pi} \langle p | [I_+, \mathcal{H}_w^{(+)}] | n \rangle \\ &= \frac{i 2^{1/2}}{F_\pi} (\langle p | \mathcal{H}_w^+ | p \rangle - \langle n | \mathcal{H}_w^+ | n \rangle). \end{aligned} \quad (62)$$

Using the static quark model calculation we find that this is related to the $\Delta S = 1$ results via

$$n_-^0 = 2^{1/2} \tan \theta_c (f + d). \quad (63)$$

Via the $SU(6)_w$ approach we find

$$\begin{aligned} p_0^+ &= 0, \\ n_-^0 &= \frac{2^{1/2}}{3} \tan \theta_c c_w \end{aligned} \quad (64)$$

which is again completely consistent with the quark model approach.

If we compare these PCAC predictions to our previous spin-sum approach which includes the pion explicitly we see (cf. Eqs. (11) and (56)) that there is complete equivalence provided we identify

$$\begin{aligned} b_u &= -\frac{1}{2} b_t, \\ b_w &= -\frac{1}{2} b_v, \\ c_w &= \frac{1}{2} c_v. \end{aligned} \quad (65)$$

That the PCAC-motivated quark model approach should give equivalent results to those generated via direct treatment of the pion can be seen from an elementary argument. In a simple quark model, such as the MIT bag picture [16–18], we can write the s -state wavefunctions

$$\psi(x) = \begin{pmatrix} iu(r) \chi \\ l(r) \hat{\sigma} \cdot \hat{r} \chi \end{pmatrix} \quad (66)$$

with $u(r)$ and $l(r)$ the upper and lower radial wavefunctions and χ the two component Pauli spinor. As a measure of the upper and lower components let us define

$$\begin{aligned} U &\equiv \int d^3x u^2(r), \\ L &\equiv \int d^3x l^2(r) \end{aligned} \quad (67)$$

These are normalized via the condition

$$U + L = 1. \quad (68)$$

We note that the s -wave quark and antiquark wavefunctions are related by

$$\gamma_5 \psi_{\text{quark}}(x) = \psi_{\text{antiquark}} + \text{terms in } l(r). \quad (69)$$

Now in the *nonrelativistic* limit we can neglect the terms in $l(r)$. Then at the $\bar{u}dW^-$ vertex in Fig. 2 we have

$$\begin{aligned} \langle \phi_{\text{quark}}^d \phi_{\text{antiquark}}^{\bar{u}} | A_\mu | 0 \rangle &\sim \bar{\psi}_{\text{quark}}^d \gamma_\mu \gamma_5 \psi_{\text{antiquark}}^{\bar{u}} \\ &= \bar{\psi}_{\text{quark}}^d \gamma_\mu \psi_{\text{quark}}^u = \langle \phi_{\text{quark}}^d | V_\mu | \phi_{\text{quark}}^u \rangle, \\ \langle \phi_{\text{quark}}^d \phi_{\text{antiquark}}^{\bar{u}} | V_\mu | 0 \rangle &\sim \bar{\psi}_{\text{quark}}^d \gamma_\mu \psi_{\text{antiquark}}^{\bar{u}} \\ &= \bar{\psi}_{\text{quark}}^d \gamma_\mu \gamma_5 \psi_{\text{quark}}^u = \langle \phi_{\text{quark}}^d | A_\mu | \phi_{\text{quark}}^u \rangle. \end{aligned} \quad (70)$$

Since the pionic wavefunction is a spin and color singlet then except for an overall normalization difference ($-i/2F_\pi$) the structure of

$$\begin{aligned} \langle B' \pi^0 | \mathcal{H}_w^{(-)} | B \rangle &\sim \langle \phi_{\text{quark}}^d \phi_{\text{antiquark}}^{\bar{u}} | V_\mu | 0 \rangle \langle \phi_{\text{quark}}^u | A^\mu | \phi_{\text{quark}}^s \rangle \\ &+ \langle \phi_{\text{quark}}^d \phi_{\text{antiquark}}^{\bar{u}} | A_\mu | 0 \rangle \langle \phi_{\text{quark}}^u | V^\mu | \phi_{\text{quark}}^s \rangle \end{aligned} \quad (71)$$

is identical to

$$\begin{aligned} \langle B' | \mathcal{H}_w^{(+)} | B \rangle &\sim \langle \phi_{\text{quark}}^d | A_\mu | \phi_{\text{quark}}^u \rangle \langle \phi_{\text{quark}}^u | A^\mu | \phi_{\text{quark}}^s \rangle \\ &+ \langle \phi_{\text{quark}}^d | V_\mu | \phi_{\text{quark}}^u \rangle \langle \phi_{\text{quark}}^u | V^\mu | \phi_{\text{quark}}^s \rangle \end{aligned} \quad (72)$$

which is the equivalence between the PCAC-derivation and direct treatment of pion emission discussed previously.

We have thus demonstrated how previous approaches to the ρ -emission and π -emission problems—PCAC, $SU(6)_w$, and the nonrelativistic quark model—are interrelated in the case of the Cabibbo nonleptonic Hamiltonian, both $\Delta S = 0$ and $\Delta S = 1$. We have given a procedure— a_t , a_v evaluated from factorization diagrams, c_v , b_v , b_t determined from hyperon decay—whereby ρ , ω , ϕ weak emission amplitudes, which are necessary components of any sensible discussion of nuclear parity violation, can be predicted. However, it is necessary to go further. The effects of the strong interactions modify the simple Hamiltonian forms we have been using. This can be estimated using renormalization group techniques and is described in detail in Section IV. In addition we must take cognizance of and correct our predictions for relativistic effects which treat ρ -emission and π -emission asymmetrically, thus breaking the approximate $SU(6)_w$ symmetry we have been using. However, before proceeding to these difficult theoretical points, it is useful to expand our considerations to include possible weak neutral current interactions since they can produce a significant modification of the simple Cabibbo model predictions we have used up until this time.

Most modern gauge theories, in particular the Weinberg–Salam model [19], have neutral currents which violate parity and hence are important here. In order to deal effectively with such models let us define the following operators

$$\begin{aligned}
 \mathcal{O}_0 &= \frac{G}{2^{1/2}} \bar{u} \gamma_\mu (1 + \gamma_5) d \bar{d} \gamma^\mu (1 + \gamma_5) u, \\
 \mathcal{O}_1 &= \frac{G}{2^{1/2}} \bar{u} \gamma_\mu (1 + \gamma_5) s \bar{s} \gamma^\mu (1 + \gamma_5) u, \\
 \mathcal{O}_2 &= \frac{G}{2^{1/2}} \bar{u} \gamma_\mu u \bar{u} \gamma^\mu \gamma_5 u, \\
 \mathcal{O}_3 &= \frac{G}{2^{1/2}} \bar{d} \gamma_\mu d \bar{d} \gamma^\mu \gamma_5 d, \\
 \mathcal{O}_4 &= \frac{G}{2^{1/2}} \bar{u} \gamma_\mu u \bar{d} \gamma^\mu \gamma_5 d, \\
 \mathcal{O}_5 &= \frac{G}{2^{1/2}} \bar{d} \gamma_\mu d \bar{u} \gamma^\mu \gamma_5 u.
 \end{aligned} \tag{73}$$

Also we use

$$\begin{aligned}
 \mathcal{O}_6 &= \frac{G}{2^{1/2}} \bar{u} \gamma_\mu u \bar{s} \gamma^\mu \gamma_5 s, \\
 \mathcal{O}_7 &= \frac{G}{2^{1/2}} \bar{s} \gamma_\mu s \bar{u} \gamma^\mu \gamma_5 u, \\
 \mathcal{O}_8 &= \frac{G}{2^{1/2}} \bar{d} \gamma_\mu d \bar{s} \gamma^\mu \gamma_5 s, \\
 \mathcal{O}_9 &= \frac{G}{2^{1/2}} \bar{s} \gamma_\mu s \bar{d} \gamma^\mu \gamma_5 d, \\
 \mathcal{O}_{10} &= \frac{G}{2^{1/2}} \bar{s} \gamma_\mu s \bar{s} \gamma^\mu \gamma_5 s.
 \end{aligned} \tag{74}$$

These are the only four-quark operators of relevance since it should be a reasonably good approximation to neglect the contributions of the heavy c and possible b , t , etc., quarks in hadronic wavefunctions.

In terms of this notation then the Cabibbo $\Delta S = 0$ Hamiltonian reads

$${}^c \mathcal{H}_w(\Delta S = 0) = \cos^2 \theta_c \mathcal{O}_0 + \sin^2 \theta_c \mathcal{O}_1 \tag{75}$$

while the Weinberg–Salam form is

$$\begin{aligned}
 {}^{ws} \mathcal{H}_w(\Delta S = 0) &= {}^c \mathcal{H}_w(\Delta S = 0) + \left\{ \frac{1}{2} (1 - \frac{8}{3} \sin^2 \theta_w) (\mathcal{O}_2 - \mathcal{O}_4 - \mathcal{O}_6) \right. \\
 &\quad \left. + \frac{1}{2} (1 - \frac{4}{3} \sin^2 \theta_w) (\mathcal{O}_3 - \mathcal{O}_5 - \mathcal{O}_7 + \mathcal{O}_8 + \mathcal{O}_9 + \mathcal{O}_{10}) \right\} \tag{76}
 \end{aligned}$$

Our quark model calculation can now proceed as before. However, an $SU(6)_w$

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approach runs into difficulty. There are two separate problems. First, we have decomposed the product

$$\mathcal{H}_w^{(-)} \propto (V_\mu + A_\mu) \times (V^\mu + A^\mu) = {}^T\mathcal{H}_w^{(+)} + {}^V\mathcal{H}_w^{(-)}$$

into a sum of $SU(6)_w$ currents of the form $(SP - PS)$ and $(RQ - QR)$. However, a Hamiltonian of the form

$$\mathcal{H}_w^{(-)} \propto V_\mu \times (V^\mu + A^\mu) \quad \text{or} \quad \mathcal{H}_w^{(-)} \propto A_\mu \times (V^\mu + A^\mu)$$

cannot be so analyzed. New $SU(6)_w$ structures and thus new reduced matrix elements come into play for $\mathcal{O}_4$ and $\mathcal{O}_5$. The second problem is that a diagonal bilinear form such as $\bar{u}u$ or $\bar{d}d$ cannot be written as a simple $SU(6)$ representation 35 but rather as a sum of singlet and 35 representation. This feature brings in new $SU(6)_w$ reduced matrix elements for $\mathcal{O}_2$, $\mathcal{O}_3$, $\mathcal{O}_4$, and $\mathcal{O}_5$. Thus for the $\Delta S = 0$ weak Hamiltonian resulting from a product of weak neutral currents it does not seem to make sense to pursue further the $SU(6)_w$ approach. Although interesting in a historical perspective and useful in understanding our charged current work, our discussion hereafter will be strictly based upon quark model calculations.

There is one new parameter which should be included. It arises from the asymmetric product of currents

$$\mathcal{H} = -\frac{1}{3} \sin^2 \theta_w (\mathcal{O}_5 - \mathcal{O}_4)$$

which can be Fierz rearranged and evaluated as

$$\begin{aligned} \langle p\pi^- | \mathcal{H} | n \rangle &= \frac{2}{3} \sin^2 \theta_w \langle p\pi^- | \bar{u}_j d_i \bar{d}_i \gamma_5 u_j - \bar{u}_j \gamma_5 d_i \bar{d}_i u_j | n \rangle \frac{G}{2^{1/2}} \\ &= \frac{2}{3} \sin^2 \theta_w \langle \pi^- | \bar{d}_i \gamma_5 u_j | 0 \rangle \langle p | \bar{u}_j d_i | n \rangle \frac{G}{2^{1/2}} \\ &= \frac{1}{3} \sin^2 \theta_w y, \end{aligned} \tag{77}$$

where y is the new parameter defined by this evaluation. In principle, y can be related to the parameter a_t with the result

$$y = -\frac{4}{3(2)^{1/2}} \frac{a_t}{\sin \theta_c \cos \theta_c}$$

but the fact that this relation may not be satisfied makes us consider y as a new parameter.

The quark equations of motion can be used in the determination of y . In this way $\bar{d}\gamma_5 u = -i(m_u + m_d)^{-1} \partial_\mu \bar{d} \gamma^\mu \gamma_5 u$, and by using PCAC and defining $S = \langle P | \bar{u}d | N \rangle$ we find

$$y = -i \frac{2}{3} \frac{GF_\pi m_\pi^2 S}{(m_u + m_d)}. \tag{78}$$

The reason we keep this term is that the denominator has small quark masses in it, and this may enhance y . Note that it is important only for pions. We shall return to its evaluation in Section IV. The presence of this term has recently been emphasized by several groups [20].

We can, for example, calculate vector meson or pion emission via any of these operator forms—Eq. (73). In Table III we tabulate the results of such a calculation. For convenience we have expressed them in terms of the $SU(6)_w$ parameters b_v, b_t, c_v, a_v, a_t using the quark model relations derived earlier.

In principle at this stage we could collect our results to make a prediction for the $\pi^-, \rho^-, \rho^0, \omega^0, \phi^0$ exchange amplitudes in any given model. Thus the Cabibbo (charged current only) Hamiltonian yields

$$\begin{aligned}\langle p\pi^- | {}^c\mathcal{H}_w^{(-)} | n \rangle &= \frac{1}{3(2)^{1/2}} \tan \theta_c c_v, \\ \langle p\rho^- | {}^c\mathcal{H}_w^{(-)} | n \rangle &= \frac{2}{9} \cot \theta_c [5a_v - 5c_v + 2b_v - b_t] - \frac{5}{9} \tan \theta_c c_v, \\ \langle p\rho^0 | {}^c\mathcal{H}_w^{(-)} | p \rangle &= \frac{1}{9(2)^{1/2}} \cot \theta_c [-10a_t - 10c_v + 3b_v - 3b_t] - \frac{8}{9(2)^{1/2}} \tan \theta_c c_v, \\ \langle p\omega^0 | {}^c\mathcal{H}_w^{(-)} | p \rangle &= \frac{1}{9(2)^{1/2}} \cot \theta_c [6a_t - 6c_v + b_v + b_t] - \frac{8}{9(2)^{1/2}} \tan \theta_c c_v, \quad (79)\end{aligned}$$

while the Weinberg–Salam model predicts

$$\begin{aligned}\langle p\pi^- | {}^{ws}\mathcal{H}_w^{(-)} | n \rangle &= \text{Cabibbo} - \frac{2}{9(2)^{1/2}} \csc 2\theta_c \sin^2 \theta_w (2c_v - b_t) + \frac{1}{3} \sin^2 \theta_w y, \\ \langle p\rho^- | {}^{ws}\mathcal{H}_w^{(-)} | n \rangle &= \text{Cabibbo} + \frac{1}{9} \csc 2\theta_c (1 - 2 \sin^2 \theta_w) [-10c_v - 10a_t + 3b_v - 3b_t], \\ \langle p\rho^0 | {}^{ws}\mathcal{H}_w^{(-)} | p \rangle &= \text{Cabibbo} + \frac{1}{9(2)^{1/2}} \csc 2\theta_c \left[20a_v (1 - 2 \sin^2 \theta_w) + 10a_t \left(1 - \frac{12}{5} \sin^2 \theta_w \right) \right. \\ &\quad \left. - 10c_v \left(1 - \frac{12}{5} \sin^2 \theta_w \right) + 3(b_v - b_t) \left(1 - \frac{16}{9} \sin^2 \theta_w \right) \right], \\ \langle \omega^0 p | {}^{ws}\mathcal{H}_w^{(-)} | p \rangle &= \text{Cabibbo} + \frac{1}{9(2)^{1/2}} \csc 2\theta_c \left[2a_t \left(3 - \frac{28}{3} \sin^2 \theta_w \right) - \frac{40}{3} \sin^2 \theta_w a_v \right. \\ &\quad \left. - 2c_v \left(3 - \frac{28}{3} \sin^2 \theta_w \right) - b_v - b_t \right]. \quad (80)\end{aligned}$$

TABLE III
 Amplitudes for the Complete set of Operators^a

i	$\langle \pi^- p \mathcal{O}_i n \rangle$	$\langle \rho^- p \mathcal{O}_i n \rangle$	$\langle \rho^0 p \mathcal{O}_i p \rangle$	$\langle \omega^0 p \mathcal{O}_i p \rangle$	$\langle p \mathcal{O}_i p \rangle$
0	0	$-\frac{2}{q}(\tilde{b}_i - 2\tilde{b}_v)$ $-\frac{10}{9}\tilde{c}_v + \frac{10}{9}\tilde{a}_v$	$-\frac{1}{3(2)^{1/2}}(\tilde{b}_i - \tilde{b}_v)$ $-\frac{10}{9(2)^{1/2}}\tilde{c}_v - \frac{10}{9(2)^{1/2}}\tilde{a}_i$	$\frac{1}{9(2)^{1/2}}(\tilde{b}_v + \tilde{b}_i)$ $-\frac{2}{3(2)^{1/2}}\tilde{c}_v + \frac{2}{3(2)^{1/2}}\tilde{a}_i$	0
1	$\frac{\tilde{c}_v}{3(2)^{1/2}}$	$-\frac{5}{9}\tilde{c}_v$	$-\frac{8}{9(2)^{1/2}}\tilde{c}_v$	$-\frac{8}{9(2)^{1/2}}\tilde{c}_v$	$\frac{8}{9}\tilde{a}_i$
2	$\frac{\tilde{c}_v}{3(2)^{1/2}} - \frac{1}{18(2)^{1/2}}(\tilde{b}_i + \tilde{b}_v)$	$\frac{1}{18}(\tilde{b}_v + \tilde{b}_i) - \frac{5}{9}\tilde{c}_v$	$\frac{2}{9(2)^{1/2}}(\tilde{b}_v + \tilde{b}_i) - \frac{8}{9(2)^{1/2}}\tilde{c}_v$ $+ \frac{8}{9(2)^{1/2}}(\tilde{a}_v + \tilde{a}_i)$	$\frac{2}{9(2)^{1/2}}(\tilde{b}_v + \tilde{b}_i) - \frac{8}{9(2)^{1/2}}\tilde{c}_v$ $+ \frac{8}{9(2)^{1/2}}(\tilde{a}_v + \tilde{a}_i)$	0
3	$-\frac{\tilde{c}_v}{3(2)^{1/2}} + \frac{1}{18(2)^{1/2}}(\tilde{b}_v + \tilde{b}_i)$	$\frac{1}{18}(\tilde{b}_v + \tilde{b}_i) - \frac{5}{9}\tilde{c}_v$	$-\frac{2}{9(2)^{1/2}}\tilde{c}_v + \frac{2}{9(2)^{1/2}}(\tilde{a}_v + \tilde{a}_i)$	$\frac{2}{9(2)^{1/2}}\tilde{c}_v - \frac{2}{9(2)^{1/2}}(\tilde{a}_v + \tilde{a}_i)$	0
4	$\frac{\gamma}{2} - \frac{1}{18(2)^{1/2}}(\tilde{b}_v - 2\tilde{b}_i)$	$\frac{1}{9}(2\tilde{b}_i - \tilde{b}_v) + \frac{5}{9}\tilde{a}_i$	$-\frac{1}{9(2)^{1/2}}(\tilde{b}_v - \tilde{b}_i) - \frac{2}{9(2)^{1/2}}\tilde{a}_v$	$-\frac{1}{9(2)^{1/2}}(\tilde{b}_v + \tilde{b}_i) - \frac{2}{9(2)^{1/2}}\tilde{a}_v$	0
5	$-\frac{\gamma}{2} + \frac{1}{18(2)^{1/2}}(2\tilde{b}_i - \tilde{b}_v)$	$\frac{1}{9}(2\tilde{b}_i - \tilde{b}_v) + \frac{5}{9}\tilde{a}_i$	$-\frac{2}{9(2)^{1/2}}(\tilde{b}_v - \tilde{b}_i) - \frac{8}{9(2)^{1/2}}\tilde{a}_v$	$\frac{2}{9(2)^{1/2}}(\tilde{b}_v + \tilde{b}_i) + \frac{8}{9(2)^{1/2}}\tilde{a}_v$	0
6	0	0	0	0	0
7	0	0	0	0	$\frac{8}{9}\tilde{a}_v$
8	0	0	0	0	0
9	0	0	0	0	$-\frac{2}{9}\tilde{a}_v$
10	0	0	0	0	0

^a Parameters with a tilde are defined by $\tilde{\alpha}_i = \alpha_i / \cos \theta_e \sin \theta_e$.

However, even if the weak charged, neutral currents are given accurately by the Cabibbo, Weinberg-Salam versions, there are good reasons to suspect that our predictions will require substantial modification due to the effects of the strong interactions. These effects will be examined in the succeeding section.

III. STRONG INTERACTION ENHANCEMENT EFFECTS

In this section we shall deal with some effects of the strong interactions on the weak nonleptonic transitions. The basic idea was suggested by Wilson [21] nearly a decade ago. The nonleptonic Hamiltonian due to the exchange of a W meson can be written

$$\begin{aligned} \mathcal{H}_w(0) = & \frac{G}{(2)^{1/2}} M_w^{-2} \int d^4x D_F(x, M_w) T(J_\mu^+(x) J^{-\mu}(0)) \\ & + \frac{G}{(2)^{1/2}} M_z^{-2} \int d^4x D_F(x, M_z) T(J_\mu^0(x) J^{0\mu}(0)) \end{aligned} \quad (81)$$

where M_w, M_z are the charged, neutral weak bosons while J_μ^+, J_μ^0 are the corresponding charged, neutral weak hadronic currents. In a free field theory the integration over the propagator may be easily done to yield the standard results quoted in the previous section. However, then the strong interactions are present, the product of currents deviates from its free field value in a manner that depends on the coordinate x , making the integration difficult. Wilson's suggestion was that renormalization effects may make certain portions (hopefully $\Delta I = \frac{1}{2}$) more singular at small x , than in the free field theory, therefore leading to an enhancement when integrated over the propagator. Such an effect is observed within QCD .

The main tool is Wilson's operator product expansion [21]. Since we expect $M_w, M_z \gtrsim 50$ GeV, the propagator is sensitive primarily to the short-distance behavior of the currents, so that the time-ordered product can be written via a Wilson expansion

$$T(J_\mu(x) J^\mu(0)) = \sum_i C_i(x) \mathcal{O}_i(0). \quad (82)$$

Here $\{\mathcal{O}_i\}$ is a complete set of operators carrying the appropriate quantum numbers, while $C_i(x)$ are c -number coefficients which contain the space-time dependence.

In asymptotically free theories [22] it is possible to calculate the coefficients $C_i(x)$ using renormalization group techniques [24]. The operators $\{\mathcal{O}_i\}$ are decomposed in terms of multiplicatively renormalizable operators, with associated dimension d_i . Then after integration over x the weak Hamiltonian has the form.

$$\mathcal{H}_w = \frac{G}{2^{1/2}} \sum_i K^{d_i} \mathcal{O}_i, \quad (83)$$

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where

$$K \equiv \left[1 + \frac{g^2(\mu^2)}{16\pi^2} b \ln \frac{M_w^2}{\mu^2} \right]. \quad (84)$$

Here g is the color gluon coupling constant, μ is the subtraction point and specifies the mass scale, $b = 11 - \frac{2}{3}N$, where N is the number of quarks, and $-bg^3/16\pi^2$ is the lowest-order approximation of the $\beta(g)$ function in the Callan–Symanzik equation.² Estimates of “reasonable” values for K vary, but if we take

$$\mu = 1 \text{ GeV}, \quad M_w = 75 \text{ GeV}, \quad \frac{g^2(\mu^2)}{4\pi} = 1 \quad (85)$$

we find $K \approx 6$ for the case $N = 4$. Other authors have used values $3 \lesssim K \lesssim 10$. In the case of $\Delta S = 1$ decays we have

$$\begin{aligned} d_- &= 0.48; \\ \mathcal{O}_- &= \frac{1}{4} \{ \bar{d}\gamma_\mu(1 + \gamma_5)u \bar{u}\gamma^\mu(1 + \gamma_5)s - \bar{d}\gamma_\mu(1 + \gamma_5)s \bar{u}\gamma^\mu(1 + \gamma_5)u \}, \\ d_+ &= -0.24; \\ \mathcal{O}_+ &= \frac{1}{4} \{ \bar{d}\gamma_\mu(1 + \gamma_5)u \bar{u}\gamma^\mu(1 + \gamma_5)s + \bar{d}\gamma_\mu(1 + \gamma_5)s \bar{u}\gamma^\mu(1 + \gamma_5)u \}, \end{aligned} \quad (86)$$

and we have for the $\Delta S = 1$ weak Hamiltonian as modified by short-distance gluon exchange effects

$$\mathcal{H}_w(\Delta S = 1) = \frac{2G}{2^{1/2}} \cos \theta_c \sin \theta_c [K^{0.48} \mathcal{O}_- + K^{-0.24} \mathcal{O}_+] + \text{h.c.} \quad (87)$$

Now $\mathcal{O}_-$ is purely $\Delta I = \frac{1}{2}$ while $\mathcal{O}_+$ is a mixture of $\Delta I = \frac{1}{2}, \frac{3}{2}$ pieces. Thus from the renormalization group enhancement, suppression factors alone we might suspect a predominance of $\Delta I = \frac{1}{2}$ over $\Delta I = \frac{3}{2}$ amplitudes of

$$\frac{A_{1/2}}{A_{3/2}} \sim \frac{K^{-0.48}}{K^{-0.24}} \sim 4. \quad (88)$$

While this is too small to explain the experimental enhancement factor of 20 or so, there exist additional $\Delta I = \frac{3}{2}$ suppressions associated with the operator matrix elements which could well provide the additional factor needed.

In the case of $\Delta S = 0$ weak transitions we can perform similar strong interaction enhancement calculations, but the situation is more complex [24]. Although many models for the weak neutral current have been proposed, we shall present our discussion from the point of view of the Weinberg–Salam model only. Indeed this model is consistent with essentially all present experiments which probe the structure of the

² The $\Delta S = 0$ calculations were done for four quarks, while it now appears that there are five or six quarks. Redoing the calculation is beyond the scope of this paper, and we will use the four-quark results. This is a small uncertainty compared to the others that we discuss.

weak neutral current. Enhancement effects for a more general class of models have been discussed by Galić *et al.* [25] and our results can straightforwardly be adapted if necessary.

It is convenient to adopt a slightly different notation in pursuing this discussion. We define a general four-quark parity violating operator as

$$\mathcal{O}(M, N) = \bar{q}\gamma_\mu\gamma_5 M q \bar{q}\gamma^\mu N q, \quad (89)$$

where M, N are arbitrary matrices in flavor and color spaces. For our purposes it is sufficient to define four matrices in flavor space. If we take as our basis, neglecting quarks heavier than the strange quark,

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \quad (90)$$

we define

$$\begin{aligned} A_1 &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & A_2 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ B_1 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, & B_2 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \end{aligned} \quad (91)$$

Then the effective Weinberg–Salam $\Delta S = 0$ weak Hamiltonian can be written as

$$\begin{aligned} {}^{ws}\mathcal{H}_w^{\text{eff}}(\Delta S = 0) &= \frac{G}{2^{1/2}} \cos \theta_c \sin \theta_c \left[\sum_{i=1}^2 (\alpha_{ii} \mathcal{O}(A_i^+, A_i) + \beta_{ii} \mathcal{O}(A_i^+ t_A, A_i t_A) + \text{h.c.}) \right. \\ &\quad \left. + \sum_{i,j=1}^2 (\gamma_{ij} \mathcal{O}(B_i, B_j) + \delta_{ij} \mathcal{O}(B_i t^A, B_j t^A)) \right], \end{aligned} \quad (92)$$

where t^A on the $SU(3)$ color matrices normalized via the condition

$$\text{Tr } t^A t^B = 2\delta^{AB} \quad (93)$$

and the parameters $\alpha, \beta, \gamma, \delta$ have the values

$$\begin{aligned} \alpha_{11} &= \cot \theta_c (K^{0.48} + 2K^{-0.24})/3, \\ \alpha_{22} &= \tan^2 \theta_c \alpha_{11}, \\ \beta_{11} &= \cot \theta_c (-K^{0.48} + K^{-0.24})/4, \\ \beta_{22} &= \tan^2 \theta_c \beta_{11}, \\ \frac{1}{2} \sin 2\theta_c \gamma_{11} &= (3 - 2 \sin^2 \theta_w) [0.0556 K^{0.48} - 0.051 K^{0.35} - 0.067 K^{-0.24} + 0.062 K^{-0.40}], \\ \frac{1}{2} \sin 2\theta_c \gamma_{12} &= -\frac{1}{3} \sin^2 \theta_w [-0.049 K^{0.85} + 0.190 K^{0.43} - 0.426 K^{-0.13} + 0.274 K^{-0.35}], \\ \frac{1}{2} \sin 2\theta_c \gamma_{21} &= -\frac{1}{3} \sin^2 \theta_w [0.086 K^{0.85} + 0.146 K^{0.43} + 0.623 K^{-0.13} + 0.151 K^{-0.35}], \end{aligned}$$

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$$\begin{aligned}
 \frac{1}{2} \sin 2\theta_c \gamma_{22} &= (1 - 2 \sin^2 \theta_w)[0.167K^{0.48} + 0.333K^{-0.28}], \\
 \frac{1}{2} \sin 2\theta_c \delta_{11} &= (3 - 2 \sin^2 \theta_w)[-0.042K^{0.48} + 0.028K^{0.35} - 0.025K^{-0.24} + 0.039K^{-0.40}], \\
 \frac{1}{2} \sin 2\theta_c \delta_{12} &= -\frac{1}{3} \sin^2 \theta_w[-0.113K^{0.85} - 0.099K^{0.43} + 0.129K^{-0.13} + 0.079K^{-0.35}], \\
 \frac{1}{2} \sin 2\theta_c \delta_{21} &= -\frac{1}{3} \sin^2 \theta_w[0.063K^{0.85} - 0.126K^{0.43} - 0.084K^{-0.13} + 0.148K^{-0.35}], \\
 \frac{1}{2} \sin 2\theta_c \delta_{22} &= (1 - 2 \sin^2 \theta_w)(-K^{0.48} + K^{-0.24})/8.
 \end{aligned} \tag{94}$$

Numerical values for these parameters are quoted in Table IV for various values of K . We can then proceed to determine the π and ρ , ω , ϕ emission amplitudes in terms of the

TABLE IV
Weinberg-Salam Model Parameters for a Variety of Strong Interaction Enhancements

	$K = 1$	$K = 4$	$K = 7$
α_{11}	$\cot \theta_c$	$1.13 \cot \theta_c$	$1.27 \cot \theta_c$
α_{22}	$\tan \theta_c$	$1.13 \tan \theta_c$	$1.27 \tan \theta_c$
β_{11}	0	$-0.31 \cot \theta_c$	$-0.48 \tan \theta_c$
β_{22}	0	$-0.31 \tan \theta_c$	$-0.48 \tan \theta_c$
γ_{11}	0	$0.08(1 - \frac{2}{3} \sin^2 \theta_w) \csc 2\theta_c$	$0.16(1 - \frac{2}{3} \sin^2 \theta_w) \csc 2\theta_c$
γ_{12}	0	0	$0.01 \sin^2 \theta_w \csc 2\theta_c$
γ_{21}	$-\frac{2}{3} \sin^2 \theta_w \csc 2\theta_c$	$-0.77 \sin^2 \theta_w \csc 2\theta_c$	$-0.90 \sin^2 \theta_w \csc 2\theta_c$
γ_{22}	$(1 - 2 \sin^2 \theta_w) \csc 2\theta_c$	$1.13(1 - 2 \sin^2 \theta_w) \csc 2\theta_c$	$1.27(1 - 2 \sin^2 \theta_w) \csc 2\theta_c$
δ_{11}	0	$-0.19(1 - \frac{2}{3} \sin^2 \theta_w) \csc 2\theta_c$	$-0.30(1 - \frac{2}{3} \sin^2 \theta_w) \csc 2\theta_c$
δ_{12}	0	$0.26 \sin^2 \theta_w \csc 2\theta_c$	$0.45 \sin^2 \theta_w \csc 2\theta_c$
δ_{21}	0	0	$-0.03 \sin^2 \theta_w \csc 2\theta_c$
γ_{22}	0	$-0.31(1 - 2 \sin^2 \theta_w) \csc 2\theta_c$	$-0.48(1 - 2 \sin^2 \theta_w) \csc 2\theta_c$

parameters a_t , a_v , b_t , b_v , c_v of the previous section and as a function of the strong interaction enhancement factor K . The results of this calculation are given in Table V. In deriving this we have used the result that for b_v and b_t diagrams the matrix element relation

$$\langle BM | \mathcal{O}(Mt^A, Nt^A) | B \rangle = -\frac{8}{3} \langle BM | \mathcal{O}(M, N) | B \rangle \tag{95}$$

is valid. In calculating the c_v contribution of $\mathcal{O}(Mt^A, Nt^A)$, there is no unique prescription which tells us how to proceed. This occurs because we really do not know the precise mechanism which generates c_v . In the next section, we discuss two models involving gluons which could produce c_v . They suggest that a color matrix t^A gets inserted whenever we contract quark lines, and this yields the procedure for calculating the c_v coefficients which we will use. The extra uncertainty induced by this choice is not large because these terms vanish if there is no strong interaction enhancement.

TABLE V
Amplitudes in the Weinberg-Salam Model, Including the Strong Interaction Enhancement

Mode	Amplitude ^a
$n \rightarrow p\pi^-$	$\begin{aligned} & \left\{ \frac{2^{1/2}}{18} b_1 \left[\left(\gamma_{12} - \frac{8}{3} \delta_{12} \right) - 3 \left(\gamma_{21} - \frac{8}{3} \delta_{21} \right) \right] - \frac{2^{1/2}}{9} b_v \left[\gamma_{12} - \frac{8}{3} \delta_{12} \right] \right. \\ & + \frac{c_v}{3(2)^{1/2}} \left[\alpha_{22} - \frac{2}{3} \beta_{22} + 2(\gamma_{12} + \gamma_{21}) - 4\delta_{22} + \frac{14}{3} (\delta_{12} + \delta_{21}) + 6\xi(\delta_{21} - \delta_{12}) \right] \left. \right\} * K^{-0.48} \\ & + \gamma \left[\gamma_{12} - \gamma_{21} + \frac{16}{3} (\delta_{12} - \delta_{21}) \right] \end{aligned}$
$n \rightarrow p\rho^-$	$\begin{aligned} & \left\{ \frac{1}{9} b_t \left[-2 \left(\alpha_{11} - \frac{8}{3} \beta_{11} \right) + 5 \left(\gamma_{11} - \frac{8}{3} \delta_{11} \right) - 3 \left(\gamma_{22} - \frac{8}{3} \delta_{22} \right) \right] + \frac{1}{9} b_v \left[4 \left(\alpha_{11} - \frac{8}{3} \beta_{11} \right) \right. \right. \\ & - \left. \left(\gamma_{11} - \frac{8}{3} \delta_{11} \right) + 3 \left(\gamma_{22} - \frac{8}{3} \delta_{22} \right) \right] + \frac{1}{9} c_v \left[-10 \left(\alpha_{11} - \frac{2}{3} \beta_{11} \right) \right. \\ & - \left. 5 \left(\alpha_{22} - \frac{2}{3} \beta_{22} \right) - 10(\gamma_{11} + \gamma_{22}) - \frac{160}{3} \delta_{11} + \frac{20}{3} \delta_{22} + 10(\delta_{12} + \delta_{21}) + 10\xi(\delta_{12} - \delta_{21}) \right] \left. \right\} * K^{-0.48} \\ & + \frac{10}{9} a_v \alpha_{11} + \frac{10}{9} a_t \left[\gamma_{11} + \frac{16}{3} \delta_{11} - \gamma_{22} - \frac{16}{3} \delta_{22} \right] \end{aligned}$

$$\begin{aligned}
 p \rightarrow p\rho^0 & \left\{ \frac{1}{9(2)^{1/2}} b_t \left[-3 \left(\alpha_{11} - \frac{8}{3} \beta_{11} \right) + 5 \left(\gamma_{11} - \frac{8}{3} \delta_{11} \right) + \left(\gamma_{12} - \frac{8}{3} \delta_{12} \right) + 3 \left(\gamma_{21} - \frac{8}{3} \delta_{21} \right) - \left(\gamma_{22} - \frac{8}{3} \delta_{22} \right) \right] \right. \\
 & + \frac{1}{9(2)^{1/2}} b_v \left[3 \left(\alpha_{11} - \frac{8}{3} \beta_{11} \right) - \left(\gamma_{11} - \frac{8}{3} \delta_{11} \right) + 3 \left(\gamma_{12} - \frac{8}{3} \delta_{12} \right) + \left(\gamma_{21} - \frac{8}{3} \delta_{21} \right) + 5 \left(\gamma_{22} - \frac{8}{3} \delta_{22} \right) \right] \\
 & + \frac{2}{9(2)^{1/2}} c_v \left[-5 \left(\alpha_{11} - \frac{2}{3} \beta_{11} \right) - 4 \left(\alpha_{22} - \frac{2}{3} \beta_{22} \right) - 5(\gamma_{11} + \gamma_{22}) - 3(\gamma_{12} + \gamma_{21}) - \frac{80}{3} \delta_{11} + \frac{28}{3} \delta_{22} - 2(\delta_{12} + \delta_{21}) + 14\xi(\delta_{12} - \delta_{21}) \right] \left. \right\} \\
 & * K^{-0.48} \\
 & + \frac{4}{9(2)^{1/2}} a_v [3\gamma_{12} + 5\gamma_{22}] \\
 & + \frac{2}{9(2)^{1/2}} a_t \left[-5 \left(\alpha_{11} + \frac{16}{3} \beta_{11} \right) + 5 \left(\gamma_{11} + \frac{16}{3} \delta_{11} \right) + 3 \left(\gamma_{12} + \frac{16}{3} \delta_{12} \right) + 3 \left(\gamma_{21} + \frac{16}{3} \delta_{21} \right) + 5 \left(\gamma_{22} + \frac{16}{3} \delta_{22} \right) \right] \\
 p \rightarrow p\omega^0 & \left\{ \frac{1}{9(2)^{1/2}} b_t \left[\left(\alpha_{11} - \frac{8}{3} \beta_{11} \right) + 3 \left(\gamma_{11} - \frac{8}{3} \delta_{11} \right) - \left(\gamma_{12} - \frac{8}{3} \delta_{12} \right) + 5 \left(\gamma_{21} - \frac{8}{3} \delta_{21} \right) + \left(\gamma_{22} - \frac{8}{3} \delta_{22} \right) \right] \right. \\
 & + \frac{1}{9(2)^{1/2}} b_v \left[\left(\alpha_{11} - \frac{8}{3} \beta_{11} \right) + 3 \left(\gamma_{11} - \frac{8}{3} \delta_{11} \right) - \left(\gamma_{12} - \frac{8}{3} \delta_{12} \right) + 5 \left(\gamma_{21} - \frac{8}{3} \delta_{21} \right) + \left(\gamma_{22} - \frac{8}{3} \delta_{22} \right) \right] \\
 & + \frac{2}{9(2)^{1/2}} c_v \left[-3 \left(\alpha_{11} - \frac{2}{3} \beta_{11} \right) - 4 \left(\alpha_{22} - \frac{2}{3} \beta_{22} \right) - 3(\gamma_{11} + \gamma_{22}) - 5(\gamma_{12} + \gamma_{21}) - 16\delta_{11} + 12\delta_{22} - \frac{26}{3}(\delta_{12} + \delta_{21}) + 18\xi(\delta_{12} - \delta_{21}) \right] \left. \right\} \\
 & * K^{-0.48} \\
 & + \frac{4}{9(2)^{1/2}} a_v [3\gamma_{11} + 5\gamma_{21}] \\
 & + \frac{1}{9(2)^{1/2}} a_t \left[6 \left(\alpha_{11} + \frac{16}{3} \beta_{11} \right) + 6 \left(\gamma_{11} + \frac{16}{3} \delta_{11} \right) + 10 \left(\gamma_{12} + \frac{16}{3} \delta_{12} \right) + 10 \left(\gamma_{21} + \frac{16}{3} \delta_{21} \right) + 6 \left(\gamma_{22} + \frac{16}{3} \delta_{22} \right) \right]
 \end{aligned}$$

^a ξ is an unknown reduced matrix element which occurs when one has a current product $1/4 A V$ containing color matrices t^A . It is included for completeness, but plays no further role in our discussion.

Since the hyperon decay amplitudes are taken from experiment they already contain the enhancement factors. This means that b_v , b_t , and c_v contain a factor $K^{0.48}$ in them. Since we are only interested in enhancement relative to that of hyperon decay, the results are given in a form $\propto b_v/K^{0.48}$. For a_v and a_t , which are calculated in absolute value via the factorization, we should not divide by $K^{0.48}$.

IV. UNCERTAINTIES IN THE PION AMPLITUDE

In this section we will discuss the uncertainties in the parity violating pion vertex. This is important because there exists the possibility of isolating the pion exchange contribution to the parity violating force. It is purely $\Delta I = 1$ and gives the only long-range effect. The discussion proceeds in two parts: first we consider the contribution of the charged currents only (i.e., the Cabibbo model), and then we add parity violating neutral currents (i.e., the Weinberg-Salam model).

One way to write the results of Section II for the Cabibbo model is as a sum rule

$$\begin{aligned} n_-^0 &= - \left(\frac{2}{3} \right)^{1/2} [2A_-^0 - \Xi_-^-] \tan \theta_c \\ &= \frac{1}{3(2)^{1/2}} \tan \theta_c c_v \end{aligned} \quad (96)$$

We will call this sum rule value g_π . This, in fact, is much stronger than the techniques of that section indicate, since it can be derived by the assumption of only $SU(3)$ symmetry, either using PCAC [4] or without [1]. As simple as this is, it is amazing that almost all attempts at dynamics violate it, always in the same way (decreasing n_-^0). Various of these considerations are:

(i) When using PCAC, a parity violating matrix element, $\langle B' \pi | \mathcal{H}_w^{P.V.} | B \rangle$, is given in the soft pion limit by a commutator term

$$\langle B | [F_5^a, \mathcal{H}_w] | B \rangle \quad (97)$$

and a possible pole contribution if there exist poles degenerate in mass with the initial or final states (i.e., other baryons). The same holds for parity conserving matrix elements, with the replacement p.c. $\leftrightarrow$ p.v. The pole terms are determined by the same matrix elements ($\langle B' | \mathcal{H}_w^{P.C.} | B \rangle$) as the commutator terms. However in the $SU(3)$ limit $\langle B' | \mathcal{H}_w^{P.V.} | B \rangle$ vanishes [26], and the p.v. amplitudes are purely commutator, while the p.c. amplitudes are solely the pole diagrams. The matrix elements $\langle B' | \mathcal{H}_w^{P.C.} | B \rangle$, when determined from p.v. data yield $f \approx -2d$ (or $b_v \approx -b_t \approx -c_v$), but when determined from p.c. data give closer to $f \approx -d$ (or $b_v \approx -b_t$, $c_v \approx 0$). Clearly the choice between these two is crucial for the determination of c_v .

(ii) To attempt to fit S and P wave data simultaneously some momentum dependence must be given to the amplitudes. One way that it is done is to allow a

K^* pole in the S wave. The weak matrix element is $\langle \pi | \mathcal{H}_w | K_m^* \rangle \propto q_\mu \cdot \epsilon_m^\mu$, which clearly vanishes as $q_\mu \rightarrow 0$. This then allows the physical hyperon amplitude to have a different $SU(3)$ structure from the soft pion result. All attempts at simultaneous fits [27–30] tend to decrease the value for c_v (i.e., $d \approx -f$). Note that we are truly interested in the $\Delta S = 0$ matrix element near the soft pion point, $q_\mu = 0$, since that is the situation for virtual pion exchange in the nucleus. If we wish to use the hyperon amplitudes in the sum rule we must subtract off any momentum dependent piece. The message of the simultaneous fits is that this reduces n_-^0 . This has been emphasized most by Andrasi *et al.* [31]. Note that these q_μ dependent portions are $SU(3)$ breaking: However, they respect the Lee-Sugawara sum rule

$$2E_-^- - A_-^0 + 3^{1/2}\Sigma_0^+ = 0. \quad (98)$$

Thus the fact that $SU(3)$ appears good in the Lee–Sugawara sum rule does not imply the validity of Eq. (96).

(iii) All quark models with baryons composed of three quarks give $n_-^0 = 0$. This is easily seen from the Hamiltonian, which is the normal ordered product of fields

$$\mathcal{H}_{w\alpha}: \bar{u}\gamma_\mu(1 + \gamma_5) s \bar{s}\gamma^\mu(1 + \gamma_5)u: \quad (99)$$

Since there are no strange quark fields in a three quark baryon (or in the pion), the matrix element vanishes [5, 32]. This is clearly independent of most quark model uncertainties.

(iv) If we assume that the operator suppressed by the strong interaction in Eq. (87) does not contribute, the Hamiltonian is an $SU(4)$ 20-plet (this actually applies to the Weinberg–Salam model’s charged current, because the Cabibbo model, per se, does not include charm). Use of $SU(4)$ then predicts [33] $n_-^0 = 0$. In models where charm plays no role, such as the naive quark model, $SU(4)$ is essentially the same as $SU(3)$ since we can take the $SU(4)$ limit without disturbing the calculation.

(v) An attempt to calculate hyperon amplitudes via the Melosh transformation [35] finds that $\Delta S = 1$ processes are proportional to $m_{B'} - m_B$. When applied to $\Delta S = 0$, one finds $n_-^0 \propto m_N - m_p \approx 0$.

(vi) However, virtual quark pairs are present in the physical hadrons. Perhaps their presence can generate a large c_v , as is suggested by the p.v. data. Donoghue and Golowich have explored this possibility using quark pairs generated by gluon interactions within the MIT bag model [33]. They find [15] that the sea generates a c_v of the right sign and approximately the magnitude to fit the p.v. hyperon amplitudes for $\Delta S = 1$. However even if this suggestion were correct, there remains some uncertainty in n_-^0 . This is because for $\mathcal{H}_w^{\Delta S=1}$ it is up quarks in the sea that produce a non-zero c_v , while for $\mathcal{H}_w^{\Delta S=0}$, it is the strange quarks. Because the strange quark is considerably more massive than the up quark its contribution to the sea is expected to be reduced. We do not have any good understanding of the size of this effect, but again it reduces n_-^0 .

(vii) Another way to deal with virtual quark pairs has been proposed by Shifman *et al.* [36]. They calculate the effects of a gluon-produced pair using free particle propagators for the quarks and gluons, and utilizing a heavy quark expansion to reduce the interaction to a local form. This is a different approach to the same diagrams that were considered in Ref. [33]. They also find that it contributes in such a way as to give the right sign of c_v . In their calculation the magnitude is found to be a factor of 1.5 to 3 too small depending on the choice of strong interaction parameters. A crucial feature in their evaluation is the use of the quark equation of motion to find

$$\langle \pi^- | \bar{d} \gamma_5 u | 0 \rangle = -i \frac{2^{1/2} F_\pi M_\pi^2}{m_u + m_d}$$

which when evaluated with $m_u \approx m_d = 5$ MeV leads to a large enhancement for the matrix element. To the extent that the size of these masses is uncertain, the contribution of this evaluation is also uncertain. We will return to this point shortly in discussing the Weinberg-Salam model.

These approaches to hyperon decay are clearly not all consistent, and do not suggest any single value for n_-^0 . However the uncertainty in n_-^0 is well defined. All methods suggest that n_-^0 lies in the range between $n_-^0 = 0$ and $n_-^0 = g_\pi$.

In the Weinberg-Salam model, the neutral current produces a p.v. effect which must be added to that of the charged currents. A portion of the neutral current Hamiltonian belongs to the same $SU(3)$ multiplets as the charged current, and is enhanced and suppressed by the strong interactions in the same way. If we assume (as is suggested by both theory and experiment) that the effect of the suppressed operator is negligible, we can display the total amplitude as

$$n_-^0 = \left([1 - 2 \sin^2 \theta_w] \left(\frac{K^{0.48} - K^{-0.24}}{K^{0.48} + K^{-0.24}} \right) + \sin^2 \theta_c \right) \frac{\rho}{\sin^2 \theta_c} g_\pi + \sin^2 \theta_w B, \quad (100)$$

where a factor ρ ($0 \leq \rho \leq 1$) has been inserted into the sum rule to account for the for the uncertainties mentioned above, and B is an unknown number which is due to the portions of the neutral current which are not related to the charged current. One immediately obvious feature is that (neglecting B temporarily) the neutral current is much stronger than the charged current by an order of magnitude

$$\frac{1 - 2 \sin^2 \theta_w + \sin^2 \theta_c}{\sin^2 \theta_c} = 10.5 \quad (101)$$

for $\sin^2 \theta_w = 0.25$ and $\sin \theta_c = 0.23$. This has been the source of speculation that the pion exchange contribution could be very large. However the uncertainties in ρ , discussed above, dilute this hope somewhat.

The parameter B is not determined by symmetry considerations and must be calculated dynamically. There are two major contributions. In one the neutral current Hamiltonian is evaluated between baryons using a quark model. The second involves a Fierz rearrangement and factorization, and is described by the previously introduced parameter y . We will treat these two contributions separately.

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The quark model evaluation is standard and is similar to the naive quark model calculation of b_v and b_t . It has been done previously [5] with the result

$$B_1 = \frac{4}{3(6)^{1/2}} \eta E(K) \frac{A_-^0}{\sin \theta_c \cos \theta_c}, \quad (102)$$

where η is a ratio of two reduced matrix elements that is unity in the nonrelativistic $SU(6)$ limit and equals 0.67 for ultrarelativistic quarks in the MIT bag model. Desplanques and Micheli have emphasized that this contribution to n_-^0 has the same sign as the sum rule, and hence the two terms add [37]. This yields an extremely important prediction of the sign of n_-^0 . $E(K)$ is an enhancement factor for the strong interaction effects of Section III

$$E(K) = -0.33K^{0.85} + 0.03K^{0.43} + 1.61K^{-0.13} - 0.31K^{-0.35} \quad (103)$$

normalized so the $E(0) = 1$. Reasonable values for the enhancement are $K = 4 \rightarrow 7$. Unfortunately, in this range there is a strong cancellation in $E(K)$ so that B is rather small ($E(4) = 0.11$).³

The second contribution is obtained by factorizing the currents as described previously and gives a result

$$B_2 = \frac{1}{3} y F(K) \quad (104)$$

with y given in Eq. (78) and

$$F(K) = 1.04K^{0.85} - 0.19K^{0.43} - 0.09K^{-0.13} + 0.24K^{-0.35}. \quad (105)$$

The greatest uncertainty in this term comes in the evaluation of y . There are two factors to be determined $\langle \pi^- | \bar{d} \gamma_5 u | 0 \rangle$ and $S \equiv \langle P | \bar{u} d | N \rangle$.

For the first, the quark equations of motion can be used to set

$$\langle \pi^- | \bar{d} \gamma_5 u | 0 \rangle = - \frac{i 2^{1/2} F_\pi m_\pi^2}{m_u + m_d}. \quad (106)$$

The uncertainty here lies in the quark masses. One approach uses current algebra [39] to determine the mass ratios $m_d/m_u = 1.8$, $m_s/m_d = 20$. To obtain absolute values one must go beyond current algebra. Weinberg has given one possibility [38]. One may assume that, in the limit of zero quark mass, the expectation value of a scalar density is given by a universal renormalization factor Z times the number of that type quark in the hadron N_k

$$\langle H | \bar{q}_k q_k | H \rangle = Z N_k. \quad (107)$$

A renormalized quark mass is then

$$m_k^* = Z m_k. \quad (108)$$

³ The expression for $E(K)$ given in Ref. [5] is incorrect.

One then assumes that mass differences between particles of different strangeness is given perturbatively by

$$\Delta\mathcal{H} = m_s \bar{s}s \quad (109)$$

so that

$$\langle H | \Delta\mathcal{H} | H \rangle = m_s^* N_s. \quad (110)$$

This leads to a determination $m_s^* = 150$ MeV, and, by use of the current algebra ratios, $m_d^* = 7.5$ MeV, $m_u^* = 4.2$ MeV. Unfortunately this does not tell us m_u or m_d unless we know the renormalization factor Z , but it does provide one estimate for the masses.⁴ Because this estimate is very small, it provides a large amplitude. Another estimate of the quark mass that uses current algebra is provided by the determination of the σ term in πN scattering [39]. This yields masses $m_u \approx m_d \approx 40$ MeV, or roughly an order of magnitude larger. A third possibility, although one which we consider not very likely, is to use "constituent" quark masses, which would yield a result yet another order of magnitude larger. Our point in this discussion is that no estimate of this term is totally reliable, and the upper bound is provided by use of $m_u^* + m_d^* = 11.7$ MeV.

A smaller uncertainty, but one which we have more control, is in the value of $S = \langle P | \bar{u}d | N \rangle$. It is a quantity which may be calculated in a quark model, yielding $S = 1$ in any nonrelativistic model, and $S = 0.5$ in the MIT bag with massless quarks [40]. A similar quantity, the axial vector coupling constant g_A , yields $g_A = \frac{5}{8}$ and $g_A = 1.1$ in these two limits. Use of $g_A = 1.25$ can be translated into $S = \frac{5}{8}$. An alternate way of using the quark equations of motion, and trying to subtract off electromagnetic effects [21], yields $S = \frac{9}{11}$. We will use $S = \frac{5}{8}$ and simply remember that other uncertainties are much larger.

If we use $m_u^* + m_d^* = 11.7$ MeV and $S = \frac{5}{8}$ we obtain

$$B_2 \sin^2 \theta_w = 2g_\pi. \quad (111)$$

The strong interaction effects enhance it further by a factor $3 \rightarrow 4$ for reasonable values of K . This is a large enough number that it has the potential of being the dominant contribution.

If all terms had their full strength ($\rho = 1$, $B_1 = 4/(3(6)^{1/2}) \sec \theta_c \csc \theta_c A_-^0$, B_2 evaluated with $m_u^* + m_d^* = 11.7$ MeV, $s = 1$ and $F(K) = 4$) they would appear with magnitudes

$$\begin{aligned} A \text{ (sum rule)} &= 10.5 g_\pi, \\ A \text{ (quark model)} &= 5.1 g_\pi, \\ A \text{ (factorized)} &= 13 g_\pi. \end{aligned} \quad (112)$$

We have presented arguments, that none of these estimates is solid, and each may be

⁴ We will shortly estimate $Z \approx \frac{5}{8}$, so that $m_u + m_d \approx 17$ MeV.

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smaller than given in Eq. (112). The largest uncertainties in n_-^0 lie in ρ , and in the quark mass determination. The spread in possible values is from $n_-^0 = 0$ up to $n_-^0 = 30 g_\pi$.⁵

Fortunately this may allow a test of whether the neutral current violates parity. If it does not, n_-^0 must lie below g_π , while if it does, n_-^0 may be much greater. An experimental measurement which lies above g_π would be clear evidence in favor of the Weinberg-Salam model, while one between 0 and g_π would be inconclusive. Clearly the former would be an exciting result. Beyond this, an experimental number would tell us valuable information about our theories of weak nonleptonic decay. The questions raised in points (i) $\rightarrow$ (vi) above cannot be answered by reference to $\Delta S = 1$ alone, but would be resolved by knowledge of n_-^0 .

V. UNCERTAINTIES IN THE VECTOR MESON AMPLITUDES

The portion of the nuclear force for which the nuclear physics is best understood is the long-range pion exchange potential. However the isospin restriction $\Delta I = 1$ implies that some processes are not sensitive to this force at all, while many more involve other isospin components as well as $\Delta I = 1$. This means that we must understand these other forces in order to compare theory with the majority of experiments. After pion exchange, the next most important single particle exchange forces are those due to vector mesons. Here the task is to compute the parity violating $\bar{N}NV$ vertex.

The standard method for calculating these $\bar{N}NV$ amplitudes has been either the factorization [6, 7] or modified factorization [5] methods. In these one inserts the vacuum intermediate state between the currents in the Hamiltonian, and the resulting expression contains only constants which are known from semileptonic processes. It was McKellar and Pick [8] who first noted that there was a disagreement between $SU(6)_w$ and factorization. In Section II we have indicated the resolution to this problem. The modified factorization method gives a part of the amplitude, that labeled by a_v and a_t . However, in addition there are new terms which do not transform in the same way. In the $SU(6)_w$ these may be calculated from $\Delta S = 1$ pionic decays of hyperons. In the Cabibbo model, they turn out to be about four times larger than the usual factorization result and of the opposite sign. They are $\Delta I = 0$, so $NN\rho^0$ also has a strong coupling. Thus it drastically changes the pattern of the vector meson amplitudes.

Such a change appears to be welcome. The experimental situation is not completely clear, but it appears that the factorization gives the incorrect sign and too small a result. Thus experiment may also be suggesting the importance of these new terms.

When we attempt to estimate the uncertainty in the above picture, we are hampered both by the impossibility of observing these decays physically, as can be done with the pions, and by the relatively small theoretical effort which has been given this

⁵ While it is not possible to give a rigorous evaluation of n_-^0 , we shall in Section VII use our calculational prejudices to propose a "best" or benchmark value.

problem. Our only handle is a quark model calculation, within the framework of the MIT bag model, given in Appendix A. In the nonrelativistic $SU(6)$ limit it reproduces the above results for b_v and b_t . Away from this limit it identifies another contribution, that of odd parity baryon poles. These add with opposite signs to π and ρ amplitudes and therefore break the $SU(6)$ predictions. The prediction of the model is that the pion pieces become larger and the vector meson pieces become smaller. However, the relative rates among pionic decays by themselves or vector meson decays by themselves are unchanged. We then have a situation where all $SU(6)_w$ NNV predictions for b_v , b_t are rescaled by some common number $\eta' < 1$. The quark model actually finds a strong cancellation so that $\eta' \approx 0$. However in a quark model as crude as this a strong cancellation is always suspect. We feel that these considerations set the range of possible variation in b_v and b_t , namely from the $SU(6)_w$ value ($\eta' = 1$) to $\eta' = 0$. Likewise, as discussed in Section IV, c_v also is uncertain. Thus the range for the full amplitudes is from the $SU(6)_w$ value to the value found in the modified factorization approach. As with the $NN\pi$ vertex, in Section VII we will suggest a benchmark for NNV amplitudes, although the range described above is more secure theoretically than any selection of a "best value."

VI. COMPARISON WITH EXPERIMENT

We intend in this section to compare with experiment the predictions that we obtain for the Weinberg-Salam model. As already mentioned, parity violating effects in nuclei are usually assumed to be due to the parity violating nucleon-nucleon interaction arising from the exchange of mesons, π , ρ , and ω . These interactions are known once we define the p.c. and p.v. meson-nucleon interactions, which we write as

$$\begin{aligned} \mathcal{H}_{MNN}^{p.c.} = & ig_{\pi NN} \bar{N} \gamma_5 \vec{\tau} \cdot \vec{\phi}^\pi N + g_\rho \bar{N} \left(\gamma_\mu + \frac{i\chi_V}{2M} \sigma_{\mu\nu} k^\nu \right) \vec{\tau} \cdot \vec{\phi}^{\mu\rho} N \\ & + g_\omega \bar{N} \left(\gamma^\mu + \frac{i\chi_S}{2M} \sigma^{\mu\nu} k_\nu \right) \phi_\mu^\omega N \end{aligned} \quad (113)$$

and⁶

$$\begin{aligned} \mathcal{H}_{MNN}^{p.v.} = & (2)^{-1/2} f_\pi \bar{N} (\vec{\tau} \times \vec{\phi}^\pi)^3 N \\ & + \bar{N} \left[h_\rho^0 \vec{\tau} \cdot \vec{\phi}_\mu^\rho + h_\rho^1 \phi_\mu^{\rho 3} + h_\rho^2 \frac{(3\tau^3 \phi_\mu^{\rho 3} - \vec{\tau} \cdot \vec{\phi}_\mu^\rho)}{2(6)^{1/2}} \right] \gamma^\mu \gamma_5 N \\ & + \bar{N} [h_\omega^0 \phi_\mu^\omega + h_\omega^1 \tau^3 \phi_\mu^\omega] \gamma^\mu \gamma_5 N \\ & - h_\rho^1 \bar{N} (\vec{\tau} \times \vec{\phi}_\mu^\rho)^3 \frac{\sigma^{\mu\nu} k_\nu}{2M} \gamma_5 N. \end{aligned} \quad (114)$$

⁶ The definition of h_ρ^2 of Ref. [41] has been changed to make it consistent with the one of Ref. [42].

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Our predictions for the weak coupling constants f_π , $h_\rho^{0,1,2}$ and $h_\omega^{0,1}$ have been described in previous sections, and will be summarized again in the next section. In order to avoid any ambiguity as to conventions involved in the definitions of $\mathcal{H}^{\text{p.c.}}$ and $\mathcal{H}^{\text{p.v.}}$, we also give the expression of the NN potential which results from the above exchanges in the nonrelativistic limit

$$\begin{aligned}
 V_{12}^{\text{p.v.}} = & \frac{f_\pi g_{\pi NN}}{2^{1/2}} i \left(\frac{\vec{\tau}_1 \times \vec{\tau}_2}{2} \right)^z (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \left[\frac{\vec{p}_1 - \vec{p}_2}{2M}, f_\pi(r) \right] \\
 & - g_\rho \left(h_\rho^0 \vec{\tau}_1 \cdot \vec{\tau}_2 + h_\rho^1 \left(\frac{\vec{\tau}_1 + \vec{\tau}_2}{2} \right)^z + h_\rho^2 \frac{(3\tau_1^z \tau_2^z - \vec{\tau}_1 \cdot \vec{\tau}_2)}{2(6)^{1/2}} \right) \\
 & \times \left((\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \left\{ \frac{\vec{p}_1 - \vec{p}_2}{2M}, f_\rho(r) \right\} + i(1 + \chi_v) \vec{\sigma}_1 \times \vec{\sigma}_2 \cdot \left[\frac{\vec{p}_1 - \vec{p}_2}{2M}, f_\rho(r) \right] \right) \\
 & - g_\omega \left(h_\omega^0 + h_\omega^1 \left(\frac{\vec{\tau}_1 + \vec{\tau}_2}{2} \right)^z \right) \\
 & \times \left((\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \left\{ \frac{\vec{p}_1 - \vec{p}_2}{2M}, f_\omega(r) \right\} + i(1 + \chi_s) \vec{\sigma}_1 \times \vec{\sigma}_2 \cdot \left[\frac{\vec{p}_1 - \vec{p}_2}{2M}, f_\omega(r) \right] \right) \\
 & - (g_\omega h_\omega^1 - g_\rho h_\rho^1) \left(\frac{\vec{\tau}_1 - \vec{\tau}_2}{2} \right)^z (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \left\{ \frac{\vec{p}_1 - \vec{p}_2}{2M}, f_\rho(r) \right\} \\
 & - g_\rho h_\rho^1 i \left(\frac{\vec{\tau}_1 \times \vec{\tau}_2}{2} \right)^z (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \left[\frac{\vec{p}_1 - \vec{p}_2}{2M}, f_\rho(r) \right], \tag{115}
 \end{aligned}$$

where

$$\begin{aligned}
 f_\pi(r) &= \frac{e^{-m_\pi r}}{4\pi r}, \\
 f_\rho(r) &= f_\omega(r) = \frac{e^{-m_\rho r}}{4\pi r}. \tag{116}
 \end{aligned}$$

The strong coupling constants are assumed to be positive.

In the comparison with experiment, corrections to the potential due to 2π exchange will be neglected. They have been found to be negligible for the $\Delta I = 0$ and 2 parts of the potential where they modify slightly the radial shape of the ρ exchange potential [41]. For the $\Delta I = 1$ part, they involve new components which correspond to the exchange of 2π in a P state, and since this is the quantum number of the ρ -particle, some double counting may occur. Examination of our results shows that the 2π exchange contribution induced by the weak πNN interaction and the ρ exchange contribution which arises from the same process at the quark level may be comparable in sign and magnitude. This result, which is not surprising in a picture where πNN and ρNN p.n.c. coupling constants have been derived as consistently as possible, indicates that the above potential probably takes into account most of the 2π exchange contributions. While it is difficult to make a precise statement about the contribution of other exchanges involving several mesons, it is reasonable to expect

they will produce effects comparable to those arising from ρ or ω exchanges as it is the case for the strong NN interaction [43].

It is well known that present data on NN phase shifts do not completely determine the NN strong interaction in the region where ρ or ω meson exchanges contribute. In the comparison with experiment, we will assume that short-range correlations are described by a super-soft-core potential, namely the deTourel-Sprung potential, since this is what is expected in a quark description of the nucleon [44].

There are presently several measurements of p.v. effects which are constraints for any theoretical understanding. They are the circular polarization, P_γ , of γ emitted in the radiative thermal capture $n + p \rightarrow d + \gamma$, the measurement of the asymmetry A_{pp} in polarized proton-proton scattering at 15 MeV and measurements in various processes involving complex nuclei. These latter processes generally depend on all of the components of the p.v. potential. However, due to uncertainties in nuclear calculations, it is difficult to use the entire information they represent. The most reliable information which can be extracted from them at the present time, or in the near future, are probably the strengths of the proton-nucleus and neutron-nucleus p.v. interaction, which can be characterized by two quantities, X_N^p and X_N^n [45].

It has already been noted that the measured value of P_γ in $n + p \rightarrow d + \gamma$ was difficult to reproduce with the usual p.v. potentials. Our values for the coupling constants h_ρ^0 , h_ρ^2 , and h_ω^0 , which are involved in this process, do not significantly alleviate the disagreement. If the Weinberg-Salam model appears to be the appropriate description of weak interactions, and if the present measurement of P_γ is confirmed, we would have to conclude that our present knowledge of processes involved in the calculation of short-range contributions is inadequate.

Discarding this possibility in the following, we are therefore left with three quantities to compare with: A_{pp} , X_N^p , and X_N^n . Because the same proton nucleus p.v. interaction gives a good description of p.v. effects in several nuclei (^{181}Ta , ^{175}Lu , ^{41}K , and ^{19}F) [46], we think that the quantity, X_N^p , which can be used to characterize the strength of this interaction, is the best determined of the above quantities. This quantity, whose "experimental" value is about 3×10^{-6} [45], can be expressed in terms of the weak coupling constants and, for the model of the NN strong interaction we considered, it is given by

$$X_N^p = 5.5f_\pi - 0.25g_\rho h_\rho^1 - 0.62g_\rho h_\rho^0 - 0.05g_\rho h_\rho^1 - 0.17g_\omega h_\omega^1 - 0.19g_\omega h_\omega^0. \quad (117)$$

A similar expression can be obtained for X_N^n by changing in (117) the sign of the isovector contributions.

Assuming that only π exchange contributes to X_N^p , experiment implies a value for f_π of about 6×10^{-7} . This value has the sign of our predictions and is between our "best value" for it and the upper estimate. We can also assume that only the isoscalar ρ exchange contributes, which would require a value of h_ρ^0 of about -1.2×10^{-6} . This value, which is opposite in sign to the factorization result, is higher than our best value but smaller than our largest estimate. These two extreme cases were chosen to show that they imply coupling constants quite consistent with our expectations. In

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between these two cases, there are several possible consistent solutions involving a contribution of π exchange, isoscalar ρ exchange, and also isoscalar ω exchange. Among these different models, the one which averages the different uncertainties and referred in Table VII as “best value” appears particularly attractive. It gives a value of X_N^p close to the one desirable to explain p.v. effects in ^{181}Ta , ^{175}Lu , ^{41}K , and ^{19}F . Furthermore, it gives a small negative value of X_N^n . This feature makes possible large cancellations of the contributions relative to X_N^p and X_N^n in the transition $\frac{1}{2}^-(2.789 \text{ MeV}) \rightarrow \frac{3}{2}^+$ in ^{21}Ne [47], and thus may explain the absence of effect at the expected level in this process [48]. Using the expression of the asymmetry A_{pp} :

$$A_{pp}(15 \text{ MeV}) = 0.01g_\omega(h_\omega^0 + h_\omega^1) + 0.03g_\rho\left(h_\rho^0 + h_\rho^1 + \frac{h_\rho^2}{6^{1/2}}\right). \quad (118)$$

We predict a value of about -1.3×10^{-7} , which is quite consistent with the recent measurement $-(1.7 \pm 0.85) \times 10^{-7}$ [49].

Apart from the circular polarization P_γ in $n + p \rightarrow d + \gamma$, we see that it appears quite possible to explain present measurements of p.v. effects in several nuclei and in polarized proton-proton scattering within the Weinberg-Salam model. While the most reasonable set of coupling constants provides a rather good description of these effects, we think it does not yet give a definitive understanding of them. The large uncertainties in nuclear calculations due to short-range correlations and to corrections to the weak potential from multi-meson exchange make it possible to accommodate the above p.v. effects with other sets of coupling constants in addition to our “best values,” and generally consistent with our expectations for the Weinberg-Salam model. Further work on reducing both the nuclear uncertainties and the experimental errors is needed before one can determine unambiguously the complete set of coupling constants.

VII. CONCLUSION

This paper is a study of the elementary particle physics aspects of the field of $\Delta S = 0$ parity violation in nuclear physics. The primary task is to calculate weak parity violating vertices for $B \rightarrow B'M$, where M is one of the mesons $\pi^\pm$, $\rho^\pm$, ρ^0 , ω^0 , ϕ^0 . An unsettling feature of previous work in this area has been the multitude of approaches that have been used. We have attempted, primarily in Section II, to tie together these approaches into a unified treatment. Past methods appear as various approximations to our framework. In our exploration we have gained deeper insight into the theoretical uncertainties inherent in the problem than would have been obvious in the study of any one of the individual methods. These have been discussed in Sections III, IV, and V. We will summarize our results below. In our discussion of the uncertainty we have tried to be generous in our estimation of the range of possible values. Therefore we feel that the boundaries of the ranges are particularly firm, and that future theoretical work will serve mainly to suggest particular values within the range.

The symmetry $SU(6)_w$ provides a relationship among all of the desired mesonic vertices. However, it contains more parameters than can be experimentally determined, so that it is not completely adequate. By noting that $SU(6)_w$ is a symmetry of the quark model in an appropriate limit, we have rephrased the $SU(6)_w$ relationship in terms of the quark model. This allows us to identify the various parameters (as in Fig. 1) and to study them separately.

At this point, we feel that it is useful to give a "most reasonable" value for the vertices in question. These are guided mainly by our own prejudices, and in light of our previous discussion may seem little more than guesses. However it is important to have "benchmark" values with which to measure experimental results as they come in. The deviation of the experimental values from the benchmark will provide useful information about our theories of nonleptonic decay and will settle some of the issues discussed above.

For the pion, we have several contributions to the total amplitude. Uncertainties in the sum rule portion arise because we do not truly understand what generates the parameter c_v . It appears related to virtual quark pairs in hadrons, and may easily have an $SU(3)$ violating component. Our guess for this piece is one-half of its canonical value. A second contribution is the factorized evaluation (104). Here the uncertainties lie in the quark masses and renormalization constants. We will take $m_u + m_d = Z_m^{-1}(m_u^* + m_d^*)$, with $m_u^* + m_d^* = 11.7$ MeV, and $Z_m = S = \frac{5}{8}$, along with a strong interaction enhancement factor of 3. Finally, the portion calculated in directly using quark model wavefunctions (10) is taken to be suppressed by the strong interactions by a factor of 2. This set of assumptions yields $n_-^0 = 12 g_\pi$ for the Weinberg-Salam model and $n_-^0 = 0.5 g_\pi$ in the Cabibbo theory. These results are listed in Tables VI and VII along with our bounds on the amplitude.

Given the importance of the pion vertex to the overall parity violation program, it is gratifying that a very important aspect of gauge theories may be tested despite

TABLE VI
Range and "Best" Estimate of Cabibbo Model Parameters^a

Cabibbo model	Reasonable range	"Best" value
f_π	$0 \rightarrow 1$	0.5
h_p^0	$15 \rightarrow -64$	-25
h_p^1	$0 \rightarrow -0.7$	-0.4
h_p^2	-58	-58
h_ω^0	$6 \rightarrow -22$	-6
h_ω^1	$0 \rightarrow -2$	-1
h_ϕ^0	0.2	0.2
h_ϕ^1	0.3	0.3

^a Amplitudes are in units of $g_\pi = 3.8 \times 10^{-8}$.

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TABLE VII
Range and "Best" Estimate of Weinberg-Salam Model Parameters^a

Weinberg-Salam model	Reasonable range	"Best" value
f_π	$0 \rightarrow 30$	12
h_ρ^0	$30 \rightarrow -81$	-30
h_ρ^1	$-1 \rightarrow 0$	-0.5
h_ρ^2	$-20 \rightarrow -29$	-25
h_ω^0	$15 \rightarrow -27$	-5
h_ω^1	$-5 \rightarrow -2$	-3
h_ϕ^0	$0.4 \rightarrow 0.2$	0.3
h_ϕ^1	$-20 \rightarrow -13$	-13

^a Amplitudes are in units of $g_\pi = 3.8 \times 10^{-8}$. Strong interaction effects discussed in Section III have been included in the estimate.

the uncertainty in the vertices. In some gauge theories, the neutral current does not violate parity which means that their predictions will be those of the Cabibbo model where the pion vertex is $\mathcal{O}(\tan \theta_c)$ relative to hyperons. On the other hand, the Weinberg-Salam model has a neutral parity violating current, and no Cabibbo angle suppression occurs. If experiment shows that the pion vertex is larger than the maximum allowed for the Cabibbo model in Table VI, then we may conclude that the neutral current *does* violate parity.

The ρ vertices are much less model dependent. The major uncertainty here is essentially the validity of the $SU(6)$ assumption. We have indicated that in fact this may not be valid and hence have assigned large errors to this vertex, as in Tables VI and VII.

In choosing a "best" value for the ρ vertex, we note that, in the Cabibbo model, the combination of b_v and c_v which enters (using $b_t = -b_v$) is $\frac{8}{9}b_v - \frac{1}{9}c_v$ (the sign is such that they add constructively) for ρ^+ or $2^{1/2}\rho^0$ emission and $(2/(3(2)^{1/2}))c_v$ for ω^0 . In quark models c_v should be larger here than in $A(N_-^0)$ because it involves a Hamiltonian with only nonstrange quarks, in contrast to $\mathcal{H}^{\rho \Delta I=1}$. We will choose c_v to be $\frac{3}{4}$ of the fit value. The quark model calculation of b_v and b_t which is given in the Appendix indicates that there may be a strong cancellation, due to baryon poles, which lowers these values. In our numbers for the best value we will include this cancellation by using an effective b_v that is $\frac{1}{4}$ of its hyperon decay value. These lead to the entries in the tables.

The uncertainties discussed in this paper make it clear that we are no longer able to expect that the parity violation in nuclear physics program will determine the detailed couplings of a weak interaction theory. However this role may no longer be needed, given the success of the deep inelastic weak interaction experiments. Instead we can see that these $\Delta S = 0$ effects are a unique window to the nonleptonic interactions. We have discussed a variety of elementary particle questions which can be

answered if future experiments and nuclear physics calculations are able to pin down the weak amplitudes. Hopefully the unified framework of this paper will be useful in analyzing those results.⁷

APPENDIX A: QUARK MODEL CALCULATION OF PION AND VECTOR MESON VERTICES

Progress in recent years has sharpened our understanding of the quark structure of hadrons. In this picture, baryons are composed of three-colored fermionic quarks, arranged completely antisymmetrically to form a color singlet. For the lowest-lying baryons the quarks are in S wave states while higher-energy baryons are radial and orbital excitations of this ground state. Forces between the quarks are only strong at large separations, and these are largely spin independent. If we use this picture to construct a model where quarks are independent, we can write down the ground state ($l = 0, s = \frac{1}{2}, j = \frac{1}{2}$) Dirac wavefunction as

$$\psi(\chi) = \frac{1}{(4\pi)^{1/2}} \begin{pmatrix} iu(r) \chi_m \\ l(r) \vec{\sigma} \cdot \hat{r} \chi_m \end{pmatrix}, \quad (\text{A1})$$

where χ_m is a two-component Pauli spinor and $u(r)$ and $l(r)$ are referred to as the upper and lower components of the wavefunction which depend on the specific model.

Excited states can also be displayed. For example the $j = \frac{1}{2} P$ wave state is given by

$$\psi(\chi) = \frac{1}{(4\pi)^{1/2}} \begin{pmatrix} il(r) \vec{\sigma} \cdot \hat{r} \chi_m \\ u(r) \chi_m \end{pmatrix}. \quad (\text{A2})$$

The specific model which we will use is the MIT bag model, and we refer the reader to the literature for details on the wavefunctions [16–18].

Calculations for hyperon decay in the soft pion limit exist in the literature and we will draw on these to develop our method. These proceed by noting (we will use $\Lambda \rightarrow N\pi^0$ as our example)

$$\lim_{q\pi \rightarrow 0} \langle N\pi^0 | \mathcal{H}_w^{\text{p.v.}} | \Lambda \rangle = \frac{-i}{F_\pi} \langle N | [F_3^5, \mathcal{H}_w^{\text{p.v.}}] | \Lambda \rangle \quad (\text{A3a})$$

$$= \frac{i}{2F_\pi} \langle N | \mathcal{H}_w^{\text{p.c.}} | \Lambda \rangle, \quad (\text{A3b})$$

where the last step follows from

$$[F_3^5, \mathcal{H}_w^{\text{p.v.}}] = [F_3, \mathcal{H}_w^{\text{p.c.}}]. \quad (\text{A4})$$

⁷ *Note added in proof.* In subsequent work (J. F. Donoghue, E. Golowich, B. R. Holstein, and W. Ponce, “Analysis of $\Delta S = 1$ Nonleptonic Weak Decays and the $\Delta I = 1/2$ Rule,” MIT preprint No. CTP-798, to be published in the *Physical Review*) it was shown that quark model calculations, such as those used in this paper, allow a satisfactory understanding of the $\Delta I = 1/2$ rule in strangeness changing nonleptonic decays of both kaons and hyperons. This reinforces our confidence in the above procedures.

The matrix element in Eq. (A3b) is easy to calculate in the quark model

$$\langle N | \mathcal{H}_w^{\text{p.c.}} | A \rangle = \frac{G}{2^{1/2}} \cos \theta_c \sin \theta_c K^{0.48} \frac{6^{1/2}}{4\pi} \int dr r^2 [u^2(r) + l^2(r)]^2 \quad (\text{A5})$$

with the parameters of Ref. [18] this yields

$$2F_\pi \langle N | \mathcal{H}_w^{\text{p.c.}} | A \rangle = 5K^{0.48} \times 10^{-8}. \quad (\text{A6})$$

Another way to proceed is to put a complete set of intermediate states in Eq. (A3a)

$$\begin{aligned} \langle N | [F_3^5, \mathcal{H}_w^{\text{p.v.}}] | A \rangle \\ = \sum_I [\langle N | F_3^5 | I \rangle \langle I | \mathcal{H}_w^{\text{p.v.}} | A \rangle - \langle N | \mathcal{H}_w^{\text{p.v.}} | I \rangle \langle I | F_3^5 | A \rangle]. \end{aligned} \quad (\text{A7})$$

The axial charge F_3^5 is given by

$$\begin{aligned} F_3^5 &= \int d^3x A_0^3(x) \\ &= \int d^3x \bar{\psi}(x) \gamma_0 \gamma_5 \tau_3 \psi(x) \end{aligned} \quad (\text{A8})$$

and can be expressed in the quark model as

$$\begin{aligned} F_3^5 &= -i \sum_{i,j} \left\{ (b_i^+ b_j - b_i b_j^+) \int dr r^2 (u_i u_j + l_i l_j) \right. \\ &\quad \left. + (d_i^+ b_j^+ - b_j d_i) \int dr r^2 (u_i u_j - l_i l_j) + \dots \right\}, \end{aligned} \quad (\text{A9})$$

where b^+ (d^+) is the creation operator for a quark (antiquark) and $\dots$ signifies terms not of interest in baryons. We see that F^5 connects a three quark state with two classes of quark model states only; (i) three-quark states where one quark is boosted to a P wave and (ii) four-quark one-antiquark states where neither or both of the new quarks are in P wave states. If we use these intermediate states we find that the result can be put in the following simple form

$$\begin{aligned} \langle N | \mathcal{H}_w^{\text{p.c.}} | A \rangle &= \frac{G}{2^{1/2}} \cos \theta_c \sin \theta_c K^{0.48} \frac{6^{1/2}}{4\pi} \sum_i \int r^2 dr (u_0(r) u_i(r) + l_0(r) l_i(r)) \\ &\quad \times \int r^2 dr (u_0^2(r) + l_0^2(r)) (u_0(r) u_i(r) + l_0(r) l_i(r)) \end{aligned} \quad (\text{A10})$$

where 0 specifies the ground state and i specifies all states satisfying

$$\tan \omega_i = \frac{\omega_i}{\omega_i + 1} \quad (\text{A11})$$

for *both* positive and negative ω_i . Positive ω_i correspond to the three-quark states and negative ω_i to the five quarks. Numerically, again with the parameters of Ref. [18], if we keep the eight lowest energy states we obtain

$$2F_\pi \langle N | \mathcal{H}_w^{\text{p.c.}} | \Lambda \rangle = 4.7 K^{0.48} \times 10^{-8} \quad (\text{A12})$$

or 95 % of the full value. Of this amount almost equal contributions come from the three-quark and five-quark intermediate states.

It is also useful to look at the nonrelativistic limit ($I \rightarrow 0$). In this case, since

$$\int r^2 dr u_0(r) u_i(r) = \delta_{0i} \quad (\text{A13})$$

the only contribution comes from the state where the wavefunction is the same as the ground state. This is the five-quark system where all the particles are in 1S states. In this limit, the orthogonality relation above tells us that Eqs. (A5) and (A10) are equivalent.

The above techniques do not directly apply to vector meson amplitudes. However we can use the same intermediate states as in the last section as a basis for a pole model calculation. In this way (here our example will be $\langle N\rho^0 | \mathcal{H}_w^{\text{p.v.}} | \Lambda \rangle$)

$$\begin{aligned} & \langle N\rho^0 | \mathcal{H}_w^{\text{p.v.}} | \Lambda \rangle \\ &= \sum_I \left[\langle N\rho^0 | I \rangle \frac{1}{E_I - E_0} \langle I | \mathcal{H}_w | \Lambda \rangle + \langle N | \mathcal{H}_w | I \rangle \frac{1}{E_I - E_0} \langle I\rho^0 | \Lambda \rangle \right]. \end{aligned} \quad (\text{A14})$$

The strong $B \rightarrow B'\rho$ amplitudes are calculated by reducing the rho via the LSZ reduction, replacing it by the vector current and comparing it to the known $NN\rho$ amplitudes. In this way

$$\frac{\langle B'\rho | B \rangle}{\langle N\rho | N \rangle} = \frac{\epsilon_\mu^* \langle B' | V^\mu(0) | B \rangle}{\epsilon_\mu^* \langle N | V^\mu(0) | N \rangle} \quad (\text{A15})$$

or

$$\langle B'\rho | B \rangle = f_\rho \epsilon_\mu^* \langle B' | V^\mu(0) | B \rangle. \quad (\text{A16})$$

This latter is easily calculated in the quark model. Combining these steps we obtain

$$\begin{aligned} & \langle N\rho^0 | \mathcal{H}_w^{\text{p.v.}} | \Lambda \rangle \\ &= \frac{G}{2^{1/2}} \cos \theta_c \sin \theta_c \frac{K^{0.46}}{4\pi} \frac{10}{2^{1/2}} f_\rho R \sum_i \frac{1}{\omega_i - \omega_0} \int dr r^2 \left(u_0 u_i - \frac{1}{3} d_0 l_i \right) \\ & \quad \times \int r^2 dr (u_0^2(r) + l_0^2(r))(u_0(r) u_i(r) + l_0(r) l_i(r)). \end{aligned} \quad (\text{A17})$$

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For other decays the answer can be obtained by using $SU(6)_w$ if we identify this with $(-1/(2(2)^{1/2}))(b_v - \frac{1}{4}b_t)$ and use $b_v = -b_t$.

If we use this in the nonrelativistic limit we find that the $SU(6)_w$ result relating π and ρ is obtained if $f_\rho/2\omega_0 R = 1/F_\pi$ which becomes the KSFR sum rule [50] if we use $m_\rho = 2\omega_0$, as is true in most quark models (although not strictly in the bag model due to the volume energy). The new terms that appear when we deviate from the non-relativistic limit correspond to odd parity baryon resonances. These cancel strongly against the $SU(6)_w$ contribution. If we put the bag parameters in we find

$$\frac{\langle N\rho^0 | \mathcal{H}_w | \Lambda \rangle}{\langle N\pi^0 | \mathcal{H}_w | \Lambda \rangle} = 0.03 \quad (\text{A18})$$

instead of the $SU(6)_w$ result for $b_v = -b_t$, $c_v = 0$.

$$\frac{\langle N\rho^0 | \mathcal{H}_w | \Lambda \rangle}{\langle N\pi^0 | \mathcal{H}_w | \Lambda \rangle} = \frac{5}{3} 2^{1/2}. \quad (\text{A19})$$

We do not know how seriously to take this cancellation since this simple quark model is only a crude representation of full dynamics in the intermediate states. However the direction of corrections to the $SU(6)_w$ result is clear.

APPENDIX B

We utilize the phase conventions

$$\begin{aligned} p &= B^{114} - B^{123}, & \pi^+ &= \frac{1}{2^{1/2}} (\phi_1^3 - \phi_2^4), \\ n &= -B^{332} + B^{134}, & \pi^0 &= \frac{1}{2} (\phi_1^1 - \phi_2^2 - \phi_3^3 + \phi_4^4), \\ \Lambda &= B^{145} - B^{235}, & \pi^- &= \frac{1}{2^{1/2}} (\phi_3^1 - \phi_4^2), \\ \Sigma^+ &= B^{116} - B^{125}, & \rho^+(J_z = 1) &= \phi_1^4, \\ \Sigma^- &= B^{336} - B^{345}, & \rho^0(J_z = 1) &= \frac{1}{2^{1/2}} (\phi_1^3 - \phi_3^4), \\ \Sigma^0 &= \frac{1}{2^{1/2}} (2B^{136} - B^{325} - B^{145}), & \rho^-(J_z = 1) &= \phi_3^2, \\ \Xi^0 &= B^{156} - B^{255}, & & \\ \Xi^- &= B^{455} - B^{356}. & & \end{aligned}$$

Note that (ρ, n) and $(\Xi^0, -\Xi^-)$ form isodoublets, while $(\Sigma^+, \Sigma^0, \Sigma^-)$, $(-\pi^+, \pi^0, \pi^-)$, and $(-\rho^+, \rho^0, \rho^-)$ form isotriplets.

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