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Factorization in non-leptonic decays of heavy mesons

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Abstract

We propose a QCD-based model for calculation of the non-perturbative corrections to the factorization approximation in the decays of heavy mesons. In the framework of the model, factorization in pseudoscalar transitions holds exactly at the leading order leaving the opportunity to calculate non-leading corrections consistently.

1. Introduction

The factorization approximation is widely used in the heavy quark physics. As a calculational scheme, it is very convenient and there are numerous models based on it. However, the theoretical basis for the approximation itself is not completely understood. The usual motivation for it is a color transparency argument due to Bjorken [1] which suggests that the quark pair produced in the decay does not have enough time to evolve to the real-size hadronic entity, being a small size bound state with a small chromomagnetic moment. That is taken to imply that the QCD interaction is suppressed. For the heavy-to-heavy-light transition (we shall hereafter refer to the color allowed decay $B \rightarrow D^+ \pi^-$ for the sake of clarity) the matrix elements in question are

$$\begin{aligned} \mathcal{M}(B(p_B) \rightarrow D(p_D) \pi(q)) &= \langle \pi D | \mathcal{H}(0) | B \rangle = \frac{G_F}{\sqrt{2}} V_{cb} V_{du} \left\{ \left(C_1 + \frac{1}{N_c} C_2 \right) \mathcal{M}_1 + 2C_2 \mathcal{M}_8 \right\} \\ &= \frac{G_F}{\sqrt{2}} V_{cb} V_{du} \left\{ \left(C_1 + \frac{1}{N_c} C_2 \right) \langle \pi D | \bar{c}(0) \gamma_\mu (1 + \gamma_5) b(0) \bar{d}(0) \gamma^\mu (1 + \gamma_5) u(0) | B \rangle \right. \\ &\quad \left. + 2C_2 \langle \pi D | \bar{c}(0) \gamma_\mu (1 + \gamma_5) t^a b(0) \bar{d}(0) \gamma^\mu (1 + \gamma_5) t_a u(0) | B \rangle \right\} \end{aligned} \quad (1)$$

Here the usual notations are used

$$\begin{aligned} \mathcal{H} &= C_1 O_1 + C_2 O_2, \quad \langle \pi D | O_1 | B \rangle = \mathcal{M}_1, \quad \langle \pi D | O_2 | B \rangle = \frac{1}{N_c} \mathcal{M}_1 + 2\mathcal{M}_8 \\ O_1 &= \bar{c}_i \gamma_\mu (1 + \gamma_5) b^i \bar{d}_k \gamma^\mu (1 + \gamma_5) u^k \quad \text{and} \quad O_2 = \bar{c}_i \gamma_\mu (1 + \gamma_5) b^k \bar{d}_k \gamma^\mu (1 + \gamma_5) u^i \end{aligned} \quad (2)$$

where we have explicitly written out color indices. In the usual factorization approximation one simply separates out currents by inserting the vacuum state and disregards any QCD interactions between them on the basis of

Bjorken's conjecture. That implies that the $\mathcal{M}_8$ contribution in (1) is dropped, as it has no factorizable color singlet form. New non-factorizable corrections appearing in the $\mathcal{M}_8$ matrix element can arise if a hard or soft gluon is emitted or absorbed by the escaping pair. Obviously, a more quantitative estimate of these contributions is needed.

The first attempt to put a factorization approximation on the solid theoretical base was made in [2]. The authors considered a limit of large energy transferred to light quarks

$$E = v \cdot q = \frac{m_b^2 - m_c^2}{2m_b} \sim m_b \quad (3)$$

as $m_b \rightarrow \infty$. Here v is the velocity of the decaying heavy quark $p_b = m_b v + k$ and q is the momentum transferred to the light quark pair. In this limit the energy scales as a heavy quark mass. Proposing a Large Energy Effective Theory (LEET) as an extension of the Heavy Quark Effective Theory and making use of a convenient gauge condition $n \cdot A = 0$ (with n being a null vector, $n^2 = 0$) they proved factorization for the physical amplitude dominated by collinear quarks. It is not clear, however, how the pair of collinear quarks hadronizes into the pion. Also, the validity of the LEET itself is not yet established.

The important step towards the sought theoretical description was made in [3]. The authors considered a specific small velocity limit where $m_b - m_c \sim \text{const}$ which implies different scaling of (3): $E \sim \text{const}$ as $m_b \rightarrow \infty$. It turns out that this limit is theoretically "clean" for the application of the QCD sum rule method and provides a theoretical justification of the "rule of discarding $1/N_c$ corrections" [4] on dynamical grounds. Unfortunately, the actual B decays are at the borderline of the formalism, bringing large uncertainties into the estimation of the matrix elements.

In what follows we develop a QCD-based model which works in the limit $E \sim m_b$ as $m_b \rightarrow \infty$. The idea is to study the propagation of the light quarks emerging to form the final hadron while passing through the (perturbative or non-perturbative) gluonic fields present in the decay. Interestingly, results can be obtained which are independent of the nature of the gluonic fields. The light quark pair, produced at $x = 0$, interacts strongly with the background gluonic field while escaping from the point they were produced and hadronizes into the pion. Since the energy transferred to the quark pair is large it is not necessary that the hadronization occurs at $x \sim 0$. As a result, it is not sufficient to take the operators of the lowest dimension in the Operator Product Expansion (OPE) but rather one must sum up a whole series of operators. The novel technical point of our method is to use introduce "generating (distribution) functions" which incorporate this infinite series of matrix elements, and then to use Heavy Quark Symmetry to restrict the form of the matrix elements enough that useful statements can be made. In principle, these functions could be modeled or perhaps fixed by other measurements. However, at the leading order in $q^2 = 0$, $x^2 \sim 0$ the color octet matrix element vanishes due to cancellations and factorization holds. The method should be able to be extended beyond leading order, allowing one to also see corrections to factorization.

2. The octet contribution

The amplitude for the singlet or octet part is

$$\mathcal{M}_{1,8} = \langle D\pi | \bar{c}(0) \gamma_\mu (1 + \gamma_5) T b(0) \bar{d} \gamma^\mu (1 + \gamma_5) T u | B \rangle = \langle D | \bar{c}(0) \gamma_\mu (1 + \gamma_5) T b(0) \mathcal{P}_\mu | B \rangle \quad (4)$$

with $T = \{1, t^a\}$ being the appropriate color matrix. The quantity $\mathcal{P}_\mu$ describes the transition of the light quarks into a pion in the presence of the gluonic fields which are either internal to the B, D hadrons or produced during the transition. The idea is to treat these gluonic fields as external fields with respect to the light quark pair produced in the decay of virtual W -meson. From the diagrams Fig. 1 one may write down the expression for one gluon matrix element of the produced quark pair in this general background field. This gives a contribution to the octet matrix element only:

Fig. 1. One gluon diagrams for the $B \rightarrow D\pi$.

$$\begin{aligned} \mathcal{P}_\mu = & -(ig_s) \int d^4x \langle \pi | \bar{d}(x) \mathcal{A}^b(x) t_{ik}^b S_F(x, 0) \gamma_\mu (1 + \gamma_5) t_{ki}^a u(0) | 0 \rangle \\ & - (ig_s) \int d^4x \langle \pi | \bar{d}(0) \gamma_\mu (1 + \gamma_5) t_{ki}^a S_F(0, x) \mathcal{A}^b(x) t_{ik}^b u(x) | 0 \rangle \end{aligned} \quad (5)$$

where we set $\mathcal{A}_\mu \equiv A_\mu$ = the external field. As one can see, in this approximation we neglect higher Fock state contributions. Our normalization is such that $\text{Tr } t^a t^b = \delta^{ab}/2$. Thus we can rewrite the Eq. (5) as

$$\mathcal{P}_\mu = -\frac{ig_s}{2} \int d^4x \left(\text{Tr } \mathcal{A}^a(x) S_F(x, 0) \gamma_\mu (1 + \gamma_5) G(0, x) + \text{Tr } \gamma_\mu (1 + \gamma_5) S_F(0, x) \mathcal{A}^a(x) G(x, 0) \right) \quad (6)$$

with non-local correlators

$$G(0, x) = \langle \pi | Tu(0) \bar{d}(x) | 0 \rangle, \quad G(x, 0) = \langle \pi | Tu(x) \bar{d}(0) | 0 \rangle \quad (7)$$

Shifting the x -dependence of the second correlator

$$\langle \pi | Tu(x) \bar{d}(0) | 0 \rangle = \langle \pi | Te^{iPx} u(0) e^{-iPx} \bar{d}(0) e^{iPx} e^{-iPx} | 0 \rangle = e^{iqx} G(0, -x) \quad (8)$$

We shall work in the Fock-Schwinger gauge [5] which gives us the advantage of expressing the A_μ in terms of gauge-invariant quantities (such as the gluon stress tensor $G_{\rho\mu}^a$):

$$x_\mu \cdot A^\mu = 0, \quad A_\mu^a = \int_0^1 d\alpha \alpha x_\rho G_{\rho\mu}^a(\alpha x) \quad (9)$$

We treat the light quarks as massless, so the bare propagator of a light quark is given by

$$S_F(x - y) = \frac{1}{2\pi^2} \frac{\not{x} - \not{y}}{((x - y)^2)^2} \quad (10)$$

This allows us to rewrite (6) as

$$\begin{aligned} -i\mathcal{P}_\mu = & -i\mathcal{P}_{1\mu} - i\mathcal{P}_{2\mu} = \frac{(-ig_s)}{4\pi^2} \int d^4x \int_0^1 d\alpha d\alpha \left[\frac{x_\rho x_t}{x^4} G_{\rho s}^a(\alpha x) \text{Tr } \sigma_{st} \gamma_\mu (1 + \gamma_5) G(0, x) \right. \\ & \left. - \frac{x_u x_\rho}{x^4} G_{\rho n}^a(\alpha x) \text{Tr } \gamma_\mu (1 + \gamma_5) \sigma_{un} G(0, -x) e^{iqx} \right] \end{aligned} \quad (11)$$

The correlator $G(0, \pm x)$ entering the expression above is non-perturbative and nonlocal but can be related to the wave function of the forming pion. Following [6] we expand it near the light cone ($x^2 \sim 0$). The general form of expansion is

$$\begin{aligned}
Tu(0)\bar{d}(x) &= \sum_n C_n (x^2 - i\epsilon) \Gamma_\alpha x^{\mu_1} \dots x^{\mu_n} (\bar{d} \Gamma^\alpha D_{\mu_1} \dots D_{\mu_n} u) \\
&+ \sum_{n,m} C_{nm} (x^2 - i\epsilon) \Gamma_\alpha x^{\mu_1} \dots x^{\mu_n} (\bar{d} \Gamma^\alpha D_{\mu_1} \dots D_{\mu_m} A_{\mu_{m+1}} \dots A_{\mu_n} u)
\end{aligned} \quad (12)$$

with Γ_α being the full set of Dirac matrices. We would like to note, however, that since $G(0, x)$ (or $G(x, 0)$) contributes to the trace, only γ_α and $\gamma_5 \gamma_\alpha$ terms enter the final expression for $G(0, x)$ ($G(x, 0)$). In Eq. (12), the non-gauge invariant terms are exactly zero by the FS gauge condition (9) which allows us to use an analysis similar to [6]. Moreover, in this gauge $x^\mu D_{\mu_i} \equiv x^\mu \partial_{\mu_i}$ in (12). Thus,

$$\langle \pi | Tu(x) \bar{d}(0) | 0 \rangle = \sum_n C_n \Gamma_\alpha x^{\mu_1} \dots x^{\mu_n} \langle \pi | \bar{d} \Gamma^\alpha D_{\mu_1} \dots D_{\mu_n} u | 0 \rangle \quad (13)$$

Parameterizing unknown matrix elements in the conventional form

$$\langle \pi | \bar{d} \Gamma^\alpha D_{\mu_1} \dots D_{\mu_n} u | 0 \rangle = B_n q^\alpha q_{\mu_1} \dots q_{\mu_n} \quad (14)$$

where q is a momentum of the pion we arrive at the simple form for the G -correlators

$$\begin{aligned}
\langle \pi | Tu(0) \bar{d}(x) | 0 \rangle &= (\Gamma_\alpha q^\alpha) \int dz e^{izxq} \phi(z) \\
\langle \pi | Tu(0) \bar{d}(-x) | 0 \rangle &= (\Gamma_\alpha q^\alpha) \int dz e^{i(1-z)xq} \phi(z)
\end{aligned} \quad (15)$$

with the function $\phi(z)$ introduced through its moments

$$\int dz z^n \phi(z) = (-i)^n n! C_n B_n \quad (16)$$

The zeroth moment gives the overall normalization of $\phi(z)$: $\int dz \phi(z) = f_\pi / (2\sqrt{N_c})$ with f_π being the decay constant of π -meson. Inserting (15) into the (6) we obtain

$$\begin{aligned}
-i\mathcal{P}_{1\mu} &= \frac{g_s}{\pi^2} \int dz d^4 x \frac{\phi(z)}{x^4} e^{izxq} \left\{ x_\mu q \cdot A^b(x) - A_\mu^b(x) (x \cdot q) - i\epsilon_{st\mu\alpha} A_s^b(x) x_t q_\alpha \right\} \\
-i\mathcal{P}_{2\mu} &= -\frac{g_s}{\pi^2} \int dz d^4 x \frac{\phi(z)}{x^4} e^{i(1-z)xq} \left\{ x_\mu q \cdot A^b(x) - A_\mu^b(x) (x \cdot q) + i\epsilon_{st\mu\alpha} A_s^b(x) x_t q_\alpha \right\}
\end{aligned} \quad (17)$$

This gives for the octet amplitude of $B \rightarrow D\pi$ transition:

$$\begin{aligned}
\mathcal{M}_8^{(1)} &= M_1 + M_2 = \langle D | \bar{c}(0) \gamma_\mu (1 + \gamma_5) t_a \left[\langle \pi | \bar{d} \gamma^\mu (1 + \gamma_5) u | 0 \rangle_8^a \right] b(0) | B \rangle \\
&= \frac{ig_s}{\pi^2} \int dz d^4 x \frac{\phi(z)}{x^4} e^{izxq} \left[x_\mu q_\nu - (x \cdot q) g_{\mu\nu} - i\epsilon_{\mu\nu\alpha\beta} x_\alpha q_\beta \right] \langle D | \bar{c}(0) \Gamma_\mu A_\nu(x) b(0) | B \rangle \\
&- \frac{ig_s}{\pi^2} \int dz d^4 x \frac{\phi(z)}{x^4} e^{i(1-z)xq} \left[x_\mu q_\nu - (x \cdot q) g_{\mu\nu} + i\epsilon_{\mu\nu\alpha\beta} x_\alpha q_\beta \right] \langle D | \bar{c}(0) \Gamma_\mu A_\nu(x) b(0) | B \rangle
\end{aligned} \quad (18)$$

where $\bar{\Gamma} = \gamma_\mu (1 + \gamma_5)$ and $t^a A^a(x) = A(x)^1$.

¹ Note that the transferred momentum q can be written as $q = En$ where n is a null vector (as in the approximation of collinear u and d -quarks) and E is energy transferred to the light quark system (it is large in the kinematical limit under consideration). Observe that contrary to [2] there are terms in (18) which are not proportional to $n \cdot A$.

This amplitude is also nonlocal, so we use the same idea as in Eq. (12) expanding it around $x^2 \sim 0$ and introducing “generating functions”. It is clear that the only matrix element to be parameterized is

$$\langle D|\bar{c}(0)\bar{\Gamma}_\mu A_\nu(x)b(0)|B\rangle \quad (19)$$

In FS gauge one can expand $A_\mu(x)$ about $x \approx 0$ using “gauge invariant” decomposition [5]

$$A_\mu(x) = \sum_{n=0}^{\infty} \frac{1}{(n+2)n!} x^{\nu_1} \dots x^{\nu_n} x^\rho \left(D_{\nu_1} \dots D_{\nu_n} G_{\rho\mu}(0) \right) \quad (20)$$

This implies the matrix elements

$$\begin{aligned} \langle D|\bar{c}(0)\bar{\Gamma}_\mu A_\nu(x)b(0)|B\rangle &= \frac{i}{2g_s} \sum_{n=0}^{\infty} \frac{1}{(n+2)n!} x^{\nu_1} \dots x^{\nu_n} x^\rho \langle D|\bar{c}(0)\bar{\Gamma}_\mu D_{\nu_1} \dots D_{\nu_n} [D_\rho, D_\nu] b(0)|B\rangle \\ &= \frac{i}{2g_s} \sum_{n=0}^{\infty} \frac{1}{(n+2)n!} x^{\nu_1} \dots x^{\nu_n} x^\rho M_{\mu\nu_1 \dots \nu_n \rho\nu} \end{aligned} \quad (21)$$

In order to parameterize the matrix elements appearing in the expression above we note that there are a total of $n+3$ indices and that the whole expression is anti-symmetric with respect to $\rho \leftrightarrow \nu$ interchange. Also, there are only two vectors (or their combinations) available, so we choose them to be p_B and q . This leads to

$$\begin{aligned} M_{\mu\nu_1 \dots \nu_n \rho\nu} &= \sum_{i=0}^n a_i p_{B\mu} p_{B\nu_1} \dots p_{B\nu_i} q_{\nu_{i+1}} \dots q_{\nu_n} (p_{B\rho} q_\nu - p_{B\nu} q_\rho) \\ &+ \sum_{i=0}^n b_i q_\mu p_{B\nu_1} \dots p_{B\nu_i} q_{\nu_{i+1}} \dots q_{\nu_n} (p_{B\rho} q_\nu - p_{B\nu} q_\rho) \end{aligned} \quad (22)$$

with a_i and b_i being unknown coefficients. Performing the contraction and dropping terms proportional to q^2 and x^2 we have for the first matrix element in (18)

$$\begin{aligned} M_1 &= -\frac{1}{2\pi^2} \int d^4x dz \frac{\phi(z)}{x^4} e^{izxq} \sum_{n=0}^{\infty} \sum_{i=0}^n \frac{1}{(n+2)n!} \\ &\times \left\{ -2a_i (p_B q) (p_B x) (xq) - b_i (xq)^2 (p_B q) + a_i m_B^2 (xq)^2 + b_i (xq)^2 (p_B q) \right\} (p_B x)^i (xq)^{n-i} \end{aligned} \quad (23)$$

Note that in this limit the dependence of the matrix element on b_i cancels out. These sums can be parameterized in terms of some distribution function $f(n, \nu)$. Using the binomial formula, we obtain

$$\sum_{i=0}^n a_i (p_B x)^i (xq)^{n-i} = \int (\nu p_B x + xq)^n f(n, \nu) d\nu, \quad \int d\nu \nu^i f(n, \nu) = \frac{a_i}{C_n^i} \quad (24)$$

where C_n^i is a binomial coefficient. This gives for the matrix element

$$M_1 = -\frac{1}{2\pi^2} \int d^4x dz d\nu e^{izxq} \left[m_B^2 (xq)^2 - 2(p_B q) (p_B x) (xq) \right] \sum_n \frac{(\nu p_B x + xq)^n}{(n+2)n!} f(n, \nu) \quad (25)$$

Performing resummation with respect to n we arrive at

$$\sum_n \frac{(\nu p_B x + xq)^n}{(n+2)n!} f(n, \nu) = \int d\alpha \psi(\alpha, \nu) e^{i\alpha(\nu p_B x + xq)}, \quad \int d\alpha \alpha^n \psi(\alpha, \nu) = \frac{(-i)^n}{n+2} f(n, \nu) \quad (26)$$

This results in

$$M_1 = -\frac{1}{2\pi^2} \int dz d\nu d\alpha \phi(z) \psi(\alpha, \nu) \left[m_B^2 q_\mu q_\nu - 2(p_B q) p_{B\mu} q_\nu \right] \int d^4 x \frac{x_\mu x_\nu}{x^4} e^{i(\alpha \nu p_B + (\alpha + z)q)x} \quad (27)$$

Similarly, for the second matrix element one obtains

$$M_2 = \frac{1}{2\pi^2} \int dz d\nu d\alpha \phi(z) \psi(\alpha, \nu) \left[m_B^2 q_\mu q_\nu - 2(p_B q) p_{B\mu} q_\nu \right] \int d^4 x \frac{x_\mu x_\nu}{x^4} e^{i(\alpha \nu p_B + (1 + \alpha + z)q)x} \quad (28)$$

At this stage we have classified the infinite number of the unknown matrix elements in terms of a few continuous functions. The strategy at this point in general would be to consider the structure of these functions and make reasonable smooth guesses as to their form. For example $\phi(z)$ is related to wavefunction used in the studies of the perturbative limit of the pion form factor and has the asymptotic form $\phi(z) = 2f_\pi z(1-z)/\sqrt{6}$ [6]. However, we find that an important cancellation occurs which is independent of the form of these structure functions. Taking $a_\beta = (\alpha + z)q_\beta + \alpha \nu p_{B\beta}$, $b_\beta = (1 + z + \alpha)q_\beta + \alpha \nu p_{B\beta}$ and integrating over x we obtain for the $\mathcal{M}_8^{(1)}$:²

$$\begin{aligned} \mathcal{M}_8^{(1)} = & -i \int d\nu d\alpha \psi_1(\alpha, \nu) \int dz \phi(z) \left\{ \frac{1}{a^4} \left[4(p_B q)(p_B a)(q a) - 2m_B^2(q a)^2 - 2a^2(p_B q)^2 \right] \right. \\ & \left. - \frac{1}{b^4} \left[4(p_B q)(p_B b)(q b) - 2m_B^2(q b)^2 - 2b^2(p_B q)^2 \right] \right\} = 0 \end{aligned} \quad (29)$$

at the leading order in q^2 for any ϕ and ψ . This implies that the semi-soft-one-gluon matrix elements do not bring new contributions to the factorization approximation result in the kinematical limit under consideration.

We would like to point out that the two-pseudoscalar final state is somewhat special: for instance, $B \rightarrow D\rho$ does not fall into the same category - there is a good chance that under the similar conditions there is no complete cancellation of the diagrams. This is due to the fact that there can be extra terms in (22) that include polarization vectors of ρ meson.

In conclusion, we have constructed a QCD-based model for the factorization approximation in the limit $E \sim m_{bm_b} \rightarrow \infty$. In the framework of the model the complete cancellation of the non-factorizable terms is observed at the leading order in q^2 , x^2 . The OPE is resummed by introducing two generating (distribution) functions that can be independently parameterized. The model gives a consistent way to estimate non-leading (non-factorizable) contributions.

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² The key integral is $I_{\mu\nu} = -\frac{\partial}{\partial a_\mu} \frac{\partial}{\partial a_\nu} \int d^4 x \frac{e^{iax}}{x^4} = 2i\pi^2 \left(-2\frac{a_\mu a_\nu}{a^4} + \frac{g_{\mu\nu}}{a^2} \right)$.