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Global anomalies and the gauge-boson equivalence theorem

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Abstract

We discuss the various resolutions which have been suggested in the literature for the way that the equivalence theorem can be satisfied in theories with global anomalies. We provide a model-independent proof for the resolution originally suggested in the context of a toy model by Kilgore.

The equivalence theorem for gauge bosons [1] states that high-energy matrix elements involving longitudinal gauge bosons are equal to similar matrix elements involving the (pseudo-) scalar Goldstone bosons of the theory, up to corrections of order $M_W/\sqrt{s}$. In a previous paper [2] we raised a concern about the equivalence theorem in theories with global anomalies, in which the anomalies are required to cancel in gauge currents but not in the coupling of the Goldstone bosons. From this we argued that, for example, the $\gamma^* \gamma Z_L$ coupling vanished, but that the $\gamma^* \gamma z$ coupling did not, where z is the Goldstone boson absorbed by the Z boson. (These could in principle be compared in the reactions $e^+ e^- \rightarrow \gamma Z_L$ and $e^+ e^- \rightarrow \gamma z$ at a high energy.) Two papers written in response proposed diametrically opposite solutions. Pal [3] has argued that the equivalence theorem is restored in this example by the vanishing of the $\gamma^* \gamma z$ coupling, so that the anomaly generates neither process. On the other hand, Kilgore [4] used a quark model for chiral symmetry breaking to argue that at the relevant energies both the $\gamma^* \gamma Z_L$ and $\gamma^* \gamma z$

couplings were nonvanishing (and equivalent). In this paper we revisit the issue and provide a model-independent derivation which supports Kilgore's result. This resolution has the interesting consequence of allowing unexpected anomaly-generated vertices of gauge bosons.

To start out, let us define a toy model where there are anomalous couplings of the Goldstone bosons, but the gauge currents have no anomalies. This requires that the Goldstone bosons be formed from only a subset of the fermions in the theory, and that other fermions exist for cancelling anomalies in the gauge currents. The analogy is with the low-energy sector of the Standard Model, where both quarks and leptons are coupled to the gauge currents, but only quarks form the pseudo-Goldstone bosons (pions). For the toy model here we consider a doublet of techniquarks and a doublet of leptons, all of which are involved in electroweak interactions. The techniquarks are also involved in technicolor interaction, which is taken to be QCD-like, i.e. confining and with dynamical symmetry breaking. If the quantum numbers of the fermions are chosen to be the same as those of the usual single family of quarks and leptons, then the Goldstone bosons will have the

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same anomalous couplings to γ , W , Z as does the pion in QCD. However, the leptons cancel all anomalies in the gauge currents. All the fermions are strictly massless, as to introduce a mass would require a new set of interactions and would seriously complicate the analysis.

In such a theory there is a window of energies where it is useful to apply the equivalence theorem. At low energies, the theorem is not useful because the corrections, of order $M_W/\sqrt{s}$, are too large, so this fact requires $s \gg M_W^2$. At the high-energy end, at energies much larger than the technicolor scale, the dynamics becomes that of the techniquarks rather than of the bound technihadrons, so that while the equivalence theorem may still be applied, we lack the tools to predict the Goldstone-boson amplitudes. (The analogy would be trying to predict pionic amplitudes in QCD at energies $E \gg 1$ GeV.) For intermediate energies, $(250 \text{ GeV})^2 \lesssim s \lesssim M_{\rho_T}^2$, where $M_{\rho_T} \simeq 2$ TeV is the technirho mass, one may use the techniques of chiral perturbation theory to easily predict Goldstone-boson amplitudes in this type of technicolor model. We therefore restrict our comments to energies high enough that the $\mathcal{O}(M_W/\sqrt{s})$ corrections can be neglected, but low enough that we can use effective Lagrangians to describe the Goldstone-boson couplings.

In this toy variant of technicolor theories, we expect that the neutral Goldstone boson, the technipion π_T^0 , would have a coupling to two photons, $\pi_T^0 \rightarrow \gamma\gamma$, similar to that which occurs in QCD and given by

$$\mathcal{M}_{\pi_T^0 \rightarrow \gamma\gamma} = -\frac{e^2 N_{TC}}{12\pi^2 F_{\pi_T}} \epsilon^{\mu\nu\alpha\beta} q_\mu q'_\nu \epsilon_\alpha^*(q) \epsilon_\beta^*(q'), \quad (1)$$

where $F_{\pi_T} \simeq 250 \text{ GeV}$ is the technipion decay constant and N_{TC} is the number of technicolors in an $SU(N_{TC})$ theory. For this toy model $N_{TC} = 3$. There are also related $\pi_T^0 \rightarrow \gamma Z$, ZZ couplings. One cannot apply the equivalence theorem in decay processes such as $\pi_T^0 \rightarrow \gamma\gamma$ (note that $Z_L \rightarrow \gamma\gamma$ is forbidden by Yang's theorem [5]) because the energy involved is too low. However, one can use this vertex within a high-energy diagram such as $e^+e^- \rightarrow \gamma^*$, $Z^* \rightarrow \gamma\pi_T^0$ or $\nu\bar{\nu} \rightarrow Z^* \rightarrow \gamma\pi_T^0$. These reactions involve an off-shell photon or Z . There will then be higher-order corrections to the amplitude (1) which depend on the $q^2 \equiv s$ of this off-shell particle, and we can represent

the change in the amplitude by a form factor $F(s)$, with $F(0) = 1$. In the QCD analogy, this form factor has been measured in the reactions $\gamma^*\gamma \rightarrow \pi^0$, η studied at e^+e^- colliders [6], and, not surprisingly, it varies with s , with a scale around 1 GeV. (More accurately, it behaves roughly as $1/(1 - s/M_V^2)$ where $M_V = M_\rho$, with M_ρ being the ρ -meson mass.) In the technicolor case, one would expect that the form factor would vary in a similar way, with the relevant scale being the mass of the technirho. This leads to an enhancement of the rate at the energies which we are considering since $F(s) > 1$ for s timelike.

The anomalous gauge couplings $\gamma^*\gamma Z_L$ and $Z^*\gamma Z_L$ do not occur at tree level, and naively one would expect them not to occur at all since all anomalies cancel in the gauge currents. However, since the π_T^0 is the Goldstone boson associated with the Z , this would contradict the equivalence theorem. In order to resolve the apparent contradiction, either the $\gamma^*\gamma\pi_T^0$ vertex has to be shown to vanish or the $\gamma^*\gamma Z_L$ vertex should be nonvanishing³.

The paper by Pal [3] claims that the first of the above possibilities is correct, i.e. that the $\gamma^*\gamma\pi_T^0$ vertex vanishes for a virtual photon with $q^2 \neq 0$. The key step in his argument is a representation of the triangle diagram for a free techniquark of mass m with one photon off-shell, at $p_{\pi_T}^2 = 0$, where p_{π_T} is the momentum of the technipion,

$$\mathcal{M}_{\pi_T^0 \rightarrow \gamma^*\gamma} = -\frac{e^2}{4\pi^2 F_{\pi_T}} \epsilon^{\mu\nu\alpha\beta} q_\mu q'_\nu \epsilon_\alpha^*(q) \epsilon_\beta^*(q') \times \left(1 + 2q^2 \int_0^1 dz \int_0^{1-z} dz' \frac{zz'}{m^2 - q^2 zz'} \right). \quad (2)$$

The idea is that at $m \neq 0$, $q^2 = 0$ (as in the case of QCD with real photons) the factor of unity reproduces the usual $\pi_T^0 \rightarrow \gamma\gamma$ result, but that at $m = 0$, $q^2 \neq 0$ the amplitude vanishes. However, it is important to note that this result is appropriate only for free, unconfined fermions. The integral has a cut starting at $q^2 = 4m^2$, which corresponds to the productions of techniquark pairs. For QCD-like theories, this representation is contradicted by the known measurements

³ In the rest of the paper we will deal with only the $\gamma^*\gamma Z_L$ and $\gamma^*\gamma\pi_T^0$ vertices because the same discussion could be given for the $Z^*\gamma Z_L$ and $Z^*\gamma\pi_T^0$ vertices.

of the form factor of the anomaly mentioned above. In QCD, the representation of Eq. (2) predicts that at $q^2 > m^2 = m_q^2$ (with $m_q \sim 10 \text{ MeV}$) the anomaly amplitude should fall off and quickly vanish as q^2 increases, i.e. that the scale of the q^2 variation of the form factor is the light-quark mass m_q . This is experimentally not correct, and the theoretical reason for this is easily understood. In a confining theory such as QCD, there is no long-range propagation of quarks, so there is little variation of the amplitude for $q^2 \simeq m_q^2$ when m_q is small compared to the energy scale of the theory. The relevant energy scale is that of the bound states, which in this channel are the vector mesons. The representation (2) is not appropriate for a confining theory, so that it does not apply to our technicolor theory.

This conclusion can be demonstrated more explicitly by a consideration of the effective Lagrangian for the anomaly. This yields an expansion of the effective action in powers of q^2 and m_q , and can be used to show that there is no problem with the $m_q \rightarrow 0$ limit. An effective Lagrangian must contain all of the fields that are light at the energy scale under discussion. In a free quark theory this would include the quark fields, but in a confining theory the quarks are integrated out and only the pion fields enter the effective Lagrangians. At lowest order the effect of the anomaly is contained in the gauged Wess-Zumino anomaly Lagrangian [8–11]. This will be written more fully later in this paper, but the relevant terms for the present discussion are

$$\begin{aligned} \mathcal{L}_{\text{WZ}} = & -\frac{ie^2}{8\pi^2} \epsilon^{\mu\nu\alpha\beta} \partial_\mu A_\nu A_\alpha \\ & \times \text{Tr} \left[Q^2 (\partial_\beta U U^\dagger + U^\dagger \partial_\beta U) \right. \\ & \left. + \frac{1}{2} (Q \partial_\beta U Q U^\dagger - Q \partial_\beta U^\dagger Q U) \right], \end{aligned} \quad (3)$$

where $U = \exp(i\tau^a \pi^a / F_\pi)$, with F_π being the pion decay constant and τ^a ($a = 1, 2, 3$) being Pauli matrices, and Q is the quark-charge matrix. The coefficient is fixed and independent of the quark mass. At next order there are many different Lagrangians which modify $\pi^0 \rightarrow \gamma\gamma$. These are not technically anomalous [7], but they do involve the $\epsilon_{\mu\nu\alpha\beta}$ tensor. Examples are

$$\begin{aligned} \mathcal{L}_\epsilon^{(6)} = & \frac{ie^2}{\Lambda_\chi^2} \epsilon^{\mu\nu\alpha\beta} \partial^\lambda F_{\lambda\mu} F_{\alpha\beta} \\ & \times \left[c_1 \text{Tr}(Q^2 U^\dagger \partial_\nu U - Q^2 U \partial_\nu U^\dagger) \right. \\ & \left. + c_2 \text{Tr}(Q U^\dagger Q \partial_\nu U - Q U Q \partial_\nu U^\dagger) \right] \\ & + \frac{ie^2}{\Lambda_\chi^2} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \\ & \times \left[c_3 \text{Tr} Q^2 \text{Tr}(m_q U - m_q U^\dagger) \right. \\ & \left. + c_4 \text{Tr}(Q^2 m_q U^\dagger - Q^2 m_q U) \right], \end{aligned} \quad (4)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The coefficients are also independent of q^2 and m_q , as all of the energy and mass dependence is accounted for explicitly in the construction of $\mathcal{L}_\epsilon^{(6)}$. The energy scale Λ_χ in these coefficients is of order the QCD scale, $\Lambda_\chi \sim M_\rho$. The matrix elements for off-shell photons can be read off of these Lagrangians. Since the coefficients are independent of m_q and there are no light particles in the theory which need to be accounted for (aside from the pions), the amplitude is smooth in the limit $m_q \rightarrow 0$.

The opposite resolution, that the $\gamma^* \gamma Z_L$ coupling is nonzero, has been suggested by Kilgore [4]. He employs a quark model in which, by a transformation on the original techniquark fields, one can generate constituent techniquarks with a large mass M_Q coupled to technipions. The anomalous $\gamma^* \gamma \pi_T^0$ coupling is determined by the techniquarks. However, in the $\gamma^* \gamma Z_L$ coupling Kilgore found that the massive techniquarks do not contribute at low energies, $M_Z^2 \ll s \ll M_Q^2$, while the leptons still do, yielding a net $\gamma^* \gamma Z_L$ coupling whose magnitude is the same as the $\gamma^* \gamma \pi_T^0$ vertex.

We would like to demonstrate that this resolution is correct, in a model-independent way. This can be accomplished by showing that, after the techniquarks have been integrated out, the $\gamma^* \gamma Z_L$ vertex comes from either both the lepton sector and the anomalous effective Lagrangian or the lepton sector only, depending on the renormalization prescription one uses. In both cases the $\gamma^* \gamma \pi_T^0$ vertex is unchanged, and is equivalent to the $\gamma^* \gamma Z_L$ vertex.

The full effective Lagrangian describing the effect of anomalies is determined by anomalous Ward identities and was first given in a power-series representation by Wess and Zumino [8]. Subsequently, Witten [9]

showed (in the context of QCD) how the pionic portion could be represented as an integral over a five-dimensional space, and also gave the four-dimensional anomalous coupling of the electromagnetic field with pions. Several authors [10,11] have corrected an error in Witten's form and have given the full anomalous Lagrangian describing the couplings of the gauge fields. The total result is very lengthy, but the portion containing one pseudoscalar meson and two electroweak gauge bosons is, in the context of technicolor,

$$\begin{aligned} \mathcal{L}_{WZ} = & -\frac{iN_{TC}}{48\pi^2} \epsilon^{\mu\nu\alpha\beta} \\ & \times \text{Tr}[(\partial_\mu \ell_\nu \ell_\alpha + \ell_\mu \partial_\nu \ell_\alpha)(\partial_\beta U U^\dagger - iU r_\beta U^\dagger) \\ & + (\partial_\mu r_\nu r_\alpha + r_\mu \partial_\nu r_\alpha)(U^\dagger \partial_\beta U + iU^\dagger \ell_\beta U) \\ & - \partial_\mu \ell_\nu \partial_\alpha U r_\beta U^\dagger + \partial_\mu r_\nu \partial_\alpha U^\dagger \ell_\beta U], \end{aligned} \quad (5)$$

where

$$\begin{aligned} \ell_\mu = & eQA_\mu + \frac{e}{c_w s_w}(T_3 - s_w^2 Q)Z_\mu \\ & + \frac{e}{2s_w}(\tau_1 W_{1\mu} + \tau_2 W_{2\mu}), \\ r_\mu = & eQA_\mu - \frac{e s_w}{c_w} QZ_\mu, \end{aligned} \quad (6)$$

with $s_w = \sin \theta_w$ (θ_w is the Weinberg angle) and $c_w = \sqrt{1 - s_w^2}$, and now $U = \exp(i\tau^a \pi_T^a / F_{\pi_T})$.

The total expression for $\mathcal{L}_{WZ}$ corresponds to an anomalous effective action whose variation under an infinitesimal chiral gauge transformation yields the so-called left-right (LR) form of the (non-Abelian) anomaly [10,11]. In deriving the LR anomaly, one employs a renormalization prescription in which the non-Abelian left- and right-handed currents are treated symmetrically, and so both chiral currents have anomalous divergences. $\mathcal{L}_{WZ}$ contains a $\gamma^* \gamma Z_L$ coupling, which is to be added to the contribution of the leptons to the $\gamma^* \gamma Z_L$ vertex. The coupling is given by

$$\begin{aligned} \mathcal{M}_{Z_L \rightarrow \gamma^* \gamma}^{WZ} = & -\frac{ie^3}{12\pi^2 c_w s_w M_Z} \epsilon^{\mu\nu\alpha\beta} q_\mu q'_\nu \epsilon_\alpha^*(q) \epsilon_\beta^*(q') \\ & + \mathcal{O}(M_Z/E_Z). \end{aligned} \quad (7)$$

In this renormalization scheme, the left- and right-handed leptons contribute separately to the $\gamma^* \gamma Z_L$ vertex. Each lepton triangle involves chiral currents at

each vertex and is required to be symmetric under interchange of any pair of vertices [12]. One then gets

$$\begin{aligned} \mathcal{M}_{Z_L \rightarrow \gamma^* \gamma}^{\text{lepton}} = & -\frac{ie^3}{24\pi^2 c_w s_w M_Z} \epsilon^{\mu\nu\alpha\beta} q_\mu q'_\nu \epsilon_\alpha^*(q) \epsilon_\beta^*(q') \\ & + \mathcal{O}(M_Z/E_Z). \end{aligned} \quad (8)$$

In both (7) and (8) we have used the fact that $\epsilon_\mu^{Z_L}(p) \simeq p_\mu/M_Z + \mathcal{O}(M_Z/E_Z)$ for $E_Z \gg M_Z$. With the replacement $M_Z = eF_{\pi_T}/2c_w s_w$, we see that at these energies the sum of the amplitudes in (7) and (8) is equal in magnitude to $\mathcal{M}_{\pi_T^0 \rightarrow \gamma^* \gamma}$, given by the right-hand side of Eq. (1).

The same result can be derived using the Bardeen form of the anomaly [13,10], and the way that this is accomplished is interesting. This scheme is equivalent to the regularization employed in Kilgore's model. In the Bardeen form, one requires that the non-Abelian vector currents be conserved, with the divergence of the non-Abelian axial-vector currents containing an anomaly. In this renormalization scheme, $\mathcal{L}_{WZ}$ has a form which differs from that in the LR scheme by a polynomial of the gauge fields and their derivatives. This can be expressed in terms of the corresponding effective actions as [10]

$$\Gamma_{WZ}^B(U, \ell, r) = \Gamma_{WZ}^{LR}(U, \ell, r) - \Gamma_{WZ}^{LR}(U = 1, \ell, r). \quad (9)$$

The result does not contain triple-gauge-boson vertices, so that the analog of Eq. (7) vanishes and the low-energy $\gamma^* \gamma Z_L$ vertex is determined by the leptons alone. However, the calculation of the lepton-triangle vertex is different in this scheme. (This is the calculation usually illustrated in textbooks.) Each lepton triangle has one axial-vector and two vector vertices, and is evaluated by imposing vector-current conservation. The result is three times larger than Eq. (8). The total $\gamma^* \gamma Z_L$ vertex at these energies is then the same as found previously, and agrees with Eq. (1).

In either form, the anomaly in the divergences of currents coupled to the electroweak gauge fields in the theory vanishes, and hence does not disrupt gauge invariance, because contributions from the strong and lepton sectors cancel, as stated earlier. This is due to the familiar fact that, for the fermion representation

considered, $\text{Tr}(T_a\{T_b, T_c\}) = 0$, where T_a , T_b , and T_c are the generators of the electroweak gauge group.

We have shown, in a model-independent way, that in a theory with global anomalies the triple-gauge-boson couplings due to the anomalies are nonvanishing at low energies and consistent with the corresponding couplings involving the Goldstone bosons, thereby satisfying the equivalence theorem, in agreement with Kilgore's result. This resolution implies the interesting possibility of having anomaly-generated gauge-boson vertices in models of dynamical electroweak symmetry breaking. In studies on the effects of new, heavy physics on gauge-boson self-interactions, it is usually assumed that there is no contribution caused by anomalies. However, there is no requirement that forbids the presence of global anomalies due to new physics, provided that there are no anomalies in the gauge currents in the theory. Hence there may be TeV-scale theories of electroweak symmetry breaking which do contain such anomalies. These could then generate contributions to gauge-boson vertices which might have detectable consequences.

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