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One of the coefficients that parametrizes the chiral Lagrangian of QCD is expressible in terms of a sum rule for which the integrand is known from experiment. We confront two model calculations of this coefficient with the behavior of the data. The model related to vector dominance reproduces the sum-rule behavior very well, whereas a one-loop chiral quark model calculation fails badly. The sum rule allows one to identify those ingredients which are needed to make the quark method more realistic.

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There are several different facets to the applications of chiral symmetry [1]. The most general aspect is chiral perturbation theory (ChPT), which makes no assumption beyond the existence of chiral symmetry [2]. This leads to chiral connections between low-energy reactions, parametrized by phenomenological coefficients in the effective Lagrangian, such as F_π and the Gasser-Leutwyler coefficients L_i ($i = 1, \dots, 10$) [3]. Another aspect which has been actively studied is the use of specific dynamical models to predict these chiral coefficients [4]. Although this gives up the rigor of the pure ChPT approach, it helps to increase our understanding of the fundamental dynamics which underlies the chiral Lagrangian of QCD. There are two types of models which have had some reasonable results. The most successful is a generalization of vector dominance, which says that the prime physical ingredient to the energy expansion is the momentum dependence generated by low-lying resonances, most notably the $\rho(770)$ and the $a_1(1260)$ [5]. The other model which comes reasonably close to reproducing the experimentally known coefficients involves quarks coupled to pions with the energy dependence being that of the free quark loop diagram [6]. For example, for the coefficient studied in this paper, the data and model predictions are

$$L_{10} = \begin{cases} -(5.2 \pm 0.3) \times 10^{-3} & (\text{expt } \mu = m_\eta), \\ -5.8 \times 10^{-3} & (\text{vectors}), \\ \frac{-N_c}{98\pi^2} = -3.2 \times 10^{-3} & (\text{quark loop}). \end{cases} \quad (1)$$

The experimental measurement comes from the form factors of radiative pion decay [7]. The present investigation was motivated by the hope that the two models might contain some of the same physics. After all, the $Q\bar{Q}$ pair which appears in the loop is bound by QCD to form a ρ meson. Perhaps some of this relationship could be seen in the details of the calculation. The coefficient L_{10} is especially suited for this comparison because it satisfies a

dispersion relation for which the integrand is well measured by experiment and which can be explicitly calculated in the models. Unfortunately, as we show below, the quark-loop model seems to have very little in common with either the data or the vector-dominance model, although the latter two correspond quite closely. We can learn from this exercise what the proper ingredients to L_{10} really are, and decide which features must be added to the quark-loop model to make it more realistic.

The chiral Lagrangian is written as an expansion in powers of the energy E , with present applications retaining the effects at order E^2 and E^4 . The terms in the most general $SU(2)_L \times SU(2)_R$ Lagrangian which are relevant for our study are [8]

$$\begin{aligned} L &= L_2 + L_4, \\ L_2 &= \frac{F^2}{4} \text{Tr}(D_\mu U D^\mu U^\dagger) + \dots, \\ L_4 &= L_{10} \text{Tr}(L_{\mu\nu} U R^{\mu\nu} U^\dagger) + \dots, \end{aligned} \quad (2)$$

with

$$\begin{aligned} U &= \exp \left[i \frac{\tau \cdot \pi}{F} \right], \\ D_\mu U &= \partial_\mu U - i l_\mu U + i U r_\mu, \\ L_{\mu\nu} &= \partial_\mu l_\nu - \partial_\nu l_\mu + [l_\mu, l_\nu], \\ R_{\mu\nu} &= \partial_\mu r_\nu - \partial_\nu r_\mu + [r_\mu, r_\nu], \\ l_\mu &= l_\mu^i \frac{\tau^i}{2}, \quad r_\mu = \frac{r_\mu^i \tau^i}{2}. \end{aligned} \quad (3)$$

These Lagrangians describe the couplings of the pions, π^i , $i = 1, 2, 3$, to each other and to external left- and right-handed currents l_μ^i, r_μ^i . After renormalization, the constant F becomes the pion decay constant $F_\pi = 92.3 \text{ MeV}$ [9].

We are interested in the $V - A$ current matrix ele-

ments which follow at low energy. Defining¹

$$\delta_{ij}\Pi_{\mu\nu}^{V-A}(q)=i\int d^4x e^{iq\cdot x}\langle 0|T(V_\mu^i(x)V_\nu^j(0)-A_\mu^i(x)A_\nu^j(0))|0\rangle \quad (4)$$

with

$$\Pi_{\mu\nu}^{V-A}=-F_\pi^2\left[g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2-m_\pi^2}\right]-(q_\mu q_\nu-g_{\mu\nu}q^2)\left\{4L'_{10}+\frac{1}{48\pi^2}\left[\ln\frac{m_\pi^2}{\mu^2}+\frac{1}{3}-H(q^2)\right]\right\}+\cdots,$$

$$H(s)=z^3\left[z\ln\left[\frac{z-1}{z+1}\right]+2\right],$$

with

$$z=\left[1-\frac{4m_\pi^2}{s}\right]^{1/2}. \quad (6)$$

The vector and axial-vector vacuum polarization functions are analytic functions, whose imaginary parts are the spectral densities ρ_V, ρ_A :

$$(q_\mu q_\nu-g_{\mu\nu}q^2)\rho_V(q^2)=\frac{1}{2\pi}\int d^4x e^{iq\cdot x}\langle 0|V_\mu(x)V_\nu(0)|0\rangle,$$

$$(q_\mu q_\nu-g_{\mu\nu}q^2)\rho_A(q^2)+q_\mu q_\nu\rho_A^{(0)}(q^2)=\frac{1}{2\pi}\int d^4x e^{iq\cdot x}\langle 0|A_\mu(x)A_\nu(0)|0\rangle, \quad (7)$$

with the relation

$$\frac{1}{\pi}\text{Im}\Pi^{V-A}(q)=\rho_V(q)-\rho_A(q). \quad (8)$$

The asymptotic behavior of ρ_V and ρ_A are determined from QCD to be [11]

$$\rho_V(s)\underset{s\rightarrow\infty}{\sim}\frac{1}{8\pi^2}\left[1+\frac{\alpha_s(s)}{\pi}\right],$$

$$\rho_V(s)-\rho_A(s)\underset{s\rightarrow\infty}{\sim}\frac{\hat{m}^2(s)}{s} \quad (9)$$

where $\hat{m}(s)$ is the running light quark current mass. This allows the use of dispersion relations to generate sum rules for the chiral coefficients:

$$F_\pi^2=\int_0^\infty ds[\rho_V(s)-\rho_A(s)], \quad (10a)$$

$$-4\bar{L}_{10}=\int\frac{ds}{s}[\rho_V(s)-\rho_A(s)]$$

where

$$\bar{L}_{10}=L_{10}^{(r)}(\mu)+\frac{1}{144\pi^2}(\ln m_\pi^2/\mu^2+1). \quad (10b)$$

This result is scale independent and has the experimental value [from Eq. (1)]

$$\bar{L}_{10}=-6.4\times 10^{-3}. \quad (11)$$

¹Purists will recognize that we are really using the covariant T^* product.

$$V_\mu^i=\frac{1}{2}(l_\mu^i+r_\mu^i),$$

$$A_\mu^i=\frac{1}{2}(l_\mu^i-r_\mu^i), \quad (5)$$

chiral perturbation theory predicts the low-energy expansion [10]

The first of these relations is the first Weinberg sum rule [12], while the second is equivalent to the Das-Mathur-Okubo sum rule [13]. The second Weinberg sum rule is only valid in QCD if the quark masses vanish, and will not be considered here. The sum rules of Eq. (10) will be our window to the physics underlying the chiral coefficients.

The spectral functions ρ_V, ρ_A can be extracted from $\tau\rightarrow\nu+\text{hadrons}$ decays, at least for energies up to $s=(m_\tau)^2$ [14]. Similar information about the ρ_V is contained in $R(e^+e^-\rightarrow\text{hadrons})$; however, in this case the $I=0$ electromagnetic current is also present and must be subtracted [15]. The high-energy limit of ρ_V , Eq. (9), is equivalent to the well-known predictions $R=\sum Q_i^2$. Figure 1 shows the extraction of $\rho_{V,A}$ from τ decay per-

FIG. 1. Experimental vector and axial-vector spectral functions ρ_V, ρ_A , from Ref. [14].

formed by Peccei and Sola [14]. Their values match smoothly to the asymptotic form near $s = 2 \text{ GeV}^2$. Small modifications in the energy range $s > 2 \text{ GeV}^2$ would have little effect on our discussion. Peccei and Sola show that the first Weinberg sum rule is approximately satisfied by the data.

The integrand for the L_{10} sum rule is shown in Fig. 2. Note that by taking the difference of ρ_V and ρ_A , the high-energy region is canceled. In addition, the factor of $1/s$ in the sum rule further suppresses large s . The result is highly convergent and depends only on the well-measured features of τ decay. Performing the sum rule results in

$$\bar{L}_{10} = -6.0 \times 10^{-3}, \quad (12)$$

in impressive agreement with the value measured in radiative π decay—Eq. (11).

It is clear from the data that vector-meson dominance will work very well, because the only significant structures in the sum-rule integrand are the $\rho(770)$ and $a_1(1260)$ resonances. In the narrow resonance approximation, we would have

$$\begin{aligned} \rho_V(s) - \rho_A(s) &= F_\rho^2 \delta(s - m_\rho^2) - F_a^2 \delta(s - m_a^2), \\ F_\pi^2 &= F_\rho^2 - F_a^2, \\ L_{10} &= \frac{F_a^2}{4m_a^2} - \frac{F_\rho^2}{4m_\rho^2}, \end{aligned} \quad (13)$$

where $F_\rho = 150 \text{ MeV}$, $F_a = (-F_\pi^2 + F_\rho^2)^{1/2}$. The result agrees reasonably with the data. The only feature needed to make this more realistic would be to allow widths for the resonances. This is a simple procedure with little numerical influence for the sum rule (there is no change to first order in Γ/m). The result is a virtually identical representative of the data, as the widths would be adjusted to fit the experimental curve.

The quark model calculation of L_{10} can be motivated in QCD by arguments of varying sophistication [16], but

FIG. 2. The integrand for the sum rule of Eq. (10) generating L_{10} .

amounts to constituent quarks coupled to pions in a chirally invariant fashion:

$$\begin{aligned} L_{QM} &= \psi(i\partial - MU_5)\psi, \\ U_5 &\equiv \frac{1}{2}(1 + \gamma_5)U^\dagger + \frac{1}{2}(1 - \gamma_5)U = \left\{ \exp \left[i \frac{\tau \cdot \pi}{F_\pi} \gamma_5 \right] \right\}. \end{aligned} \quad (14)$$

To calculate the effective pion Lagrangian, one integrates out the quarks, leaving a result in the standard Gasser-Leutwyler form with a coefficient L_{10} given in Eq. (1) [17]. This procedure is most compactly accomplished by the heat-kernel method, which yields the whole Lagrangian at once [18], (as shown in the Appendix). However, the physics behind the model is better exposed if we calculate L_{10} using the sum rule, Eq. (10). The process of integrating out the quarks corresponds to the calculation of single free quark-loop diagrams. For the correlation functions, this yields the vacuum-polarization diagram of Fig. 3. The imaginary parts are found to be

$$\begin{aligned} \rho_V(s) &= \frac{1}{8\pi^2} \left[1 - \frac{4m^2}{s} \right]^{1/2} \left[1 + \frac{2M^2}{s} \right] \theta(s - 4m^2), \\ \rho_V(s) - \rho_A(s) &= \frac{3M^2}{4\pi^2 s} \left[1 - \frac{4M^2}{s} \right]^{1/2} \theta(s - 4M^2). \end{aligned} \quad (15)$$

Several aspects of these results deserve comment. First, the high-energy behavior of $\rho_V - \rho_A$ does not vanish as fast as is required by QCD. This is due to the constituent quark mass, which must vanish at short distance scales in QCD. It might then be reasonable to disregard the high-energy tail of $\rho_V - \rho_A$. Second, we see that the curves are structureless. Despite the fact that S wave $q\bar{q}$ has the quantum numbers of a ρ , and the p wave those of a_1 , there are no interactions in the model which can produce any peaks. They are featureless, aside from a trivial threshold kinematic factor. In particular the difference $\rho_V - \rho_A$ does not change sign over the entire range of integration.

In Fig. 4, the integrand of the L_{10} sum rule is shown for the quark model. The integrated value reproduces Eq. (1), independent of M . Even though the predicted value of L_{10} is only a factor of 2 away from the correct answer, the integrand bears no resemblance to the data. Its magnitude is too small and the sum rule picks up much of its strength from high values of s , where the model should break down. If we were to cut off the integral at $s = 2 \text{ GeV}^2$, the value of L_{10} would be even smaller (i.e., $L_{10} = -2.2 \times 10^{-3}$ for $M = 330 \text{ MeV}$). We take this lack of similarity for the integrand to imply that the approximate agreement between the resultant values for L_{10} is an *accident*, and that the model does not provide an adequate representation of QCD for the dynamics of this problem.

FIG. 3. Quark-loop vacuum polarization diagram.

FIG. 4. The integrand for the sum rule of Eq. (10) in the quark model.

One possible way that this conclusion might be avoided is if there exists some deeper relation in QCD that requires that even though this model misrepresents the shape of this sum rule, the integrated magnitude is that of QCD. This might conceivably occur due to the analytic properties of the sum rule, such that if a contour is closed as $s \rightarrow \infty$, one relates the integral over the real axis to that over the contour at infinity where QCD perturbation theory can be applied. However, this seems unlikely as the model does not have the proper high-energy behavior to match QCD. The model is also similar to the $1/M$ expansion for heavy quarks in QCD sum rules which use perturbation theory for large M [19]. However, in the present context M is a light-quark mass and so $1/M$ expansion cannot be used to connect to QCD.

Having compared the model with the data, we also learn that ingredients need to be added in order to make the model correspond better to reality. Basically, we need to modify the *free* quark loop into one with resonances at the appropriate energies. This means that the binding interaction between quarks needs to be appended. We know that the quark model, with constituent quark masses, will easily produce the ρ and the a_1 when confinement is incorporated, for example, with an interparticle potential or a bag, as in Fig. 5(a). Such models also do a reasonable job of predicting the couplings of the $Q\bar{Q}$ bound states to currents, i.e., the decay constants F_ρ, F_a . Thus in such a framework, the shape of the integrand would certainly resemble the data. The difficulty comes in actually carrying out such a calculation. In the literature, the effect of the vacuum condensate of gluons has been included [20], as in Fig. 5(b). This might be con-

FIG. 5. Corrections to the free quark-loop diagram: (a) binding interaction between quark, (b) effect of vacuum gluon condensate, (c) quark self-energy.

sidered to be the beginning of a binding interaction, although the propagation of quarks in the condensate at low order in α_s is not enough in itself to provide confinement. Another suggestion in the literature is the inclusion of the p^2 -dependent mass [21], large at small p^2 and vanishing as $p^2 \rightarrow \infty$ as in Fig. 5(c). This provides an improvement in the high-energy behavior, and emphasizes low s in the integrand, but does not address the need to produce resonances in the ρ and a_1 channel. Lattice calculations *can* treat this issue by the calculation (in Euclidean space) of correlation functions such as Eq. (10). Binding corrections are automatic in such an approach, and presumably they *would* reproduce the importance of the particle spectrum in the sum-rule integrand. However, such calculations have not yet been performed.

In conclusion, we have studied the integrand of a sum rule which yields the chiral expansion parameter L_{10} . The vector dominance model contains the same physics as seen in the data, while the quark loop approach fails to reproduce the required structure and requires modification if it is to capture the correct physics. In particular, the effect of $Q\bar{Q}$ binding to form resonances needs to be incorporated. The coefficient L_{10} in QCD is related to the parameter S in the study of the origin of electroweak symmetry breaking [22]. The various estimates of S for technicolor in the literature also correspond to vector models or quark models. Our work here suggests that the methods based on the spectrum of low-lying mesons (mainly vector mesons) are more reliable than those based on quark loops.

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APPENDIX

The quark loop calculation can be performed by a number of different techniques. Most straightforward is a direct perturbative evaluation of the vector and axial-vector two-point functions via the effective action $\Gamma(U, V_\mu, A_\mu)$:

$$\exp[i\Gamma(U, V_\mu, A_\mu)] = \frac{\int [d\psi][d\bar{\psi}] \exp \left[i \int d^4x \bar{\psi}(x) [i\mathcal{D} - M(x)] \psi(x) \right]}{\int [d\psi][d\bar{\psi}] \exp \left[i \int d^4x \bar{\psi}(x) (i\mathcal{D} - m) \psi(x) \right]} \quad (\text{A1})$$

where

$$i\mathcal{D}_\mu = i\partial_\mu + V_\mu + A_\mu \gamma_5 \quad (\text{A2})$$

is the covariant derivative and

$$M(x) = m \exp \left[i \sum_{j=1}^8 \lambda_j \pi_j(x) \gamma_5 \right] \quad (\text{A3})$$

represents the coupling to the Goldstone degrees of freedom. Integrating out the quark degrees of freedom, we have

$$\Gamma(U, V_\mu, A_\mu) = -i \ln \det \frac{i \not{D} - M(x)}{i \not{D} - m} = -i \text{Tr} \ln \left[1 + \frac{i}{i \not{D} - m} (-i) \left\{ V + A \gamma_5 - m \left[\exp \left[i \sum_{j=1}^8 \lambda_j \pi_j(x) \gamma_5 \right] - 1 \right] \right\} \right], \quad (\text{A4})$$

which can be evaluated by expanding out the logarithm. For example, the two-point functions become

$$\int d^4x e^{iq \cdot x} \langle 0 | T [V_\mu^a(x) V_\nu^b(0)] | 0 \rangle = N_c \int \frac{d^4k}{(2\pi)^4} \text{tr} \gamma_\mu \frac{\lambda_a}{2} \frac{1}{k-m} \gamma_\nu \frac{\lambda_b}{2} \frac{1}{k+q-m} = (q^2 \eta_{\mu\nu} - q_\mu q_\nu) \delta^{ab} \Pi_V(q^2) \quad (\text{A5})$$

where

$$\begin{aligned} \Pi_V(q^2) &= \frac{4\Gamma(2-d/2)}{(4\pi)^{d/2}} (m^2)^{d/2-2} \int_0^1 dy y(1-y) \left[1 - \frac{q^2}{m^2} y(1-y) \right]^{d/2-2} \\ &= N_c \frac{2}{3} \frac{\Gamma(2-d/2)}{(4\pi)^{d/2}} (m^2)^{d/2-2} + N_c \frac{2}{15} \frac{1}{(4\pi)^2} \frac{q^2}{m^2} + O(q^4), \\ \int d^4x e^{iq \cdot x} \langle 0 | T [A_\mu^a(x) A_\nu^b(0)] | 0 \rangle &= +F_\pi^2 \frac{q_\mu q_\nu}{q^2} N_c \int \frac{d^4k}{(2\pi)^4} \text{tr} \gamma_\mu \gamma_5 \frac{\lambda_a}{2} \frac{1}{k-m} \gamma_\nu \gamma_5 \frac{\lambda_b}{2} \frac{1}{k+q-m} \\ &= (q^2 \eta_{\mu\nu} - q_\mu q_\nu) \delta^{ab} \Pi_V(q^2) \\ &\quad + \delta^{ab} N_c 4m^{d/2} \eta_{\mu\nu} \frac{\Gamma(2-d/2)}{(4\pi)^{d/2}} \int_0^1 dy \left[1 - \frac{q^2}{m^2} y(1-y) \right]^{d/2-2} \\ &= (q^2 \eta_{\mu\nu} - q_\mu q_\nu) \delta^{ab} \Pi_V(q^2) + \delta^{ab} N_c \eta_{\mu\nu} \left[4m^{d/2} \frac{\Gamma(2-d/2)}{(4\pi)^{d/2}} + \frac{2}{3} \frac{q^2}{m^2} + O(q^4) \right]. \end{aligned} \quad (\text{A6})$$

Comparing with the chiral expansion we find

$$\begin{aligned} \int d^4x e^{iq \cdot x} \{ \langle 0 | T [A_\mu^a(x) A_\nu^b(0)] | 0 \rangle - \langle 0 | T [V_\mu^a(x) V_\nu^b(0)] | 0 \rangle \} \\ = \delta^{ab} \left[(q_\mu q_\nu - q^2 \eta_{\mu\nu}) 4L_{10} + \left[\frac{q_\mu q_\nu}{q^2} - \eta_{\mu\nu} \right] F_\pi^2 + O(q^4) \right]. \end{aligned} \quad (\text{A7})$$

In a one-loop model we identify

$$F_\pi^2 = 4m^{d/2} \frac{\Gamma(2-d/2)}{(4\pi)^{d/2}} N_c, \quad (\text{A8})$$

so that in comparing (A6) and (A7), the only mystery then lies with the finite component of the quark loop result $\frac{2}{3} N_c q^2 \eta_{\mu\nu} / (4\pi)^2$ which does not appear to match onto the required gauge invariant form $4L_{10}(q_\mu q_\nu - q^2 \eta_{\mu\nu})$. However, since both procedures represent chiral expansion, we know that they might be equivalent. The resolution of this apparent paradox lies in recognizing that in obtaining the Gasser-Leutwyler effective Lagrangian one makes use of the equation of motion:

$$D^\mu (U^\dagger D_\mu U) = 0. \quad (\text{A9})$$

Then, for example, the term

$$\text{Tr} D_\mu D_\nu U^\dagger D^\mu D^\nu U \quad (\text{A10})$$

can be rewritten as

$$\text{Tr} [D_\mu, D_\nu] U^\dagger [D^\mu, D^\nu] U + \text{Tr} D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U. \quad (\text{A11})$$

The latter has an explicitly gauge-invariant contribution to the axial-vector two-point function while the form contains only the term $q^2 \eta_{\mu\nu}$. We see then that a proper gauge invariant form arises *only* if the equations of motion are employed and that we can relate Eqs. (A6) and (A7) by simply equating the coefficient of the $q^2 \eta_{\mu\nu}$ terms, yielding

$$\frac{2}{3} N_c \frac{1}{(4\pi)^2} = -4L_{10}, \quad (\text{A12})$$

which is the result quoted in the text.

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