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Two-Component Semileptonic Form Factors

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In the semileptonic decays of a heavy quark to a light quark, present models disagree in an essential fashion concerning the shape of the form factors. Both the heavy quark limit of QCD and dispersion relations suggest that the correct form factors contain two distinct contributions. We discuss these as applied to the $B\to\pi e\nu$ and $B\to\rho e\nu$ decays. We also describe the inconsistencies of available phenomenological models and discuss how they must be modified.

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Semileptonic decays of heavy quarks provide measurements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and probe the dynamics of heavy quarks. Predictions for exclusive decays (such as $B\to\pi e\nu$, which will be our main example) rely on quark models and vector dominance techniques. These approaches have been developed in the studies of weak decays of light quarks where they provide compatible and reasonably effective descriptions. It is important to realize that both of these techniques are being extended well outside their known range of validity when they are being applied to heavy quark decays, and that important discrepancies exist in the results. For example, in Fig. 1 we show the f_+ form factor (defined below) for $B\to\pi e\nu$ predicted in two of the leading models, GIWS [1] (Isgur, Scora, Grinstein, and Wise) and BSW [2] (Bauer, Stech, and Wirbel). The differing behavior at low hadronic energy is symptomatic of a fundamental logical conflict between the models, which we will explain below. The resolution of this conflict will lead to a quite different composition for these semileptonic form factors.

Weak form factors are a function of $q^2=(p-p')^2$. In heavy quark decay in the B rest frame, $q^2=0$ corresponds to maximum recoil for the pion, while maximum $q^2=t_m=(m_B-m_\pi)^2$ corresponds to the pion being at rest.

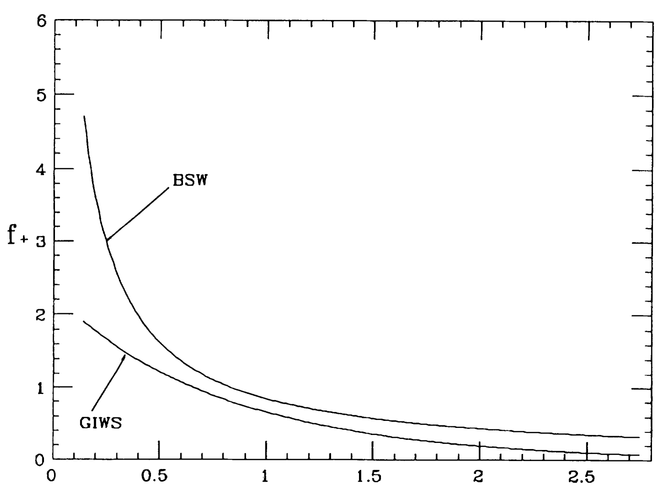

FIG. 1. $B\to\pi$ form factors from the GIWS and BSW models.

An equivalent kinematic variable is the pion energy related to q^2 by

$$E_\pi=(t_m-q^2)/2m_B+m_\pi. \tag{1}$$

The extreme cases here are

$$\begin{aligned} q^2=0 &\Rightarrow E_\pi=E_\pi|_{\max}=\frac{m_B^2+m_\pi^2}{2m_B}=2.64\ \text{GeV}, \\ q^2=t_m &\Rightarrow E_\pi=E_\pi|_{\min}=m_\pi. \end{aligned} \tag{2}$$

We will give results in terms of both E_π and q^2.

The $B\to\pi$ form factors are

$$\langle\pi(p')|\bar{u}\gamma_\mu b|\bar{B}(p)\rangle =f_+(q^2)(p+p')_\mu+f_-(q^2)(p-p')_\mu. \tag{3}$$

Only $f_+(q^2)$ contributes to $B\to\pi e\nu$ if one neglects m_e. The GIWS model involves calculating the normalization of the matrix element at zero recoil ($q^2=t_m$), and then using a recoil form factor which emerges from quark model calculations. This recoil dependence is governed by typical hadronic scales, and we find that their form factor is fitted [3] by the form

$$f_+^{\rm GIWS}=b/(1+E_\pi/\overline{M})^2, \tag{4a}$$

with $b=2.4$, $\overline{M}=1.1$ GeV. In terms of the variable q^2, this has the following form:

$$\begin{aligned} f_+^{\rm GIWS}&=\frac{c}{[1-q^2/(m_B^2+2m_B\overline{M}+m_\pi^2)]^2}, \\ c&=b\left[\frac{2m_B\overline{M}}{m_B^2+2m_B\overline{M}+m_\pi^2}\right]^2=0.21, \\ m_B^2&+2m_B\overline{M}+m_\pi^2=(m_B+1.0\ \text{GeV})^2. \end{aligned} \tag{4b}$$

In contrast BSW use a quark model to calculate at the opposite extreme, maximum recoil or $q^2=0$. They then extend the results to other values of q^2 by the assumption, motivated by vector dominance ideas [4], that the form factor contains the B^* pole;

$$f_+^{\rm BSW}=c^*/(1-q^2/m_{B^*}^2), \tag{5a}$$

with $c^*=0.33$, $m_{B^*}\approx m_B+50$ MeV. Because in $B\to\pi$ decay one can get very close to the B^* pole, i.e., $(1-t_m/m_{B^*}^2)^{-1}=14.4$, this produces the rapidly rising form factor of Fig. 1. Transformed to the variable E_π, this is the same as

$$f_+^{\rm BSW}=b^*_{\rm BSW}/(1+E_\pi/\overline{M}^*)\,,$$
$$b^*_{\rm BSW}=17.6\,, \tag{5b}$$
$$\overline{M}^*=(m^2_{B^*}-m^2_B-m^2_\pi)/2m_B\approx 0.050\ {\rm GeV}\,.$$

The logical conflict between these models lies in the assumptions for the energy behavior of the form factors. The GIWS model finds that the energy variation is governed by a typical hadronic scale $\overline{M}=1.1$ GeV. BSW [and also KS (Körner and Schuler) [5]] have a much more rapid variation because they build in a B^* pole form, and the physically allowed region includes energies very near the pole. This then leads to a very nontypical energy scale $\overline{M}^*=0.050$ GeV. It is important to emphasize that BSW do not calculate in the region of the pole, but only at the point of maximum E_π. The rapid variation near the pole (low E_π) is not a feature of the light-cone quark model, but is an artifact of the assumption of a vector dominance form. In light quark decays, both quark models and vector dominance produce q^2 variations with similar typical hadronic scales, such as m_ρ. In B decays, the energy scales differ quite strongly. We need to decide which approach (if any) is right.

The heavy quark limit of QCD can be used to sort out the ingredients to these form factors. We present a slight generalization of the counting rules of Isgur and Wise [6]. In the limit $m_Q\to\infty$, the structure of the heavy hadron becomes independent of m_Q. Consider the weak transition of a heavy meson to a light meson (π) where the light hadron emerges at a given fixed energy E_π. In the heavy mass limit, the dynamics of this transition becomes independent of the mass and the amplitude is then described by a function of E_π. For states conventionally normalized by

$$\langle Q\bar q(p')|Q\bar q(p)\rangle=2M_{Q\bar q}\delta^3(p-p')\,, \tag{6}$$

we must divide the matrix element of Eq. (3) by $(2M_{Q\bar q})^{1/2}$ before applying the heavy quark symmetry. Including the time and space components of the matrix element, we obtain

$$\frac{\langle\pi(p')|\bar u\gamma_0 b|B(p)\rangle}{(2m_B)^{1/2}}=\frac{(f_++f_-)m_B+(f_+-f_-)E_\pi}{(2m_B)^{1/2}}=g(E_\pi)\,, \tag{7}$$
$$\frac{\langle\pi(p')|\bar u\boldsymbol{\gamma} b|B(p)\rangle}{(2m_B)^{1/2}}=\frac{(f_+-f_-)\mathbf{p}_\pi}{(2m_B)^{1/2}}=2h(E_\pi)\mathbf{p}_\pi\,,$$

where $g(E_\pi)$ and $h(E_\pi)$ are universal functions not predicted by the symmetry, but which are independent of the heavy quark mass. Thus one obtains $f_+=-f_-$, with $f_+(q^2)=(2m_B)^{1/2}h(E_\pi)$. If both B and D mesons are heavy enough for these methods to apply, one can obtain the form factor for $B\to\pi$ from that for $D\to\pi$ by scaling $f_+^{B\to\pi}(E_\pi)=(m_B/m_D)^{1/2}f_+^{D\to\pi}$ at fixed E_π (not fixed q^2). However, note the kinematic range of $B\to\pi$ extends to much higher values of E_π than that of $D\to\pi$, so this relation is not by itself enough to predict $B\to\pi$.

The above analysis has implicitly assumed that E_π is not so large that perturbative hard gluon exchange could give the heavy quark a momenta comparable to its mass. Such very hard gluons could introduce off-shell effects depending on the mass of the heavy quark. In practice, perturbative off-shell effects seem small throughout the kinematic range of D and B decays [7,8].

The quark model result of Eq. (4) will clearly satisfy the scaling law if $b\propto(2m_B)^{1/2}$. We have checked that this indeed turns out to be the case. A B^* pole contribution will also satisfy the scaling behavior for large values of E_π; however, it does so in a unique fashion. The key feature is that as $m_b\to\infty$, the B and B^* become degenerate, since their spin splitting vanishes as $1/m_b$, i.e., $m_{B^*}-m_B\sim 1/m_B$, $m^2_{B^*}-m^2_B\approx{\rm const}\approx 0.5\ {\rm GeV}^2$. This puts the B^* pole very close to the physical region, a property not shared by any other resonance. The residue of the pole may be calculated. The scaling behavior of the relative couplings are [6] $g_{B^*B\pi}\sim m_B$ and $f_{B^*}\sim m_B^{1/2}$. Thus near the B^* pole the effect is

$$f_+(q^2)=\frac{g_{B^*B\pi}f_{B^*}}{m^2_{B^*}-q^2}=\frac{b^*}{1+E_\pi/\overline{M}^*}\,,$$
$$b^*=\frac{g_{B^*B\pi}f_{B^*}}{m^2_{B^*}-m^2_B-m^2_\pi}\,, \tag{8}$$

with $\overline{M}^*$ given in Eq. (5b). Unlike the expected scaling pattern [such as Eq. (4) with $b\sim(2m_B)^{1/2}$, $\overline{M}\sim{\rm const}$] this form has $b^*\sim m_B^{3/2}$, $\overline{M}^*\sim 1/m_B$. Because of the very tiny value of $\overline{M}^*$, for most values of E_π the form factor is approximated by $f_+\approx b^*\overline{M}^*/E_\pi$, which does scale properly. However, the pole contribution is logically distinct, since one could consider the $m_\pi\to 0$ limit of QCD in which case the form factor at $E_\pi=0$ has a different scaling property [6]. In a more practical setting, the scaling rules between $D\to\pi$ and $B\to\pi$ are violated by 50% at the zero-recoil point when one uses the physical masses.

As a by-product of this analysis, one can show that the BSW model is incompatible with the assumption of B^* pole dominance. This occurs because their wave function overlap calculated at $q^2=0$ scales as $c\sim m_B^{-1}$ for large m_B. Translated to the B^* pole by the assumption of a monopole form, this produces a residue at the pole which scales as m_B^1 rather than the $m_B^{3/2}$ behavior required by QCD.

The two components described above can also be identified in the context of dispersion relations. The form factor f_+, considered as an analytic function of q^2, has an isolated pole at $q^2=m^2_{B^*}$ and a cut starting at $q^2=(m_B+m_\pi)^2$. (If one imagines smaller values of m_π or larger values of $m^2_{B^*}-m^2_B$, the pole could occur above the start of the cut. This situation occurs in the $D\to\pi$ case where the D^* is above the $D^*\to D\pi$ threshold.) In QCD, the form factor vanishes sufficiently fast as q^2

$\to\infty$ that one may write an unsubtracted dispersion relation

$$f_+(q^2)=\frac{g_{B^*B\pi}f_B}{m_{B^*}^2-q^2}+\frac{1}{\pi}\int_{(m_B+m_\pi)^2}^{\infty}dt\,\frac{\mathrm{Im}f_+(t)}{t-q^2-i\varepsilon}. \tag{9}$$

[In the $D\to\pi$ case, the first term is absent, and the D^* pole appears above threshold as a narrow resonance in $\mathrm{Im}f_+(t)$. The net physics is of course the same.] Contributions to $\mathrm{Im}f_+(t)$ would include higher B^* resonances plus the $B\pi$ and $B^*\pi$ continuum. These contributions are located in energy a fixed distance away from the B mass ($\Delta E\sim\frac{1}{2}$-1 GeV), and do not approach the B mass as $m_b\to\infty$. This energy gap then allows them to generate $h(E_\pi)$. A change of variables can help to demonstrate this. Consider the variables E_π and τ with the latter defined by $t=t_m+2m_B\tau$. With these variables,

$$f_+(E_\pi)=\frac{b^*}{1+E_\pi/\overline{M}^*}+\frac{1}{\pi}\int_{2m_\pi}^{\infty}d\tau\,\frac{\mathrm{Im}f_+(t=t_m+2m_B\tau)}{\tau+E_\pi}. \tag{10}$$

Roughly, τ is the energy above m_B for a given contribution, $t\approx(m_B+\tau)^2$. For all contributions located a fixed distance away from m_B, the dispersion integral will generate a function of E_π which varies as E_π/μ with μ being a typical hadronic scale. Thus in a dispersive analysis, $h(E_\pi)$ reflects the continuum and higher resonance contributions to the form factor.

However, as we have shown above, the B^* contribution is only logically distinct near the pole. The monopole form is not likely to remain unchanged far from the pole. One expects an extra damping of this contribution at larger recoil, and this is found in quark model calculations. The physics of this is illustrated in Fig. 2. The b quark is static and the spectator d quark has small momentum. The production of a higher recoil pion is suppressed not only by the B^* propagator, but also by the following factors. We know from other hadron phenomenology that the dominant quark pair creation mechanism is a soft process; there is not much probability of creating a very hard $u\bar{u}$ pair. In addition, once produced, a hard u quark has little amplitude for being found in the pion. Similarly, the hard $\bar{u}$ is not likely to form only a B^* meson. Each of these factors damps the B^* contribution at large E_π, beyond the factor already contained in the B^* propagator. Jaffe [9] has shown how related quark model behavior can be manifest in the context of dispersion relations from the contribution of continuum and pole effects above the cut. Phenomenologically the extra damping can be accommodated by allowing the B^* coupling to depend on E_π. Thus we are led to a two-component model of the semileptonic form factor,

$$f_+=\frac{b^*(E_\pi)}{1+E_\pi/\overline{M}^*}+(2m_B)^{1/2}h(E_\pi), \tag{11}$$

where $b^*(E_\pi)$ includes the damping described above. In model calculations, these two ingredients will generally be separately identifiable; both need to be included.

Existing models do not contain this two-component form. It is, however, easy to so modify the GIWS model by adding the B^* contribution calculated in Ref. [6], with the result shown in Fig. 3 (labeled $\overline{\text{GIWS}}$). Also shown (labeled BD) is a two-component model which we will present more fully elsewhere [10]. Briefly, the B^* contribution is fixed using the heavy quark symmetry to relate f_{B^*} to F_B (we use $F_B=200$ MeV) and extracting $g_{B^*B\pi}$ from the $B^*\to B\pi/B^*\to B\gamma$ ratio. We have estimated $b^*(E_\pi)$ by including monopole suppression for each of the effects described above. The function $h_\pi(E_\pi)$ has been obtained by matching the GIWS calculation at $E_\pi=0$ to the BSW one at $q^2=0$, using the unique multipole form interpolating between them which satisfies the proper heavy quark scaling laws, i.e., of the form of $f_+=b(1+E_\pi/\overline{M})^{3/2}$ with $b=2.3$, $\overline{M}=0.93$ GeV. In both models, we note the striking dominance of the B^* at small E_π, but at larger values of E_π, the non-B^* component is dominant.

Despite the large modification of the form factor due to

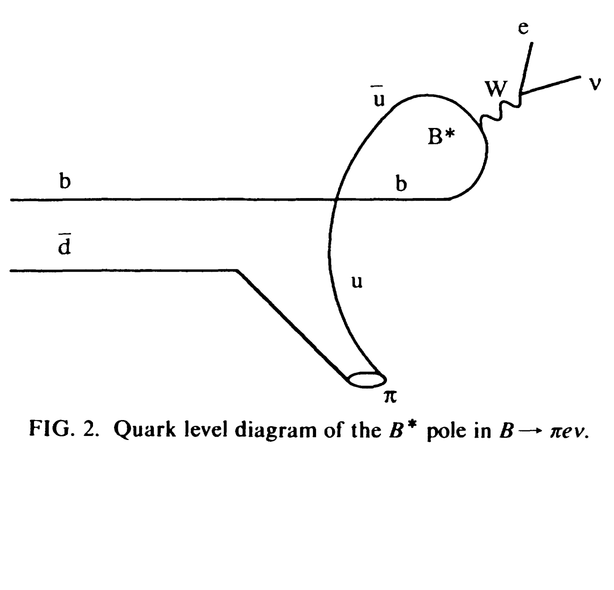

FIG. 2. Quark level diagram of the B^* pole in $B\to\pi e\nu$.

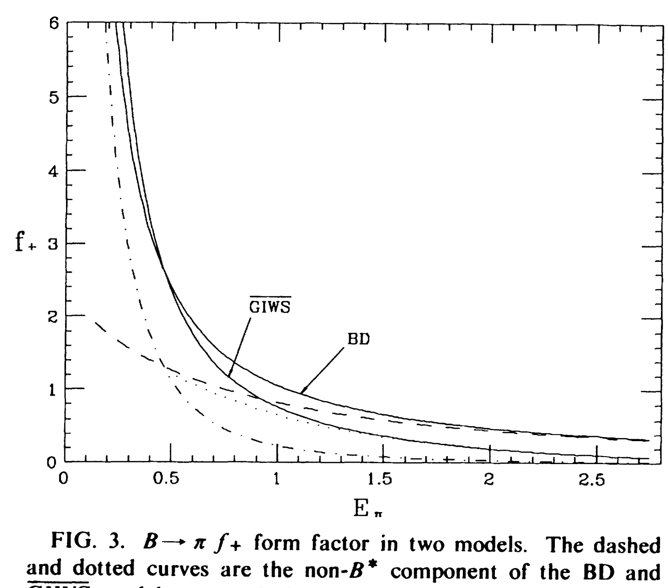

FIG. 3. $B\to\pi\,f_+$ form factor in two models. The dashed and dotted curves are the non-B^* component of the BD and $\overline{\text{GIWS}}$ models, respectively. The dot-dashed curve is the B^* effect in the BD model.

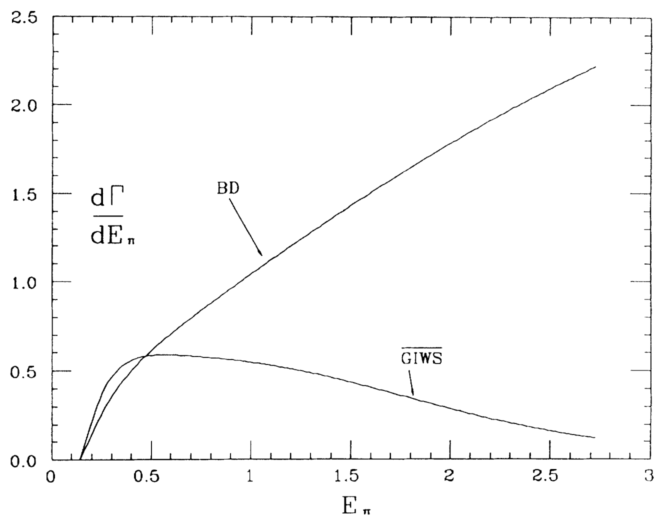

FIG. 4. $B \to \pi$ energy distribution in the two models of Fig. 3.

the B^*, the effects on the decay rate are not as dramatic. This is due to a helicity suppression mechanism of the matrix element as $\mathbf{p}_\pi \to 0$. When the pion is at rest, angular momentum conservation requires that the $e\bar{\nu}$ be in a spin-zero combination. However, as $m_e \to 0$, the weak currents produce a left-handed particle and a right-handed antiparticle, which is spin one. Thus the matrix element must vanish as $p_\pi \to 0$, $m_e \to 0$. This is seen in the differential decay rate (with $m_e = 0$)

$$\frac{d\Gamma}{dE_\pi} = \frac{G_F^2 |V_{ub}| m_B^2}{12\pi^3} p_\pi^3 |f_+(E_\pi)|^2 . \tag{12}$$

In Fig. 4, we show this distribution. The remaining sizable disagreement between the models arises at large recoil, where the rising phase space competes in the GIWS model with an exponentially falling form factor. The high recoil region clearly deserved further study.

Although we have concentrated on the most dramatic case, $B \to \pi e \nu$, our comments are clearly relevant for all other heavy-to-light weak decays. In the case of $B \to \rho e \nu$, the three axial vector matrix elements have only smooth scaling behavior, while the single vector form factor has both a smooth contribution and a B^* pole. Similar results hold in $D \to K, K^*, \pi, \rho$ transitions, and we will detail these elsewhere [10]. The only present data on the q^2 variation of the form factors exist in $D \to K e \nu$ [11]. We have checked that a two-component form is consistent with observations. Experimental analyses of semileptonic D decays have in general assumed a single-component vector dominance shape in the extraction of the magnitudes of form factors. It needs to be checked if the quoted magnitudes are modified when this assumption is not made.

The basic idea that the B^* pole is important at low E_π and that it should be added to the GIWS model was discussed in Ref. [6]. We have made explicit several aspects which are not stated in that work and have extended the phenomenology of this idea. All of the existing phenomenological models have inconsistencies as presently applied. The remedy in the GIWS model is relatively simple, but that for the BSW and KS models may require more work. These issues are important for the theoretical consistency of the models. They also have implications for the attempt to extract V_{ub} from a comparison of $D \to \pi$ and $B \to \pi$. Since the pion energy in $D \to \pi$ is relatively modest, the reaction occurs largely in the pole-dominated region, whereas in $B \to \pi$ the two components enter with different weight. One cannot simply compare integrated rates, nor extrapolate to zero recoil. In this regard using $B \to \rho$ and $D \to \rho$ at a common E_ρ should lead to a more reliable estimate of the CKM elements. Even after modifications, the modes are not likely to agree in detail, and more understanding is needed before we may have any hope of "reliable" models. However, the identification of the proper ingredients to the models is a necessary step.

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