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Union of chiral and heavy quark symmetries

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We describe the chiral symmetric couplings of pions to heavy mesons (B or D), valid in the portion of phase space where the pions have low momentum. In order to include consistently all low energy excitations, the vector mesons (B^* or D^*) must appear explicitly in the effective lagrangian. The result is then invariant under both the chiral and heavy quark symmetries. We include matrix elements relevant for various weak decays.

The chiral symmetries of QCD constrain the interactions of pions at low energy [#1]. Traditionally, the greatest number of applications have been found in the strong, weak and electromagnetic interactions of pions and kaons among themselves without heavy particles. However, chiral predictions exist also in the presence of heavy particles, such as baryons [2,4], as long as the pion momentum is low. In heavy meson decays (we will use the B as our main example), pions typically emerge carrying a large momentum. However, in many processes there is a corner of the allowed phase space where the pion momentum is small. In these regions, chiral symmetry can be used to relate processes. Any such technique, which is well founded theoretically, enhances our capabilities of dealing with weak decays. In this paper we describe the chiral couplings of heavy mesons.

Before proceeding we would like to note that one is required to include the B^* in the chiral description. The construction of effective lagrangians allows all degrees of freedom which require a large excitation energy to be integrated out from the theory, leaving only the low energy degrees of freedom explicit in the lagrangian. For example, in standard applications, the ρ meson is integrated out, but pions are explicitly kept [5]. Heavy particles are treated as nearly static sources. However, as the heavy quark mass increases, the B^* and B become degenerate, i.e., $m_{B^*}-m_B \sim \mathrm{O}(1/m_B)$. In practice, $m_{B^*}-m_B \sim 50$ MeV. Therefore the excitation of a B^* requires little energy, less than the emission of a pion, and the B^* must be explicitly included in the effective lagrangian.

There are then two symmetries which must be dealt with simultaneously. As the quark mass increases to infinity [6], there is a heavy quark spin symmetry which relates the couplings of B and B^*. The heavy quark symmetry also implies relations between different heavy mesons, i.e., B and D, valid to the extent that the charm quark may be treated as heavy. Thus the heavy quark symmetry will relate the B and B^* portions of the effective lagrangian, and will imply an ordering of coefficients in powers of the heavy quark mass. At the same time, the chiral symmetry will relate processes involving different numbers of pions, and will imply an ordering of the coefficients in powers of the pion momenta. It is amusing that the same effective lagrangian will be constrained by both the $m_q \to 0$ and $m_q \to \infty$ limits of QCD.

The b quark is a singlet under the light quark symmetry so that (B_u, B_d) transform as a doublet under isospin and (B_u, B_d, B_s) is a triplet under flavor SU(3),

$$B \to VB ,$$

with V in vectorial SU(2) or SU(3). We follow the Particle Data Group's conventions, such that $B_u = \bar{b}u$. The weak current

$$J_\mu^{bu} = \bar{b}\gamma_\mu(1+\gamma_5)u \qquad (1)$$

is a member of left handed chiral doublet (triplet for

[#1] For reviews see for example refs. [1–3].

SU(3)), which transforms purely left-handedly,

$$J_\mu=\begin{pmatrix}J^{bu}_\mu\\ J^{bd}_\mu\\ J^{bs}_\mu\end{pmatrix},\quad J_\mu\to LJ_\mu\,, \tag{2}$$

with L in $\mathrm{SU}(2)_\mathrm{L}$ (or $\mathrm{SU}(3)_\mathrm{R}$). The pions can be described [7] [#2] by chiral matrices ξ and U, with $U=\xi\xi$ and

$$\xi=\exp\left(\mathrm{i}\frac{\lambda^A\phi^A}{F}\right),$$

$$\xi\to L\xi V^\dagger=V\xi R^\dagger\,,$$

$$U\to LUR^\dagger\,, \tag{3}$$

with L, V as above and R in $\mathrm{SU}(2)_\mathrm{R}$ (or $\mathrm{SU}(3)_\mathrm{R}$) and where F is the pion decay constant at lowest order in chiral symmetry. Note that although L, R are global transformations, the transformation (3) requires V to be position dependent. This then requires a covariant derivative

$$\mathrm{d}_\mu=\partial_\mu+\mathrm{i}V_\mu\,,$$

$$V_\mu=-\tfrac{1}{2}\mathrm{i}(\xi^\dagger\partial_\mu\xi+\xi\partial_\mu\xi^\dagger)\,,$$

$$\mathrm{d}_\mu B\to V(\mathrm{d}_\mu B)\,. \tag{4}$$

For completeness we also define an axial current

$$A_\mu=-\tfrac{1}{2}\mathrm{i}(\xi^\dagger\partial_\mu\xi-\xi\partial_\mu\xi^\dagger)\,,$$

$$A_\mu\to VA_\mu V^\dagger\,, \tag{5}$$

with a vectorial transformation property.

The kinetic energy for the B mesons is chirally invariant when d_μ is used:

$$L_\mathrm{kin}=(\mathrm{d}_\mu B)^\dagger(\mathrm{d}^\mu B)-m_B^2B^\dagger B\,. \tag{6}$$

If desired, the B_s–B_d mass splitting can be put in with the lagrangian

$$L_{\Delta m}=\alpha_1\,\mathrm{Tr}(mU+U^\dagger m)B^\dagger B$$

$$+\alpha_2B^\dagger(\xi^\dagger m\xi^\dagger+\xi m\xi)B\,. \tag{7}$$

The weak chiral couplings will be arranged in an expansion in the pion energy, which at the lagrangian level means derivatives acting on the pion field or factors of the quark mass. Note that derivatives acting on B meson fields are not small. At order E_π^0 there is a unique lagrangian with the correct transformation properties

$$L_0=-f_B(\mathrm{d}_\mu B)^\dagger\xi^\dagger J^\mu\,. \tag{8}$$

The coefficient has been labeled f_B, because this generates the matrix element

$$\langle 0|\bar{b}\gamma_\mu(1+\gamma_5)u|B(p)\rangle=\mathrm{i}f_Bp^\mu\,, \tag{9}$$

in a normalization such that $f_\pi=\sqrt{2}\,F_\pi=131$ MeV. At $\mathrm{O}(E_\pi^1)$ there are two more chirally invariant terms

$$L_1=\beta_1B^\dagger\xi\partial_\mu U^\dagger J^\mu+\beta_2(\mathrm{d}_\mu\mathrm{d}_\nu B)^\dagger\xi\partial^\nu U^\dagger J^\mu\,, \tag{10}$$

plus two mass terms [#3]

$$L_m=-[\beta_3(\mathrm{d}_\mu B)^\dagger\xi mJ^\mu+\beta_4(\mathrm{d}_\mu B)^\dagger\xi^\dagger mU^\dagger J^\mu]\,, \tag{11}$$

where m_q is the quark mass matrix. These lagrangians have the same form in chiral SU(2) or chiral SU(3).

Heavy quark symmetry can be included using a compact notation [9]. One defines fields and current

$$\mathscr{B}=\left(\frac{\mathrm{i}\not{\partial}+m_B}{2m_B}\right)(B\gamma_5+\gamma^\mu B^*_\mu)\,,$$

$$J=\gamma_\mu(1+\gamma_5)J^\mu\,,$$

$$\bar{\mathscr{B}}=(B^\dagger\gamma_5-\gamma^\mu B^{*\dagger}_\mu)\left(\frac{-\mathrm{i}\overleftarrow{\not{\partial}}+m_B}{2m_B}\right). \tag{12}$$

In this notation, the kinetic energy terms are simply

$$L_\mathrm{kin}=m_B\,\mathrm{Tr}[\bar{\mathscr{B}}(\mathrm{i}\not{\partial}-m_B)\mathscr{B}]\,. \tag{13}$$

The weak couplings are also compactly written:

$$L_\mathrm{w}=L_0+L_1+L_m$$

$$=\frac{-\mathrm{i}m_Bf_B}{2}\mathrm{Tr}(\bar{\mathscr{B}}\xi^\dagger J)+\frac{\beta_1}{2}\mathrm{Tr}(\bar{\mathscr{B}}\xi\partial\!\!\!/U^\dagger J)$$

$$+\mathrm{i}\frac{\beta_2m_B}{2}\mathrm{Tr}[(\mathrm{d}_\nu\bar{\mathscr{B}})\xi\partial^\nu U^\dagger J]-\mathrm{i}\frac{\beta_3m_B}{2}\mathrm{Tr}(\bar{\mathscr{B}}\xi mJ)$$

$$-\mathrm{i}\frac{\beta_4m_B}{2}\mathrm{Tr}(\bar{\mathscr{B}}\xi^\dagger mU^\dagger J)\,. \tag{14}$$

[#2] Reviews of the method are found in refs. [3,2].

[#3] In the notation of ref. [8], these would read

$$-L_m=\bar{\beta}_3(\mathrm{d}_\mu B)^\dagger\xi\chi^\dagger J^\mu+\bar{\beta}_4(\mathrm{d}_\mu B^\dagger)\xi^\dagger\chi U^\dagger J^\mu$$

with $\beta_i=2B_0\bar{\beta}_i$.

To be more specific, the B^* couplings which are contained in this have the form

$$
\begin{aligned}
L_{\rm w} = {} & \mathrm{i} f_B m_B B_\mu^{*\dagger} \xi^\dagger J^\mu \\
& + \mathrm{i} \frac{\beta_1}{m_B} \left(B_{\mu\nu}^{*\dagger} \xi \, \partial^\mu U^\dagger J^\nu + \tfrac{1}{2} \mathrm{i} \epsilon^{\mu\nu\alpha\beta} B_{\alpha\beta}^{*\dagger} \xi \, \partial_\mu U^\dagger J_\nu \right) \\
& - \mathrm{i} \beta_2 m_B (\mathrm{d}^\nu B_\mu^*)^\dagger \xi \, \partial_\nu U^\dagger J^\mu + \mathrm{i} \beta_3 m_B B_\mu^{*\dagger} \xi m J_+^\mu \\
& + \mathrm{i} \beta_4 m_B B_\mu^{*\dagger} \xi^\dagger m U^\dagger J^\mu , \\
& B_{\mu\nu}^* \equiv \mathrm{d}_\mu B_\nu^* - \mathrm{d}_\nu B_\mu^* .
\end{aligned}
\tag{15}
$$

We have checked these relations using the commutator method of Isgur and Wise [6].

The quark mass effects in L_m are not likely to be significant in B_u and B_d decays. They mainly provide a small shift in the B decay constants, i.e.,

$$
\begin{aligned}
F_{Bu} &= F_B \left(1 + \frac{\beta_3 + \beta_4}{F_B} m_u \right) , \\
F_{Bd} &= F_B \left(1 + \frac{\beta_3 + \beta_4}{F_B} m_d \right) , \\
F_{Bs} &= F_B \left(1 + \frac{\beta_3 + \beta_4}{F_B} m_s \right) .
\end{aligned}
\tag{16}
$$

Most estimates put $F_{Bs}/F_{Bd} \approx 1.2$, which would imply a small isospin breaking of the size $F_{Bu}/F_{Bd} \approx 1.005$. We have included these terms for completeness thus far, but drop them from consideration below.

In order to treat the weak decays for $B \to$ vacuum, $B \to \pi$ and $B \to 2\pi$, one requires these couplings plus those of $B^* \to$ vacuum and $B^* \to \pi$. The latter are required in the pole diagrams of fig. 1. Note the unusual double pole diagram in $B \to \pi\pi e\nu$; however, the effect of all B poles vanish when contracted with the lepton current L_μ, because $q_\mu L^\mu = 0$ if lepton masses are neglected. The various form factors are given by eq. (9) plus

$$
\begin{aligned}
& \langle 0 | J_\mu | B^* \rangle = \mathrm{i} f_B m_B \epsilon_\mu , \\
& \langle \pi^0 | J_\mu | B^- \rangle = f_+ (p_B + p_\pi)_\mu + f_- (p_B - p_\pi)_\mu ,
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
& f_+ = \frac{1}{4F_\pi} (f_B + 2\beta_1 - 2\beta_2 m_B E_\pi) + B^* \text{ pole} , \\
& f_- = f_+ - \beta_1 / F_\pi - B^* \text{ pole} , \\
& \langle \pi^+ \pi^- | J_\mu | B^- \rangle = -\mathrm{i} [S(p_+ + p_-)_\mu + P(p_+ - p_-)_\mu \\
& \qquad + R(p_B - p_+ - p_-)_\mu + \mathrm{i} H \epsilon_{\mu\nu\alpha\beta} p_B^\nu p_+^\alpha p_-^\beta] , \\
& S = \frac{1}{4F_\pi^2} (f_B + 2\beta_1 - 4\beta_2 m_B E_\pi) + B^* \text{ pole} , \\
& P = \frac{1}{4F_\pi^2} (f_B + 2\beta_1) + B^* \text{ pole} , \\
& R = \frac{1}{4F_\pi^2} (f_B - 4\beta_2 m_B E_\pi) + B^* \text{ pole} + B \text{ poles} , \\
& H = 0 + B^* \text{ pole} , \\
& \langle \pi^+ | J_\mu | B^* \rangle = \mathrm{i} D \epsilon_{\mu\nu\alpha\beta} p_\pi^\nu p_B^\alpha \epsilon^\beta \\
& \qquad + E \epsilon^\mu + F \epsilon \cdot p_\pi p_B^\mu + G \epsilon \cdot p_\pi p_\pi^\mu , \\
& E = \sqrt{2} \left(\frac{f_B}{2F_\pi} m_{B^*} - \frac{m_{B^*}^2 E_\pi}{F_\pi} \beta_2 + \frac{E_\pi \beta_1}{F_\pi} \right) , \\
& F = -\frac{\sqrt{2}\, \beta_1}{m_B^* F_\pi} , \\
& G = 0 , \\
& D = \frac{\sqrt{2}\, \beta_1}{m_B^* F_\pi} .
\end{aligned}
\tag{17 cont'd}
$$

In the $B^{(*)} \to \pi$ transitions, $m_B E_\pi = \frac{1}{2}(t - m_B^2 - m_\pi^2)$, $t = q^2 = (p_B - p_\pi)^2$. In forming the B^* poles, one needs the $B^* B \pi$ coupling which follows from

$$
\begin{aligned}
L_{B^* B\pi} &= -g_{B^* B\pi} F_\pi \operatorname{Tr} (\bar{\mathscr{B}} \not{A} \gamma_5 \mathscr{B}) \\
&= 2 g_{B^* B\pi} B_\mu^{*\dagger} \, \partial_\mu \pi \, B + \dots .
\end{aligned}
\tag{18}
$$

The B^* pole in f_+, S, P, H has the form

$$
f_+ \Big|_{\text{pole}} = \frac{g_{B^* B\pi} m_B f_B}{m_{B^*}^2 - t} ,
\tag{19}
$$

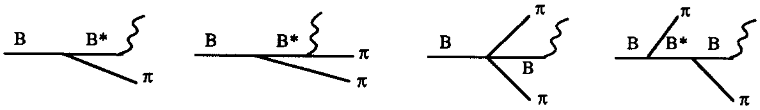

Fig. 1. Pole diagrams in semileptonic $B \to \pi$ and $B \to \pi\pi$ transitions.

$$S\Big|_{\text{pole}} = \frac{g_{B^*B\pi} m_B / F_\pi}{m_{B^*}^2 - (p_B - p_+)^2}$$
$$\times \left[\left(f_B + \frac{2\beta_1 E_-}{m_B} - 2\beta_2 m_B E_- \right) - \frac{2\beta_1}{m_B^2} (p_+ \cdot p_- - E_+ E_-) \right],$$

$$P\Big|_{\text{pole}} = \frac{g_{B^*B\pi} m_B / F_\pi}{m_{B^*}^2 - (p_B - p_+)^2}$$
$$\times \left[\left(f_B + \frac{2\beta_1 E_-}{m_B} - 2\beta_2 m_B E_- \right) + \frac{2\beta_1}{m_B^2} (p_+ \cdot p_- - E_+ E_-) \right],$$

$$H\Big|_{\text{pole}} = -\frac{4 g_{B^*B\pi}}{m_B F_\pi} \frac{\beta_1}{m_{B^*}^2 - (p_B - p_+)^2}, \qquad \text{(19 cont'd)}$$

where these forms are valid only near zero recoil ($E_\pi = m_\pi$). It is interesting that at these energies $B \to (\pi\pi)_{\text{S-wave}}$ is as strong as $B \to (\pi\pi)_{\text{P-wave}}$, as the former is never included in models of $b \to u$ semileptonic decay [10].

The coefficients can be ordered according to both the chiral and heavy quark symmetries. If we call Λ_χ the scale of chiral symmetry breaking, and label f_B as order Λ_χ^0, the $O(E^1)$ coefficients β_i are all of order $1/\Lambda_\chi$, such that the chiral expansion is in powers of E_π/Λ_χ. The heavy quark symmetry leads to well defined powers of m_B, with the well known result $f_B \sim [\alpha_s(m_B)]^{-6/25}/m_B^{1/2}$. The factor continuing α_s arises from matching the full theory of QCD onto an effective field theory for the heavy mesons [11]. For the overall lagrangian to have a common scaling behavior, one obtains the dependence

$$\beta_1 \sim [\alpha_s(m_B)]^{-6/25} m_B^{1/2},$$
$$\beta_2 \sim [\alpha_s(m_B)]^{-6/25} m_B^{-3/2}. \qquad (20)$$

These two behaviors can be combined to yield

$$f_B \sim \frac{[\alpha_s(m_B)]^{-6/25}}{m_B^{1/2}},$$
$$\beta_1 \sim \frac{m_B^{1/2} [\alpha_s(m_B)]^{-6/25}}{\Lambda_\chi}, \qquad (21)$$
$$\beta_2 \sim \frac{[\alpha_s(m_B)]^{-6/25}}{m_B^{3/2} \Lambda_\chi}. \qquad \text{(21 cont'd)}$$

This leads to interesting consequences in the matrix elements. Recall that when contracted with the lepton current L_μ, one has $p_B^\mu L_\mu = p_\pi^\mu L_\mu$, such that the advantage of the heavy mass in p_B is lost. For example, $B \to \pi e\nu$ decay is sensitive to only f_+. The B^* pole is the dominant coupling in f_+, S, P, H, while in the remainder β_1 is more important than f_B in eq. (17) even though β_1 is not leading in the chiral expansion.

Some of the relations in the matrix elements are familiar from the kaon sector. The result

$$f_+ + f_- = \frac{f_B}{F_\pi} + B^* \text{ pole} \quad \text{(at zero recoil)} \qquad (22)$$

is the analog of the Callan–Treiman relation aside from the B^* pole. Similarly the $B \to \pi\pi$ form factors are similar to the K_{e4} form factors predicted by Weinberg. The extra input of the heavy quark symmetry provides other constraints, such as $f_+ - f_-$ being larger than $f_+ + f_-$ by a factor of m_B, which were previously derived without consideration of the chiral properties [6]. The effective lagrangian above unifies all these ingredients.

One might worry that the chiral expansion in heavy mesons might have a much smaller radius of convergence than in light mesons. The energy expansion of purely pionic processes is in powers of q^2 with a scale of 1 GeV, is $A = A_0[1 + q^2/(1\ \text{GeV})^2]$. On kinematic grounds, one might expect the expansion in $B \to \pi$ to involve $(t - t_m) = 2m_B(E_\pi - m_\pi)$ with $t = (p_B - p_\pi)^2$, $t_m = (m_B - m_\pi)^2$. An expansion of the form $1 + (t - t_m)/(1\ \text{GeV})^2$ would be disastrous because the factor of m_B would magnify energy effects, i.e., $2m_B m_\pi = (1.2\ \text{GeV})^2$. Fortunately the existence of the smooth heavy quark limit proves that this cannot happen. Form factors are well behaved as $m_B \to \infty$, requiring an expansion of the form

$$A = A_0 \left(1 + \frac{t - t_m}{2m_B \Lambda} \right) = A_0 \left(1 + \frac{E_\pi - m_\pi}{\Lambda} \right), \qquad (23)$$

with $\Lambda \sim O(1\ \text{GeV})$. This can be explicitly seen in the expansion for $f_+ + f_-$ in which the β_2 correction is suppressed by the requisite power of m_B. The only difference in the energy expansion in comparison with light mesons is that here it is linear in E_π rather than quadratic.

Overall, the result of the combined constraints is simply summarized. The B^* pole is the dominant component of $B\to\pi$ and $B\to\pi\pi$, while the next leading correction is set by β_1. One can envision several applications of these results. The $B\to\pi\pi e\nu$ reaction is required at low recoil by the chiral symmetry, and we are investigating its phenomenological importance. The constraints on form factors, such as eq. (17), are interesting tests of the consistency of phenomenological models, and also of lattice calculational methods. Some of these relations may be experimentally testable, for example in D decays at a τ-charm factory.

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