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Charmed nonleptonic decay in asymptotically free theories*

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The nonleptonic decay of charmed states is studied by means of renormalization-group techniques in a class of asymptotically free models. An SU(3) sextet rule is shown to result. The enhanced piece of the Hamiltonian belongs to a 20-dimensional representation of SU(4), which distinguishes the renormalization-group approach from some alternate octet-enhancement schemes.

I. INTRODUCTION

The recent discovery of $\psi(3105)$ at SLAC and BNL has promoted renewed interest in the possible existence of a charmed partner— $\mathcal{P}'$ —for the conventional quark triplet— $\mathcal{P}, \mathcal{N}, \lambda$.¹ In this picture the fundamental symmetry is SU(4) and the pseudoscalar and vector mesons constitute a pentadecuplet. The SU(3) decomposition of this representation is

$$15 = 8 + 3 + \bar{3} + 1,$$

so that in addition to the usual octet (nonet) there are predicted to exist a charmed triplet and antitriplet for both 0^- and 1^- states. Nonleptonic weak decay will presumably be the dominant decay channel for these states and it is our purpose here to study such processes.

II. ENHANCEMENT EFFECTS

Nonleptonic decays have always been enigmatic in that the simple Cabibbo picture cannot account for the empirically observed $\Delta I = \frac{1}{2}$ rule in $\Delta S = 1$ processes.² Recently, a promising new approach—the renormalization-group—has been applied to these cases with some (though not total) success.³ In this note we shall study nonleptonic charmed decay via these methods and will later point out striking differences with some alternate enhancement approaches.

We work in an asymptotically free model with colored SU(4). The strong interactions are mediated by massless, non-Abelian neutral gauge fields with the gauge group being SU(N). The weak interactions are described by a Weinberg-Salam type gauge theory.⁴ The strangeness-conserving and strangeness-changing charged currents are given by

$$J_{\lambda}^{\Delta S=0} = \sum_i (\bar{\mathcal{P}}_i \cos \theta - \bar{\mathcal{P}}'_i \sin \theta) \gamma_{\lambda} (1 + \gamma_5) \mathcal{N}_i, \quad (1)$$

$$J_{\lambda}^{\Delta S=1} = \sum_i (\bar{\mathcal{P}}_i \sin \theta + \bar{\mathcal{P}}'_i \cos \theta) \gamma_{\lambda} (1 + \gamma_5) \lambda_i.$$

Here θ is the Cabibbo angle and $i = 1, 2, \dots, N$ designates the color degree of freedom so that both currents are color singlets. Writing

$$J_{\lambda} = J_{\lambda}^{\Delta S=0} + J_{\lambda}^{\Delta S=1},$$

the effective nonleptonic Hamiltonian is written as⁵

$$H_{\text{eff}} = f^2 \int d^4x D_F(x, M_w^2) T(J_{\lambda}^{\dagger}(x) J^{\lambda}(0)), \quad (2)$$

where f is the fundamental weak coupling constant and $D_F(x, M_w^2)$ represents the propagator for the W bosons which mediate the weak interaction.

In the absence of strong interaction renormalization effects the charm-changing Hamiltonian consists of three pieces

$$\begin{aligned} H^{\Delta S=\Delta C=\pm 1} &= \frac{G}{\sqrt{2}} \cos^2 \theta \bar{\mathcal{N}}_i \gamma_{\lambda} (1 + \gamma_5) \mathcal{P}_i \bar{\mathcal{P}}'_j \gamma^{\lambda} (1 + \gamma_5) \mathcal{N}_j + \text{H.c.}, \\ H^{\Delta S=0, \Delta C=\pm 1} &= \frac{G}{\sqrt{2}} \cos \theta \sin \theta [\bar{\lambda}_i \gamma_{\lambda} (1 + \gamma_5) \mathcal{P}_i \bar{\mathcal{P}}'_j \gamma^{\lambda} (1 + \gamma_5) \lambda_j - \bar{\mathcal{N}}_i \gamma_{\lambda} (1 + \gamma_5) \mathcal{P}_i \bar{\mathcal{P}}'_j \gamma^{\lambda} (1 + \gamma_5) \mathcal{N}_j] + \text{H.c.}, \\ H^{-\Delta S=\Delta C=\pm 1} &= -\frac{G}{\sqrt{2}} \sin^2 \theta \bar{\lambda}_i \gamma_{\lambda} (1 + \gamma_5) \mathcal{P}_i \bar{\mathcal{P}}'_j \gamma^{\lambda} (1 + \gamma_5) \mathcal{N}_j + \text{H.c.}, \end{aligned} \quad (3)$$

where we have written $G/\sqrt{2} = f^2/M_w^2$. The dominant term—not suppressed by factors of $\tan \theta$ —favors decays in which the change in strangeness

and change in charm are identical. For example, representing the charmed pseudoscalar triplet as (d^0, d^+, s^+) the two-body channels

$$\begin{aligned}
s^+ \rightarrow \eta^0 \pi^0 & \quad d^0 \rightarrow \pi^+ K^- & d^+ \rightarrow \pi^+ \bar{K}^0 \\
& \rightarrow \bar{K}^0 K^+, & \rightarrow \eta^0 \bar{K}^0 \\
& & \rightarrow \pi^0 \bar{K}^0,
\end{aligned}$$

proceed without suppression, while rates for processes

$$\begin{aligned}
s^+ \rightarrow K^+ \pi^0 & \quad d^0 \rightarrow K^0 \bar{K}^0 & d^+ \rightarrow K^+ \bar{K}^0 \\
& \rightarrow K^+ \eta^0 & \rightarrow K^+ K^- & \rightarrow \pi^+ \pi^0 \\
& \rightarrow K^0 \pi^+, & \rightarrow \pi^+ \pi^- & \rightarrow \pi^+ \eta^0, \\
& & \rightarrow \eta^0 \pi^0 \\
& & \rightarrow \pi^0 \pi^0 \\
& & \rightarrow \eta^0 \eta^0,
\end{aligned}$$

wherein strangeness is conserved but charm is not are down by a factor $\tan^2 \theta \sim \frac{1}{20}$ and rates for channels such as

$$\begin{aligned}
s^+ \rightarrow K^+ K^0, & \quad d^0 \rightarrow \pi^- K^+ & d^+ \rightarrow \pi^0 K^+ \\
& \rightarrow \pi^0 K^0 & \rightarrow \eta^0 K^+ \\
& \rightarrow \eta^0 K^0, & \rightarrow \pi^+ K^0,
\end{aligned}$$

wherein the strangeness change is opposite to the charm change are suppressed by $\tan^4 \theta \sim \frac{1}{400}$. Via renormalization-group techniques we can examine the effects of the strong interactions upon these conclusions. Working in the 't Hooft gauge, and neglecting the effects of Higgs scalars,³ we find a Wilson expansion of the time-ordered product in Eq. (2),⁶

$$T(J_\lambda^\dagger(x) J^\lambda(0)) = \sum_i C_i(x) \Theta_i(0). \quad (4)$$

Here $\{\Theta_i\}$ is a complete set of operators carrying the appropriate quantum numbers while $C_i(x)$ are c -number coefficients which contain the space-time dependence. Operators of lowest dimension will dominate the result. Forms in which only two-fermion operators appear are purely $\Delta I = \frac{1}{2}$ but can always be transformed away by the mass and wave-function renormalizations of the quark fields.⁷ The operators of lowest dimension which survive are four-quark terms of dimension six,

$$\Theta_i \sim : \bar{q} \gamma_\lambda (1 + \gamma_5) q \bar{q} \gamma^\lambda (1 + \gamma_5) q :. \quad (5)$$

The corresponding coefficients $C_i(x)$ obey the Callan-Symanzik relation⁵ and, following Lee and Gaillard,³ are found to be

$$C_i(x, g) \simeq C_i(x, 0) \left[1 - \frac{g^2}{8\pi^2} b \ln(\mu(-x^2)^{1/2}) \right]^{d_i/2b}. \quad (6)$$

Here g is the color gluon coupling constant, μ is related to the subtraction point and specifies the

mass scale, $b = (11N - 8)/3$, where $-bg^3/16\pi^2$ is the lowest-order approximation of the $\beta(g)$ function in the Callan-Symanzik equation,⁸ and d_i is a parameter related to the anomalous dimension of operator Θ_i .

Treating first the $\Delta S = \Delta C$ operator we form operators of definite dimension by separating⁹

$$H^{\Delta S = \Delta C} = H_+^{\Delta S = \Delta C} + H_-^{\Delta S = \Delta C}, \quad (7a)$$

where

$$\begin{aligned}
H_+^{\Delta S = \Delta C} = \frac{G}{2\sqrt{2}} \cos^2 \theta [& \bar{\mathcal{U}}_i \gamma_\lambda (1 + \gamma_5) \mathcal{O}_i \bar{\mathcal{D}}_j' \gamma^\lambda (1 + \gamma_5) \lambda_j \\
& \pm \bar{\mathcal{U}}_i \gamma_\lambda (1 + \gamma_5) \lambda_i \bar{\mathcal{D}}_j' \gamma^\lambda (1 + \gamma_5) \mathcal{O}_j] \\
& + \text{H.c.} \quad (7b)
\end{aligned}$$

To lowest order in g^2 both H_+ and H_- are renormalized multiplicatively and

$$d_{\pm} = \mp 6 \frac{N \mp 1}{N}. \quad (8)$$

Then using $\mu \simeq 1$ GeV, $M_W \simeq 100$ GeV, $g^2/4\pi \simeq 1$ we find

$$\begin{aligned}
H_{\text{eff}}^{\Delta S = \Delta C} \simeq & F^{-9(N-1)/N(11N-8)} H_+^{\Delta S = \Delta C} \\
& + F^{9(N+1)/N(11N-8)} H_-^{\Delta S = \Delta C},
\end{aligned} \quad (9)$$

with

$$F = 1 + \frac{g^2}{12\pi^2} (11N - 8) \ln \frac{M_W}{\mu}.$$

For $N=3$

$$H_{\text{eff}}^{\Delta S = \Delta C} \simeq (7)^{-0.24} H_+^{\Delta S = \Delta C} + (7)^{0.48} H_-^{\Delta S = \Delta C}. \quad (10)$$

Thus H_- is enhanced and H_+ is suppressed by the same factors as in the calculation of Lee and Gaillard for the charm-conserving strangeness-changing case.³ There are two features of nonleptonic K decay which are difficult to understand in the conventional Cabibbo model:

- (i) the suppression of the charged mode $K^+ \rightarrow \pi^+ \pi^0$ with respect to the neutral $K^0 \rightarrow \pi^+ \pi^-$ by a factor of about 500;
- (ii) the apparent lack of suppression by $\cos \theta \sin \theta$ of the nonleptonic amplitude.

The renormalization effects of the strong interactions assist on both scores, but in neither case is the effect large enough to be consistent with experiment. There are two ways out of this dilemma. One is the realization that in other models—e.g. that of Lee, Prentki, and Zumino¹⁰—wherein there are a large number of basic fermion quarks, b is reduced and enhancement effects become larger. The other is the fact that renormalization-group ideas give only the effective nonleptonic Hamiltonian operator. Matrix elements must still be taken, and at this point previously discussed

octet-enhancement ideas—current algebra, tadpole dominance, etc.—come into play. The $\Delta I = \frac{1}{2}$ enhancement actually observed presumably involves some combination of these mechanisms.

We shall take the view here that however the further enhancement arises in the case of the $\Delta S = 1, \Delta C = 0$ decay modes, effects of a similar size appear in charm changing decays. What then are the consequences?

First we shall look on the SU(2) level. Both $H_+^{\Delta S = \Delta C}$ and $H_-^{\Delta S = \Delta C}$ carry $I = 1, I_z = 1$. Thus, even without enhancement effects we automatically have a $\Delta I = 1$ rule. For the doublet pseudoscalar meson decays this reads

$$A(d^+ \rightarrow \pi^+ \bar{K}^0) = A(d^0 \rightarrow \pi^+ K^-) + \sqrt{2} A(d^0 \rightarrow \pi^0 \bar{K}^0). \quad (11)$$

The SU(3) consequences are more interesting. The original Hamiltonian $H^{\Delta S = \Delta C}$ was composed of a combination of the SU(3) representations $\bar{15} + 15, 6 + \bar{6}$. The enhanced operator $H_-^{\Delta S = \Delta C}$ is composed, however, solely of $6 + \bar{6}$ while the suppressed operator $H_+^{\Delta S = \Delta C}$ is purely a member

$$H^{\Delta S = 0, \Delta C = \pm 1} = H_+^{\Delta S = 0, \Delta C = \pm 1} + H_-^{\Delta S = 0, \Delta C = \pm 1}, \quad (13a)$$

where

$$H_+^{\Delta S = 0, \Delta C = \pm 1} = \frac{G}{2\sqrt{2}} \cos \theta \sin \theta [\bar{\lambda}_i \gamma_\lambda (1 + \gamma_5) \mathcal{P}_i \bar{\mathcal{P}}'_j \gamma^\lambda (1 + \gamma_5) \lambda_j \pm \bar{\lambda}_i \gamma_\lambda (1 + \gamma_5) \lambda_i \bar{\mathcal{P}}'_j \gamma^\lambda (1 + \gamma_5) \mathcal{P}_j \\ - \bar{\lambda}_i \gamma_\lambda (1 + \gamma_5) \mathcal{P}_i \bar{\mathcal{P}}'_j \gamma^\lambda (1 + \gamma_5) \mathcal{P}_j \mp \bar{\lambda}_i \gamma_\lambda (1 + \gamma_5) \mathcal{P}_i \bar{\mathcal{P}}'_j \gamma^\lambda (1 + \gamma_5) \mathcal{P}_j] + \text{H.c.} \quad (13b)$$

We find after turning on the strong interactions the same enhancement (suppression) as before for the antisymmetric (symmetric) combinations

$$H_{\text{eff}}^{\Delta S = 0, \Delta C = \pm 1} \cong (7)^{-0.24} H_+^{\Delta S = 0, \Delta C = \pm 1} \\ + (7)^{0.48} H_-^{\Delta S = 0, \Delta C = \pm 1}. \quad (14)$$

Here $H_-^{\Delta S = 0, \Delta C = \pm 1}$ is purely $\Delta I = \frac{1}{2}$ while $H_+^{\Delta S = 0, \Delta C = \pm 1}$ is a mixture of $\Delta I = \frac{1}{2}, \frac{3}{2}$. Thus on the SU(2) level we find that such decays are guided by a $\Delta I = \frac{1}{2}$ rule. Their rates are, of course, still suppressed, however, by a factor of $\tan^2 \theta$ with respect to the $\Delta S = \Delta C$ modes. In the case of the pseudoscalar two-body modes, this $\Delta I = \frac{1}{2}$ rule imposes the restrictions

$$\sqrt{2} A(s^+ \rightarrow K^+ \pi^0) = A(s^+ \rightarrow K^0 \pi^+), \\ A(d^+ \rightarrow \pi^+ \pi^0) = 0, \\ A(d^0 \rightarrow \pi^+ \pi^-) = A(d^0 \rightarrow \pi^0 \pi^0), \\ A(d^+ \rightarrow \pi^+ \eta^0) = \sqrt{2} A(d^0 \rightarrow \pi^0 \eta^0). \quad (15)$$

of the $15 + \bar{15}$ representation. Thus we predict a sextet rule in the case of $\Delta S = \Delta C$ decays. Using such a rule we may relate, for example, the various charmed pseudoscalar modes

$$\begin{aligned} \left(\frac{3}{2}\right)^{1/2} A(s^+ \rightarrow \eta^0 \pi^+) &= A(s^+ \rightarrow \bar{K}^0 K^+) \\ &= A(d^0 \rightarrow \pi^+ K^-) \\ &= -\sqrt{2} A(d^0 \rightarrow \pi^0 \bar{K}^0) \\ &= -\sqrt{6} A(d^0 \rightarrow \eta^0 \bar{K}^0), \end{aligned} \quad (12)$$

$$A(d^+ \rightarrow \pi^+ \bar{K}^0) = 0.$$

Thus the d^+ cannot decay via a two-body channel in the pure sextet scheme. This is clear since $\pi^+ \bar{K}^0$ carries isospin $\frac{3}{2}$ and must be part of a 27 representation which is not contained in $\bar{3} \times \bar{6}$. Realistically, as in the case of $K^+ \rightarrow \pi^+ \pi^0$, we might expect the d^+ decay to occur at the 10% level compared to the neutral channels.

We may perform a similar renormalization-group analysis for the strangeness-conserving, charm-changing Hamiltonian. Forming symmetric and antisymmetric combinations as before

In the SU(3) picture $H_-^{\Delta S = 0, \Delta C = \pm 1}$ is again a member of the $6 + \bar{6}$ representation while $H_+^{\Delta S = 0, \Delta C = \pm 1}$ is part of a $15 + \bar{15}$ dimensional representation. We envision a sextet rule then, which gives the predictions in Eq. (15) plus additional relations among the $\Delta S = 0, \Delta C = \pm 1$ amplitudes themselves,

$$\begin{aligned} A(d^0 \rightarrow \pi^+ \pi^-) &= -\sqrt{3} A(d^0 \rightarrow \pi^0 \eta^0) \\ &= \sqrt{2} A(s^+ \rightarrow \pi^0 K^+) \\ &= -\sqrt{6} A(s^+ \rightarrow \eta^0 K^+) \\ &= -A(d^0 \rightarrow \eta^0 \eta^0), \end{aligned} \quad (16)$$

and between strangeness conserving and nonconserving decays, e.g.

$$A(s^+ \rightarrow \pi^+ \eta^0) = 2 \cot \theta A(s^+ \rightarrow K^+ \eta^0). \quad (17)$$

Finally, for completeness, we consider the $\Delta S = -\Delta C$ channels, although these are suppressed with respect to $\Delta S = \Delta C$ decays by a factor of $\tan^4 \theta \sim \frac{1}{400}$ in the rate. We find

$$H_{\text{eff}}^{-\Delta S = \Delta C = \pm 1} \cong (7)^{-0.24} H_+^{-\Delta S = \Delta C = \pm 1} + (7)^{0.48} H_-^{-\Delta S = \Delta C = \pm 1}, \quad (18a)$$

with

$$H_{\pm}^{-\Delta S = \Delta C = \pm 1} = -\frac{G}{2\sqrt{2}} \sin^2 \theta [\bar{\lambda}_i \gamma_\lambda (1 + \gamma_5) \mathcal{P}_i \bar{\mathcal{P}}'_j \gamma^\lambda (1 + \gamma_5) \mathcal{R}_j \pm \bar{\lambda}_i \gamma_\lambda (1 + \gamma_5) \mathcal{R}_i \bar{\mathcal{P}}'_j \gamma^\lambda (1 + \gamma_5) \mathcal{P}_j] + \text{H.c.} \quad (18b)$$

Now the enhanced operator $H_-^{-\Delta S = \Delta C = \pm 1}$ is purely $\Delta I = 0$, while the suppressed operator $H_+^{-\Delta S = \Delta C = \pm 1}$ is an isotopic vector. Thus at the SU(2) level we have a $\Delta I = 0$ rule, which gives

$$\begin{aligned} A(s^+ \rightarrow K^+ K^0) &= 0, \\ A(d^+ \rightarrow \pi^+ K^0) &= \sqrt{2} A(d^+ \rightarrow \pi^0 K^+), \\ &= -A(d^0 \rightarrow \pi^- K^+) = \sqrt{2} A(d^0 \rightarrow \pi^0 K^0), \\ A(d^+ \rightarrow K^+ \eta^0) &= -A(d^0 \rightarrow K^0 \eta^0). \end{aligned} \quad (19)$$

In an SU(3) picture the enhanced (suppressed) operator belongs to the representation $6 + \bar{6}$ ($15 + \bar{15}$) so that we predict a sextet rule, which gives the above predictions plus an additional relation between the $K\eta$ and $K\pi$ decay schemes

$$\sqrt{3} A(d^+ \rightarrow K^+ \eta^0) = -A(d^+ \rightarrow K^+ \pi^0), \quad (20)$$

and among these and the other charm nonconserving modes, e.g.

$$A(d^+ \rightarrow K^+ \eta^0) = \cot \theta A(s^+ \rightarrow K^+ \eta^0). \quad (21)$$

If we examine nonleptonic decay from the standpoint of SU(4) symmetry, each of the enhanced operators

$$H_-^{\Delta S = \Delta C}, H_-^{\Delta S = 0, \Delta C = \pm 1}, H_-^{\Delta S = \Delta C = \pm 1}, H_-^{\Delta S = \pm 1, \Delta C = 0}$$

is a member of a 20 dimensional representation of SU(4) while the suppressed operators

$$H_+^{\Delta S = \Delta C}, H_+^{\Delta S = 0, \Delta C = \pm 1}, H_+^{\Delta S = \Delta C = \pm 1}, H_+^{\Delta S = \pm 1, \Delta C = 0}$$

are part of an 84 dimensional representation.

A generalization of Gell-Mann's $K \rightarrow \pi\pi$ (Ref. 11) theorem *forbids any two-body pseudoscalar decays in a strict SU(4) limit*. We can attempt to account for SU(4) breaking effects in a simple quark-model calculation, which gives,¹² e.g.

$$\frac{A(K^0 \rightarrow \pi^+ \pi^-)}{A(s^+ \rightarrow K^+ K^0)} \cong \cot \theta \frac{M_K^2 - M_\pi^2}{M_s^2 - M_K^2}. \quad (22)$$

Note that both amplitudes vanish if pseudoscalar masses are set equal, in accord with the Gell-Mann theorem. However, SU(4) breaking effects are considerable and by no means is the relation in Eq. (22) necessarily trustworthy. For this

reason we have in this note emphasized the SU(2) and SU(3) relations which follow in our model.

III. CONCLUSION

We now have sketched the complete picture of charmed nonleptonic decay. Although, for the sake of sanity, we quoted specific relations between decay amplitudes only for pseudoscalar two-body decay, similar relations obtain for two-body vector-meson decay channels and, of course, the group structure of the Hamiltonian must be the same for any charmed decay mode.

One of the most interesting features of this result is that the enhanced nonleptonic charm changing operator belongs to a 20 dimensional representation of SU(4). The tadpole method of octet enhancement and perhaps some other mechanisms¹³ predict that the enhanced operator belongs to the adjoint or 15 dimensional representation. Of course, the 15 is not present in the original Hamiltonian but is produced when the large SU(4) breaking effects are inserted. In this case the operator responsible for charm-changing decay transforms like $d^0, \bar{d}^0$ and thus *favours non-strangeness-changing decay*, whereas the renormalization-group enhancement method predicts a predominance of $\Delta S = \Delta C$ decay channels, as would be expected from the naive Hamiltonian.¹⁴ Experimental study of such charmed decays thus might enable the differentiation between octet enhancement schemes—each of which would work perfectly well as long as only $\Delta S = 1, \Delta C = 0$ channels can be studied. The existence of charm then, while expanding the already crowded list of particle states, might enable some light to be shed on the long-standing $\Delta I = \frac{1}{2}$ enigma.

Note. After the completion of this manuscript, we were informed of a report by Cabibbo, Altarelli, and Maiani¹⁵ in which similar results were obtained.

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