2013

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# CONCEPTUALIZING CCSSM MATHEMATICAL PRACTICES ON TASKS AND EMPIRICAL WORKS

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*The Common Core State Standards for Mathematics contains both mathematical content and practices for grades K-12. Many researchers focus on the content, but overlook how the practices are embedded within tasks. Each practice includes a brief description that people interpret multiple ways. This study conceptualizes the practices and uses the framework to code two textbook series on bivariate data tasks to determine if students are exposed to the mathematical practices within the textbooks. Initial results indicate that the tasks provide opportunities for students to attend to precision but not to look for and express regularity in repeated reasoning.*

Keywords: Curriculum analysis, Data Analysis and Statistics

## Purposes of the Study

*The Common Core State Standards for Mathematics* (CCSSM) was released in 2010 followed by adoption in 45 states and five territories in the U.S. The CCSSM has the potential to shift the landscape of education in the U.S. particularly with the change of content and sequence (Heck, Weiss, & Pasley, 2011). Most discussions concerning the CCSSM have focused on the content standards (e.g., Conley, Drummond, de Gonzalez, Rooseboom, & Stout, 2011; Schmidt & Houang, 2012). However, one important issue related to the CCSSM – the eight mathematical practices (SMPs), has been overlooked. Some researchers and practitioners have sought to elaborate the practices and provided scenarios to illustrate them (e.g., Koestler et al., 2013). However, there have not been any systematic frameworks to code for the SMPs in curriculum materials and no formal research about that work. In this study, we describe how we conceptualize the practices in curriculum materials and provide our initial results about empirical work using examples from one high school textbook series for the content related to bivariate data.

## Perspectives and Coding Scheme

In this study, we look at the nature of tasks found in curriculum materials. That is, we seek to determine if the tasks offer the potential for students to utilize the practices. Admittedly, the tasks might offer potential for students to access the practices, but the practices might not be implemented in the same way in each task (Stein & Lane, 1996). Within this study, we highlight the characteristics of tasks that provide the opportunity for students to implement the practices. The performance expectation framework developed in the TIMSS study (Schmidt et al., 1997) was used to provide examples of tasks that offer the opportunities to look for the eight practices. Following is a summary of the coding scheme for SMPs.

1. Make sense and persevere in solving problems (MP1).

Persevering in solving problems is only observed in practice, not in the task itself. Therefore, the framework focuses on the former component. Particularly, we look to determine if the task requires students to formulate and clarify problems and situations such as to: (a) construct a verbal or symbolic statement or a question in which a mathematical problem goal can be specified, (b) design an appropriate statistical experiment to solve a stated problem or to specify the data and range of data needed.

2. Reason abstractly and quantitatively (MP2).

We consider if the task provides opportunities for students to: (a) make sense of quantities (not merely numbers), (b) reason about the relationship between two quantities, and (c) reason abstractly with symbols and formulas.

3. Construct viable arguments and critique the reasoning of others (MP3).

We consider if the task provides opportunities for students to argue or critique others' arguments. Particularly, we look for performance expectations from the task that ask students to: (a) verify the computational correctness of a solution, or justify a step in the solution, (b) identify information relevant to verify or disprove a conjecture, (c) argue the truth of a conjecture or construct a plausible argument, (d) identify a contradiction (something that is never true), (e) critique a written or spoken mathematical idea, solution, result, or method for solving a problem and the efficiency of the method or similarly critique an algorithm and its efficiency.

4. Model with mathematics (MP4).

We consider if the tasks ask students to: (a) construct a verbal or symbolic statement of a real world or other situations, (b) simplify a real world or other problem situation by selecting aspects and relationships to be captured in a representation modeling the situation, (c) select or construct a mathematical representation of a problem (real-world or other problem situation plus a related question/goal), and (d) develop notations or terminologies to record actions and results of real-world or other mathematizable situations.

5. Use appropriate tools strategically (MP5).

We examine if the task mentions something about the selection of tools used to solve the task or if the task inherently requires students to use a technological tool, but not merely for calculation. In this situation, the technological tool is used to deepen understanding.

6. Attend to precision (MP6).

We determine if the tasks ask students to: calculate, measure, or use specialized terms and symbols. Particularly, performance expectations from the task might ask the student to: (a) use equipment to measure, (b) compute/calculate with or without instruments, (c) graph with scale with or without technology/device, (d) collect data by surveys, samples, measurements, etc., (e) develop or select, using notations, terminologies to record actions and results in dealing with real-world or other mathematizable situations, and (f) describe the characteristics of a formal algorithm or solution procedure.

7. Look for and make use of structure (MP7).

We consider if the task provides opportunities for students to look for a structure such as: (a) fit a curve of given type to a set of data (only if students are not told what kind of curve to fit), (b) classify mathematical objects by implicit criteria (e.g., geometric shapes), (c) predict a number, pattern, outcome, etc., that will result from an operation, procedure or experiment before it is actually performed.

8. Look for and express regularity in repeated reasoning (MP8).

We examine if the task offers students opportunities to abstract from a series of similar situations, a general technique, strategy, or algorithm to use in a class of problems. In particular,

we look to see if some of the performance expectations of the task ask students to: (a) describe the effect of a change in a situation (e.g., the effect on its graph of changing a parameter), (b) develop a formal algorithm for computation or a formal solution procedure for problems of a specified class or type, (c) identify a class of problems for which a formal solution procedure is appropriate, (d) generalize the solution, the strategy, or the algorithm of a specific problem, and (e) abstract the common elements from multiple related situations.

## Methods

We examined coding schemes and specific examples from the performance expectation framework of the TIMSS study (Schmidt et al., 1997) and adapted the scheme to fit in the context of bivariate data. In particular, we highlighted the performance expectations providing potential for students to access the practices. We then used the framework to code an investigation from the teacher's edition of the *Core-Plus Mathematics* (CPMP) (Hirsch, Fey, Hart, Schoen, Watkins, Ritsema, et al., 2008) to determine if the practices appeared in the set of tasks and revised the coding scheme. In another round, we independently coded the practices for all the tasks related to bivariate data from three teacher's editions: CPMP series (published by Glencoe McGrawHill, 2008, 2009, 2010), The University of Chicago School Mathematics Project (published by Wright Group McGrawHill, 2008, 2009, 2010) and Holt McDougal Larson (HML) series (published by Holt McDougal, 2012). For reliability, pairwise agreements for all eight practices were more than 70%.

## Results

Table 1 summarizes the percentages of tasks that address the mathematical practices across the three series. For the set of bivariate data tasks, the percentages vary by series and by practices. CPMP offers the most tasks attending to the mathematical practices, followed by UCSMP and HLM series. Most of the tasks offered students the potential to *model with mathematics* (MP4) and *attend to precision* (MP6) with the focus on computation and calculation. In contrast, very few tasks offer expectation for students to access *make sense and persevere in solving problems* (MP1) and *look for and express regularity in repeated reasoning* (MP8).

**Table 1: Percentages of Tasks that Address the Mathematical Practices**

| | MP1 | MP2 | MP3 | MP4 | MP5 | MP6 | MP7 | MP8 |
|---|---|---|---|---|---|---|---|---|
| HML (N=122) | 3.3 | 57.4 | 54.1 | 76.2 | 37.7 | 81.1 | 12.3 | 0 |
| UCSMP (N=214) | 5.1 | 76.2 | 43 | 86.9 | 52.8 | 84.6 | 36 | 1.9 |
| CPMP (N=246) | 10.6 | 82.4 | 83.3 | 89 | 44.1 | 87.8 | 55.9 | 7.8 |

Following is the coding of a task taken from CPMP series.

Make a model bungee jump by attaching a weight to an elastic cord or to a chain of rubber bands.

a. Use your model to collect test data about bungee cord stretch for at least five weights. Record the data in a table and display it as a *scatterplot* on a graph.

b. Use the pattern in your experimental data to predict length of the stretched bungee cord for weights different from those already tested. Then test the accuracy of your predictions.
c. Compare your results to those of others doing the same experiment. What might explain any differences in results? (Hirsch et al., 2008, p. 5)

When solving this task, students have chances to (2a) make sense of quantities such as weight and length and (2b) reason about the relationship between the two quantities; (3b) identify information relevant to verify or disprove a conjecture, (c) argue the truth of a conjecture or construct a plausible argument, and (e) critique a written solution, result; (4b) simplify a real world situation by selecting aspects and relationships to be captured in a representation modeling the situation, (4c) select or construct a mathematical representation of a problem (in this case, a graph); (6a) use equipment to measure, (6c) graph with scale with or without technology/device, (6d) collect data by surveys, samples, measurements, etc.; and (7c) predict a number, pattern, outcome, etc., that will result from an operation, procedure or experiment before it is actually performed.

## Discussion

The framework for coding mathematical practices shows to be applicable to code for bivariate data or statistical tasks. The tasks' low presence of *look for and express regularity in repeated reasoning* within tasks is significant because educators may need to modify the tasks if they want to address the mathematical practice. It is conceivable that the variation across textbooks illustrates curriculum developers' different interpretation of how to embed the mathematical practices. In addition, the framework clarifies and operationalizes how the written curriculum embeds the SMPs. Further research needs to examine the framework in different domains. Furthermore, when coding tasks from the textbooks, we look specifically at the teacher's edition to see how the authors expect the tasks to be implemented. This might be different from the tasks as implemented in classroom. Hence, future research needs to develop frameworks for coding tasks in implemented phase.

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