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A short remark on Chien’s variational principle of maximum power losses for viscous fluids

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Abstract

Purpose – The purpose of this paper is to point out a paradox in variational theory for viscous flows. Chien (1984) claimed that a variational principle of maximum power losses for viscous fluids was established, however, it violated the well-known Helmholtz’s principle.

Design/methodology/approach – Restricted variables are introduced in the derivation, the first order and the second order of variation of the restricted variables are zero.

Findings – An approximate variational principle of minimum power loses is established, which agrees with the Helmholtz’s principle, and the paradox is solved.

Research limitations/implications – This paper focusses on incompressible viscous flows, and the theory can be extended to compressible one and other viscous flows. It is still difficult to obtain a variational formulation for Navier-Stokes equations.

Practical implications – The variational principle of minimum power loses can be directly used for numerical methods and analytical analysis.

Originality/value – It is proved that Chien’s variational principle is a minimum principle.

Keywords Variational theory, Navier-Stokes equation, Semi-inverse method, Viscous flow

Paper type Research paper

1. Introduction

The variational theory for viscous flow has been caught much attention, and various variational formulations were established for some special cases (Bogner, 2008; Chen et al., 2010; Ecer, 1980; He, 1998; Petrov, 2015; Fei et al., 2013; Jia et al., 2014; Li et al., 2013; Li and Liu, 2013; Tao and Chen, 2013). As early as 1868, Helmholtz proposed a minimum principle for viscous fluids neglecting kinetic effect (Finlayson, 1972), and it was once proved that there exists no variational representation for Navier-Stokes equations (Finlayson, 1972). However, Chien (1984) claimed that a variational principle of maximum power loses for viscous fluids was established, which contradicted the well-known Helmholtz’s principle.

In this paper, we will re-illustrate the derivation process to reveal the paradox and prove that Chien’s principle is actually an approximate minimum principle.

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2. Chien's principle
Considering the incompressible viscose flow, we have the following equations:

\[ u_{k,k} = 0, \quad (1) \]

\[ \sigma_{ij} = -p\delta_{ij} + \mu(u_{i,j} + u_{j,i}), \quad (2) \]

\[ \rho \frac{Du_i}{Dt} = \rho \left( \frac{\partial u_i}{\partial t} + u_k u_{i,k} \right) = \sigma_{ij,j} + \rho F_i. \quad (3) \]

The boundary conditions are:

\[ u_i = 0, \text{ (on solid boundary } \Gamma_s) \quad (4) \]

\[ u_i = \overline{u}_i, \text{ (on inlet and outlet boundary } \Gamma_u) \quad (5) \]

\[ \sigma_{ij} n_j = f_i \text{ (on boundary acted by the given forces } \Gamma_e) \quad (6) \]

Chien (1984) proposed the following principle for incompressible viscous fluids.

Among all possible \( u_i, \sigma_{ij}, p \), which satisfy first, condition of incompressibility \( u_{k,k} = 0 \); second, \( u_i = 0 \) on solid boundary surface \( \Gamma_s \); third, \( u_i = \overline{u}_i \) on boundary section where the water is supplied (\( \Gamma_u \)), and forth, stress-flow velocity relations \( \sigma_{ij} = -p\delta_{ij} + \mu(u_{i,j} + u_{j,i}) \), the actual \( u_i, \sigma_{ij}, p \) solutions in the problem of incompressible viscose flow make the total power losses of fluid flow \( J(u_i, \sigma_{ij}, p) \) maximum:

\[ J(u_i, \sigma_{ij}, p) = J_0 + \iiint \left\{ \frac{1}{4\mu} \left[ \sigma_{ij} \sigma_{ij} - 3p^2 \right] - \rho u_i F_i \right\} d\tau - \iint_{\Gamma_e} f_i u_i dS, \quad (7) \]

where:

\[ \delta J_0 = \iiint \rho \left( \frac{\partial u_i}{\partial t} + u_k u_{i,k} \right) \delta u_i d\tau. \quad (8) \]

3. Restricted variables
It should be pointed out that there exists no \( J_0 \). Equation (8) can be explained by the principle of virtual work. Applying the concept of restricted variables (He, 2012), which are widely used in the variational iteration method (He, 2012; Zhang et al., 2015; Noor and Noor, 2014), we can write \( J_0 \) in the form:

\[ J_0 = \iiint \rho \left( \frac{Du_i}{Dt} \right) u_i d\tau, \quad (9) \]

where the superscript “—” denotes restricted variable, i.e.:

\[ \delta \left( \frac{Du_i}{Dt} \right) = 0, \quad (10) \]
thus we have:

\[
\delta J_0 = \delta \int \int \int \left( \rho \frac{Du_i}{Dt} \right) u_i d\tau = \int \int \int \left\{ \left( \rho \frac{Du_i}{Dt} \right) \delta u_i + u_i \delta \left( \rho \frac{Du_i}{Dt} \right) \right\} d\tau
\]

\[
= \int \int \int \left\{ \left( \rho \frac{Du_i}{Dt} \right) \delta u_i \right\} d\tau.
\]

Chien’s variational principle can be approximately written in the form:

\[
J(u_i, \sigma_{ij}, p) = \int \int \int \left\{ \left( \rho \frac{Du_i}{Dt} \right) u_i + \frac{1}{4\mu} \left[ \sigma_{ij} \sigma_{ij} - 3\rho \delta \right] - \rho u_i F_i \right\} d\tau - \int \int \int \int_{\Gamma} \bar{f}_i u_i dS
\]

(12)

4. Approximate minimum principle

We can see clearly now that Chien (1984) used unintentionally the concept of restricted variables. It should be specially pointed out the higher orders of variation of restricted variables should vanish completely, i.e.:

\[
\delta^2 J_0 = \int \int \int \left\{ \delta \left( \rho \frac{Du_i}{Dt} \right) \delta u_i \right\} d\tau = 0.
\]

(13)

Using the constraint, Equation (7), the second order of variation of \( J \) is

\[
\delta^2 J = \int \int \int \frac{1}{4\mu} \left[ \delta \sigma_{ij} \delta \sigma_{ij} - 3\delta \rho \delta \right] d\tau = \int \int \int \frac{\mu}{4} \left( \delta u_{ij} + \delta u_{ji} \right) \left( \delta u_{ij} + \delta u_{ji} \right) d\tau > 0.
\]

(14)

So it is a “minimum principle,” not a “maximum principle” at all. Due to the application of restricted variables, we obtained actually an approximate variational principle, so the principle should be called approximate minimum principle.

5. Conclusion

In this paper, we clarify the misunderstandings of Chien’s maximum principle of viscous flow, it actually an approximate minimum principle, which agrees with the Helmholtz’s principle.

References


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