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Short communication

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The Kubelka–Munk equation is solved approximately for paint films with finite thickness, an inequality is induced in the solution process, and a minimal Kubelka–Munk absorption coefficient is obtained.

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1. Introduction

Computer colour matching of paints is based on the determination of the Kubelka–Munk absorption (K) and scattering coefficients (S) of pigments [1–4]. When studying absorption properties, the extinction coefficient E can be expressed as the sum of absorption coefficient K and scattering coefficient S

\[ E = K + S \] (1)

For the determination of K and S in some practical dyeing systems, the Kubelka–Munk theory is obviously not valid [5]:

\[ \frac{K}{S} = \frac{(R_{\infty} - 1)^2}{2R_{\infty}} \] (2)

where \( R \) is the reflectance of sheet, \( R = R_{\infty} \) when the thickness of the sheet tends to infinity.

The Kubelka–Munk theory was originally developed for paint films having a thickness much smaller than the total thickness [1–4]. This paper focuses on a film with finite thickness.

2. The Kubelka–Munk absorption

Consider light of intensity \( I_0 \) incident on a non-glossy piece of paper of thickness \( L \) and reflectance \( R \). Behind this piece of paper is a surface of reflectance \( R_0 \). The light which re-emerges from the top surface of the paper after scattering, absorption or transmission has intensity \( I \). At a distance \( x \) from the bottom surface of the paper there is a thin lamina of thickness \( dx \) and scattered light is incident on it which is travelling both upwards and downwards through it with intensities \( i_R \) and \( i_T \), respectively. The Kubelka–Munk equation can be written in the form [1–4]:

\[ -\frac{di_T}{i_T} = -(S + K)i_T \, dx + i_S \, S \, dx \] (3)

\[ \frac{di_R}{i_R} = -(S + K)i_R \, dx + i_I \, I \, dx \] (4)

Eqs. (3) and (4) can be solved analytically by the variational iteration method [5,6]. This paper provides a simple analytical approach to the system.

Divide Eq. (3) by \( i_T \) and Eq. (4) by \( i_R \) and add together:

\[ \frac{di_R}{i_R} - \frac{di_T}{i_T} = -2(S + K)i_T \, dx + S \left( \frac{i_R}{i_T} + \frac{i_I}{I} \right) \, dx \] (5)

Define \( R = i_R/i_0 \) as reflectance of sheet and \( r = i_I/i_T \) as reflectance of increment and consider the following inequality:
\[
\frac{i_R}{i_I} \geq 2 \quad \text{(6)}
\]

Eq. (5) can be simplified as

\[
d\ln r \geq -2Kdx \quad \text{(7)}
\]

Integration of Eq. (7) results in

\[
\ln R - \ln R_0 \geq -2KL \quad \text{(8)}
\]

Kubelka–Munk absorption coefficient reads

\[
K \geq \frac{\ln R - \ln R_0}{2L} \quad \text{(9)}
\]

The reflectance \( R \) can be calculated using Saunderson correction \[5\].

\[
R = \frac{k_1 + (1 - k_1 - k_2)R}{1 - k_2R} \quad \text{(10)}
\]

where \( k_1 \) is the fraction of incident light externally specularly reflected upon entering the material, \( k_2 \) is the fraction of light internally diffusely reflected upon leaving the material at the front surface.

Saunderson correction for matt film \[3,7\] is

\[
R = \frac{R - 0.04}{0.4R + 0.56} \quad \text{(11)}
\]

The minimal absorption coefficient is

\[
K_{\text{min}} = \frac{\ln \left( \frac{k_1 + (1 - k_1 - k_2)R}{1 - k_2R} \right) - \ln R}{2L} \quad \text{(12)}
\]

3. Conclusion

This paper gives a formulation for the minimal Kubelka–Munk absorption coefficient for paint films with finite thickness.

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References