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Hierarchy of Wool Fibers and Fractal Dimensions

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Abstract

Wool fiber shows excellent advantages in warmth-retaining and many other practical properties possibly due to its hierarchical structure. Its fractal dimension of wool fiber is calculated which is very close to the Golden Mean, 1.618. The present study might provide a new interpretation for the reason why wool fiber has so many excellent properties.

Keywords: wool fiber, fractal dimension, metabolic rate, sheep

1. Introduction

Wool fiber, one of the most widely used natural fibers from animal origin in textile industry, is a renewable, sustainable resource. After million years of evolution, Nature endows wool with genius of warmth-retaining property. Even in the modern times, it is difficult to synthesize a material like wool fiber having advantages of perspiration and moisture absorption besides warm-retaining ability. Much attention has been directed towards the micro-structure assembly in wool fibre which indicates its mechanical properties [1-3], however, the fractal structure of the wool fibre, which has a close relationship with many other properties, was rarely investigated previously.

2. Hierarchical Structure of Wool Fiber

The structure of the wool fiber is rather complex with many hierarchic levels [4], as shown in Fig.1. The primary component of wool fiber is low-sulfur α-keratins which are part of superfamily of protein. Three α-helix macromoleculars twist compactly together to form an elementary structure of wool fibre, which is called protofibril, of diameter about 2nm [5-6]. It is widely believed that eleven protofibrils assemble a rod-like microfibril with diameter of 7~8nm [3,7-8]. Distinguishable arrangements of microfibrils assemble different cortical cells in wool cortex (classified as ortho- and para-), about 4μm in diameter [9]. Microfibrils helically wind to be a macrofibril of diameter some 0.4μm in orthocortical cells, whereas this structural elaboration is unconspicuous in paracortical cells[2]. Finally, the cortex composed of cortical cells with a sheath of cuticle layer forms the body of wool fibre with diameter of 20μm[10].

3. Fractal Dimension of Wool Fiber

The fractal dimension is defined as [11]

$$D_f = \log M / \log N \qquad (1)$$

where M is the number of new units within the original unit with a new dimension, N is the ratio of the original dimension to the new dimension.

In this work, we investigate three self-similar hierarchic structures beginning with the basic building blocks of wool fibre, the α-helix.

The original hierarchic level, the protofibril, consists of three α-helix macromolecules, as

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was shown in Fig. 2(a).

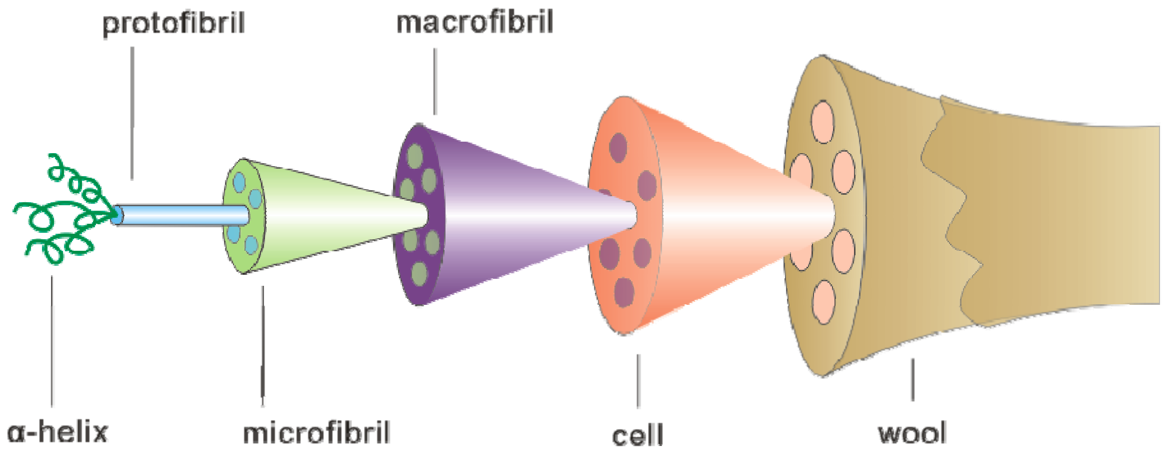
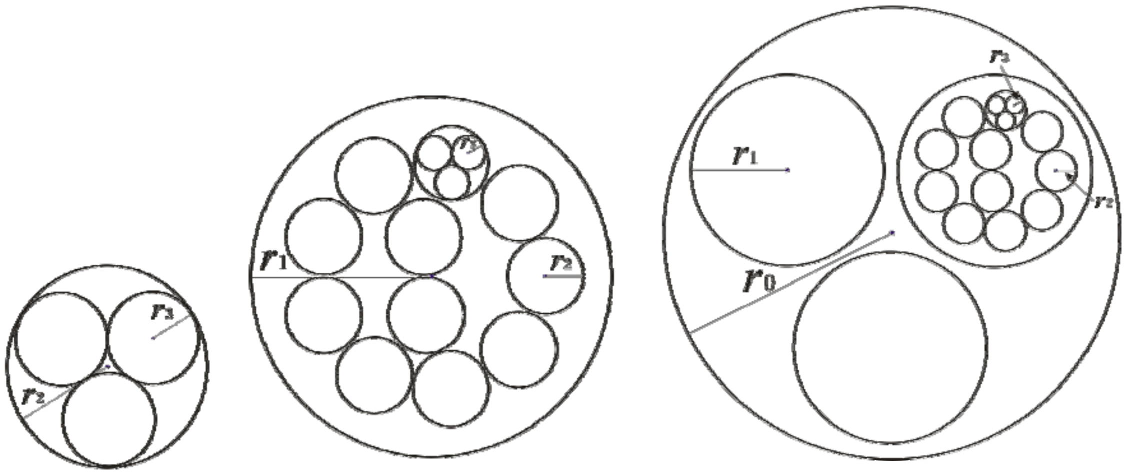


Fig.1 schematic diagram of the structure of wool fiber



(a) (b) (c)

Fig. 2 diagrams of three hierarchic levels of wool fibre
(a) the first hierarchic level, (b) the second hierarchic level, (c) the third hierarchic levels.

The proportion between the diameter of protofibril and the α -helix is

$$N_1 = \frac{r_2}{r_3} = 2.155, \qquad M_1 = 3$$

Thus, the cross-section area of three α -helix molecules occupies 64.6% of the corresponding area of protofibril, and the fractal dimension D_1 can be calculated as follows:

$$D_1 = \log M_1 / \log N_1 = \log 3 / \log 2.155 = 1.431 \qquad (2)$$

The elementary fractal dimension D_1 originates from such a microcosmic structure that it can not express accurately the fractal dimension of wool fibre in the whole range of cortex.

For the second hierarchic level, there are 11 protofibrils which assemble one microfibril, shown in Fig. 2(b). The average diameter of protofibril and the microfibril is 2nm and 8nm, respectively. In this hierarchic level, all the protofibrils take up 68.8% of the cross-section of one microfibril. Generally, there are 33

α -helixes within a single microfibril. So the ratio of microfibril diameter to α -helix is

$$N_2 = \frac{r_1}{r_3} = 8.619, \quad M_2 = 33$$

The fractal dimension D_2 is

$$D_2 = \frac{\log M_2}{\log N_2} = \frac{\log 33}{\log 8.619} = 1.623 \qquad (3)$$

The fractal dimension in the third hierarchic level can be obtained based on the real configuration of wool fibre cortex. The orthocortex and paracortex are bilaterally arranged in the cross-section of cortex. The area proportion between the two portions is about 2:1 [10]. The arrangement of microfibrils is different in the two sections, with compact assemblage in orthocortex but loose assemblage in paracortex [12]. The ratio of microfibrils to matrix in orthocortex is about 4:1, and in paracortex the proportion is 1:1[13]. Cell membrane complex (CMC) between cortical cells was ignored during hereinafter calculation, since it just occupies 3% of the total area of the cortex cross-section [14]. The following assumptions are made:

1) Cortex is directly composed of microfibrils.
2) Microfibrils evenly disperse in the whole cross-section of cortex.

As a result, the microfibrils take up 70% of the total cross-section area of the cortex according to the analysis above. Herein, we employ the same generating unit as in the original hierarchic level, shown in Fig.2(c). The smallest circle represents the α -helixes, the middle circle represents the microfibril. The largest circle represents a small portion taken out from the cortex with three microfibrils, and the microfibrils occupy 70% area within the largest circle, i.e.

$$\frac{3\pi r_1^2}{\pi r_0^2} = 0.7$$

So we get

$$N_3 = \frac{r_0}{r_3} = 17.842, \quad M_3 = 99$$

The fractal dimension is

$$D_3 = \frac{\log N_3}{\log M_3} = \frac{\log 99}{\log 17.842} = 1.595 \qquad (4)$$

D_2 and D_3 are very close to the gold mean.

4. Discussions

The fractal dimension values, D_2 and D_3 , very close to the Golden Mean value[15-16], derived from the latter two hierarchic levels show the optimal fractal character of wool fibre, which has an inevitable connection with the prominent properties of this natural fiber.

It is well known that golden mean has been found in many natural phenomena and played very significant roles [17-20]. After many generations of evolution, it is both surprising and understandable that wool fiber has acquired an interesting fractal dimensional structure. The significance of the value D=1.618 for sheep's survival is quite self-explanatory. Because the possible range for the fractal dimension values is $0 \le D \le 2$ so that two extreme conditions will be: 1) $D = 0$, that means wool fiber turn to be hollow fiber with good heat retention but bad heat dispersing ability and the sheep could not survive during summer; 2) $D = 2$, that means wool is just like the man-made fiber, heat will disperse fast and directly along the fiber length. Thus the fiber can not provide a good heat insulating ability for sheep in winter. This present contribution is concerned with the fractal dimension of wool fiber associated with its good properties.

When a sheep is in an extreme motion to escape from a hunt, its body temperature will increase sharply due to higher metabolic rate:

$$B \propto M^{\frac{3}{4}+\frac{T-30}{40}} \qquad (4)$$

where B is the metabolic rate, *M* body mass, *T* temperature in ^{o}C.

In order to maintain its body temperature as low as possible, the hierarchical structure of wool fiber provides an optimal way to transfer heat.

5. Conclusions

The fractal dimension of wool fibre indicates that the fractal structure of wool fibre is very close to the optimal fractal dimension. This important result provides a brand-new horizon for us to re-understand the essence of good warmth retention property and many other prominent properties of wool fibre. The idea of Golden Mean value can be further invited to textiles design process to make excellent textile products with full utilization of the good properties of wool fibre.

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