Research highlights in this issue (Nonlinear Science Letters A)

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This issue witnesses the last development of advanced analytical methods for nonlinear problems especially for nonlinear vibrations and fractional differential equations, including García’s amplitude-period formula for nonlinear oscillators plus Suárez-Antola’s remark, El-Dib’s modification of the homotopy perturbation method, the optimal homotopy asymptotic method, the variational iteration method, Singh’s numerical method for fractional calculus, Guner’s shockwave ansatz for solitary waves, Otto’s golden mean quantum mechanics. This issue also publishes an invited article by famous mathematician, Prof. Ganji, where the Akbari-Ganji method is systematically elucidated, and it is accessible to all non-mathematicians, making the nonlinear science much less mysterious and much more attractive for students and engineers to unveil hidden phenomena in a practical problem. Here are highlights for each article.


This paper suggests a simple estimation of the period for a nonlinear oscillator, and an open problem for his period formula is proposed. García’s amplitude-period formula is extremely simple for use with relatively high accuracy, and for some special case, it can lead an extremely accurate solution. Any comments on García’s open problem is welcome.


This is an invited article to make a remark on García’s amplitude-period formula, and partly solves García’s open problem. In this paper, Suárez Antola also suggests some amplitude-period formulae, easy and effective.


The multiple scales method is a well-known method in the perturbation theory, and it is effective for weakly nonlinear oscillators. However, the combination of the homotopy perturbation method with the multiple scales method yields an unexpected result and allows to study the stability behavior for all strongly nonlinear oscillators. El-Dib’s modification of the homotopy perturbation method is also valid and effective for other nonlinear problems, and such submissions are welcome.

The optimal homotopy asymptotic method was originally proposed to solve differential equations. Recently there is a strong tendency toward the applications of analytical methods to fractional calculus. The optimal homotopy asymptotic method is a development of the homotopy perturbation method by embedding an auxiliary function

\[ H(p) = ap + bp^2 + cp^3 \]

When \( a=1, b=c=0 \), it becomes the homotopy perturbation method. The main difficulty in the optimal homotopy asymptotic method is how to optimally and easily identify the values of the auxiliary constants, \( a, b \) and \( c \) in the auxiliary function \( H(p) \). We are happy to see mathematicians from Pakistan have succeeded in solving this problem. This paper applies the collocation and Galerkin’s methods to optimally determine the parameters involved in the auxiliary homotopy function.


Coating using molten polymers is a complex process, and it affects greatly safety and service life of the coated wires. How to optimize the coating process is still an open problem, this paper solves the problem partly by establishing a mathematical model for the coating process, and an analytical solution with physical understanding is obtained by the optimal homotopy asymptotic method.


In this paper, an auxiliary parameter is involved in the variational iteration algorithm, which is then optimally identified. This modification of the variational iteration method can be used for all kinds of nonlinear problems. Additionally an auxiliary parameter can be also adopted in the variational iteration algorithm-II and the variational iteration algorithm-III, see He JH, et al., The variational iteration method which should be followed, Nonlinear Science Letters A, 1(1)(2010)1-30. Submissions on the auxiliary parameter for the variational iteration algorithm-II and the variational iteration algorithm-III are welcome.


This paper suggests an effective numerical method for fractional differential equations using Chebyshev operational matrix of differentiation and the collocation method. Due to the extreme simplicity of construction of an operational matrix, Singh’s numerical method for fractional calculus is very much attractive and challenging in fractional calculus.


A shock wave is a special kind of solitary waves. When a wave propagates faster than the local speed of sound in a fluid, a shock wave might occur. A shock wave is very important for engineers to design, for example, a supersonic aircraft. But no one knows when and where a shock wave will appear exactly. This paper suggests a shockwave ansatz to search for shock waves and find necessary conditions for shock waves. Guner’s shockwave ansatz has therefore both theoretical importance and practical applications.

This paper discovers an impressive relationship between the golden mean ($\varphi =0.6180\ldots$) and $\pi =3.14159\ldots$. The two numbers are familiar to all people, but it will be astonished if you find that dark energy density can be accurately expressed by either $\pi$ or $\varphi$

$$E\text{(dark)} \approx \frac{3}{\pi}mc^2 \approx \frac{21}{22}mc^2$$

and the ordinary energy density

$$E\text{(ordinary)} \approx (1-\frac{3}{\pi})mc^2 \approx \frac{1}{22}mc^2$$


What is the gyromagnetic factor or $g$-factor? Can we predict its value?

The $g$-factor is related to the magnetic moment of an electron, its value is twice what it should be according to the Dirac equation in classical mechanics, a fundamental equation connecting the electron's spin with its electromagnetic properties. However, the most accurate experimental value so far according to *Fundamental Physical Constants: Electron g Factor* (NIST) is

$$g=2.00231930436182(52)$$

see https://physics.nist.gov/cgi-bin/cuu/Value?gem

The value deviates from 2. This paper gives a very impressive formula for predicting its value:

$$g = 2 + \ln(1 + \frac{\varphi^5}{24}) = 2.002319312$$

where $\varphi$ is the gold mean. Otto’s $g$-factor formulation elucidates the importance of the gold mean in quantum mechanics.


This an invited article. The Akbari-Ganji method was first proposed by Dr. Ganji and further developed by Mr. Akbari, and now it has become a matured method for almost all nonlinear problems, and especially for nonlinear vibration as this paper unveils. The Akbari-Ganji method is a simple but sophisticated modification of the collocation method. The basic properties of the method are:

1) A suitable trial function with $n$ unknown parameters is chosen as an approximate solution for a nonlinear problem. For a nonlinear vibration, the trial function is always chosen as a series of sine functions.

2) Differentiating the governing equation to obtain additional boundary and initial conditions for any higher order derivatives of the solution.

3) The $n$ unknown parameters are determined by solving $n$ algebraic equations simultaneously.

The Akbari-Ganji method is of extremely simplicity, and the accuracy of the approximate solution can be achieved to any order. This makes most impossible things in nonlinear science possible.

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