Investigation of Compression Strength of Bliss Style Corrugated Fiberboard Boxes

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In the global packaging industry, there is a significant demand for corrugated fiberboard. Based on end-use, the packaging for processed and fresh food categories accounted for approximately 39% of the overall consumption of corrugated fiberboard in 2015. Bliss style boxes are prominent in the agriculture sector due to key advantages such as providing an uninterrupted bottom, laminated corners for higher stacking strength, increased material use efficiency, side cutout options for display at retail, and a wide range of styles over other styles of containers. While numerous predictive strength models associating corrugated fiberboard material specifications to the box compression strength (BCT), and ultimately the stacking strength of corrugated containers, have been developed over the past century, there is a considerable lack of studies that include Bliss style containers. The overall aim of this empirical study was to develop a mathematical relationship based on the simplified McKee formula towards predicting BCT of four styles of Bliss boxes. Effects of box styles, length of load-bearing walls and number of internal corners on the overall BCT were explored using data collected from lab-based testing. The proposed mathematical model includes a box design constant (k), edge crush test values, board thickness, and three lengths of load-bearing walls (total, single-wall, and double-wall) of the containers. The k-values for each bliss box design, explored through linear regression analyses, explain up to 98.1% of the differences in BCT between the styles. The proposed mathematical model can assist practitioners with accelerating packaging development cycle times and optimizing packaging designs.

**KEY WORDS**

Corrugated Fiberboard, Bliss Boxes, Box Compression Strength, McKee Formula, Mathematical Model

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INTRODUCTION

Driven by an anticipated increase in near future demand, the global corrugated fiberboard industry is forecast to reach a market size of $336.26 billion by 2023. Increasing from $268.50 billion in 2017, this represents a compound annual growth rate of 3.82 % [1]. The key drivers of this increased demand of corrugated fiberboard includes significant investments in environmental protection and energy optimization programs by producers, increased “right-weighting” (light-weighting) solution initiatives, rapidly evolving e-commerce business, surge in digital (interactive) print options for traditional and e-commerce packaging, demand for more recyclability in containerboard stocks and industry consolidation [2].

Based on the end-use, this corrugated packaging market can be divided into the following sectors: processed food, fresh food and produce, beverages, personal and household care, chemicals, paper products, electrical goods, glassware and ceramics, tobacco, etc. (Figure 1). “Processed food” and “Fresh food and produce” accounted for a market share of approximately 39 % in 2015 and are projected to continue their dominance during the forecast period 2016-2021 [3].

The most common styles of corrugated fiberboard forms can be categorized as slotted boxes, telescope boxes, folders, rigid boxes (e.g., bliss boxes), self-erecting boxes and interior forms (Figure 2) [4]. The construction, as well as applications of these styles, vary. With all four of its vertical edges reinforced, the bliss box provides greater board-usage to stacking-strength ratio than the other styles of corrugated boxes. In comparison with slotted boxes, bliss boxes require less fiberboard since its bottom consists of a single thickness resulting in flap area reduction. While relatively stronger and material efficient, bliss boxes are constructed with three die-cut pieces which require specialized set-up equipment at the point of use including hot melt adhesive.

![Bar chart: Global consumption of corrugated board packaging by end use, 2015](image)

*Fig. 1: Global consumption of corrugated board packaging by end use, 2015 [3].*
Bliss boxes lend themselves to many different uses and are primarily find considerable application in the agricultural sector for packing fresh fruits and vegetables, dairy, meat and prepared foods. In addition, bliss boxes can be used towards other applications that require higher stacking strength for storage and handling during their distribution, such as bottled products industry and the construction industry [5]. Also, explosives, articles of concentrated weight, etcetera can be packed in bliss boxes [6], [7].

Bliss-style boxes, while technically considered as a tray, are unique because they are constructed from three pieces of corrugated fiberboard (Figure 3). Similar to a tray, the body of the bliss boxes is made of a single piece of fiberboard forming the bottom, large sides and major flaps on top. Each end panel, however, is made from a separate piece of fiberboard that may include minor flaps. The bottom and large sides also constitute overlapping flanges into which the end panels are glued towards attachment with the main body.

The three-piece construction of bliss boxes allows for customized board grades to be used towards providing optimization between material usage and strength [7].
While Regular Slotted Containers (RSC) are the most commonly used style of corrugated fiberboard containers globally, bliss-style boxes provide the following unique opportunities by comparison [8]:

- **Continuous bottom**: the uninterrupted bottom provides a flat surface for the product to rest on thereby reducing potential damage resulting from friction (vibration).

- **Laminated corners**: Since a bulk of the compressive load is supported by the corners of a box, the reinforcement provided through the lamination of flanges of the main body and end panels improve the stacking strength of bliss boxes [9]–[12].

- **Material efficiency**: Due to the ability to construct bliss boxes using varying board grades resulting in improved stacking strength, the overall amount of material usage is lowered.

- **Side cutout options**: Meet requirements of display/retail-ready packaging for big-box stores.

- **Wide range of styles**: Unlike RSCs, bliss boxes can be created in a variety of styles optimized for shipping and retail-ready demands of various product categories.

While the pallets commonly used for distribution of goods of retail has been standardized by the Grocery Manufacturers Association (GMA) to be 101.60 cm x 121.92 cm, the corrugated packaging footprint for fresh produce varies substantially [13]. Towards improving stackability; lowering shipping costs; improving product protection; driving consistency in production; meeting display and marketing demands of retailers; and promoting the recyclable/renewable benefits of corrugated fiberboard, a Corrugated Common Footprint (CCF) standard has been established by the industry [14].

The CCF standard provides specifications towards a full size (“5-down”, 40.64 cm x 30.48 cm) and half size (“10-down”, 60.96 cm x 40.64 cm) footprints (Figure 4) [14]. The recommended uniform footprint dimensions and interstacking features for corrugated containers facilitate efficient loading, handling, storage and shipment of produce (i.e., fresh fruits and vegetables) on standardized GMA pallets (Figure 5). Studies cited by Fibre Box Association claim larger payloads, reduced bruising, similar cooling rates and cost savings in annual handling and long-distance trucking costs for CCF containers as compared to reusable plastic containers [15].

*Fig. 4: (a) Half-size and (b) full-size Corrugated Common Footprint (CCF) containers (dimensions in cm).*
Box Compression Strength Models

Since the landmark decision reached via the Pridham case in 1914, corrugated fiberboard containers have become the preferred shipping container to move goods in the various distribution strategies [16]. Numerous predictive strength models associating corrugated fiberboard material specifications to the compression strength (lab based observation using standard methodologies), and ultimately the stacking strength (field use approximation), of corrugated containers have been developed over the past century [17]–[27]. The numerous currently existing models and software towards predicting corrugated board and box properties range from molecular length scale to unitized loads [28].

Most packaging professionals associated with the development process of protective packaging solutions related to RSC style corrugated fiberboard boxes are familiar with the McKee formula reported in 1963 (Equation 1) [24]. It may be noted that this equation applies only to RSC style boxes having lengths of less than three times the width, and a perimeter of less than seven times the depth [27], [29].

\[
BCT = 2.028 \ ECT^{0.746} \left( \sqrt{D_{MD}D_{CD}} \right)^{0.254} P^{0.492} \tag{1}
\]

Where,

\[
BCT = \text{Box compression strength (lb)} \\
ECT = \text{Edge crush test value of board (lb/in)} \\
D_{MD} = \text{Machine-direction flexural stiffness (lb in)} \\
D_{CD} = \text{Cross-direction flexural stiffness (lb in)} \\
P = \text{Box perimeter (in)}
\]

The simplified form of the McKee formula (Equation 2) is most commonly used by practitioners [24]. Since flexural stiffness is not frequently performed, it is substituted with board thickness \( t \), in. It has been observed that the simplified McKee formula overestimates the original McKee formula [30].

\[
BCT = 5.87ECT\sqrt{tP} \tag{2}
\]

Batelka & Smith (1993) expanded the original McKee box compression model (Equation 1) to include bliss boxes and die cut boxes towards improving the prediction accuracy [29]. The modified equation (Equation 3) included the \( ECT \) and flexural stiffness \( (RMS \text{ Bending}) \) and added a “depth” term, \( d \). The modified model is claimed to have improved the prediction accuracy for all structures included in the study by 68% and for all structures within the limiting boundaries of the original model by 42% [29].

\[
BCT = 1.014 \ ECT^{0.746} \left( D_{MD} D_{CD} \right)^{0.127} \left( \Sigma W^{0.492} \right) 1.593d^{-0.236} \tag{3}
\]

Where,

\[
BCT = \text{Box compression strength (lb)} \\
ECT = \text{Edge crush test value of board (lb/in)} \\
D_{MD} = \text{Machine-direction flexural stiffness (lb in)} \\
D_{CD} = \text{Cross-direction flexural stiffness (lb in)} \\
W = \text{Width of each panel (in)} \\
d = \text{Box depth (in)}
\]
Batelka & Smith provided an example of the possible application of the modified McKee equation for a bliss box constructed with two different grades of corrugated fiberboard [29]. The total predictive value of compression strength ($BCT_T$) is calculated using Equation (4) as the sum of contributions from the body ($BCT_B$) and end panels ($BCT_E$). It may be noted that this predictive value of the compression strength was not compared with experimental results in this study.

$$BCT_T = BCT_B + BCT_E$$  \hspace{1cm} (4)

More recently, Ge & Goodwin (2010) developed and reported equations to calculate top-to-bottom and side-to-side compression strengths for FEFCO styles 0601 bliss-style cases and 0406 wrap-around style boxes [27]. Ge & Goodwin’s equation (Equation 5) for predicting bliss-style top-to-bottom compression strength is based on the simplified McKee formula and includes specific mathematical relationships to account for the bliss box perimeter and board thickness.

$$BCT = 5.87ECT \sqrt{(t + \frac{3m}{2L + 2W}) \times (2L + 2Wm)}$$  \hspace{1cm} (5)

Where,

- $BCT =$ Box compression strength (lb)
- $ECT =$ Edge crush test value of board (lb/in)
- $t =$ Board thickness (in)
- $m =$ Manufacturer’s joint width (in)
- $L =$ Box length (in)
- $W =$ Box width (in)

Batelka & Smith and Ge & Goodwin’s models are well suited to predict compression box strength of bliss boxes with a standard geometry (i.e., end panels are rectangular pieces with no creases glued to the body) but there are other bliss box styles in which the cross-sections of the box include creases and flanges creating triangular columns and edges. These features are known in the industry to increase box compression strength significantly [31]. Consequently, the overall objective of this study was to develop a new mathematical relationship based on the simplified McKee formula that accounted for variations in cross-section geometry of bliss-style boxes.

**MATERIALS AND METHODS**

The methodology used for this investigation included the following sequential phases:

- Measuring compression strength of four different bliss-style boxes of two sizes systematically.
- Analyzing how design characteristics affect compression strength in these types of boxes.
- Developing a mathematical equation based on the simplified McKee formula that allows predicting box compression strength of bliss-style boxes with reasonable accuracy and ease.
- Fitting the mathematical model to the experimental data.

**Containers**

Four different bliss box designs were included in this study: standard bliss box ($S$); standard bliss box with internal flanges ($SIF$); standard bliss box with internal flanges and diagonal corner ($SIFDC$); and V-bliss box ($V$) (Figure 6). Each design of bliss box was constructed to meet the full size (“5-down”, 40.64 cm x 30.48 cm x 23 cm) and half size (“10-down”, 60.96 cm x 40.64 cm x 23) CCF footprints. Length of the internal flanges included for $SIF$, $SIFDC$, and $V$ designs were 5.08 cm (2 in), 7.62 cm (3 in), 10.16 cm (4 in) and 12.70 cm (5 in). 5-down boxes had an approximate internal volume of 53,101 cm$^3$ (3,240 in$^3$) while 10-down boxes had an approximated internal volume of 25,425 cm$^3$ (1,552 in$^3$). C-flute corrugated fiberboard with an $ECT$ value of 3.62 kN/mm (32 lb/in) and caliper of 3.97 mm (0.16 in) was used for the construction of all test samples.
Boxes were designed using ArtiosCAD® software [32] and cut/creased using a Kongsberg table model 1930 (Esko Graphics, Ludlow, Massachusetts, USA). Each box was identified by unique code comprised of four sections as follows:

\[ A - B - C - D \]

Where, 
- **A** = Size of the bliss box  
  [5-down=5; 10-down=10]  
- **B** = Bliss box design  
  \[ S, SIF, SIFDC, V \]  
- **C** = Length of internal flange  
  \[ 0 \text{ cm} = 0; 5.08 \text{ cm (in)} = 2; 7.62 \text{ cm (in)} = 3; 10.16 \text{ cm (in)} = 4; 12.70 \text{ cm (in)} = 5 \]  
- **D** = Replicate number  
  \[ \text{first}=1; \text{second}=2; \text{third}=3; \text{fourth}=4; \text{fifth}=5 \]

As an example, “10-SIFDC-4-2” represents “10-down standard bliss box with internal flanges and diagonal corner with a 4-inch (10.16 cm) flange length and replicate number two.”

**Methods**

**Conditioning**

Prior to all testing, the prototyped boxes were conditioned at standard temperature and humidity (23 ± 1 °C and 50 ± 2 % relative humidity) in a walk-in environmental chamber (Darwin Chambers Company, Saint Louis, MO, USA) per ASTM D4332 [33].

**Compression Testing**

ASTM D642 (Standard Test Method for Determining Compressive Resistance of Shipping Containers, Components, and Unit Loads) was used to test the compression strength [34]. This procedure is commonly used for measuring the ability of the container to resist external compressive loads applied to its faces, to diagonally opposite edges, or to corners. This test method is also used to compare the characteristics of a given design of container with a standard, or to compare the characteristics of containers differing in construction. This test method is related to TAPPI T804 om-12 [35]. The tests were conducted using a fixed platen arrangement on a Lansmont compression tester Model 152-30K (Lansmont Corporation, Monterey, CA, USA), with a platen speed of 1.3 cm/minute and a pre-load of 22.68 kgf for zero-deflection in accordance with the standard.

**DATA ANALYSIS**

Data were analyzed using JMP® Pro (Cary, North Carolina, USA) [36] and MATLAB® (Natick, Massachusetts, USA) [37]. One-way analyses of variance (ANOVA) were carried out to investigate whether and how box compression strength is related to four different bliss-style box designs of two sizes. Linear regression analyses were computed to fit experimental data to an equation.

![Fig. 6: Bliss box designs used in the study. Body parts are depicted in gray and end panels in red. Box's dimensions shown correspond to 10-down sizes.](image-url)
RESULTS AND DISCUSSION

A total of 65 5-down and 55 10-down bliss-style boxes were used in the study. Five attributes were collected to characterize the geometry of each design (Table 1):

- Box compression strength (BCT)
- Total length of load-bearing walls (PT)
- Total length of load-bearing walls using single-wall corrugated board (P\textsubscript{SW})
- Total length of load-bearing walls using double-wall board (P\textsubscript{DW})
- Number of internal corners (C) in the wrap and side components of the box.

**Effect of Bliss Box Style on BCT**

Four separate one-way analyses of variance followed by Tukey's HSD multiple comparison tests were performed using JMP\textsuperscript{®} Pro. The first two ANOVA examined the nature of differences in average BCT for each of the four design styles (i.e., S, SIF, SIFDC, V) for 10-down and 5-down boxes separately. Significant differences were identified for average BCT of both 10-down boxes (F(3,51) = 64.1, \(p < 0.0001\)) and 5-down boxes (F(3,61) = 116.3, \(p < 0.0001\)). Tukey's HSD post-hoc test was used to make comparisons between BCT data within 5-down and 10-down boxes. Notably, for both sizes, the average BCT of each design style was significantly different from the others (Figure 7).

The other two ANOVA analyses examined the differences in average BCT for each dimensional variation within the four bliss-styles. Significant differences were identified between dimensional variation with regard to average BCT of both 10-down boxes (F(10,44) = 38.6, \(p < 0.0001\)) and 5-down boxes (F(12,52) = 110.6, \(p < 0.0001\)). Tukey’s HSD post-hoc test was used to make comparisons between BCT data within 5-down and 10-down boxes. For 5-down size boxes, this analysis revealed seven homogenous subsets. For 10-down size boxes, the analysis revealed nine homogenous subsets. For both sizes, standard bliss box designs (S-5 and S-10) were significantly different from the other design variations (Figure 8).

From the four analyses, it can be seen that, as expected, BCT increases significantly when the total length of load-bearing walls increases by adding length through flanges and by adding triangular columns. For both 5-down and 10-down boxes, the four design styles can be ordered as follows, from lowest BCT to highest BCT: S, SIF, SFIDC, and V.

**Effect of Length of Load Bearing Walls on BCT**

Bliss-style boxes comprise of two parts: a body and two end panels. As a result, the total length of load-bearing walls (PT) has sections with single-wall corrugated board and sections with double-wall corrugated board. This is one of the reasons why the simplified McKee's equation (Equation 2) does not predict BCT of a bliss box accurately. Figure 9 depicts experimental BCT of the four bliss-style boxes as a function of total length of load-bearing walls. The cloud of points on the right represents 5-down boxes; larger boxes with more wall length. The group of points on the left represents 10-down boxes; smaller boxes with less wall length. The red line on the bottom part of the chart represents BCT predicted by the simplified McKee’s equation used for RSC containers. This equation yields approximated BCT values for the standard bliss box (S) but it underestimates BCT for SIF, SFIDC, and V styles.

In a bliss box, single-wall and double-wall load-bearing walls length have an inverse relationship with BCT. As length of walls with single-wall board (P\textsubscript{SW}) decreases, the length of walls with double-wall board (P\textsubscript{DW}) increases and so does BCT. This relationship can be seen in Figure 10.
Table 1: Summary of attributes used to characterize each bliss box design.

<table>
<thead>
<tr>
<th>Size</th>
<th>Design</th>
<th>ID</th>
<th>( BCT ) (N)</th>
<th>( P_t ) (cm)</th>
<th>( P_{sw} ) (cm)</th>
<th>( P_{dw} ) (cm)</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>S-10-2</td>
<td></td>
<td>2533.4 ± 90.6</td>
<td>162.7</td>
<td>103.4</td>
<td>29.7</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>SIF-10-2</td>
<td></td>
<td>5196.4 ± 103.4</td>
<td>183.8</td>
<td>82.2</td>
<td>50.8</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>SIF-10-3</td>
<td></td>
<td>6393.9 ± 96.6</td>
<td>194.0</td>
<td>72.1</td>
<td>61.0</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>SIF-10-4</td>
<td></td>
<td>7108.3 ± 495.2</td>
<td>204.2</td>
<td>61.9</td>
<td>71.1</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>SIF-10-5</td>
<td></td>
<td>7513.9 ± 1065.7</td>
<td>214.3</td>
<td>51.8</td>
<td>81.3</td>
<td>8</td>
</tr>
<tr>
<td>10-Down</td>
<td>SIFDC-10-2</td>
<td></td>
<td>7145.6 ± 881.3</td>
<td>190.5</td>
<td>119.5</td>
<td>35.5</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>SIFDC-10-3</td>
<td></td>
<td>8786.1 ± 150.8</td>
<td>200.6</td>
<td>109.3</td>
<td>45.6</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>SIFDC-10-4</td>
<td></td>
<td>7966.8 ± 1334.1</td>
<td>210.8</td>
<td>99.2</td>
<td>55.8</td>
<td>12</td>
</tr>
<tr>
<td>V</td>
<td>V-10-2</td>
<td></td>
<td>8911.6 ± 870.9</td>
<td>194.4</td>
<td>123.4</td>
<td>35.5</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>V-10-3</td>
<td></td>
<td>9311.9 ± 1170.4</td>
<td>204.5</td>
<td>113.2</td>
<td>45.6</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>V-10-4</td>
<td></td>
<td>10382.1 ± 739.2</td>
<td>214.7</td>
<td>103.1</td>
<td>55.8</td>
<td>18</td>
</tr>
<tr>
<td>S</td>
<td>S-5</td>
<td></td>
<td>2755.9 ± 483.3</td>
<td>225.0</td>
<td>165.7</td>
<td>29.7</td>
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<tr>
<td></td>
<td>SIF-5-2</td>
<td></td>
<td>6679.4 ± 121.9</td>
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<td>144.5</td>
<td>50.8</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>SIF-5-3</td>
<td></td>
<td>7119.8 ± 329.4</td>
<td>256.3</td>
<td>134.4</td>
<td>61.0</td>
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<tr>
<td></td>
<td>SIF-5-4</td>
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<td>7994.3 ± 641.3</td>
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<td>124.2</td>
<td>71.1</td>
<td>8</td>
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<tr>
<td></td>
<td>SIF-5-5</td>
<td></td>
<td>8495.2 ± 596.7</td>
<td>276.6</td>
<td>114.0</td>
<td>81.3</td>
<td>8</td>
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<tr>
<td>5-Down</td>
<td>SIFDC-5-2</td>
<td></td>
<td>8368.0 ± 821.8</td>
<td>252.8</td>
<td>181.8</td>
<td>35.5</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>SIFDC-5-3</td>
<td></td>
<td>8696.3 ± 540.4</td>
<td>262.9</td>
<td>171.6</td>
<td>45.6</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>SIFDC-5-4</td>
<td></td>
<td>9821.7 ± 464.6</td>
<td>273.1</td>
<td>161.5</td>
<td>55.8</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>SIFDC-5-5</td>
<td></td>
<td>11113.4 ± 806.0</td>
<td>283.2</td>
<td>151.3</td>
<td>66.0</td>
<td>12</td>
</tr>
<tr>
<td>V</td>
<td>V-5-2</td>
<td></td>
<td>10359.0 ± 605.2</td>
<td>256.7</td>
<td>185.7</td>
<td>35.5</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>V-5-3</td>
<td></td>
<td>11251.3 ± 358.7</td>
<td>266.8</td>
<td>175.5</td>
<td>45.6</td>
<td>18</td>
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<tr>
<td></td>
<td>V-5-4</td>
<td></td>
<td>11857.2 ± 305.7</td>
<td>277.0</td>
<td>165.4</td>
<td>55.8</td>
<td>18</td>
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<tr>
<td></td>
<td>V-5-5</td>
<td></td>
<td>12743.3 ± 750.9</td>
<td>287.1</td>
<td>155.2</td>
<td>66.0</td>
<td>18</td>
</tr>
</tbody>
</table>

\( BCT \): Box compression strength  
\( P_t \): Total length of load-bearing walls  
\( P_{sw} \): Length of load-bearing walls with single-wall corrugated board  
\( P_{dw} \): Length of load-bearing walls with double-wall corrugated board  
C: Number of internal corners
Effect of Number of Internal Corners on BCT

In addition of having sections of load-bearing walls with single-wall and double-wall corrugated board, the transversal cross-sections of the different bliss boxes styles have an increasing number of internal corners or angled bends (Figure 11). In the $S$ bliss-style, the body component of the box forms four corners and the end panels do not have any corners (i.e., they are rectangular pieces). In the $SIF$ style, body and ends form four corners each totaling eight internal corners. The $SIFDC$ style has four corners provided by the body and eight additional corners provided by the end panels and their diagonal features completing a total of 12 internal corners.

The $V$ style has a total of 18 internal corners. The same 12 corners than the $SIFDC$ style plus six corners provided by the triangular columns in the mid-part of the sides.

Internal corners create vertical edges and triangular columns that can increase box compression strength significantly. This is a very well-known design strategy used by practitioners in the corrugated box industry [31]. Figure 12 shows how BCT tends to increase with increased number of internal corners. Moreover, the relationship seems to be non-linear.

Fig. 7: Box compression strength for the four bliss-style box designs for 10 down (left) and 5-down (right) sizes. Error bars represent standard errors. Levels not connected by the same letter are significantly different.
Fig. 8: Box compression strength for each design variation for (a) 10-down size and (b) 5-down size. Error bars represent standard deviations. Levels not connected by the same letter are significantly different.
Fig. 9: Box compression strength as a function of total length of load-bearing walls of all samples tested. The red line represents BCT calculated using the simplified McKee equation.

Fig. 10: Box compression strength as a function of the length of load-bearing walls with single-wall and double-wall corrugated boards.
Fig. 11: Transversal cross sections of the four bliss-style boxes tested. Body parts are depicted in black and end panels in red. Box’s dimensions shown correspond to 10-down sizes.

Fig. 12: Box compression strength versus number of internal corners of each bliss-style box tested.
Mathematical Model

In an attempt to explain the experimental data, we explored several equations and developed one that predicts BCT with accuracy and practicality. We propose a mathematical model (Equation 6) based on the simplified version of McKee’s equation (Equation 2) and the analyses described in the previous sections. The model takes into consideration the following variables:

- The total length of load-bearing walls using single-wall corrugated board.
- The total length of load-bearing walls using double-wall corrugated board.
- The total number of internal corners.
- The ratio between the length of load-bearing walls using double-wall corrugated board and the total length of load-bearing walls.
- The edge crush test value of single-wall corrugated board.
- The edge crush test value of two single-wall corrugated boards.
- The thickness of the corrugated board.
- A unitless constant that accounts for other design differences.

\[
BCT = k \left( \frac{C}{10} + \frac{P_{DW}}{P_T} \right) \left( ECT_{SW} \sqrt{tP_{SW}} + ECT_{DW} \sqrt{2tP_{DW}} \right)
\]

Where,

- \( BCT \) = Box compression strength (lb)
- \( k \) = Box design constant
- \( ECT_{SW} \) = Edge crush test value of single-wall board (lb/in)
- \( ECT_{DW} \) = Edge crush test value of single-wall board (lb/in)
- \( t \) = Board thickness (in)
- \( P_T \) = Length of load-bearing walls of the box (in)
- \( P_{SW} \) = Length of load-bearing walls with single-wall board (in)
- \( P_{DW} \) = Length of load-bearing walls with double-wall board (in)
- \( C \) = Number of corners in the transversal cross section

The experimental data collected used boxes fabricated with one type of board (C flute board). Therefore, the model assumes that the same corrugated board is used for the body and end panels of the bliss box.

As body and end panels are glued to form the bliss box, the \( ECT \) value for double-wall was measured using glued and unglued standard test samples. There were no significant differences between glued and unglued samples. An edge crush test value of 6.33 kN/mm (56 lb/in) was used for the sections of load-bearing walls with double-wall board (\( ECT_{DW} \)).
**Curve Fitting**

Linear regression analyses for each design style were used to fit the experimental data to the proposed model. Experimental data showed that, as total length of load-bearing walls increases, BCT increases in a linear fashion (Figure 10). MATLAB® software was used to compute iterations and find the k-values for each design style that minimized the sum of squared errors (SSE) between observed and predicted BCT values (Equation 7): 

$$SSE = \sum_{i=1}^{n} (BCT_{Experimental} - BCT_{Predicted})^2$$  \hspace{1cm} (7)

One regression analysis that treated bliss styles as a block explained about 95.5 % of the differences of BCT between bliss styles tested ($R^2 = 0.9549$). This analysis originated four different $k$-values (i.e., one for each design style), they are summarized in Table 2. Figure 14 shows how experimental data and model predictions are in close agreement for a model with four $k$-values constants. A second regression analysis was run, but this time the combination of design and size (i.e., 5-down and 10-down) was used as a block. This analysis calculated eight $k$ values (Table 2) and explained 98.1 % of the differences in BCT between bliss styles tested ($R^2 = 0.9811$). Figure 15 shows how experimental data and model predictions are in close agreement for this model using eight $k$-values.

---

**Table 2: Summary of k-values for each design style and size calculated with two linear regression analyses.**

<table>
<thead>
<tr>
<th></th>
<th>$R^2$</th>
<th>S-10</th>
<th>S-5</th>
<th>SIF-10</th>
<th>SIF-5</th>
<th>SIFDC-10</th>
<th>SIFDC-5</th>
<th>V-10</th>
<th>V-5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.9549</td>
<td>5.330</td>
<td>6.113</td>
<td>6.113</td>
<td>6.113</td>
<td>6.095</td>
<td>6.095</td>
<td>5.140</td>
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<tr>
<td></td>
<td>0.9811</td>
<td>5.504</td>
<td>5.150</td>
<td>5.778</td>
<td>6.421</td>
<td>5.757</td>
<td>6.308</td>
<td>5.328</td>
<td>4.836</td>
</tr>
</tbody>
</table>

*Fig. 14: (a) Comparison between experimental data and model predictions when design style is used as a block. (b) Corresponding linear regression analysis ($R^2 = 0.9549$).*
CONCLUSIONS

The global corrugated fiberboard industry is experiencing an increased demand [2]. Corrugated packaging for processed food, fresh food, and produce accounted for a market share of approximately 39% in 2015 and are projected to continue their dominance in the immediate future [3]. Slotted boxes, telescope boxes, folders, rigid boxes (i.e., bliss boxes), self-erecting boxes, and interior forms are the most common styles of corrugated packaging [4]. In particular, bliss-style boxes find prominence in the agriculture sector [5]. Their key advantages include providing a continuous bottom, laminated corners for higher stacking strength, increased material use efficiency, side cutout options for display at retail, and a wide range of styles over other styles of containers.

Box compression strength is a critical characteristic for any corrugated box. Minimizing material use and caliper while maximizing box strength is an important industry concern as it can be directly linked to money savings by decreasing product loss, material costs, and manufacturing costs. While numerous predictive strength models [17]–[27] associating corrugated fiberboard material specifications to the box compression strength ($BCT$) and ultimately the stacking strength of corrugated containers have been developed over the past century, there is a considerable lack of studies that include bliss-style containers.

The overall aim of this empirical study was to develop a mathematical relationship for predicting compression strength of four bliss box styles. The project comprised the following phases:

1. Collecting compression strength data in a systematic manner of four different bliss-style boxes of two sizes were collected
2. Performing analyses of how box design characteristics affect box compression strength
3. Developing a mathematical relationship based on the simplified McKee equation
4. Fitting the mathematical model to the experimental data

Fig. 15: (a) Comparison between experimental data and model predictions when design style and size are used as a block. (b) Corresponding linear regression analysis ($R^2 = 0.9811$).
Different mathematical equations were explored and tested. The proposed equation includes the following variables:

- The total length of load-bearing walls using single-wall corrugated board.
- The total length of load-bearing walls using double-wall corrugated board.
- The total number of internal corners.
- The ratio between the length of load-bearing walls using double-wall corrugated board and the total length of load-bearing walls.
- The edge crush test value of single-wall corrugated board.
- The edge crush test value of two single-wall corrugated boards.
- The thickness of the corrugated board.
- A unitless constant that accounts for other design differences.

Linear regression analyses were used to fit the experimental data to the proposed model with great accuracy. One approach considered each bliss style as a block and yielded a constant k for each of the four design styles (i.e., S, SIF, SIFDC, V). This model explained 95.5% of the experimental data ($R^2 = 0.9549$). An alternative approach considered the combination of bliss box style and size as a block. This analysis yielded eight constants k, one for each design size combination (i.e., S-5, S-10, SIF-5, SIF-10, SIFDC-5, SIFDC-10, V-5, V-10). This model explained 98.1% of the experimental data ($R^2 = 0.9811$).

We believe that the first approach provides more flexibility than the second one as it does not constrain the user to a size range while having a good fit with respect to the experimental data. As it can be observed in Table 1, BCT measurements for some of the boxes such as S-5, SIF-10-5, SIFDC-10-2, SIFDC-10-4, and V-10-3 had high variability (i.e., a coefficient of variation between 10% and 20%). Despite these exceptional cases, the model is able to account for more than 95% of the observed BCT measurements.

The proposed mathematical relationship may allow practitioners to predict box compression strength of four different bliss-style boxes with reasonable accuracy and ease. It has the potential to accelerate packaging development cycle times and to optimize packaging design. Further research steps may include additional testing to verify the model with more box sizes and design variations, additional testing to determine whether the model can predict BCT well when using different boards for body and end panels, and investigating the effect of box height on BCT.
REFERENCES


